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# **HORIZONTAL MERGER AND VERTICAL DIFFERENTIATION**

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# HORIZONTAL MERGER AND VERTICAL DIFFERENTIATION

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## ABSTRACT

The effects of merger have usually been examined in the context of homogeneous goods, and are unambiguously established. This paper deals with merger in vertically differentiated industries, presenting two models, one with two firms, and one with three firms. Results depend on the number of previous firms in the industry, and on the qualities produced by the merging firms. However, some results about the welfare effects of merger differ from the standard ones, which is mainly due to the nature of competition in a vertical differentiation set.

**Keywords:** *Merger, vertical differentiation.*

## RESUMO

Os efeitos das fusões têm sido normalmente analisados no contexto de bens homogéneos, e estão estabelecidos de forma não ambígua. Este artigo debruça-se sobre fusões em indústrias com diferenciação vertical, apresentando dois modelos, um com duas empresas, e um outro, com três empresas. Neste contexto, alguns resultados sobre os efeitos a nível do bem estar diferem dos habituais, o que se deve fundamentalmente à natureza da concorrência em diferenciação vertical.

**Palavras chave:** *Fusões, diferenciação vertical*

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# HORIZONTAL MERGER AND VERTICAL DIFFERENTIATION

## 1. INTRODUCTORY NOTE

Theoretical concerns with horizontal merger have often been addressed to its welfare effects. They usually bring out two fundamental issues, which are an increase in market power, by the reduction of the number of firms, and the possibility of efficiency gains, due to lower unitary costs, mainly provided by economies of scale and by some rationalisation of the production process.

These expected results of merger have generally been established for industries with homogeneous or horizontally differentiated goods. But, as far as I know, the case of merger in a vertical differentiation context has never been examined.

In industries with homogeneous goods, market power is an obvious result. Efficiency gains, by means of economies of scale are also more likely to happen in this kind of industries, provided that the mergers aren't operating in an upwards sloping zone of their unitary cost curves.

If goods are horizontally differentiated, and if they are close substitutes, market power will certainly be a result of merger. Here, efficiency gains may be a result of economies of scale, now also provided by lower unit sunk costs (as the costs of product engineering, or of introductory advertisement), as some previously rival products, which are quite similar, may be eliminated. Economies of scope, by the joint production of old and new goods after the merger, may also appear, and cause efficiency gains. Though this is not the concern of this paper, the case of merger in a horizontal differentiation environment arises interesting questions.

Following Kuhn and Motta (1999)'s model, unilateral effects of horizontal mergers can be established in five lemmas, which are, and in the authors' words:

- A merger increases prices and decreases consumer surplus;
- A merger always benefits the merging firm;
- A merger increases outsiders' profits;
- A merger increases producer surplus;
- A merger reduces net welfare.

This paper examines if these results hold in an industry with vertically differentiated goods, in the context of two very simple models. The first model, presented in Section 2, includes only two goods and two firms, with positive cost functions. In this case, merger leads to a monopoly situation, and the issues here become the comparison of welfare between a duopoly and a monopoly, as well as the possibility of efficiency gains.

Section 3 deals with the second model, much similar to the first one, in what concerns demands and costs. The difference is that here there are three goods and three firms, and the idea is to take account of the effects both for the merging firm and for the outsider. In fact, the presence of an outsider adds some interest to the issues presented in the two firms' model.

In both models, some results differ from Khun and Motta (1999)'s ones, which is mainly due to the nature of competition in vertically differentiated markets. In particular, welfare effects are, in one type of merger, substantially different from those established by these authors.

Finally, in section 4, I draw some conclusions, summarising the results of this work.

## 2. THE MODEL WITH TWO FIRMS

In this section, I consider first an industry composed by two firms, producing two vertically differentiated goods, with qualities  $\mathbf{q}$  and  $\mathbf{r}$ , with  $\mathbf{r} > \mathbf{q}$ , and prices, respectively,  $\mathbf{p}$  e  $\mathbf{s}$ . The model I use was purposed by Motta (1993)<sup>1</sup>. Utility function is  $U = \mathbf{v} \mathbf{q}_k - \mathbf{p}_k$ , where  $\mathbf{q}_k$  represents the utility of consuming one unit of the good with quality  $\mathbf{q}_k$ ,  $\mathbf{p}_k$  its price, and  $\mathbf{v}$  the marginal valuation of utility. The parameter  $\mathbf{v}$  is uniformly distributed in the interval  $[0,1]$ , as in Scarpa (1998)<sup>2</sup>.

Firms bear quality costs, which are assumed to be fixed and growing with the quality of each good. So, cost function is  $C_k = \mathbf{q}_k^2 / 2$ .

$\mathbf{Y}_1$  is the quantity of the good with quality  $\mathbf{q}$ , and  $\mathbf{Y}_2$  the quantity of the good with quality  $\mathbf{r}$ . These quantities are determined in the usual way. Consumers indifferent

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<sup>1</sup> As I use Motta (1993)'s model, the explanation is very brief. The main difference I introduce is in the calculation of the solutions. Readers may get better acquainted with the model by consulting its original presentation.

<sup>2</sup> Motta (1993) has introduced this model for two firms, and his  $\mathbf{v}$  lies between a superior limit,  $\mathbf{v}$ , and an inferior one,  $\mathbf{v}$ . Scarpa (1998) uses the same model for three firms,  $\mathbf{v}$  lying between zero and the unit.

between buying nothing and buying one unit of  $q$ , and between buying one unit of  $q$  and one unit of  $r$  are, respectively, represented by:

$$v_1 q - p = 0 \quad (1)$$

$$\text{and } v_2 q - p = v_2 r - s \quad (2)$$

Solving for  $v_1$  and  $v_2$ , we find the demands of the two goods:

$$Y_1 = (pr - qs) / q (q - r) \quad (3)$$

$$Y_2 = 1 - (s - p) / (r - q) \quad (4)$$

The game has two stages, as usual. First, firms choose qualities, and, on the second stage, they compete on prices. Starting with this last stage, and following Motta (1993), firms maximise short run profits and find their reaction functions:

$$p = sq / 2r \quad (5)$$

$$s = 0.5 (p + r - q) \quad (6)$$

Reaction functions are upwards sloped, as expected. Solving them, firms may then know the expressions of their long run profits,  $\Pi_1$  and  $\Pi_2$ , and of their prices and their demands, depending uniquely on the qualities  $q$  and  $r$ :

$$P_1 = rq (r - q) / (4r - q)^2 - q^2 / 2 \quad (7)$$

$$P_2 = (8r - 8q - 16r^2 + 8rq - q^2) r^2 / 2(4r - q)^2 \quad (8)$$

$$Y_1 = r / (4r - q) \quad (9)$$

$$Y_2 = 2r / (4r - q) \quad (10)$$

$$p = q(r - q) / (4r - q) \quad s = 2r(r - q) / (4r - q) \quad (11)$$

Then, in the first stage, qualities are simultaneously chosen. Each firm maximises its long run profit:

$$\partial P_1 / \partial q = (-7qr^2 + 4r^3 - 64qr^3 + 48q^2r^2 - 12qr^3 + q^4) / (4r - q)^3 = 0 \quad (12)$$

$$\partial P_2 / \partial r = r (16r^2 - 12rq - 64r^3 + 48qr^2 - 12q^2r + 8q^2 + q^3) / (4r - q)^3 = 0 \quad (13)$$

To solve these expressions, it is enough that numerators are zero. Besides,  $r$  must be positive. So only the expressions in brackets, in both numerators, are set equal to zero.

There is a unique real solution for  $q$  and  $r$ , which proves to be a maximum by second order conditions:

$$q = 0.0482 \quad r = .25331$$

These are Motta (1993)'s solutions for his upper marginal valuation of income equalised to the unit, as we did. Then, long run profits, demands and prices are:

$$P_1 = 0.0015 \quad P_2 = 0.0244$$

$$Y_1 = 0.2625 \quad Y_2 = 0.525$$

$$p = 0.0102 \quad s = 0.1076 \quad \text{uncovered market} = 0.2125$$

Let's see now what happens if both firm merge, leading to a monopoly situation. One should expect that the merger, eliminating competition and leading to monopoly power, would decrease consumer surplus, by the limitation of quantities, by the increase of prices, and by the growth of the uncovered market (which is the number of consumers who don't buy any unit of the good).

Demand functions don't depend on the industry's structure, and remain the same. The game turns now to be a "one man's decision process", namely the decision on prices and quantities.

The monopoly will take its decisions in the following way:

1° Maximisation of joint profits, which is the same than the collusion solution:

$$\frac{\partial (P_1 + P_2)}{\partial p} = 0$$

$$\frac{\partial (P_1 + P_2)}{\partial s} = 0$$

The resulting expressions are a kind of "reaction functions", showing how each price depends on the other, though both products are made by the same firm:

$$p = sq / r \tag{14}$$

$$s = p + 0.5 (r - q) \tag{15}$$

It is interesting to notice that merger increases product interdependence, as reaction functions present a slope which is the double of the one in the pre merger case. It is easier for the merger to change prices in a higher degree, as there is no firm

competition. It may even eliminate one of the goods, if joint profit maximisation leads to that decision.

Next, we find the expressions for the most concerning variables, by solving the “reaction functions” and substituting in the previous expressions. Solutions for prices, demands and long run profits, in the merger situation, are:

$$P_1 + P_2 = 0.25(1-2r)r$$

$$Y_1 = 0 \quad Y_2 = 0.5$$

$$p = 0.5q \quad r = 0.5s$$

Whatever qualities may be chosen in the first stage, joint profit maximisation leads to the production of only one good, the best quality one. Then, long run joint profits don't depend on  $q$ . So, in the first stage, the merger has only to select a value for  $r$ . Maximisation of  $\Pi_1 + \Pi_2$  is easily done and leads to  $r = 0.25$ , and any  $q$ , the value of which has no meaning. Second order conditions, in this case, are straightforward, and prove this is a maximum. With  $r = 0.25$ ,

$$P_1 + P_2 = 0.0313$$

$$Y_1 = 0 \quad Y_2 = 0.5$$

$$s = 0.125 \quad \text{uncovered market} = 0.5$$

Comparison of the two situations may be presented by means of some propositions, the proof of which is easily done by the mere inspection of the results for both cases.

**Proposition 1:** The merger of two firms producing two vertically differentiated goods leads to the elimination of the worse quality good, whatever the qualities may be. This is another case of good quality expelling bad quality. The merger prefers to produce only the best quality, and, by capturing some of the worse quality ancient consumers, it is able to rise its price (in 16%), though not losing a significant demand, as  $Y_2$  decreases in 4%.

**Proposition 2:** With the merger of both firms, the sum of profits is larger, and consumers are worse off, as they buy less at a higher price. Then, the number of consumers who don't buy any unit of the good is larger.

This result was expected, because of monopoly power. And, obviously, the sum of profits should be higher, as this is also the collusion solution. To check more properly changes in welfare, it is interesting to compute consumer surplus (CS) and total welfare (TW) in both situations. CS is calculated as in Motta (1993)<sup>3</sup>.

$$CS_{pre\ merger} = 0.0432 \quad CS_{merger} = 0.0313$$

$$TW_{pre\ merger} = 0.0692 \quad TW_{merger} = 0.0625$$

***Proposition 3:*** Consumer surplus is larger in the pre merger situation, so it decreases with monopoly, and total welfare decreases too. These results are according to the general results for merger, as enunciated by Khun and Motta (1999).

***Proposition 4:*** The merger brings some efficiency gains. Indeed, costs are reduced in 6%, by the elimination of the costs with the worse quality, and the reduction of the costs with the best quality. Then, though firms benefit of cost reductions, society is also benefited, as a smaller amount of resources is used in this industry.

The reader should notice, however, that, in a vertical differentiation context, efficiency gains have a different source. As stated above, in the case of horizontal merger without product differentiation, economies of scale are evident, whenever the cost function allows for them. Horizontal differentiation may enable the merger to eliminate goods that are much alike. Here, there is no point for economies of scale, as products are different, and, as we have seen, the whole amount of production is smaller after the merger. In fact, there aren't properly efficiency gains, but resource savings on sunk quality costs, which, anyway, make society better off.

Finally, it is interesting to mention that these results are quite similar to those of Mussa and Rosen (1978). These authors compare pure competition and monopoly solutions in a market with a vertically differentiated good. It is true that they use quite different tools (as their paper came to day a little before the core of vertical differentiation theory was published), and that they examine pure competition and not duopoly as a starting point.

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<sup>3</sup> See Motta (1993), in his note 4.



But they also find that the monopolist is interested in reducing the lower quality, in order to set a higher price for the better quality.

### 3. THE MODEL WITH THREE FIRMS

Let's suppose now that the industry has three firms, each one producing a different good, with a different quality. The model is the same of the previous section, and, so, costs are set equal to  $q_k^2/2$ ,  $q_k = q$ ,  $m$ , and  $r$ . The three qualities are represented by  $q$  (the lowest one, produced by firm 1),  $m$  (the intermediate one, produced by firm 2) and  $r$  (the highest one, produced by firm 3). Notice that each firm has already its rank of quality previously set, which is a restrictive hypothesis.

Like it was done for the situation with two firms, I use the model by Motta (1993), which was extended to the case of three firms by Scarpa (1998).

Utility function also takes the form of  $U = vq_k - p_k$ , where  $v$  represents the marginal valuation for quality  $k$ . Prices are expressed by  $p$ ,  $z$  and  $s$ , and demands by  $Y_1$ ,  $Y_2$  and  $Y_3$ , respectively for firms 1, 2 and 3.

As before, the game is developed in two stages: in the second stage firms compete on prices, setting its demands and profits depending only on qualities, and, in the first stage, they choose their qualities.

#### 3.1. Pre Merger Situation

Pre merger situation was already developed by Scarpa (1998). I computed, with his solutions for qualities, the other variables of the model:

$$\begin{aligned}
 q &= 0.0095 & m &= 0.0497 & r &= 0.2526 \\
 P_1 &= 0.00005 & P_2 &= 0.0012 & P_3 &= 0.0235 \\
 Y_1 &= 0.1136 & Y_2 &= 0.2721 & Y_3 &= 0.5225 \\
 p &= 0.0009 & z &= 0.0091 & s &= 0.1060 \\
 & & & & & \text{uncovered market} = 0.0919
 \end{aligned}$$

The introduction of an intermediate quality slightly decreases the best one, but causes a strong reduction of the worst one. Thus, quality space is enlarged, as firm 1 is now capturing some of the consumers who bought nothing at all when only two goods were supplied.

The amount of the uncovered market means that about 9% of the consumers don't buy any variety of the vertically differentiated good. It is much smaller than in any of the cases of the previous situation, which enhances the importance of another firm in the industry for consumer welfare.

### 3.2. Merger

In this model, market changes from three to two firms, so is interesting to examine market power and the effects for the outsider. Results depend on the firms that merge. Theoretically, it may happen with any pair of the three firms, leaving one firm as the outsider. In real world, it is more natural that the neighbouring qualities firms merge. I shall develop the model for two possible cases, the merger of firms 1 and 2, and the merger of firms 2 and 3.

#### 3.2.1. Merger of Firms 1 and 2

First, let's see the effects of the lower qualities firms merger, being the higher quality one left as the outsider. On the demand side, everything remains as before. On the supply side, and in the first stage, the merging firm will maximise its joint short run profits, there resulting "reaction functions" between their goods, and between these and the other one. As before, short run profits are equal to revenues, as quality costs are already chosen in this stage. The outsider also maximises his own short run profit. As expected, "reaction functions" are all positively sloped, and represented by:

$$p = zq / m \quad (16)$$

$$z = 0.5 (2p(r-m)) + s (m-q) / (r-q) \quad (17)$$

$$s = 0.5z + 0.5(r-m) \quad (18)$$

It is useful to recall the expressions of reaction functions for the pre merger case, as calculated by Scarpa (1998):

$$p = 0.5zq / m \quad (19)$$

$$z = 0.5 (p(r-m)) + s (m-q) / (r-q) \quad (20)$$

$$s = 0.5z + 0.5(r-m) \quad (21)$$

In both cases, reaction functions depend positively on the neighbouring prices, as expected, and also depend positively on some differential between their own product's qualities and their neighbouring ones.

Firm 3's reaction function is obviously the same. As for the internal logic of the merged firm, if it rises  $z$ , it will also rise  $p$ , but more intensely than before. The best reply to an increase in  $p$  will also be followed by a higher increase in  $z$ , which means that product interdependence is stronger. The best reply of  $z$  to  $s$  is the same.

Solving these reaction functions, we'll get values for all expressions, depending on qualities.

$$P_1 + P_2 = rm(r-m) / (4r-m)^2 \quad (22)$$

$$P_3 = 4r^2(r-m) / (4r-m)^2 \quad (23)$$

$$Y_1 = 0 \quad Y_2 = r / (4r - m) \quad Y_3 = 2r / (4r - m) \quad (24)$$

$$p = q(r-m) / (4r-m) \quad (25)$$

$$z = m(r-m) / (4r-m) \quad (26)$$

$$s = 2r(r-m) / (4r-m) \quad (27)$$

Prices depend on the intermediate and the best qualities' differentials. Also, short run profits never depend on the value of the lowest quality,  $q$ .

To get long run profits, it is only necessary to deduce quality costs to these expressions. Then, these profits are maximised in the first stage, in order to obtain the values for qualities, as follows:

$$\partial(P_1 + P_2) / \partial q = 0$$

$$\partial(P_1 + P_2) / \partial m = 0$$

$$\partial P_3 / \partial r = 0$$

As  $\partial(P_1 + P_2) / \partial q = 0$  for  $q=0$ , I take this solution to compute the other first order conditions, resulting:

$$q = 0 \quad m = 0.0482 \quad r = 0.25331$$

Second order conditions<sup>4</sup> show that, for these values, profits are maximised. For this choice of  $\mathbf{q}$ ,  $\mathbf{m}$  and  $\mathbf{r}$ , the main variables of the model take the following values:

$$P_1 + P_2 = 0.0015 \quad P_3 = 0.0244$$

$$Y_1 = 0 \quad Y_2 = 0.2625 \quad Y_3 = 0.5249$$

$$p = 0 \quad z = 0.0102 \quad s = 0.1076$$

$$\text{uncovered market} = 0.2125$$

Clearly, this is Motta (1993)'s solution for two firms, as described in section 2, the merger replacing the lower quality firm, and producing only one good. It is natural that both solutions are the same, as the industry changes from a three firms industry to a two firms one, producing only two goods.

From these solutions, the following propositions may be established:

***Proposition 5:***

A) When the lowest quality firms merge, the resulting firm produces only the intermediate quality, though in a smaller quantity. The merger of the lower quality firms has the effect of decreasing this intermediate quality, and increasing the highest one.

B) All profits are higher, meaning that both the merger and the outsider benefit with merger.

The value of  $\mathbf{q}$  becomes the lowest possible (indeed, zero), as the merged firm is interested in reducing costs. With this good out of the market, it is possible for the merger to set a reduction on its other quality,  $\mathbf{m}$ , capturing some of those consumers who bought  $\mathbf{q}$  before. Then, firm 3 takes profit of the reduction in  $\mathbf{m}$ , and, though it increases slightly  $\mathbf{r}$ , it captures some of the previous consumers of  $\mathbf{m}$ , even selling at higher price. In general, the quality of the vertically differentiated good is improved, as  $\mathbf{q}$  is eliminated,  $\mathbf{m}$  slightly decreases (in 3%), and  $\mathbf{r}$  increases. So, quality rises with the diminution of the number of firms.

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<sup>4</sup> Second order conditions are taken as follows: 1) the second derivative of the outsiders' profit negative; 2) the hessian of the joint profits function semi definite negative. In the next case, I used the same second order conditions.

Notice that maximal differentiation isn't the best choice for the merger, owing to competition from the outsider.

Again, better qualities drive the worst quality away. By eliminating this latter, the merger may sell the intermediate quality at higher prices, though leaving some of his previous consumers to the outsider. The best quality firm increases slightly its quantity, as it becomes farther apart from the intermediate one. Thus, the outsider is highly benefited, as it sells more at a higher price, by means of the worsening of **m**. Finally, the number of consumers who don't buy any unit of the good increases.

With higher prices and smaller quantities, consumers should be worst off. And, then, will the rise in profits offset the expected decrease in consumer surplus? It is possible to compute consumer surplus (**CS**) in the pre merger and in the merger situation, using the method indicated in the previous section. Then, by adding long run profits, we get total welfare (**TW**).

$$CS_{pre\ merger} = 0.0443 \quad CS_{merger} = 0.0432$$

$$TW_{pre\ merger} = 0.0691 \quad TW_{merger} = 0.0692$$

**Proposition 6:** When firms 1 and 2 merge, consumer surplus decrease, but total welfare increases. Besides, there are some efficiency gains with the merger.

Indeed, profits increase in 4.7%, and consumer surplus decreases in 2.5%. Higher profits are the result of higher revenues and lower costs. So, there is a point here for efficiency gains. The elimination of the worst quality, and the fact that the intermediate one becomes worse, mean lower costs for the merger firm, as well as for the two firms together, and, so, there result some efficiency gains. Again, these efficiency gains result from resource savings.

However, the interesting point here is that total welfare increases<sup>5</sup>, which changes general results for horizontal merger, when vertical differentiation is present. Thus, society is better off with two firms than with only one. For this, accounts the elimination of the worse quality, chosen by the merger. As this latter can manipulate qualities, and

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<sup>5</sup> I checked this result using another method of calculating consumer surplus, by deducing the expense on each good to the definite integral of the demand function, and adding for the three goods. Results don't change.

competes only with the highest one, it prefers to produce only one good, the quality of which is placed between the ancient worse and intermediate ones.

### 3.2.2. Merger of Firms 2 and 3

Now, suppose that the two firms with the highest qualities merge. In the second stage, maximisation of short run profits for firm 1, and of short run joint profits for the merger, yield the new “reaction functions”, which are:

$$p = 0.5 zq/m \quad (28)$$

$$z = 0.5 (2s(m-q) + p(r-m)) / (r-q) \quad (29)$$

$$s = 0.5 (r-m) + z \quad (30)$$

They are positively sloped, and each one depends on the neighbouring quality's price. The outsiders' reaction function is the same, as expected. As for the merger, its intermediate quality product best reply is the same in  $p$ , and steeper in  $s$ . The price of the best quality product exhibits a reaction function which is steeper in  $z$ . So, we also find, in this case, stronger interdependence between the mergers' products.

Now, in the next step of the second stage, firms find the expressions of their most concerning decision variables, as functions of the three quality levels:

$$P_1 = mq(m-q) / (4m-q)^2 \quad (31)$$

$$P_2 + P_3 = 0.25 (4mr - qr - 3qm) / (4m-q) \quad (32)$$

$$Y_1 = m / (4m - q) \quad Y_2 = q / 0.5 (4m - q) \quad Y_3 = 0.5 \quad (33)$$

$$p = q(m-q) / (4m-q) \quad (34)$$

$$z = 2m(m-q) / (4m-q) \quad (35)$$

$$s = 0.5 (4mr - qr - 3qm) / (4m-q) \quad (36)$$

Unlike the precedent case, now the mergers' profits depend on the three qualities, while the outsider only takes account, for his profits, of his own and his neighbouring quality. Besides, the amount of  $Y_3$  is already set, whatever may be the quality values.

In the first stage, quality will be chosen, by maximising long run profits, which are equal to the previous ones deduced of quality costs. First order conditions are now:

$$\partial P_1 / \partial q = 0$$

$$\partial (P_2 + P_3) / \partial m = 0$$

$$\partial (P_2 + P_3) / \partial r = 0$$

Solving these conditions, there result solutions for the qualities. It is interesting to point out that the choice of  $r$  is independent of the other qualities. This happens because  $\partial (P_2 + P_3) / \partial r = 0$  results immediately in  $r = 0.25$ , and, therefore,  $\partial^2 (P_2 + P_3) / \partial r^2 < 0$ . The merger will choose first  $r$ , and, then, as it also produces the quality  $m$ , plays a followers' game with the outsider for the choice of  $m$  and  $q$ .

Taking  $r = 0.25$ , we may determine solutions for the other two qualities. Checked the second order conditions, these solutions prove to be a maximum, and are:

$$q = 0.0136 \quad m = 0.0280 \quad r = 0.25$$

Now, all the three goods will be available to consumers. Back to the second stage, the other variables are then determined:

$$P_2 + P_3 = 0.0279 \quad P_1 = 0.0005$$

$$Y_1 = 0.2846 \quad Y_2 = 0.0693 \quad Y_3 = 0.5$$

$$p = 0.0019 \quad z = 0.0082 \quad s = 0.1192$$

$$\text{uncovered market} = 0.1461$$

Again, it is possible now to establish some propositions to compare the two situations.

***Proposition 7:***

A) If the best qualities' firms merge, the highest quality is almost the same, while the intermediate quality gets much worse, and the worst one is noticeably improved. Now, the merger prefers a greater differentiation between its own qualities, practically maintaining  $r$  and, by lowering much  $m$ , it gets to rise the best quality's price in 12% without loosing much demand (only 4%). The outsider sees its demand increased in 150%, due to the decrease in  $m$ , as  $\partial Y_1 / \partial m < 0$ . Anyway, the whole demand for the vertically differentiated good is smaller.

B) All profits are higher, but the outsider is much better off than the merger. Indeed, the mergers' profits increase in 13%, while the other firm experiences a 778% increase in its profits. With a much better quality, this firm may set a higher

price, and, though its neighbouring quality is worse and sold at a lower price, it gets to rise profits in such a way.

As in the previous section, it is easy to compute welfare indicators:

$$\begin{aligned}
 CS_{pre\ merger} &= 0.0443 & CS_{merger} &= 0.0351 \\
 TW_{pre\ merger} &= 0.0691 & TW_{merger} &= 0.0635
 \end{aligned}$$

**Proposition 8:** When the best quality firms merge, consumer surplus decreases, and is lower than in the case of firms 1 and 2 merger. Total welfare also decreases. As the whole demand diminishes, and two of the prices are higher, consumers are worst off.

Changes in welfare are more drastic than in the previous case. Profits rise is of 15%, not offsetting the decrease of 21% in consumer surplus. Indeed, this situation is much worse for consumers than the previous one, and, conversely, much better for firms.

Besides, cost decrease in 5%, mainly due to the intermediate quality costs, and leading to efficiency gains.

#### 4. Concluding Remarks

Maybe that the most striking result of this essay is the increase in total welfare, in the case of three firms, and when the lowest and intermediate quality firms merge. In fact, this is the only case that doesn't match the general results of merger. It is true that the reduction in consumer surplus is small, as well as the rise in profits. So, the merger doesn't make too much difference to anyone. However, total welfare is higher.

As in the general results for merger, it always happens that after the merger consumer surplus decrease, profits increase and the outsider is better off. Nevertheless, what really changes is how much this latter increases his profits. When the outsider is the lowest quality firm, its profits rise in 778%, but if it happens to be the better quality firm, profits only get 4% higher.

Another important conclusion is that, in the context of vertical differentiation, it matters who was first. One cannot say that a market with two firms and two qualities has always the same solution. Indeed, the same solution is found for one *a priori* two firms situation, and for the merger of firms with the worst and intermediate qualities. But if,



instead, the intermediate and better quality firms merge, the market solution is different. In this case, the outsider has a much higher quality than if there were *a priori* two firms, though the merger worsens a little its better quality, which means that the quality space is significantly narrowed.

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