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OBJECTIVES OF PUBLIC FIRMS AND ENTRY

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Abstract

Regulators and policy makers nowadays see entry of new firms in markets where there is a public or regulated incumbent firm as a way of promoting competition and enhancing welfare. In this paper we show how, when the incumbent firm is public or regulated, the number of entrants in the market and the resulting welfare depend on the objectives and behavior of the incumbent firm. We use the regime types of a public firm and the model of De Fraja and Delbono (1989) and extend their model to consider entry of private firms in the market. We rank the regimes by order both of entry promotion and of welfare enhancing.

Resumo

É frequente, hoje em dia, os reguladores apreciarem positivamente a entrada de novas empresas em mercados onde existe uma empresa pública ou regulada com o objectivo de aumentar a concorrência e o bem-estar. Neste trabalho, demonstramos que em mercados em que existe uma empresa incumbente pública ou regulada, o número de entrantes e o nível de bem-estar associado dependem dos objectivos e comportamento da empresa pública. Para obter estes resultados utilizamos o modelo de De Fraja e Delbono (1989) e os regimes que eles admitem para o comportamento da empresa pública. Todavia, em vez de considerar, como De Fraja e Delbono, estruturas de mercado exógenas, admitimos a possibilidade da entrada de empresas privadas no mercado, sendo o número total de empresas privadas dependente da possibilidade das empresas estarem presentes no mercado sem prejuízos. Com esta extensão ao modelo de De Fraja e Delbono conseguimos ordenar os diversos regimes das empresas públicas em termos de possibilidade de promover a entrada e de melhorar o bem-estar.

Keywords: public firms, entry
JEL classification: L32, L13

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1 Introduction

De Fraja (1997) mentions the fact that "competitive entry in regulated industries, once usually forbidden, is nowadays often actively sought by regulators and industrial policy makers" and that "when designing the regulatory mechanism for an industry where entry might occur, the public authority may therefore wish to keep an eye, as it were, on the possibility of potential competition becoming actual and choose policies different from those which would be selected were the possibility of entry ruled out". Thereafter, the author refers several practical situations where "entrants are left unregulated, while the incumbent is subject to the standard regulatory constraints ... ". Entry in the markets depends on several structural factors of the markets, on the behavior of the incumbents and expectations of the entrants. When the incumbent firms are public or regulated, the possibility of entry and the number of entrants depends also on the regulator's policy and on the strategy and objectives of the incumbents. The objectives of the incumbent firms depend of course a lot on the regulator. If the regulator wants to promote entry he must contemplate an appropriate instrument of regulation of the incumbent firms but also the objective and behavior of the firms.

Our main concern in this paper is just how the objectives of regulated or public firms can influence entry and the resulting welfare. In order to deal with this problem we will use the typology of the objectives of the public firms as in De Fraja and Delbono (1989). The authors consider "four regimes, which differ in the behavior of the public firm" that they call Stackelberg, Nash, Entrepreneur and Public Monopoly and they build a model to analyze the welfare in each of the regimes. In this paper we use their model to investigate a different issue: the influence of those regimes on entry and the resulting welfare. As we are mainly interested on entry, we rule out the public monopoly regime. Instead of considering, as De Fraja and Delbono, the existence of n private firms in the market we suggest the case where there is a public incumbent firm and we look for the number of firms that can enter the market. To concentrate on the influence of each regime on entry we rule out the existence of other impediments to entry that are not related with the behavior of the incumbent firm.

We start by giving a quick summary of the model by De Fraja and Delbono (1989). In the third section, we find the number of entrants allowed under each strategy, followed by the ordering of the strategies according to entry promotion. This section provides some detail about the relation between the costs of the firms and the number of firms that can enter the market. In the penultimate section, we compare the welfare under the different entry policies. In the final section, we make some concluding remarks and present a more dynamic interpretation of our setting.

2 The Model

The model in De Fraja e Delbono (1989) can be summarized in the following way: there are $n+1$ firms, one of which is public and the remaining n are private, having the same technology and possibly different payoffs. The index 0 in what follows refers to the public firm. The inverse demand function is linear in the total output $Q = q_0 + nq$, $p = a - Q$. The cost function is given by $c(q) = c + kq^2/2$, where k is positive. The welfare is given by the sum of consumer's and producer's surplus:

$$W(q_0, q) = \int_0^Q (a-t)dt - nc(q) - c(q_0).$$

The three strategies in De Fraja and Delbono (1989) which interest us here are

- Stackelberg (indexed by S) where the public firm acts as a Stackelberg leader and maximizes the welfare after having incorporated q as a function of q_0;
- Nash (indexed by N) where the public and private firms simultaneously maximize welfare and profit, respectively;
- Entrepreneur (indexed by E) where all the firms simultaneously maximize profit.

De Fraja and Delbono (1989) establish that the welfare is always greater if the public firm uses strategy S and that there exists $m > 0$ such that

$$n < m \Rightarrow W_E < W_N.$$

Instead of considering the existence of n private firms in the market we suggest the case where there is a public incumbent firm and look for the number of firms that can enter the market.

3 The number of entrants

The number of entrants allowed by each regime is the number n of private firms for which the profit becomes zero. Being indivisible entities, the number of entrants is the integer part of the numbers obtained when solving for profit equal to zero. We trust the reader will remember this and, in general, do not mention the integer part of the numbers we obtain. Before proceeding with the more cumbersome calculations, we simplify them by scaling the parameter a to 1.

The profit in De Fraja and Delbono (1989) is given by

$$\begin{aligned}
\pi_S &= (1+\frac{k}{2})\frac{k^2\beta^2}{(t+k\beta^2)^2} - c \\
\pi_E &= (1+\frac{k}{2})\frac{1}{(1+\beta)^2} - c \\
\pi_N &= (1+\frac{k}{2})\frac{k^2}{t^2} - c
\end{aligned}$$

where $t = (1+k)^2 + nk$ and $\beta = 1 + k + n$. Since these expressions represent the maximum profit for each strategy, we find out when they are zero in terms of n. Note that $\pi_S = 0$ will correspond to finding the zeros of a degree 4 polynomial in n. For the other two, the polynomials are quadratic.

Solving the equation $\pi_i = 0$ for n where $i = N, E, S$ indicates the different strategies, we obtain

$$\begin{aligned}
&\pi_N = 0 \quad \Leftrightarrow \quad n_N = -\frac{(k+1)^2}{k} \pm \frac{\sqrt{2c(2+k)}}{2c} \\
&\pi_E = 0 \quad \Leftrightarrow \quad n_E = -2 - k \pm \frac{\sqrt{2c(2+k)}}{2c} \\
&\pi_S = 0 \quad \Leftrightarrow \quad n_S = \begin{cases} -k - \frac{3}{2} + \frac{\sqrt{2}}{4}\sqrt{\frac{2+k}{c}} \pm \frac{\sqrt{2}}{4}\sqrt{\Delta_{12}} \\ -k - \frac{3}{2} - \frac{\sqrt{2}}{4}\sqrt{\frac{2+k}{c}} \pm \frac{\sqrt{2}}{4}\sqrt{\Delta_{34}} \end{cases}
\end{aligned}$$

where

$$\begin{aligned}
\Delta_{12} &= -6 - \frac{8}{k} + \frac{2+k}{c} - 2\sqrt{2}\sqrt{\frac{2+k}{c}} \\
\Delta_{34} &= -6 - \frac{8}{k} + \frac{2+k}{c} + 2\sqrt{2}\sqrt{\frac{2+k}{c}}.
\end{aligned}$$

Notice that only one of the values obtained for n_N and n_E may be positive while two of those obtained for n_S may be positive (the values of n_S which depend on Δ_{34} being negative). Hence, we have four values of n to consider and they are

$$
\begin{aligned}
n_N &= -\frac{(k+1)^2}{k} + \frac{\sqrt{2c(2+k)}}{2c} \\
n_E &= -2 - k + \frac{\sqrt{2c(2+k)}}{2c} \\
n_{S1} &= -k - \frac{3}{2} + \frac{\sqrt{2}}{4}\sqrt{\frac{2+k}{c}} + \frac{\sqrt{2}}{4}\sqrt{\Delta_{12}} \\
n_{S2} &= -k - \frac{3}{2} + \frac{\sqrt{2}}{4}\sqrt{\frac{2+k}{c}} - \frac{\sqrt{2}}{4}\sqrt{\Delta_{12}}.
\end{aligned}
$$

For Nash and Entrepreneur, the profit is positive for $n \in [0, n_i)$, for $i = N, E$. Note however that the interval is empty for $n_i \leq 0$, in which case there is no entry. We may interpret a negative n_i as firms exiting the market (see the Conclusions). While n_N and n_E are always real, n_{Si} may be complex. If all n_{Si} are complex, due to the expression of π_S (a quartic with negative leading coefficient), the profit is always negative therefore forbidding entrants. This extreme case occurs for values of c and k which make Δ_{12} negative, which are

$$
\textbf{Case 1:} \quad c > c_- = \frac{1}{2}\frac{k(2+k)(5k+4-4\sqrt{k(1+k)})}{(3k+4)^2}.
$$

It is only for

$$
\textbf{Case 2:} \quad c < c_-
$$

that π_S may have two real positive roots. The profit is positive for

$$
n_{S2} < n < n_{S1},
$$

which means that, for fixed values of c and k the number of entrants is between n_{S2} and n_{S1} rather than being lower than a given bound.

For the strategies of Nash and Entrepreneur, numbers n_N and n_E provide upper bounds for the quantity of entrants allowed by each strategy.

4 Ranking the regimes by order of entry promotion

Entry promotion can be ranked in several ways. We may say that a strategy favours entry if, for fixed values of the parameters, it allows more private firms to enter the economy. Or, we may say that entry inducing means that entry is allowed for a higher value of the fixed cost c or of k. We deal with these issues in this section, starting with the latter.

Obviously, entry becomes blocked when $n_i = 0$ in each of the different regimes. This corresponds to a situation of natural monopoly. Solving the above equations for n_i produces the following results

$$\begin{aligned} n_N = 0 \quad &\Leftrightarrow \quad c_N = \frac{1}{2}\frac{k^2(2+k)}{(1+k)^4} \\ n_E = 0 \quad &\Leftrightarrow \quad c_E = \frac{1}{2(2+k)} \\ n_{Si} = 0 \quad &\Leftrightarrow \quad c_S = c_N. \end{aligned}$$

Note that for both Nash and Entrepreneur, n is positive for $c < c_i$, for $i = N, E$. The functions c_N and c_E are plotted in Figure 1 where it can be seen that, for all values of k, $c_E > c_N$. This means that the strategy Entrepreneur will allow entry at a higher cost than the two other strategies. Analitically, we have

$$c_E - c_N = \frac{k^4 + 3k^3 + 4k^2 + 4k + 1}{2(1+k)^4(2+k)},$$

which is positive for all values of $k > 0$. From this expression, we also realize that the difference between c_E and c_N increases as k becomes small so that, for small values of k, Entrepreneur will allow entry for a much higher value of c than any of the other regimes. Hence, a situation of natural monopoly is more likely under the Nash than under the Entrepreneur regime.

A note of caution is now due since, for the Stackelberg regime, entry may be blocked or the policy altered not only by making $n_{Si} = 0$ but by making n_{Si} become complex. The relative position of c_S and the zeros of Δ_{12} is depicted in Figure 2, where it can be seen that c_S is always below or tangent (at $k = (-1+\sqrt{5})/2$) to the line representing c_-. This means that entry will be blocked by $n_{Si} = 0$ only in case 2. In case 1, entry is blocked because two of the roots of π_S become complex.

If we are interested in ranking according to the number of private firms which are allowed to enter the market, we have to compare the values of n_i obtained in the previous section. We have

$$n_E - n_N = \frac{1}{k}$$

for all values of c. This means that Entrepreneur favours entry over Nash and that the difference between the number of entrants allowed grows to infinity as k approaches zero. It is independent of c.

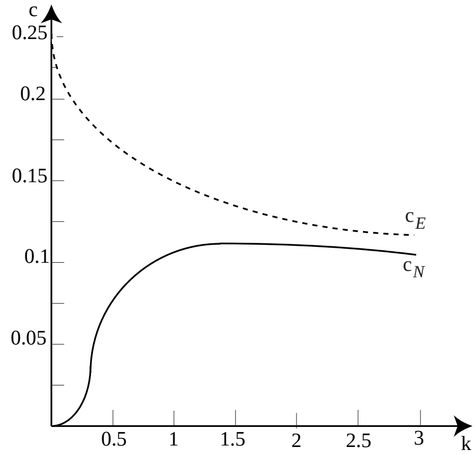

Figure 1: The curves illustrate the relationship between c and k when the number of entrants is zero. The subscripts E and N denote Entrepreneur and Nash regime, respectively.

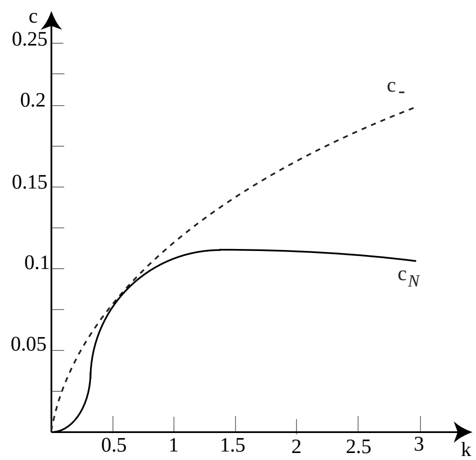

Figure 2: The curves illustrate the relative position of values of c and k for which the number of entrants is zero. The index N refers to the Nash regime while the index $-$ indicates that the quantity corresponding to the number of entrants in the Stackelberg regime becomes complex.

Tedious but standard calculations show that

$$n_E > n_{S1} > n_{S2} \qquad \forall c, k > 0$$

Invoking the continuity of the roots of each n_i for $i = N, E, S1, S2$, we present in Figures 4 and 5 the relative position of each n_i for various regions in Figure 2. Because of continuity, values of c and k in the same connected component in Figure 2 produce qualitatively equivalent results. We use representative paths to illustrate the way in which a variation in c and k will affect the values of n_i. The paths we use in the construction of the diagrams in Figure 4 are drawn in Figure 3(a). The paths are obtained by shifting the curve $c_N(k)$ by 1/2 and 1.2, respectively, and therefore do not intersect this curve - the curves we plot are $c_N(k)/2$ and $1.2c_N(k)$. These constant values are not entirely arbitrary: any constant value less than 1 will produce a curve below $c_N(k)$, closer to the horizontal axis for smaller values of the constant. The reverse happens for constants greater than 1. For high values of the constant, the path goes entirely above c_- and n_{Si} take complex values while n_N is negative. This does not interest us. In Figure 3(b), we draw two paths that intersect the curve $c_N(k)$. These are given by $c = \delta k$ with $\delta = 1/30, 1/10$, respectively, are labelled (e1) and (e2). The main difference between these two curves is the number of intersections they have with c_N and which is one for (e1) and two for (e2).

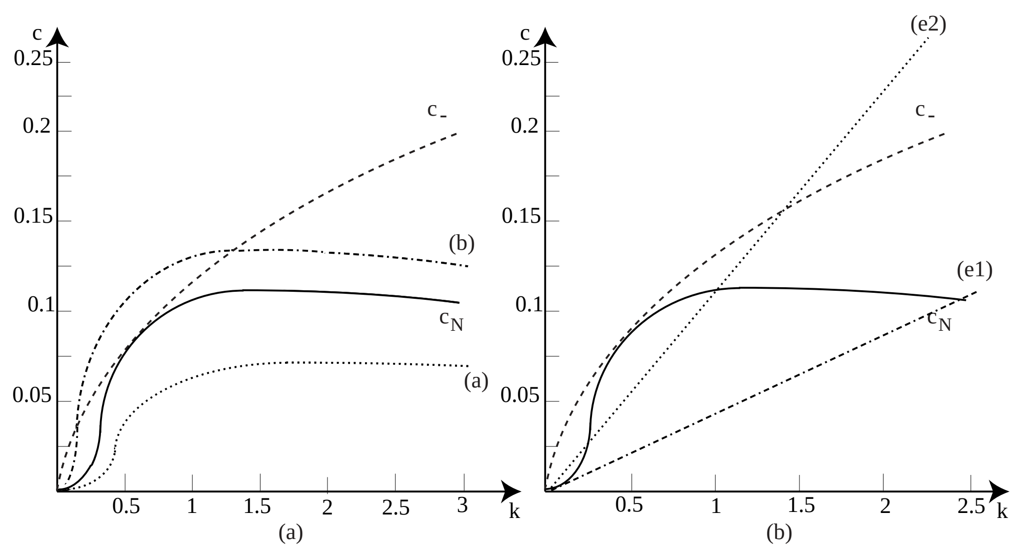

Figure 3: Together with the curves of Figure 2, we plot the paths along which we study the variation of entrants under Nash and Stackelberg strategies. Path (a) corresponds to shifting the curve c_N by half and path (b) to a shift of the sane curve by 1.2. Path (e1) is $c = k/30$ while path (e2) is $c = k/10$.

As mentioned above, figures 4 and 5 present the relative position of each n_i, for $i = N, S1, S2$, for different families of values of c and k. The families are described by the paths in Figure 3. We do not draw the curve for n_E as its relative position does not change. It is always above the other curves.

For values of c and k in the sub-region where $c_N(k)/2$ lies, we have $n_{S2} < 0$

(see Figure 4(a)). Thus, the number of entrants in Stackelberg is bounded above by n_{S1} and below by zero. Similarly, for Nash. Although the number of entrants allowed by the Nash regime is always lower than that allowed by Stackelberg, the values converge as k increases.

Figure 4(b) corresponds to a region where $n_N < 0$, that is, the Nash regime allows no entrants. Under the Stackelberg regime, for small values of k entrants are allowed and their number, while bounded above by n_{S1} as before, is not bounded below by zero but by the integer greater than n_{S2}. The numbers n_{S1} and n_{S2} converge for a value of k approximately equal to 0.3 where they become complex and the Stackelberg regime blocks entry.

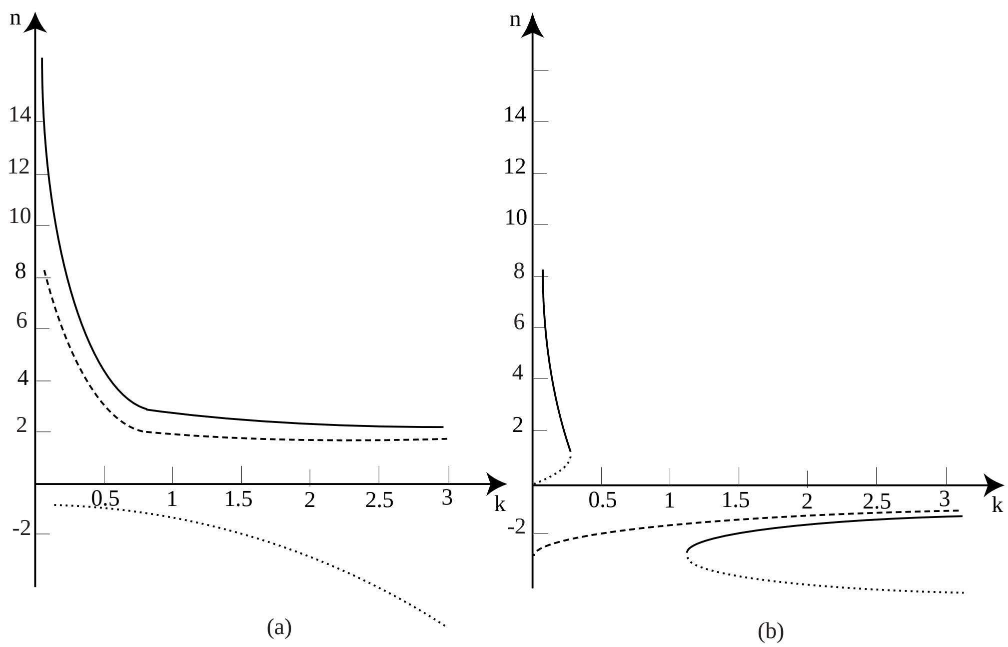

Figure 4: The labels (a) and (b) indicate that the results are obtained along the equally labelled paths in Figure 3(a). The full line corresponds to the solution n_{S1}, the dotted line to n_{S2} and the dashed line to n_N. Along path (a) entry is allowed for a number of private firms between zero and the value on the curve for each value of k. For path (b) and small values of k, under Stackelberg, entry is allowed for a number between the full and the dotted lines.

Before analysing Figure 5, recall that the slope of the paths in Figure 3(b) changes the number of intersections of the path with the curve $c_N(k)$. Note also that a very low value of the slope will make the intersection fall outside the scope of the values of k we are considering. In Figure 5(e1), corresponding to one intersection, we can see that both regimes allow entrants up to a value of k just above 2.5 and that the number of entrants is bounded below by zero. Although not visible in Figure 5(e1), numerical simulations show that by reducing the value of the slope we increase the maximum number of entrants allowed.

In Figure 5(e2), the Nash regime only allows entrants for values of k around 0.2 and the maximum number allowed is one. The results for Stackelberg are similar to those obtained in Figure 5(e1). We note that increasing the value of the slope in this region takes the path above c_- where no regime allows entrants.

We may interpret the variation of the slope as follows. The decrease in the value of the slope in (e1) corresponds to increasing k while keeping a low value of the fixed cost c. This will obviously make entry more attractive. Note that an increase in the slope of (e1) will lead to (e2) whereas an increase in the slope of (e2) will lead to values of c and k where there is no entry.

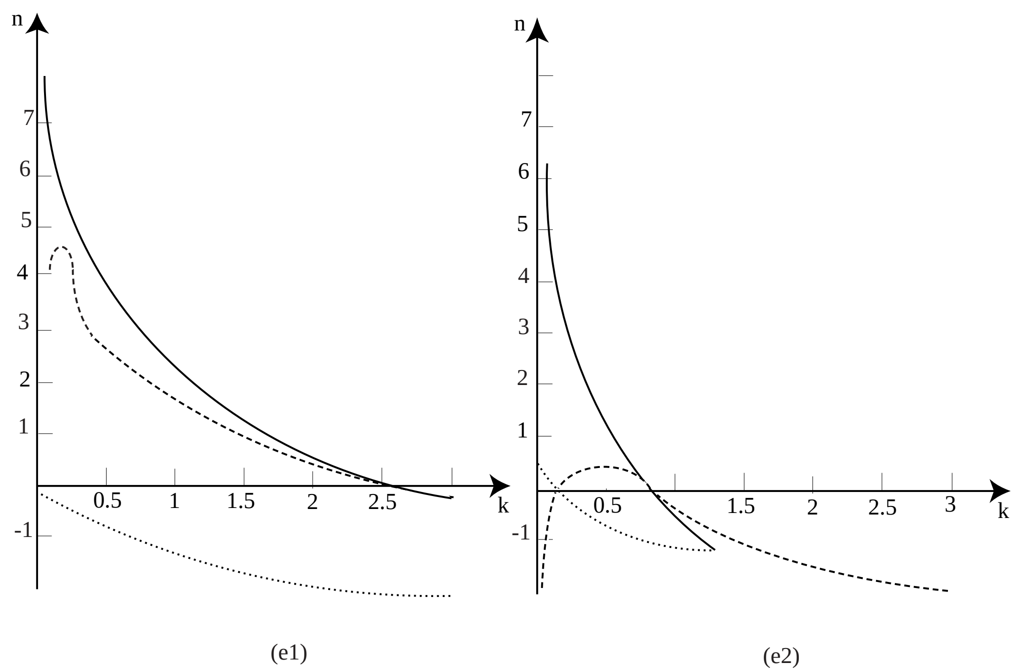

Figure 5: The labels (e1) and (e2) indicate that the results are obtained along the equally labelled paths in Figure 3(b). The full line corresponds to the solution n_{S1}, the dotted line to n_{S2} and the dashed line to n_N. Both strategies allow zero entrants for the same value of k. For small values of k along (e2), the Stackelberg regime again expects a positive number of entrants.

5 Welfare

The interest in studying the welfare resides in the comparison between W_N and W_E as we know from De Fraja and Delbono (1989) that W_S is greater than the welfare in the other two regimes for all values of n, c and k. We calculate the difference between W_N and W_E to find that it is positive, that is $W_N > W_E$, for

$$n < m = \frac{1}{2}\frac{-k + \sqrt{k(4k^3 + 12k^2 + 13k + 4)}}{k}.$$

We are interested in establishing the relative position of the welfare under the two regimes, when the entry is blocked. We have $n_N < m$ when

$$c > f(k) = \frac{k^2(2+k)}{2 + 8k + 15k^2 + 12k^3 + 3kR + 2R},$$

with $R = \sqrt{k(4k^3 + 12k^2 + 13k + 4)}$. We also have $n_E < m$ for

$$c > g(k) = \frac{k(2+k)}{2 + 11k + 12k^2 + 4k^3 + 3R + 2kR}.$$

It is $g(k) > f(k)$ for all values of $k > 0$ and therefore, since $n_e > n_N$ for all c and k positive,

$$n_E < m \Rightarrow n_N < m \Rightarrow W_E < W_N,$$

which means that, in this case, entry is blocked for both strategies when the welfare is bigger for the Nash regime. The reverse is obtained if $n_N > m$. This means that for values of c and k corresponding to low values of n_E and n_N (that is, to a market that does not admit many entrants), the welfare will be bigger under the Nash regime. If, on the other hand, the market admits a number of entrants bigger than m in both regimes, the Entrepreneur strategy will be better for promoting welfare. This supports the results obtained by De Fraja and Delbono (1989).

Suppose now that c and k are such that $n_N < m < n_E$. In this case, n_N corresponds to a number of entrants for which the Nash regime has a greater welfare. Analogously, n_E will correspond to a number of entrants for which the welfare is greater under Entrepreneur. Hence, both strategies block entry at values which guarantee that they optimize the welfare.

Thus, we can see that different values of c and k give different support to the various regimes. Influence on these quantities can promote one or the other strategy.

6 Conclusion

It is clear that Entrepreneur is the strategy that most welcomes entrants, both in terms of the number of entrants and in terms of allowing entry at higher costs. Recall that under this regime the public firm acts just as any other private firm and the profits of all (private and public) firms are equal. Hence, saying that the profit of the private firms is zero for $n = n_E$ is also saying that the public firm has zero profits. The question we are answering in this case is better formulated as "how many identical (private) firms can be allowed in the market before all profits become zero?". The answer is $n_E(c, k)$ and points to a point less clear in De Fraja and Delbono (1989), namely, their absence of comment on the fact that for certain values of n, c and k the expression for the profit they present is negative. In their setting, leaving the market is not an option and therefore, the firms facing this negative profit will stop production and support only the fixed costs.

For the remaining two regimes, the profit of the public firm is always greater than that of any private firm (see De Fraja and Delbono (1989), (8b)) and will be positive for the value of n for which entry is blocked. As for the number of entrants, Stackelberg allows a larger number of private firms to enter the market even though the values in both regimes converge for high values of k. Both strategies block entry for the same value of costs. Note however (see Figures 4(b) and 5(e2)) that Stackelberg may require that there always are entrants. This is in the nature of the strategy itself as, when it requires no entrants the strategy coincides with Nash.

In what concerns the welfare, Stackelberg is always the best strategy and we do not consider it further. We concentrate on the other two regimes. If entry is blocked after only a small number of private firms has entered the market then the Nash regime will produce a higher welfare. The opposite occurs if many private firms are allowed to enter the market, in which case, from the point of view of the welfare, the Entrepreneur regime is preferable. An interesting occurrence takes place when the number of firms allowed in the market is low under Nash and high under Entrepreneur. In this case, each strategy is the best option for the given number of private firms in the market. This leaves room for some policy considerations from the government which we do not pursue further.

Finally, we give a more dynamic interpretation of our results which can be illustrated by the telecommunications market. When costs were very high there was no entry of private firms in this market and there was a natural monopoly. As costs decrease, we see the number of private telecommunications firms increase in the market. This corresponds to reading Figures 4 and 5 from right to left (a decrease in k also produces a decrease in c). It is conceivable that costs might become high again following some natural disaster or some economical crisis. Under this setting, unless the government is able to change strategy to one that is more favourable to entry for the same level of cost, n_i may become negative meaning that private firms exit the market. This may or may not reach the situation of natural monopoly. It will reach a natural monopoly if all the firms which entered in the first phase now exit. If some firms have become powerful enough to support the new higher costs, they may choose to remain in the market.

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