THE IMPACT OF MONETARY SHOCKS ON PRODUCT AND WAGES

A neoclassical aggregated dynamic model

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A NEOCLASSICAL AGGREGATED DYNAMIC MODEL

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ABSTRACT

In this work, I present an extension of Kydland and Prescott’s (K&P) model. The extension is to consider that prices and wages are nominal and that economic agents anticipate the probability of occurrence of exogenous shocks. Once calibrated the model, I compute the nominal and real dynamic response to a monetary shock. The results observed are in agreement with the economic stylised facts: there are permanent nominal effects but only short-term real effects.

An interesting issue to pursue in future research would be to quantify the influence of money in the business cycle and to compare the results obtained with those in the real model of K&P.

Keywords: Business Cycle, Economic nominal variable, Monetary shocks, Micro-foundation.

RESUMO

Neste meu trabalho, apresento uma extensão ao modelo de Kydland e Prescott (K&P). A extensão é considerar que os preços e os salários são nominais e que os agentes económicos antecipam a probabilidade de ocorrência de choques exógenos. Calibrando o modelo, uso computação para simular a resposta nominal e real a um choque monetário. Os resultados observados estão de acordo com os factos estilizados: existem efeitos nominais permanentes mas efeitos reais apenas de curto prazo.

Interessante será, numa investigação futura, comparar o modelo nominal que eu apresento com o modelo real de K&P no estudo das flutuações macro-económicas.

Palavras chave: Ciclos económicos, Variáveis económicas nominais,Choques monetários, fundamentação microeconómica.
1. INTRODUCTION

In this work I intend to extend Kydland and Prescott's (1982) model by introducing nominal prices and wages. Considering economic agents that maximise an inter-temporal utility function, I compute the evolution of real and nominal prices and wages in response to a non-anticipated monetary shock. In this way, the model may be classified as a structural general equilibrium aggregated dynamic model.

I use a structural model that underpins on the individual behaviour of economic agents as "[w]e need a deeper idea of what we mean by ‘structure’, not because ‘depth’ is desirable in itself […], but because a model has to be able to isolate those aspects of behavior that remain invariant to policy shifts from those that do not if it is to be of any use in assessing the consequences of the shift.” (Lucas Jr., 1987, pp. 11-12).

Although the use of models with a microeconomic foundation is theoretically appealing, it is difficult to obtain algebraic results with them. The solution is to use computational methods on a previously formalised and calibrated model, but this makes generalisation difficult (e.g., Vercelli, 1992).

As it is standard in neo-classical models, I consider rational economic agents. In this way, the exogenous shocks must be anticipated in the best possible way. In the model, economic agents assume that potential shocks is an aleatory extraction of a known probability function.

Finally, the markets are assumed to be always open to trade and any instantaneous imbalance in goods and labour markets is considered to generate a variation on nominal prices and wages. The use of these imbalances as the explanation for economic dynamics is consensual and central to the economic explanation of the determination of market prices equilibrium (e.g, Negishi, 1961).

In addition to the inclusion of nominal prices and wages, my model further extends that of Kydland and Prescott (1982) by considering that economic agents anticipate the probability of occurring exogenous shocks.
2. MODEL OF THE FAMILIES

During period \( t \), the market hourly nominal wage is \( W_t \) and the market goods unitary nominal price is \( P_t \).

At the beginning of period \( t \), a family knows \( W_t \) and \( P_t \); she has a sum \( r_t \) of money in its pocket, she supplies the amount of work \( l_t \), and she demands the amount \( c_t \) of goods. At the end of the period, the budget constrain implies that the sum \( r_{t+1} = r_t + l_t W_t - c_t P_t \) remains in the family’s pocket.

The utility function that the family maximises in the present is the expected value of an inter-temporal function separable in time that considers all the periods in the future (e.g., Varian, 1999). In this utility function, the family's variables of decision are the level of consumption and the amount of work it supplies, while the variables of state are the money in its pocket, the nominal wages and the nominal prices, for all periods ahead.

\[
U = u(c_t, l_t) + \mathbb{E} \left[ \beta u(c_{t+1}, l_{t+1}) + \ldots + \beta^{T-t} u(c_T, l_T) \right],
\]

s.a \( r_{t+1} = r_t + l_t W_t - c_t P_t; \ldots \); \( r_{T+1} \geq 0 \)  

This model is easier to understand if rewritten in the recursive form, becoming explicit the influence of present decisions in the future (e.g., Stokey, Lucas Jr. and Prescott, 1989).

\[
U(r_t, W_t, P_t) = u(c_t, l_t) + \mathbb{E} \left[ U(r_{t+1}, W_{t+1}, P_{t+1}) \right],
\]

s.a \( r_{t+1} = r_t + l_t W_t - c_t P_t \)

The family supplies the quantity of labour and demands the quantity of goods that maximise its utility function. This problem was first formalised by Bellman (1957) and is also designated in the literature as Stochastic Dynamic Programming. The result of this maximisation is the demand of goods and the supply of labour inverse functions.

\[
V(r_t, W_t, P_t) = \max_{c_t, l_t} \{ u(c_t, l_t) + \mathbb{E} [V(r_{t+1}, W_{t+1}, c_t, l_t, P_{t+1})] \}
\]

The previous equation states that the family has rational expectations a la Muth (1961), using the very model of decision in the forecast of the future values of the endogenous variables.
As the family knows that the non-anticipated shock is a statistical extraction with 
\( G(w, p) \) distribution function, the future is included in the present by using the expected 
effects of present decisions.

\[
V(r_t, W_t, P_t) = \max_{c_t, l_t} \left\{ u(c_t, l_t) + \beta \int_0^\infty \int_0^\infty V(r_t + l_t, W_t - c_t, P_t, w, p) g(w, p) \right\}
\]  

(4)

The distribution function \( G(w, p) \) is exogenous to the model because it is not observable 
by the economic agents and is a subjective anticipation of the exogenous shocks that 
may occur. However, as economic agents are rational in the model, the steady state 
equilibrium values must be used as estimates for the first moment of this distribution 
function.

Condensing the expected term, it becomes clear that only nominal wealth influences the 
expected value of the inverse utility function. Although this expected value is 
influenced by expectations about \( G \), I don’t explore that in this work:

\[
V(r_t, W_t, P_t) = \max_{c_t, l_t} \left\{ u(c_t, l_t) + \beta V_\beta(r_{t+1}) \right\}
\]  

(5)

In aggregated terms, the labour supply and the goods demand functions result from the 
sum of the family functions to all the \( N \) families. In a stationary equilibrium situation, 
where new families are included to substitute those extinguished, the aggregated 
functions are invariant over time.

\[
C(W_t, P_t, G) = \sum_{n=1}^{N} c(r_{t,n}, W_t, P_t, G); \quad L_S(W_t, P_t, G) = \sum_{n=1}^{N} l(r_{t,n}, W_t, P_t, G)
\]  

(6)

Additionally, in a situation of stationary equilibrium, the two previous aggregated 
functions can be calculated considering the sum of just one family for all the periods, as 
in each period the family represents a different family type (e.g., Blanchard e Fischer, 
1989, p.92):

\[
C(W_t, P_t, G) = \sum_{s=1}^{T} c(r_{s}, W_s, P_s, G); \quad L_S(W_t, P_t, G) = \sum_{s=1}^{T} l(r_{s}, W_s, P_s, G)
\]  

(7)

3. MODEL OF THE FIRMS

Firms also maximise an inter-temporal function, i.e. the net present value function. 
Furthermore, they are considered to have an infinite time horizon (as in Solow's model, 
1957). Firms choose next period's capital level and this period's demand of labour as to
solve the following dynamic optimisation problem, where \( R \) is the market rate of nominal interest:

\[
PV(k_t, W_t, P_t) = \max_{k_{t+1}, l_t} \left\{ \pi(k_t, l_t) + \int_{p=0}^{\infty} \int_{w=0}^{\infty} PV(k_{t+1}, w, p) g(w, p) \right\}
\]

with \( \pi(l_t) = [f(l_t, k_t) - k_{t+1} + (1 - \delta) k_t]P_t - l_t, W_t \)

In a way similar to that of the families, only the quantity of capital influences the firms' expected present value:

\[
PV(k_t, W_t, P_t) = \max_{k_{t+1}, l_t} \left\{ \pi(k_t, l_t) + R \cdot PV_f(k_{t+1}) \right\}
\]

In aggregated terms, the labour demand and the goods supply functions result from the sum of the firm's functions to all \( M \) firms. In a stationary equilibrium situation, those aggregated functions are invariant over time.

\[
Y(W_t, P_t, G) = \sum_{m=1}^{M} \left[ f(l_{t,m}, k_{t,m}) - k_{t+1,m} + (1 - \delta) k_{t,m} \right]
\]

\[
L_D(W_t, P_t, G) = \sum_{n=1}^{N} l(W_t, P_t, G)
\]

As it is common in macroeconomic models, firms' profits are spent in the acquisition of consumption goods.

4. MODEL OF THE MARKET

In the neo-classical perspective, economic agents are optimisers and, in non-equilibrium situations, where nominal wages and prices do not guarantee that supply equals demand, the market closes for transactions (non-tâtonnement).

\[
Y(W_t, P_t, G) = C(W_t, P_t, G) + \pi / P_t; \quad L_D(W_t, P_t, G) = L_D(W_t, P_t, G)
\]

However, non-tâtonnement is a mechanism of resources allocation only when there is perfect and common knowledge, which is not the general case.

If there is imperfect knowledge, the dynamics of prices in response to market imbalances is an important mechanism of exchange of information between economic agents in their adjustment towards equilibrium. This is a feedback mechanism of trial and error that economic agents use to find out the equilibrium prices (Negishi, 1961).
Adjustment mechanism in the labour market

A measure of imbalance of the labour market is the excess of labour supply - unemployment \((U)\). The market is in an equilibrium situation when unemployment is zero (or equal to a fixed value, \(U - U^* = 0\), without loss):

\[ U = L_s - L_d \]  

(12)

The adjustment mechanism I suggest is one of “Phillips curve” type, where the speed of wages' variation, \(\dot{W}\), is a decreasing function of unemployment:

\[ \dot{W} = h_w(U), \quad \text{with} \quad \frac{d h_w}{d U} < 0 \]  

(13)

Adjustment mechanism in the goods market

In way similar to the labour market, in aggregated terms, I use the supply surplus of goods, \(SS\), as the measure of the market imbalance:

\[ SS = Y - C - \pi / P \]  

(14)

Additionally, I consider that the speed of price variation, \(\dot{P}\), is a decreasing function of the surplus:

\[ \dot{P} = h_p(SS), \quad \text{with} \quad \frac{d h_p}{d SS} < 0 \]  

(15)

Quantities traded out of equilibrium

I consider, without loss, that the quantity the economic agents intend to supply or demand is always achieved. Note that the accumulated surplus can be negative.

5. Model of the economic circuit

At least since Adam Smith, it is acknowledged that the economy is a system with two components: real and monetary. In the real component circulates, in one way, goods and, in the other way, labour. In the monetary component circulates money for payments of supplied goods and for payments of supplied labour (Lekachman, 1960, p.274).
Economic agents interact in one market where goods and labour are traded for money, building an economic circuit.

![Fig. 1 - Representation of the economic circuit](image)

### 6. Formalisation, Calibration and Computation of the Model

The formalisation of the model encompasses the following functional forms:

\[
\begin{align*}
\text{utility function:} & \quad u(l,c) = \ln(K_c + c) + \ln(K_l - l) \\
\text{demand function:} & \quad G(w, p) \sim N(\mu_w, \sigma^2_w, \sigma^2_p) \\
\text{production function:} & \quad f(l, k) = A l^\alpha k^{1-\alpha} \\
\text{money-wage relationship:} & \quad W_{t+1} = W_t(1 - U_t, K_w) \\
\text{money-price relationship:} & \quad P_{t+1} = P_t(1 - SS_t, K_p)
\end{align*}
\]

I consider that time is divided in periods. The duration of each of these periods is unknown. Nonetheless, it would represent a time scale and it is related to the time that each economic agent needs to re-do the allocation of his/her scarce resources in non-equilibrium situations. The division of time in periods is essential to make possible the computation of the model’s properties.

Although the use of computational methods permits to calculate the model with non-linear functional forms, it implies the previous calibration of its parameters.

The calibration of the model I suggest does not result from the observation of any economic series. The aim is simply to make the model stable.
Labour supply and goods demand functions

By calibrating the parameters of the model with $T = 70$, $\beta = 0.9$, $\mu_W = \mu_P = 10$, $\sigma^2_W = \sigma^2_P = 5$, $K_c = 0$, $K_l = 10$, $r_1 = 23$ (the long-term equilibrium result), we obtain the following values: $l_t = 5$, $c_t = 5$, $r_t = 23$ if $t \in [1, 65]$. It should be noted that families worry with the "end of life" only in the last 5 periods.

![Graph showing Labour and Consumption functions](image1)

**Fig. 2** - Family's goods demand and labour supply functions near the "end of life".

Using the same calibration, the next figure represents the labour supply function in both short- and long-term (without loss, the quantity of families is normalised to one).

![Graph showing Short-term and Long-term Labour Supply](image2)

**Fig. 3** - Aggregated short-term and long-term labour supply function.

Assuming, once again, the previous calibration, the following illustration presents the goods demand function in the short-term (the long-term is nearly equal).

![Graph showing Short-term Goods Demand](image3)

**Fig. 4** - Aggregated short-term goods demand function.
Computation of the economic circuit

In order to simplify the calibration of the model, I consider that firms’ capital is constant and exogenous. Additionally, I consider that all families have the same time horizon.

The first step in the simulation was to calculate the steady state expected value of price and wages. With this objective, I first used $\beta = 0.9$, $\sigma^2_w = \sigma^2_p = 5$, $\mu_w = \mu_p = 10$, $K_c = 0$, $K_l = 5$, obtaining, approximately, $\mu_w = 6.75$ and $\mu_p = 11$. Next, I calculated the steady state quantity of money in the families' pockets: which turned to be $r_1 = 11$. Finally, I experimented several values for the parameter that characterises the velocity of adjustment of prices and wages with supply surplus, and I found $K_p = 0.5$ e $K_w = 0.1$ to be acceptable. It is an economic stylised fact that prices adjust more rapidly than wages.

7. Monetary shocks - a dynamical analysis of the response to a monetary shock

Being the market in a steady state equilibrium situation, a given sum of money that corresponds to 5% of the aggregate product will be equally distributed between all families’ pockets. Then, I present the impact of this monetary shock on product and wages.

Nominal effects

I illustrate in the next two figures the dynamic response of nominal prices and nominal wages to the monetary shock. I show that prices adjust more rapidly than wages with an overshoot and, more importantly, that there is a long-term effect on both prices and wages.

![Fig. 5 - Prices Dynamics](image-url)
Real Effects

Real effects are measured by the variation of real wages and product level. These real effects caused by the monetary shock are dependent upon the relative speed of the response of prices and wages to the supply surplus induced by that monetary shock. As the prices' response is faster than the wages' response, I observe, in the short-term, a decrease of the real wage and an increase of the product level. In the long-term, however, there is no effect.
8. CONCLUSION

The model presented here represents an extension to that of Kydland and Prescott (1982). In this work, I present a dynamic model where prices and wages are nominal, each family has a given sum of money in its pocket and there is an attempt to anticipate eventual monetary exogenous shocks.

The model is calibrated in a way close to economic reality, simulating the nominal and real dynamic response to a monetary shock. The main outcomes of this exercise are: 1) in nominal terms, prices and wages increase in a persistent way; 2) in the short term, real wages decrease and product level increases; 3) in the long term, there is no real effects. These results are in accordance to the economic stylised facts.

It would be interesting in future research to use the model presented here to quantify the influence of money in the business cycle and to compare the results obtained with those in the real model of Kydland and Prescott (1982).

BIBLIOGRAPHY


