

# **Local and global dominance conditions for the weighted earliness scheduling problem with no idle time**

**Jorge M. S. Valente**



FACULDADE DE ECONOMIA

UNIVERSIDADE DO PORTO

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# Local and global dominance conditions for the weighted earliness scheduling problem with no idle time

Jorge M. S. Valente\*

Faculdade de Economia da Universidade do Porto, Portugal

September 14, 2004

## Abstract

In this paper, we present several local and global dominance conditions for the single machine weighted earliness scheduling problem with no idle time. We also propose an improvement algorithm that uses these conditions and can be applied to improve the sequence given by a heuristic procedure. This algorithm can be used to improve not only upper bounds for the weighted earliness criterion, but also lower bounds for the earliness/tardiness scheduling problem. The computational tests show that, in both of these cases, the improvement algorithm is superior to an initial heuristic schedule, as well as an existing adjacency condition.

**Keywords:** scheduling, weighted earliness, dominance rules

## Resumo

Neste artigo apresentamos diversas condições de optimalidade local e global para um problema de sequenciamento com um único processador, custos de posse e inexistência de tempo morto. Um algoritmo que utiliza estas condições e que pode ser utilizado para melhorar a sequência gerada por uma heurística é também apresentado. Este algoritmo pode ser utilizado para melhorar não apenas limites superiores para o critério do custo de posse, como igualmente limites inferiores para o problema de sequenciamento com

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\*Address: Faculdade de Economia do Porto, Rua Dr. Roberto Frias, 4200-464 Porto, Portugal. Telephone: +351 225 571 100. Fax: +351 225 505 050. E-mail: jvalente@fep.up.pt.

custos de posse e de atraso. Os testes computacionais mostram que, em qualquer destes dois casos, o algoritmo proposto supera quer uma sequência heurística inicial, quer uma condição já existente relativa a trabalhos adjacentes.

**Palavras-chave:** sequenciamento, custos de posse, condições de optimalidade

## 1 Introduction

In this paper, we consider a single machine scheduling problem with earliness costs that can be stated as follows. A set of  $n$  independent jobs  $\{J_1, J_2, \dots, J_n\}$  has to be scheduled without preemptions on a single machine that can handle at most one job at a time. The machine and the jobs are assumed to be continuously available from time zero onwards and machine idle time is not allowed. Job  $J_j, j = 1, 2, \dots, n$  has a processing time  $p_j$ , a due date  $d_j$  and a weight  $h_j$ . For any given schedule, the earliness of  $J_j$  can be defined as  $E_j = \max\{0, d_j - C_j\}$ , where  $C_j$  is the completion time of  $J_j$ . The objective is then to find a schedule that minimizes the sum of the earliness costs of all jobs  $\sum_{j=1}^n h_j E_j$ .

The earliness cost may represent the cost of completing a project early in PERT-CPM analyses, deterioration of perishable goods or a holding cost for finished goods. The objective function is appropriate in situations where the holding or earliness costs are the main concern, such as in the production of deteriorating goods. The assumption that no machine idle time is allowed represents a type of production environment where the machine idleness cost is higher than the cost incurred by completing a job early, or the machine is heavily loaded, so it must be kept running in order to satisfy the demand. Korman (1994) and Landis (1993) give some specific examples of production settings where idle time is not allowed. The weighted earliness problem is also important as a subproblem in early/tardy scheduling. The lower bounds presented by Li (1997) and Liaw (1999) for the earliness/tardiness scheduling problem require initial sequences for an earliness subproblem, and Valente and Alves (2003) show that using better initial sequences can improve these lower bounds.

In this paper, we present dominance rules that provide local and global optimality conditions. We also give an algorithm that uses the proposed dominance rules. This algorithm can be used to improve the sequences given by a heuristic procedure, and it generates a schedule that cannot be improved by adjacent interchanges. The dominance rules and the improvement algorithm were developed using an approach

similar to the one previously employed by Akturk and Yildirim (1998) for the single machine weighted tardiness problem.

The remainder of the paper is organized as follows. The local and global dominance rules are presented in sections 2 and 3, respectively. In section 4 we describe the improvement algorithm and the computational results are presented in section 5. Finally, some concluding remarks are given in section 6.

## 2 Local dominance conditions

In this section, we present several local dominance conditions. Throughout the rest of the paper, assume the jobs have been renumbered according to an earliest due date (EDD) indexing convention, so that for all  $i$  and  $j$ , with  $i < j$ , we have  $d_i < d_j$ , or  $d_i = d_j$  and  $p_i > p_j$ , or  $d_i = d_j$ ,  $p_i = p_j$  and  $h_i \leq h_j$ . Let  $E(S)$  denote the total weighted earliness of a schedule  $S$ . Also consider two schedules  $S_1 = Q_1ijQ_2$  and  $S_2 = Q_1jiQ_2$  that only differ in the ordering of the adjacent jobs  $i$  and  $j$ , with  $Q_1$  and  $Q_2$  being two disjoint subsequences of the remaining  $n - 2$  jobs. Finally, let  $t = \sum_{k \in Q_1} p_k$  denote the completion time of  $Q_1$ . The interchange function  $\Delta_{ij}(t)$  can then be used to specify the local or adjacent dominance properties. This function gives the change in the objective function value when two adjacent jobs  $i$  and  $j$ , whose processing time starts at time  $t$ , are swapped, that is,  $\Delta_{ij}(t) = E(S_1) - E(S_2)$ . It is convenient to express the interchange function as  $\Delta_{ij}(t) = f_{ij}(t) - g_{ij}(t)$ , where  $f_{ij}(t)$  and  $g_{ij}(t)$  are defined as follows:

$$f_{ij}(t) = \begin{cases} h_i p_j & t \leq d_i - (p_i + p_j), \\ h_i (d_i - t - p_i) & d_i - (p_i + p_j) \leq t \leq d_i - p_i, \\ 0 & d_i - p_i \leq t, \end{cases}$$

$$g_{ij}(t) = \begin{cases} h_j p_i & t \leq d_j - (p_i + p_j), \\ h_j (d_j - t - p_j) & d_j - (p_i + p_j) \leq t \leq d_j - p_j, \\ 0 & d_j - p_j \leq t. \end{cases}$$

We remark that the interchange function  $\Delta_{ij}(t)$  does not depend on how the jobs in  $Q_1$  and  $Q_2$  are arranged, but it depends on the start time  $t$  of the adjacent pair of jobs  $i$  and  $j$ . We then have:

- if  $\Delta_{ij}(t) < 0$ ,  $i$  should precede  $j$  at time  $t$ ;

- if  $\Delta_{ij}(t) > 0$ ,  $j$  should precede  $i$  at time  $t$ ;
- if  $\Delta_{ij}(t) = 0$ , it is indifferent whether  $i$  or  $j$  is scheduled first at time  $t$ .

We now present some definitions concerning local precedence relations between pairs of adjacent jobs. We say that  $i$  *unconditionally* precedes  $j$ , denoted as  $i \rightarrow j$ , when  $i$  always precedes  $j$  when these jobs are adjacent. A *breakpoint* is defined as a critical start time for a pair of adjacent jobs at which the ordering relation between the two jobs changes, that is, if  $i$  should precede  $j$  (or  $j$  should precede  $i$ ) when  $t \leq$  breakpoint, then  $j$  should precede  $i$  (or  $i$  should precede  $j$ ) when  $t \geq$  breakpoint. When there is at least one breakpoint for a pair of jobs  $i$  and  $j$ , we say that  $i$  *conditionally* precedes  $j$ , denoted as  $i \prec j$ , when  $i$  should precede  $j$  if they are adjacent. In this situation, the precedence relation between these two jobs is different in the two sides of the breakpoint.

When all possible situations are considered, every job pair can be classified into one of the following mutually exclusive cases:

**Case 1.**  $h_i p_j \leq h_j p_i$ ,  $d_i - p_i \leq d_j - p_j$ ,

**Case 2.**  $h_i p_j \leq h_j p_i$ ,  $d_i - p_i > d_j - p_j$ ,

**Case 3.**  $h_i p_j > h_j p_i$ ,  $d_i - p_i \geq d_j - p_j$ ,  $h_i (d_i - d_j + p_j) \geq h_j p_i$ ,

**Case 4.**  $h_i p_j > h_j p_i$ ,  $d_i - p_i \leq d_j - p_j$ ,  $h_i (d_i - d_j + p_j) < h_j p_i$ ,

**Case 5.**  $h_i p_j > h_j p_i$ ,  $d_i - p_i < d_j - p_j$ ,  $h_i (d_i - d_j + p_j) \geq h_j p_i$ ,

**Case 6.**  $h_i p_j > h_j p_i$ ,  $d_i - p_i > d_j - p_j$ ,  $h_i (d_i - d_j + p_j) < h_j p_i$ .

We remark that  $h_i (d_i - d_j + p_j)$  and  $h_j p_i$  are the values of  $f_{ij}(t)$  and  $g_{ij}(t)$ , respectively, when  $t = d_j - p_i - p_j$ . Given the EDD indexing convention, we also have  $d_j - p_i - p_j \geq d_i - p_i - p_j$  (with the equality only being possible in some situations included in cases 1 and 5). When these six cases are analysed, it can be seen that there are two different breakpoint types:

$$t_{ij}^1 = \frac{h_i (d_i - p_i) - h_j (d_j - p_j)}{h_i - h_j} \quad (1)$$

$$t_{ij}^2 = d_i - p_i - \frac{h_j p_i}{h_i} \quad (2)$$

These breakpoints are determined by the intersections of the  $f_{ij}(t)$  and  $g_{ij}(t)$  functions. A  $t_{ij}^1$  or type 1 breakpoint occurs when the sloped middle segments of the  $f_{ij}(t)$  and  $g_{ij}(t)$  functions intersect. A  $t_{ij}^2$  or type 2 breakpoint results from an intersection of the middle segment of  $f_{ij}(t)$  with the horizontal first segment of  $g_{ij}(t)$ .

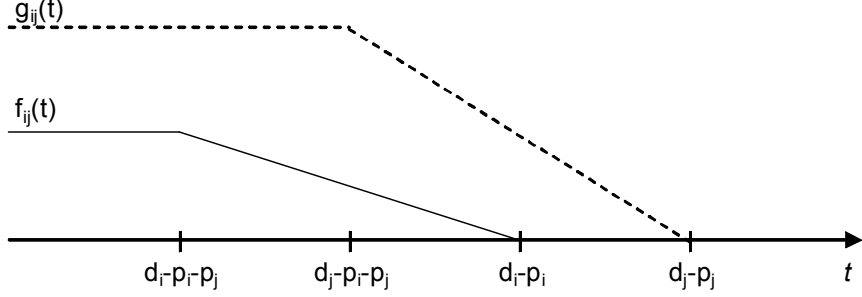


Figure 1: Case 1

In case 1, as shown in figure 1, we have  $f_{ij}(t) \leq g_{ij}(t), \forall t$ , so  $i$  unconditionally precedes  $j$ .

**Proposition 1** *If  $h_i p_j \leq h_j p_i$  and  $d_i - p_i \leq d_j - p_j$ , then job  $i$  unconditionally precedes job  $j$ .*

**Proof.** When  $t = d_j - p_i - p_j$ , we have  $\Delta_{ij}(t) = f_{ij}(t) - g_{ij}(t) \leq 0$ , since  $f_{ij}(t) \leq h_i p_j$  (because  $d_j - p_i - p_j \geq d_i - p_i - p_j$ ) and  $g_{ij}(t) = h_j p_i \geq h_i p_j \geq f_{ij}(t)$ . When  $t = d_i - p_i$ , we also have  $\Delta_{ij}(t) \leq 0$ , since  $f_{ij}(t) = 0$  and  $g_{ij}(t) \geq 0$  (given that  $d_j - p_j \geq d_i - p_i$ ). It is then straightforward to conclude that  $\Delta_{ij}(t) \leq 0$  for all  $t$ , and  $i \rightarrow j$ . ■

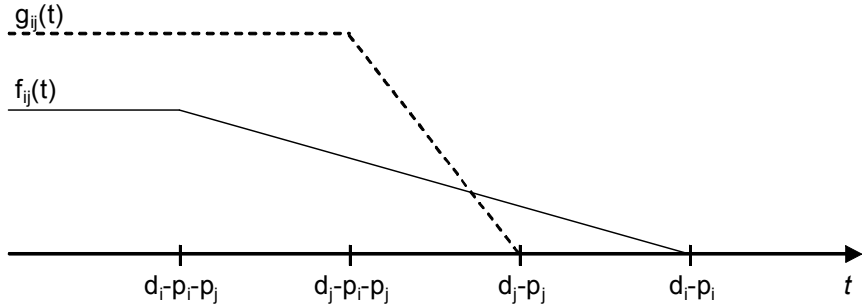


Figure 2: Case 2

In case 2 (see figure 2), we have a type 1 breakpoint, with  $i$  conditionally preceding  $j$  ( $j$  conditionally preceding  $i$ ) when the processing of these jobs starts before (after) the breakpoint.

**Proposition 2** *If  $h_i p_j \leq h_j p_i$  and  $d_i - p_i > d_j - p_j$ , there is a  $t_{ij}^1$  breakpoint and  $i \prec j$  for  $t \leq t_{ij}^1$ , while  $j \prec i$  for  $t \geq t_{ij}^1$ .*

**Proof.** When  $t = d_j - p_i - p_j$ , we have  $\Delta_{ij}(t) = f_{ij}(t) - g_{ij}(t) < 0$ , since  $g_{ij}(t) = h_j p_i$  and  $f_{ij}(t) < h_i p_j \leq h_j p_i = g_{ij}(t)$ . When  $t = d_j - p_j$ , we have  $\Delta_{ij}(t) > 0$ , since  $f_{ij}(t) > 0$  (because  $d_i - p_i > d_j - p_j$ ) and  $g_{ij}(t) = 0$ . Therefore, the non constant middle segments of  $f_{ij}(t)$  and  $g_{ij}(t)$  must intersect, and we have a  $t_{ij}^1$  breakpoint. When  $t < t_{ij}^1$ , we have  $\Delta_{ij}(t) < 0$  and  $i \prec j$ . If  $t > t_{ij}^1$ , then  $\Delta_{ij}(t) > 0$  and  $j$  conditionally precedes  $i$ . ■

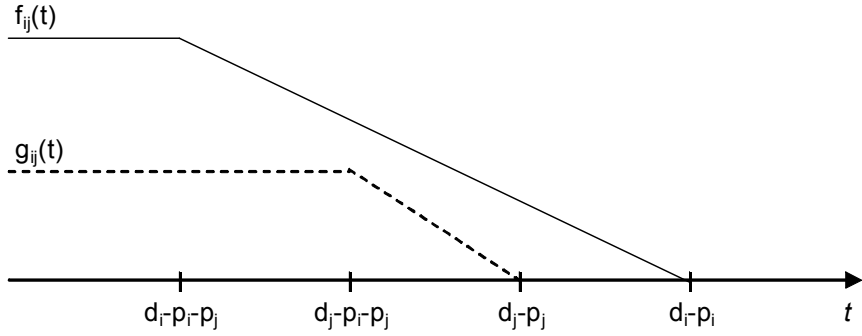


Figure 3: Case 3

In case 3 (see figure 3 for an example), we have  $\Delta_{ij}(t) \geq 0$  for all  $t$ , since  $f_{ij}(t) \geq g_{ij}(t), \forall t$ . Therefore,  $j$  unconditionally precedes  $i$ .

**Proposition 3** *If  $h_i p_j > h_j p_i$ ,  $d_i - p_i \geq d_j - p_j$  and  $h_i (d_i - d_j + p_j) \geq h_j p_i$ , then job  $j$  unconditionally precedes job  $i$ .*

**Proof.** When  $t = d_j - p_i - p_j$ , we have  $\Delta_{ij}(t) = f_{ij}(t) - g_{ij}(t) = h_i (d_i - d_j + p_j) - h_j p_i \geq 0$ . When  $t = d_j - p_j$ , we have  $g_{ij}(t) = 0$  and  $f_{ij}(t) \geq 0$ , since  $d_i - p_i \geq d_j - p_j$ . It can then be seen that  $\Delta_{ij}(t) \geq 0, \forall t$ , and  $j \rightarrow i$ . ■

In case 4, as shown in figure 4, we have a type 2 breakpoint, with  $j$  conditionally preceding  $i$  ( $i$  conditionally preceding  $j$ ) when the processing of the jobs starts before (after) the breakpoint.

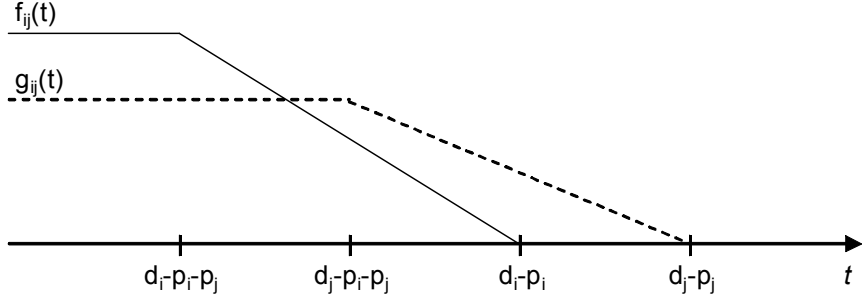


Figure 4: Case 4

**Proposition 4** *If  $h_i p_j > h_j p_i$ ,  $d_i - p_i \leq d_j - p_j$  and  $h_i (d_i - d_j + p_j) < h_j p_i$ , there is the breakpoint  $t_{ij}^2$  and  $j \prec i$  for  $t \leq t_{ij}^2$ , while  $i \prec j$  for  $t \geq t_{ij}^2$ .*

**Proof.** When  $t = d_i - p_i - p_j$ , we have  $\Delta_{ij}(t) > 0$ , since  $g_{ij}(t) = h_j p_i < h_i p_j = f_{ij}(t)$ . When  $t = d_j - p_i - p_j$ , we have  $\Delta_{ij}(t) = f_{ij}(t) - g_{ij}(t) = h_i (d_i - d_j + p_j) - h_j p_i < 0$ . Therefore, the non constant middle segment of  $f_{ij}(t)$  must intersect the constant first segment of  $g_{ij}(t)$ , and we have a  $t_{ij}^2$  breakpoint. Since  $d_i - p_i \leq d_j - p_j$ , no other breakpoint exists. When  $t < t_{ij}^2$ , we then have  $\Delta_{ij}(t) > 0$  and  $j \prec i$ . If  $t > t_{ij}^2$ , we have  $\Delta_{ij}(t) < 0$  and  $i \prec j$ . ■

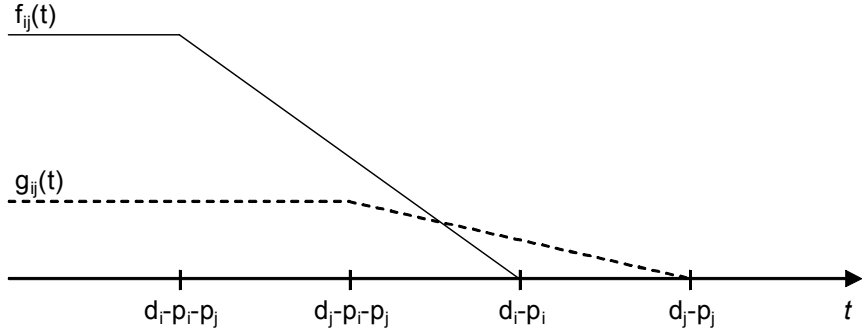


Figure 5: Case 5

In case 5 (see figure 5), there is a  $t_{ij}^1$  breakpoint, and  $j \prec i$  ( $i \prec j$ ) when the first of these jobs starts before (after) the breakpoint.

**Proposition 5** *If  $h_i p_j > h_j p_i$ ,  $d_i - p_i < d_j - p_j$  and  $h_i (d_i - d_j + p_j) \geq h_j p_i$ , there is the breakpoint  $t_{ij}^1$  and  $j \prec i$  for  $t \leq t_{ij}^1$ , while  $i \prec j$  for  $t \geq t_{ij}^1$ .*

**Proof.** When  $t = d_j - p_i - p_j$ , we have  $\Delta_{ij}(t) = f_{ij}(t) - g_{ij}(t) = h_i (d_i - d_j + p_j) - h_j p_i \geq 0$ . Also, for  $t < d_j - p_i - p_j$  we have  $f_{ij}(t) > g_{ij}(t)$ , since  $h_i p_j > h_j p_i$  and



$f_{ij}(t) \geq g_{ij}(t)$  when  $t = d_j - p_i - p_j$ . When  $t = d_i - p_i$ , we have  $\Delta_{ij}(t) < 0$ , since  $g_{ij}(t) > 0$  (because  $d_j - p_j > d_i - p_i$ ) and  $f_{ij}(t) = 0$ . Therefore, the non constant middle segments of  $f_{ij}(t)$  and  $g_{ij}(t)$  must intersect (this intersection can occur at  $t = d_j - p_i - p_j$ ), and there is a  $t_{ij}^1$  breakpoint. If  $t < t_{ij}^1$ , we have  $\Delta_{ij}(t) > 0$  and  $j$  conditionally precedes  $i$ . If  $t > t_{ij}^1$ , then  $\Delta_{ij}(t) < 0$  and  $i \prec j$ . ■

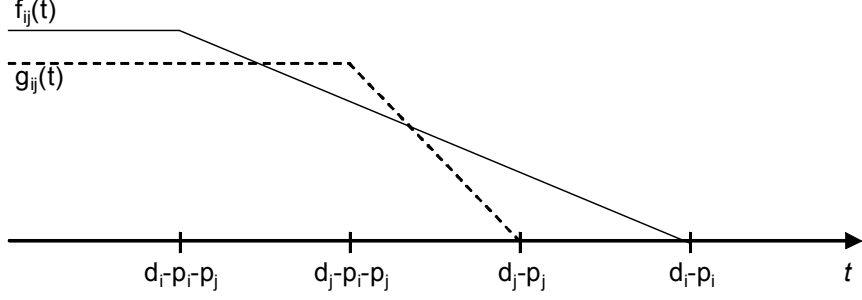


Figure 6: Case 6

In case 6 (see figure 6 for an example), we have both types of breakpoint. Job  $i$  conditionally precedes job  $j$  when their processing starts between these two breakpoints. When  $t < t_{ij}^2$  or  $t > t_{ij}^1$ , we have  $j \prec i$ .

**Proposition 6** *If  $h_i p_j > h_j p_i$ ,  $d_i - p_i > d_j - p_j$  and  $h_i (d_i - d_j + p_j) < h_j p_i$ , there are the two breakpoints  $t_{ij}^1$  and  $t_{ij}^2$ . We then have  $j \prec i$  for  $t \leq t_{ij}^2$ ,  $i \prec j$  for  $t_{ij}^2 \leq t \leq t_{ij}^1$  and  $j \prec i$  for  $t \geq t_{ij}^1$ .*

**Proof.** When  $t = d_i - p_i - p_j$ , we have  $\Delta_{ij}(t) > 0$ , since  $g_{ij}(t) = h_j p_i < h_i p_j = f_{ij}(t)$ . When  $t = d_j - p_i - p_j$ , we have  $\Delta_{ij}(t) = f_{ij}(t) - g_{ij}(t) = h_i (d_i - d_j + p_j) - h_j p_i < 0$ . Therefore, the non constant middle segment of  $f_{ij}(t)$  intersects the constant segment of  $g_{ij}(t)$ , and we have a type 2 breakpoint. When  $t = d_j - p_j$ , we have  $\Delta_{ij}(t) > 0$ , since  $f_{ij}(t) > 0$  (because  $d_i - p_i > d_j - p_j$ ) and  $g_{ij}(t) = 0$ . Therefore, the non constant middle segments of  $f_{ij}(t)$  and  $g_{ij}(t)$  must intersect, and there is a  $t_{ij}^1$  breakpoint. We then have  $\Delta_{ij}(t) < 0$  and  $i \prec j$  for  $t_{ij}^2 < t < t_{ij}^1$ , and  $\Delta_{ij}(t) > 0$  and  $j$  conditionally precedes  $i$  when  $t < t_{ij}^2$  or  $t > t_{ij}^1$ . ■

### 3 Global dominance conditions

In this section, we present two global dominance conditions. Let  $\Delta_{iQj}(T)$  denote the change in the objective function value that results from interchanging two jobs

$i$  and  $j$  whose processing completes at time  $T$ , when these jobs are not necessarily adjacent but separated by a (possibly empty) subsequence  $Q$ . Notice that when  $Q = \emptyset$ ,  $\Delta_{iQj}(T)$  then reduces to the adjacent interchange function  $\Delta_{ij}(T - p_i - p_j)$  presented in the previous section. Also let  $E_{iQj}(T)$  and  $E_Q(T)$  denote the total weighted earliness of all jobs in subsequences  $iQj$  and  $Q$ , respectively, when their processing completes at time  $T$ . We then have:

$$\begin{aligned}\Delta_{iQj}(T) &= E_{iQj}(T) - E_{jQi}(T) \\ &= h_i \max\left(0, d_i - T + p_j + \sum_{k \in Q} p_k\right) + E_Q(T - p_j) + h_j \max(0, d_j - T) \\ &\quad - h_j \max\left(0, d_j - T + p_i + \sum_{k \in Q} p_k\right) - E_Q(T - p_i) - h_i \max(0, d_i - T).\end{aligned}$$

Job  $i$  is said to *globally* precede job  $j$ , denoted as  $i \Rightarrow j$ , when  $i$  precedes  $j$  even if these two jobs are not adjacent. We can now present the two global precedence conditions.

**Theorem 7** *If  $d_i \leq d_j$ ,  $p_i \geq p_j$  and  $h_i \leq h_j$ , then job  $i$  globally precedes job  $j$ .*

**Proof.** Consider any schedule where  $j$  precedes  $i$  and a (possibly empty) subsequence of jobs  $Q$  is performed between  $j$  and  $i$ . Let  $S$  be the start time of  $j$  and  $F$  be the completion time of  $i$ . If  $j$  and  $i$  are swapped, the total cost of all jobs in  $Q$  must decrease or remain the same, since  $p_i \geq p_j$ . We recall  $E_k(T)$  denotes the cost of job  $k$  when it is completed at time  $T$ . We then have

- (a)  $E_i(S + p_j) \geq E_i(S + p_i)$ , since  $p_i \geq p_j$ ;
- (b)  $E_i(T) - E_j(T) = h_i(d_i - T)^+ - h_j(d_j - T)^+$  is a non decreasing function of  $T$ , since  $h_j - h_i \geq 0$  and  $d_i \leq d_j$ .

From (b), we have  $E_i(F) - E_j(F) \geq E_i(S + p_j) - E_j(S + p_j)$ . Using the result in (a), we then have  $E_i(F) - E_j(F) \geq E_i(S + p_i) - E_j(S + p_j)$  or  $E_i(F) + E_j(S + p_j) \geq E_i(S + p_i) + E_j(F)$ . Therefore, the joint cost of jobs  $i$  and  $j$  is lower (or equal) when  $i$  is scheduled before  $j$ , which concludes the proof. ■

**Theorem 8** *If  $d_i < d_j$ ,  $p_i < p_j$  and  $h_i > h_j$ , then job  $j$  globally precedes job  $i$  when*

- a) jobs  $i$  and  $j$  are in case 3 and  $T < d_i$ ;
- b) jobs  $i$  and  $j$  are in case 4 and  $T < t_{ij}^2 + p_i + p_j$ ;

c) jobs  $i$  and  $j$  are in case 5 and  $T < t_{ij}^1 + p_i + p_j$ ;  
where  $T$  is the completion time of the last of these two jobs.

**Proof.** We need to prove, for each of these three situations, that  $\Delta_{iQj}(T) > 0$ , that is, the total weighted earliness is lower when  $j$  is scheduled before  $i$ . In situation a), both jobs are always early, since  $T < d_i < d_j$ . We then have:

$$\begin{aligned}\Delta_{iQj}(T) &= h_i \left( d_i - T + p_j + \sum_{k \in Q} p_k \right) + E_Q(T - p_j) + h_j(d_j - T) \\ &\quad - h_j \left( d_j - T + p_i + \sum_{k \in Q} p_k \right) - E_Q(T - p_i) - h_i(d_i - T) \\ &= h_i \left( p_j + \sum_{k \in Q} p_k \right) + E_Q(T - p_j) - h_j \left( p_i + \sum_{k \in Q} p_k \right) - E_Q(T - p_i) \\ &= E_Q(T - p_j) - E_Q(T - p_i) + h_i p_j - h_j p_i + (h_i - h_j) \sum_{k \in Q} p_k.\end{aligned}$$

Since  $p_j > p_i$ , it follows that  $E_Q(T - p_j) - E_Q(T - p_i) \geq 0$ . We also have  $h_i p_j - h_j p_i > 0$  (because  $i$  and  $j$  are in case 3), and  $(h_i - h_j) \sum_{k \in Q} p_k \geq 0$  (since  $h_i > h_j$ ). Therefore,  $\Delta_{iQj}(T) > 0$  and  $j \Rightarrow i$ .

In situation b),  $j$  is always early, but  $i$  can be tardy, because  $d_i - p_i - p_j < t_{ij}^2 < d_j - p_i - p_j$ , since a type 2 breakpoint occurs when the middle segment of  $f_{ij}(t)$  intersects the horizontal segment of  $g_{ij}(t)$ . When  $i$  is always early, it can be shown that  $j \Rightarrow i$ ; the proof is identical to the one presented for situation a). When  $i$  is tardy, we have:

$$\begin{aligned}\Delta_{iQj}(T) &= h_i \left( d_i - T + p_j + \sum_{k \in Q} p_k \right) + E_Q(T - p_j) + h_j(d_j - T) \\ &\quad - h_j \left( d_j - T + p_i + \sum_{k \in Q} p_k \right) - E_Q(T - p_i) - 0 \\ &= h_i \left( d_i - T + p_j + \sum_{k \in Q} p_k \right) + E_Q(T - p_j) - E_Q(T - p_i) - h_j \left( p_i + \sum_{k \in Q} p_k \right) \\ &= (h_i - h_j) \sum_{k \in Q} p_k + E_Q(T - p_j) - E_Q(T - p_i) + h_i(d_i - T + p_j) - h_j p_i.\end{aligned}$$

Just as in situation a), we have  $(h_i - h_j) \sum_{k \in Q} p_k \geq 0$  and  $E_Q(T - p_j) -$

$E_Q(T - p_i) \geq 0$ , so we just need to consider the expression

$$h_i(d_i - T + p_j) - h_j p_i. \quad (3)$$

We have  $T = t_{ij}^2 + p_i + p_j - \epsilon$ , with  $\epsilon > 0$ . Replacing  $t_{ij}^2$  with the expression given in (2), we then have:

$$\begin{aligned} T &= d_i - p_i - \frac{h_j p_i}{h_i} + p_i + p_j - \epsilon \\ &= d_i + p_j - \frac{h_j p_i}{h_i} - \epsilon. \end{aligned} \quad (4)$$

When  $T$  is replaced in equation (3) by the expression (4), we have:

$$\begin{aligned} &h_i \left( d_i - d_i - p_j + \frac{h_j p_i}{h_i} + \epsilon + p_j \right) - h_j p_i \\ &= h_j p_i - h_j p_i + h_i \epsilon \\ &= h_i \epsilon \end{aligned}$$

Since  $\epsilon > 0$ , we have  $h_i \epsilon > 0$ . Therefore,  $\Delta_{iQj}(T) > 0$  and  $j$  globally precedes  $i$ .

In situation c), it can be shown that  $i$  or  $j$  (but not both simultaneously) can be tardy. When both jobs are indeed tardy when scheduled last, we have:

$$\begin{aligned} \Delta_{iQj}(T) &= h_i \left( d_i - T + p_j + \sum_{k \in Q} p_k \right) + E_Q(T - p_j) + 0 \\ &\quad - h_j \left( d_j - T + p_i + \sum_{k \in Q} p_k \right) - E_Q(T - p_i) - 0 \\ &= (h_i - h_j) \sum_{k \in Q} p_k + E_Q(T - p_j) - E_Q(T - p_i) \\ &\quad + h_i(d_i - T + p_j) - h_j(d_j - T + p_i). \end{aligned}$$

Just as before, we have  $(h_i - h_j) \sum_{k \in Q} p_k \geq 0$  and  $E_Q(T - p_j) - E_Q(T - p_i) \geq 0$ , so we just need to consider the expression

$$h_i(d_i - T + p_j) - h_j(d_j - T + p_i). \quad (5)$$

We have  $T = t_{ij}^1 + p_i + p_j - \epsilon$ , with  $\epsilon > 0$ . Replacing  $t_{ij}^1$  with the expression given

in (1), we then have:

$$T = \frac{h_i(d_i - p_i) - h_j(d_j - p_j)}{h_i - h_j} + p_i + p_j - \epsilon. \quad (6)$$

When  $T$  is replaced in equation (5) by the expression (6), we have:

$$\begin{aligned} & h_i \left( d_i - \frac{h_i(d_i - p_i) - h_j(d_j - p_j)}{h_i - h_j} - p_i - p_j + \epsilon + p_j \right) \\ & - h_j \left( d_j - \frac{h_i(d_i - p_i) - h_j(d_j - p_j)}{h_i - h_j} - p_i - p_j + \epsilon + p_i \right) \\ & = (h_i - h_j) \epsilon + h_i \left( d_i - p_i - \frac{h_i(d_i - p_i) - h_j(d_j - p_j)}{h_i - h_j} \right) \\ & - h_j \left( d_j - p_j - \frac{h_i(d_i - p_i) - h_j(d_j - p_j)}{h_i - h_j} \right). \end{aligned}$$

We have  $(h_i - h_j) \epsilon > 0$ , since  $h_i > h_j$  and  $\epsilon > 0$ , so we just need to consider the other two terms in the previous expression. We have:

$$\begin{aligned} & h_i \left( \frac{h_i(d_i - p_i) - h_j(d_i - p_i) - h_i(d_i - p_i) + h_j(d_j - p_j)}{h_i - h_j} \right) \\ & - h_j \left( \frac{h_i(d_j - p_j) - h_j(d_j - p_j) - h_i(d_i - p_i) + h_j(d_j - p_j)}{h_i - h_j} \right) \\ & = \frac{h_i h_j(d_j - p_j) - h_i h_j(d_i - p_i)}{h_i - h_j} - \frac{h_i h_j(d_j - p_j) - h_i h_j(d_i - p_i)}{h_i - h_j} = 0. \end{aligned}$$

We then have  $\Delta_{iQ_j}(T) > 0$  when both  $i$  and  $j$  are tardy in the last position. Using arguments similar to those applied before, it can also be shown that  $\Delta_{iQ_j}(T) > 0$  when both jobs are early, or when  $j$  is always early and  $i$  is tardy. Therefore,  $j$  globally precedes  $i$  in situation c), which concludes the proof. ■

## 4 Improvement algorithm

In this section, we present a procedure that can be used to improve the sequence generated by any heuristic for the weighted earliness problem. We also describe the heuristics and tests that were used to evaluate the effectiveness of this procedure. The improvement algorithm is quite similar to a procedure presented by Akturk and Yildirim (1998) for the weighted tardiness problem. Let  $[k]$  denote the EDD index of the job in the  $k$ th position in the sequence. The improvement algorithm can then

be described as follows.

Step 1: Determine the EDD indexes of the jobs.

Step 2: Calculate the case and breakpoint data for all job pairs.

Step 3: Apply the global dominance conditions:

For  $k = 1$  to  $n - 1$

For  $l = k + 1$  to  $n$

If  $[l]$  globally precedes  $[k]$ , swap  $[l]$  and  $[k]$ .

Step 4: Set  $k = 1$ .

Step 5: Apply the local dominance conditions:

While  $k < n$

If  $[k + 1]$  locally precedes  $[k]$

swap  $[k]$  and  $[k + 1]$ ;

If  $k > 1$ , set  $k = k - 1$ .

Else, set  $k = k + 1$ .

The time complexity of the algorithm is  $O(n^3)$  (see Akturk and Yildirim (1998) for details). This procedure can be used to improve any heuristic sequence. On the one hand, it can be used to improve heuristic schedules for the weighted earliness problem. On the other hand, the weighted earliness problem is also important as a subproblem in early/tardy scheduling. As previously mentioned, the lower bounds developed by Li (1997) and Liaw (1999) for the earliness/tardiness scheduling problem require initial sequences for an earliness subproblem, and Valente and Alves (2003) have shown that these bounds can be improved by using better initial sequences. Therefore, the improvement procedure can also be used to improve the initial earliness subproblem sequences required by the earliness/tardiness lower bounds.

The effectiveness of the improvement algorithm was evaluated for each of these two settings. The following dispatching heuristics were used to generate the initial

schedules that are then improved by the proposed procedure. The weighted longest processing time (WLPT) rule, presented by Smith (1956), sorts the jobs in non-increasing order of  $p_j/h_j$ . The EDD rule schedules the jobs in increasing order of the EDD indexes described in section 2. Finally, the apparent earliness cost (AEC) heuristic, proposed by Valente and Alves (2003), is a backwards scheduling heuristic, since at each iteration it selects the job that will be scheduled just before the current partial sequence. At each iteration, the AEC heuristic selects the unscheduled job with the highest priority index

$$\frac{h_j}{p_j} \exp\left(-\frac{(T - d_j)^+}{k\bar{p}}\right),$$

where  $\bar{p}$  is the average processing time of the remaining jobs,  $k$  is an empirical parameter and  $T$  is the time at which the next selected job will be completed.

The lower bound presented by Li is based on a decomposition of the earliness/tardiness scheduling problem into weighted earliness and weighted tardiness subproblems. The lower bound for the original early/tardy problem is then the sum of the lower bounds for the two subproblems. The procedure developed by Akturk and Yildirim (1998) has already been shown to yield improved lower bounds for the weighted tardiness criterion, and therefore we will just analyse the effect of the improvement procedure on the value of the weighted earliness subproblem lower bound. The lower bound proposed by Liaw uses a sequence for the weighted earliness subproblem when the lateness factor of a problem is low ( $\leq 0.5$ ), and a weighted tardiness sequence when the lateness factor is high ( $\geq 0.5$ ). Therefore, the improvement procedure will only be used in Liaw's lower bound for problems with a low lateness factor. The improvement algorithm will be compared with the original heuristic result, as well as with a rule developed by Valente and Alves. This rule is a simple adjacency condition that can be applied repeatedly to improve a schedule (see Valente and Alves (2003) for details).

## 5 Computational Results

In this section, we present the results from the computational tests. A set of problems with 15, 20, 25, 30, 40, 50, 100, 200, 250, 300, 400, 500 and 1000 jobs was randomly generated as follows. For each job  $J_j$ , an integer processing time  $p_j$ , an integer earliness penalty  $h_j$  and an integer tardiness penalty  $w_j$  (required for

the early/tardy lower bounds tests) were generated from one of the two uniform distributions  $[1, 10]$  and  $[1, 100]$ , to create low and high variability, respectively. For each job  $J_j$ , an integer due date  $d_j$  is generated from the uniform distribution  $[P(1 - LF - RDD/2), P(1 - LF + RDD/2)]$ , where  $P$  is the sum of the processing time of all jobs,  $LF$  is the lateness factor, set at 0.0, 0.2, 0.4, 0.6, 0.8 and 1.0, and  $RDD$  is the range of due dates, set at 0.2, 0.4, 0.6 and 0.8. For each combination of instance size, processing time and penalty variability,  $LF$  and  $RDD$ , 50 instances were randomly generated. All the algorithms were coded in Visual C++ 6.0 and executed on a Pentium IV 1.7 Ghz personal computer. Throughout this section, and in order to avoid excessively large tables, we will present results only for some representative cases.

In table 1, we present the weighted earliness problem average upper bound values. For each dispatch rule, we give the original upper bound value (H), as well as the objective function value obtained after applying the existing adjacency condition (AC) or the proposed improvement algorithm (IA). The number of times the improvement algorithm performs better ( $<$ ), equal ( $=$ ) or worse ( $>$ ) than the original heuristic schedule and the adjacency condition is given in table 2. Since the improvement algorithm cannot lead to a worse schedule than the original heuristic procedure, the worse ( $>$ ) column is omitted in this case. We also performed a test to determine if the difference between the results obtained with the improvement algorithm and the original heuristic procedure or the adjacency condition is statistically significant. Given that the exact same instances were used, a paired-samples test is appropriate. Since not all the hypothesis of the paired samples t-test were met, the non-parametric Wilcoxon test was selected. The significance values of this test, that is, the confidence level values above which the equal distribution hypothesis is to be rejected, were always equal to 0.000.

From these results, we can conclude that the improvement algorithm is clearly superior to both the original heuristic procedure and the adjacency condition, since the average upper bound value is lower and better results are obtained for most, or in some cases even all, of the test instances. The Wilcoxon test values also indicate that these differences are statistically significant. The reduction in the upper bound value provided by the improvement procedure is larger for the EDD rule, but even the AEC dispatch rule, which gives the best heuristic results, benefits from the application of the improvement algorithm. The proposed improvement procedure is also particularly effective for the WLPT heuristic, since it reduces the objective



$n$	Heur	low variability			high variability		
		H	AC	IA	H	AC	IA
25	AEC	1453	1432	1429	111664	110024	109787
	EDD	2789	1720	1571	240689	138435	122997
	WLPT	1734	1546	1444	132306	117096	111118
50	AEC	5922	5880	5874	452932	449257	448833
	EDD	11572	7502	6527	993004	600305	504544
	WLPT	7100	6444	5908	542977	486990	452374
100	AEC	23969	23876	23864	1839415	1831522	1830663
	EDD	47177	31681	26455	3990351	2543354	2043535
	WLPT	28819	26618	23782	2202935	2008650	1826466
250	AEC	152450	152201	152166	11787305	11765524	11763331
	EDD	298728	209064	167544	25197317	16894062	12894262
	WLPT	181439	171720	149814	13984077	13056878	11542636
500	AEC	613078	612561	612493	47465554	47419994	47415208
	EDD	1198176	855428	674175	101363082	69785470	51677495
	WLPT	729205	700738	600305	56188802	53279627	46169738
1000	AEC	2460792	2459728	2459589	189001908	188908501	188898794
	EDD	4807648	3489956	2708404	404495190	283007168	205459683
	WLPT	2920567	2839610	2402903	223768583	214833044	183392717

Table 1: Upper bound values

$n$	Heur	low variability					high variability				
		IA vs H		IA vs AC			IA vs H		IA vs AC		
		<	=	<	=	>	<	=	<	=	>
25	AEC	771	429	270	914	16	838	362	287	913	0
	EDD	813	387	503	652	45	831	369	547	591	62
	WLPT	1108	92	819	373	8	1122	78	798	402	0
50	AEC	813	387	390	764	46	859	341	436	763	1
	EDD	838	362	694	481	25	848	352	711	459	30
	WLPT	1160	40	1029	169	2	1170	30	1020	180	0
100	AEC	857	343	560	580	60	874	326	623	572	5
	EDD	841	359	772	420	8	841	359	780	410	10
	WLPT	1191	9	1115	85	0	1196	4	1134	66	0
250	AEC	862	338	735	431	34	882	318	787	409	4
	EDD	829	371	810	387	3	849	351	809	386	5
	WLPT	1200	0	1177	23	0	1200	0	1181	19	0
500	AEC	873	327	790	397	13	892	308	827	371	2
	EDD	836	364	814	385	1	846	354	826	374	0
	WLPT	1200	0	1198	2	0	1200	0	1197	3	0
1000	AEC	879	321	827	370	3	889	311	849	351	0
	EDD	839	361	816	384	0	842	358	817	383	0
	WLPT	1200	0	1200	0	0	1200	0	1200	0	0

Table 2: Comparison of upper bound values

function value for most, or in several cases even all, of the test instances.

The results for the early/tardy problem lower bounds are given in tables 3 and 4. As mentioned in the previous section, only Li's weighted earliness subproblem lower bound was analysed, while Liaw's lower bound was calculated just for instances with  $LF < 0.5$ . Since the value of Li's weighted earliness subproblem bound was nearly always equal to 0 for instances with a high lateness factor ( $LF > 0.5$ ), we will only present results for instances with  $LF < 0.5$ . In table 3, we present the average lower bound values. For each heuristic and lower bound type, we give the original lower bound (H), as well as the value obtained after applying the adjacency condition (AC) and the improvement algorithm (IA). The number of times the improvement algorithm performs better ( $>$ ), equal ( $=$ ) or worse ( $<$ ) than the original heuristic sequence and the adjacency condition is given in table 4. A test was also performed to determine if the differences are statistically significant, and the Wilcoxon procedure was once again chosen. The significance values of this test were usually equal to 0.000, and only in a very small number of cases (concerning Liaw's lower bound and the AEC dispatch rule) were these values larger than 0.05.

From the results presented in tables 3 and 4, we can once again conclude that the improvement algorithm is superior to the original heuristic procedure and the adjacency condition, since the average lower bound value is higher and the results are better or equal for most of the test instances. The Wilcoxon test values also indicate that these differences are statistically significant (except for a very small number of cases). The increase in the lower bound value given by the improvement algorithm is much larger for the WLPT and (especially) the EDD heuristics.

In table 5 we give the weighted earliness problem upper bound runtimes (in seconds) for instances with 500 and 1000 jobs, since they could hardly be measured for smaller problems. The computation times of the lower bounding procedures are not presented since they are quite similar to the upper bound runtimes. From these results, we can see that the improvement algorithm requires a higher computation time. However, the additional computational effort is not particularly significant, since the application of the improvement algorithm only takes less than 0.3 seconds for even the largest instances.

$n$	LB	Heur	low variability			high variability		
			H	AC	IA	H	AC	IA
25	Li	AEC	2602	2604	2604	199984	200194	200199
		EDD	1143	1969	2350	69832	141665	175223
		WLPT	2564	2601	2606	196311	200009	200308
	Liaw	AEC	2643	2645	2646	203114	203316	203316
		EDD	1238	2041	2410	77027	147335	179963
		WLPT	2604	2641	2646	199422	203103	203385
100	Li	AEC	42626	42628	42628	3276958	3277426	3277437
		EDD	11733	18048	33788	425807	1014945	2453856
		WLPT	42603	43358	43631	3261979	3335102	3351735
	Liaw	AEC	42795	42798	42800	3290610	3291090	3291128
		EDD	12163	18430	34067	458890	1045495	2476200
		WLPT	42726	43480	43755	3272619	3345780	3362343
500	Li	AEC	1034207	1034210	1034210	78324429	78324935	78324943
		EDD	237931	299877	758249	4154432	9746452	54767538
		WLPT	1081654	1097012	1111235	82760897	84554654	85608938
	Liaw	AEC	1035236	1035239	1035243	78405253	78405851	78405903
		EDD	240209	302012	759844	4327264	9913805	54891073
		WLPT	1082263	1097623	1111856	82812452	84606513	85660861
1000	Li	AEC	4128440	4128443	4128443	310008336	310008902	310008909
		EDD	931238	1105121	2960850	12172468	31409970	211369831
		WLPT	4335455	4389456	4460046	328820063	335576410	340816440
	Liaw	AEC	4130495	4130499	4130510	310169233	310169653	310170019
		EDD	935816	1109455	2964108	12517506	31745038	211620705
		WLPT	4336656	4390664	4461279	328922658	335679777	340919890

Table 3: Lower bound values

$n$	LB	Heur	low variability						high variability					
			IA vs H			IA vs AC			IA vs H			IA vs AC		
			>	=	<	>	=	<	>	=	<	>	=	<
25	Li	AEC	228	372	0	20	580	0	271	328	1	34	565	1
		EDD	502	97	1	290	272	38	488	110	2	311	236	53
		WLPT	311	289	0	154	446	0	334	266	0	160	440	0
	Liaw	AEC	294	235	69	94	449	55	272	313	14	39	550	10
		EDD	502	57	39	316	224	58	486	102	11	317	228	54
		WLPT	387	126	85	241	277	80	331	240	28	162	417	20
100	Li	AEC	329	271	0	54	545	1	352	248	0	129	471	0
		EDD	489	111	0	413	179	8	530	70	0	437	149	14
		WLPT	359	241	0	349	251	0	360	239	1	350	250	0
	Liaw	AEC	432	32	136	244	194	162	408	139	53	172	381	47
		EDD	529	5	66	495	11	94	539	37	24	470	76	54
		WLPT	479	5	116	481	18	101	408	140	52	406	141	53
500	Li	AEC	220	380	0	15	584	1	306	294	0	35	565	0
		EDD	445	154	1	429	170	1	530	69	1	476	122	2
		WLPT	357	243	0	359	241	0	365	235	0	365	235	0
	Liaw	AEC	324	93	183	235	139	226	401	77	122	166	284	150
		EDD	461	1	138	498	2	100	550	7	43	530	5	65
		WLPT	497	3	100	505	2	93	483	24	93	470	27	103
1000	Li	AEC	205	392	3	17	580	3	291	309	0	6	594	0
		EDD	406	190	4	397	196	7	546	54	0	488	111	1
		WLPT	366	232	2	367	232	1	377	223	0	377	223	0
	Liaw	AEC	345	86	169	271	117	212	404	74	122	206	268	126
		EDD	398	1	201	486	2	112	558	0	42	535	1	64
		WLPT	513	7	80	516	6	78	475	22	103	476	30	94

Table 4: Comparison of lower bound values

$n$	Heur	low variability			high variability		
		H	AC	IA	H	AC	IA
500	AEC	0.016	0.017	0.072	0.016	0.017	0.072
	EDD	0.000	0.006	0.057	0.000	0.007	0.057
	WLPT	0.000	0.002	0.056	0.000	0.002	0.055
1000	AEC	0.067	0.068	0.295	0.069	0.070	0.299
	EDD	0.001	0.027	0.231	0.000	0.027	0.231
	WLPT	0.001	0.007	0.226	0.001	0.007	0.234

Table 5: Upper bound runtimes (s)

## 6 Conclusion

In this paper, we considered a single machine scheduling problem with weighted earliness penalties and no idle time. This problem is relevant for production settings where the earliness costs are the main concern, such as in the production of perishable goods, and is also important as a subproblem in early/tardy scheduling. We developed several dominance rules that provide local and global optimality conditions. We also presented an improvement algorithm that uses these rules and can be applied to improve the schedule generated by a heuristic procedure. This algorithm can be used to improve both upper bounds for the weighted earliness criterion and lower bounds for the early/tardy scheduling problem. The computational tests show that, in both cases, the improvement algorithm is superior to the initial heuristic schedule, as well as an existing adjacency condition.

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