

**COMPUTATIONAL RESULTS FOR  
CONSTRAINED MINIMUM  
SPANNING TREES IN FLOW  
NETWORKS**

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# COMPUTATIONAL RESULTS FOR CONSTRAINED MINIMUM SPANNING TREES IN FLOW NETWORKS\*

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## ABSTRACT

In this work, we address the problem of finding a minimum cost spanning tree on a single source flow network. The tree must span all vertices in the given network and satisfy customer demands at a minimum cost. The total cost is given by the summation of the arc setup costs and of the nonlinear flow routing costs over all used arcs. Furthermore, we restrict the trees of interest by imposing a maximum number of arcs on the longest arc emanating from the single source vertex. We propose a dynamic programming model and a solution procedure to solve this problem exactly. Intensive computational experiments were performed using randomly generated test problems and the results obtained are reported. From them we can conclude that the method performance is independent of the type of cost functions considered and improves with the tightness of the constraints.

**Keywords:** Dynamic programming, network flows, constrained trees, general nonlinear costs.

**J.E.L. Classification.** CO2, C61.

## 1 INTRODUCTION

We consider a problem which is an extension of the classical Minimum Spanning Tree problem (MST). As in the MST problem we want to find a minimum cost tree, rooted at the single source,

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spanning all other vertices in a given network. However, we consider that all vertices, except for the source vertex, have an integer nonnegative flow requirement and thus we must also find the flow that must be routed along each used arc. A nonlinear flow dependent cost function is associated with each arc. Furthermore, we also consider a limite on the maximum number of arcs permitted on any of path emanating form the single source.

Network flow problems arise frequently in several application areas (Guisewite 1994): transportation, communication, network design and distribution, production and inventory planning, facility location, scheduling and air traffic control.

A limite on the number of arcs in any path from the root vertex is imposed to guarantee a specified level of service, for example to guarantee a prescribed level of reliability to potential arc or vertex failure (see e.g. (Woolston & Albin 1988)) or to avoid excessive delay of sending a message since this delay is roughly proportional to the number of arcs the message has to traverse (see e.g. (Chepoi & Vaxes 2002)).

The problem we address here is *NP-Hard*, which is not surprisingly since the problem the problem of finding optimal trees for concave minimum cost network flow problems is also *HP-Hard*, even for the simplest version (Guisewite & Pardalos 1991).

Some authors have looked at constraint versions of classical MST and Steiner tree problems, see for example (Gouveia, Magnanti & Requejo 2004) and the references therein. Many other authors have looked at Minimum Cost Network Flow Problems (MCNFPs): for a recent discussion on general concave MCNFPs, see for example (Burkard, Dollani & Thach 2001, Fontes, Hadjiconstantinou & Christofides 2003) for approximate methods and (Fontes, Hadjiconstantinou & Christofides 2006) for exact methods. However, as far as the authors are aware of, no previous work has been reported on path constrained minimum spanning tree problems involving flow supply and general nonlinear cost functions. The dynamic programming model and solution algorithm given here are an extension of the work given in (Fontes 2007).

The computational results have shown the method to be rather robust, since its performance does not depend on the type of cost functions. Moreover, the computational results have also shown that the methods performance increases with the tightness of the constraints.

## 2 PROBLEM DESCRIPTION AND FORMULATION

Let  $G = (W, A)$  denote a directed network with a set  $W$  of  $n + 1$  vertices (the source vertex and  $n$  demand vertices) and with a set  $A$  of  $m$  directed arcs. Vertices 1 to  $n$  have associated a nonnegative integer demand  $r_i$ , which must be satisfied. The total cost, to be minimized, is given by the summation of all costs incurred by both using an arc and routing flow through it, since each arc  $(i, j) \in A$  has associated a general nonlinear and nonnegative cost function

$g_{ij}$ . The cost of sending  $r$  units of flow through an arc, say  $(i, j)$  is given by a monotonously increasing function  $g_{ij}(r)$  which satisfies  $g_{ij}(0) = 0$ . (The flow that can be routed through each arc  $(i, j)$  may have upper  $u_{ij}$  and lower  $l_{ij}$  limits.) The arcs limit  $p$  forces all paths of the minimum cost tree to have no more than  $p$  arcs.

For such a problem the state variable is defined as a triplet  $(S, x, p)$  where  $S$  is the set of vertices to be supplied and hence spanned,  $x$  is the vertex acting as a source and  $p$  is the maximum number of arcs in any path. Therefore, at this state we want to find a minimum cost tree rooted at vertex  $x$  that supplies all vertices in set  $S$  and has no more than  $p$  arcs in any of its paths. Define  $f(S, x, p)$  to be the minimum cost of such a tree.

At each state, the set of vertices  $S$  is to be partitioned into two subsets,  $\{S', \bar{S}'\}$  where  $S' \subseteq S \setminus \{x\}$  and  $\bar{S}'$  is the complement of  $S'$  in the set  $S$ , that is  $\bar{S}' = S \setminus S'$ . Then an immediate decision on a vertex to act as a source for set  $S'$  (receiving the necessary commodity from vertex  $x$ ) is made. Therefore, three costs are incurred: one associated with supplying set  $\bar{S}' = S \setminus S'$  from vertex  $x$  using at most  $k$  arcs  $f(S \setminus S', x, k)$ , another associated with supplying set  $S'$  from the chosen vertex, say  $z$  using at most  $p - 1$  arcs  $f(S', z, p - 1)$  (since arc  $(x, z)$  has already been used), and finally a cost associated with making the flow required by the vertices in  $S'$ , say  $r$ , available at vertex  $z$   $g_{xz}(r)$ .

We can then model the above problem as follows:

$$f(S, x, p) = \min_{S' \subseteq S \setminus \{x\}} \left[ f(S \setminus S', x, p) + \min_{\substack{z \in S' \\ l_{xz} \leq r \leq u_{xz}}} [f(S', z, p - 1) + g_{xz}(r)] \right]. \quad (1)$$

An illustration, for  $p = 3$ , is given in Figure 1.

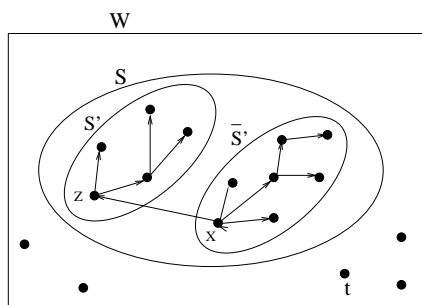


Figure 1: Possible directed trees with arcs limit  $p = 3$ .

Recursion (1) applies for all  $S \subseteq W$  and all  $x \in S$ . Hence, the cost of an optimal tree supplying all demand vertices in set  $W$  from the source vertex  $t$  with limit  $p$ , is given by  $f(W, t, P)$ , if one exists.

$$f^* \equiv f(W, t, P) = \min_{S' \subseteq W \setminus \{t\}} \left[ f(W \setminus S', t, P) + \min_{\substack{z \in S' \\ l_{tz} \leq \sum_{i \in S'} r_i \leq u_{tz}}} \left[ f(S', z, P - 1) + g_{tz} \left( \sum_{i \in S'} r_i \right) \right] \right]$$

Initial conditions are provided by

$$f(S, x, k) = \begin{cases} 0, & \text{if } S = \{x\} \text{ for all } k \\ \infty, & \text{if } S \neq \{x\} \text{ for all } k. \end{cases} \quad (2)$$

### 3 THE DYNAMIC PROGRAMMING ALGORITHM

In an initial procedure we label all states as not computed and then initialize states as given by equation (2). The optimal tree  $f(W, t, P)$  is obtained by calling the recursive function  $Compute(W, t, P)$ .

*Compute*( $S, x, k$ )

If state  $(S, x, k)$  has already been computed then return  $f(S, x, k)$

Set  $min = \infty$

For each  $S' \subseteq S$  /\* recall that a set is represented by an integer, therefore to consider all subsets it is enough to do a for  $i = 1$  to  $2^{|S|} - 1$  \*/

Call *Compute*( $S \setminus S', x, k$ )

If  $f(S \setminus S', x, k) \geq min$  then get another  $S'$

For each  $z \in S'$  /\* here a cycle for  $z = i$  to  $n$  followed by a bit test is performed \*/

If  $(x, z) \notin A$  then get another  $z$

$$r = \sum_{i \in S'} r_i$$

If  $r > u_{xz}$  or  $r < l_{xz}$  then get another  $z$

If  $f(S \setminus S', x, k) + g_{x,z}(r) \geq min$  then get another  $z$

Call *Compute*( $S', z, k - 1$ )

If  $f(S \setminus S', x, k) + g_{x,z}(r) + f(S', z, k - 1) \geq min$  then get another  $z$

$$min = f(S \setminus S', x, k) + g_{x,z}(r) + f(S', z, k - 1)$$

Store information on:

$subset=S', \text{ vertex}=z, \text{ flow}=r, \text{ and } f(S, x) = \min.$

End for

End for

Return  $f(S, x, k)$

At the end of the procedure, if  $f(W, t, P) = \infty$  then no tree network exists satisfying the arcs limit  $P$  and the flow limits; otherwise  $f(W, t, P)$  gives the cost associated with an optimal path constrained tree. The solution structure, i.e. the arcs used and the amount of flow routed through these arcs, is obtained by a recursive routine that backtracks through the information stored (subset, vertex, and flow) during the computation of intermediate states.

The complexity of the DP algorithm, as expected, increases exponentially with problem size. On the other hand, the DP model performance does not deteriorates with the type, nature, or form of the cost functions used and its performance actually improves with the tightness of the arcs limit constraints.

## 4 COMPUTATIONAL RESULTS

The algorithm presented in this paper was implemented in Fortran 90 and computationally evaluated by solving a set of randomly generated test problems.

The problems considered are amongst the most difficult problems as all arcs have cost functions that are neither convex nor concave. The problems data can be downloaded from the OR-Library (Beasley n.d.) and a thorough description of the generation procedure is provided in (Fontes et al. 2003).

Three different types of cost functions are considered: type G1 and type G2 are variations of the fixed-charge cost function where discontinuities other than at the origin are introduced and type G3, for which we consider that arc costs are initially concave and then convex having a discontinuity at the break point. The discontinuity point,  $\bar{R}$  was set to 50% of the total demand  $R$ .

Types G1 and G2 correspond, respectively, to the so called staircase and sawtooth cost functions, see (Kim 2003), in our case with two segments.

$$g_{ij}(r) = \begin{cases} 0, & \text{if } r = 0, \\ -a_{ij}r^2 + b_{ij}r + c_{ij} & \text{if } r \leq \bar{R}, \\ a_{ij}r^2 + b_{ij}r + c_{ij} + k & \text{otherwise,} \end{cases}$$

where  $a_{ij} = 0$  for G1 and G2,  $k = b_{ij}$  for G1, and  $k = -b_{ij}$  for G2 and G3.

In tables 1 to 3 we summarize the results obtained for uncapacitated problems involving cost functions of types G1, G2, and G3 with the discontinuity point occurring at 50% of  $R$ , and considering eight different limites on the number of arcs in each path  $P = 3, 4, \dots, 9$ , or 10. We report on the average, maximum, and minimum computational time and on the standard deviation, in minutes, required to solve the problems. For each size, cost function type and  $P$  value we solve 30 problem instances. Thus, overall we have solved 450 problem instances for each limite value  $P$ .

In order to better analyse the results obtained we also give their graphical representation. It should be noticed that each of the figures shown in the table and in all graphs were obtained as averages over 30 problem instances of a given problem size, cost function type, and arcs limit value. The computational time is reported in minutes and also in logarithmic scale. The latter one is provided since the range magnitude of the computational times is quite large.

As it can be seen, for all cost functions, the computational time required increases slowly with the increase of the arcs limit value, except for  $P = 5$ , which typically is much larger than for any other arcs limit value. For very small  $P$  values, in particular for  $P = 3$  the computational time is smaller since the constraint are very restrictive, possibly eliminating many solutions otherwise feasible. For large  $P$  values the computational time increases since the constrained problem becomes harder due to the enlargement of the solution domain. However, for much larger values,  $P$  being greater than or equal to 8, the computational time remains basically constant, which is probably do to the fact that for such values the constrains are no longer effective.

To illustrate that the methods performance is independent on the cost function type we have plotted the computational time, again in logarithmic scale and in minutes, for problems of all sizes and for all three cost type functions in figures 3 to 7.

Unlike diameter-constrained minimum spanning trees, path constrained minimum cost flow spanning trees do not seem to be more difficult for odd constraint values then for even values.

$N$	$P$	Aver	Max	Min	StDev
	3	0.001	0.017	0.000	0.004
	4	0.002	0.017	0.000	0.005
	5	0.002	0.017	0.000	0.006
10	6	0.002	0.017	0.000	0.005
	7	0.002	0.017	0.000	0.005
	8	0.002	0.017	0.000	0.005
	9	0.002	0.017	0.000	0.006
	10	0.001	0.017	0.000	0.004
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	3	0.009	0.017	0.000	0.008
	4	0.012	0.017	0.000	0.007
	5	0.029	0.029	0.029	0.012
12	6	0.018	0.033	0.000	0.009
	7	0.019	0.033	0.000	0.008
	8	0.020	0.033	0.000	0.009
	9	0.021	0.033	0.017	0.007
	10	0.019	0.033	0.000	0.009
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	3	0.225	0.333	0.167	0.045
	4	0.363	0.550	0.250	0.072
	5	0.755	0.755	0.755	0.160
15	6	0.501	0.767	0.367	0.110
	7	0.558	0.850	0.400	0.121
	8	0.567	0.867	0.417	0.127
	9	0.603	0.917	0.450	0.139
	10	0.572	0.867	0.417	0.132
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	3	2.191	3.683	1.317	0.598
	4	3.647	5.433	2.050	0.973
	5	8.081	8.081	8.081	2.238
17	6	5.543	8.467	3.133	1.593
	7	6.353	9.683	3.583	1.876
	8	6.518	10.033	3.633	2.003
	9	6.939	10.717	3.783	2.168
	10	6.780	10.550	3.600	2.159
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	3	19.724	33.433	9.733	5.550
	4	32.304	55.183	14.833	9.078
	5	68.093	68.093	68.093	23.176
19	6	50.032	89.583	22.183	14.730
	7	55.684	102.867	24.417	17.129
	8	58.486	109.800	24.550	18.520
	9	59.748	113.383	24.683	19.318
	10	61.289	117.600	25.200	20.228

Table 1: Computational performance for problems with cost functions of type G1.

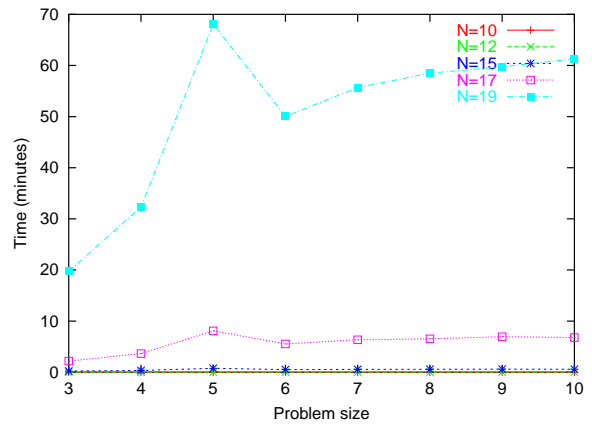
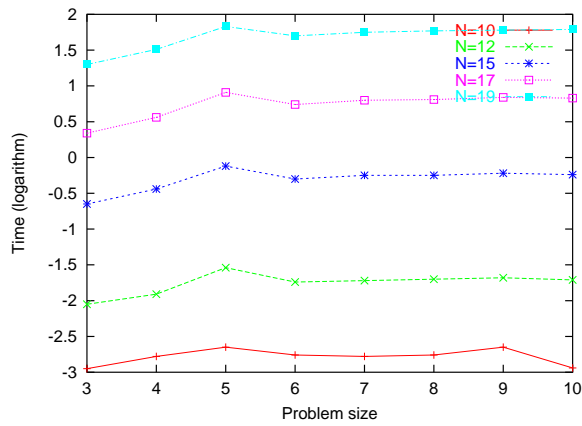


$N$	$P$	Aver	Max	Min	StDev
	3	0.001	0.017	0.000	0.004
	4	0.001	0.017	0.000	0.004
	5	0.003	0.017	0.000	0.006
10	6	0.002	0.017	0.000	0.005
	7	0.003	0.017	0.000	0.006
	8	0.002	0.017	0.000	0.006
	9	0.002	0.017	0.000	0.006
	10	0.002	0.017	0.000	0.006
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	3	0.009	0.017	0.000	0.008
	4	0.013	0.017	0.000	0.007
	5	0.028	0.050	0.017	0.009
12	6	0.018	0.033	0.000	0.009
	7	0.020	0.033	0.000	0.010
	8	0.018	0.033	0.000	0.010
	9	0.019	0.033	0.000	0.010
	10	0.020	0.033	0.000	0.008
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	3	0.227	0.333	0.167	0.044
	4	0.363	0.533	0.250	0.073
	5	0.753	1.117	0.500	0.157
15	6	0.515	0.783	0.350	0.114
	7	0.558	0.883	0.400	0.130
	8	0.562	0.850	0.400	0.133
	9	0.571	0.867	0.417	0.131
	10	0.586	0.900	0.417	0.135
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	3	2.182	3.667	1.267	0.606
	4	3.585	5.333	2.067	0.959
	5	8.025	11.950	4.350	2.174
17	6	5.637	8.583	3.133	1.608
	7	6.356	9.900	3.583	1.899
	8	6.562	10.067	3.683	1.995
	9	6.814	10.550	3.700	2.136
	10	6.812	10.533	3.733	2.134
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	3	19.926	33.383	10.100	5.543
	4	32.112	54.133	14.267	8.919
	5	53.940	107.400	19.167	20.091
19	6	51.138	89.967	22.017	14.639
	7	56.766	105.033	23.550	17.585
	8	58.878	109.033	24.650	18.519
	9	60.971	115.267	25.167	19.841
	10	61.416	115.633	25.450	19.972

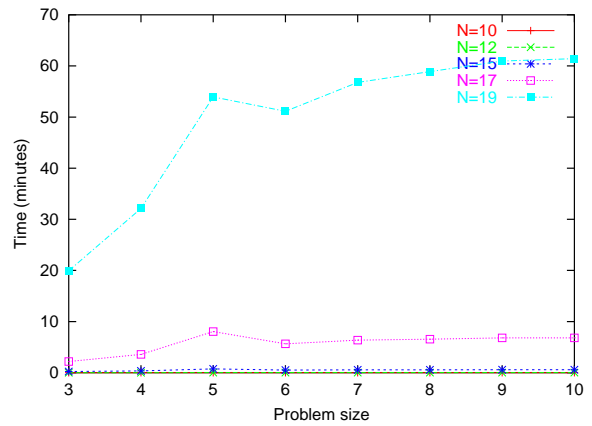
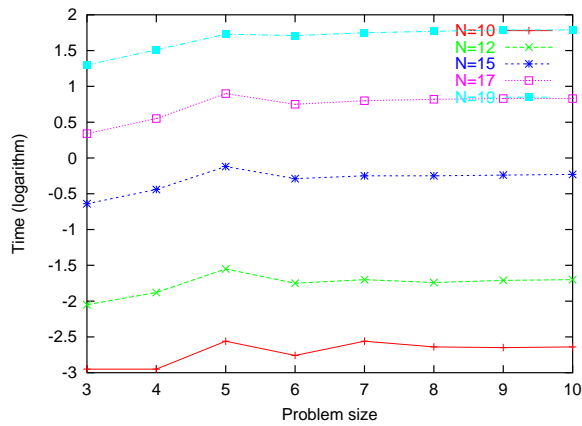
Table 2: Computational performance for problems with cost functions of type G2.

$N$	$P$	Aver	Max	Min	StDev
	3	0.001	0.017	0.000	0.004
	4	0.001	0.017	0.000	0.004
	5	0.003	0.017	0.000	0.006
10	6	0.002	0.017	0.000	0.005
	7	0.001	0.017	0.000	0.004
	8	0.002	0.017	0.000	0.006
	9	0.002	0.017	0.000	0.006
	10	0.001	0.017	0.000	0.004
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	3	0.009	0.017	0.000	0.008
	4	0.012	0.017	0.000	0.008
	5	0.027	0.033	0.017	0.008
12	6	0.018	0.033	0.000	0.008
	7	0.020	0.033	0.000	0.008
	8	0.019	0.033	0.000	0.008
	9	0.019	0.033	0.000	0.010
	10	0.019	0.033	0.000	0.010
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	3	0.231	0.333	0.167	0.044
	4	0.347	0.533	0.250	0.069
	5	0.752	1.167	0.517	0.159
15	6	0.504	0.783	0.350	0.111
	7	0.546	0.833	0.400	0.123
	8	0.558	0.883	0.417	0.127
	9	0.572	0.900	0.433	0.131
	10	0.571	0.900	0.433	0.131
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	3	2.178	3.700	1.300	0.606
	4	3.493	5.367	1.983	0.963
	5	7.972	12.400	4.450	2.271
17	6	5.497	8.700	3.117	1.655
	7	6.083	9.617	3.483	1.908
	8	6.376	10.117	3.633	2.050
	9	6.647	10.667	3.717	2.170
	10	6.635	10.800	3.617	2.215
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	3	19.853	33.617	9.950	5.600
	4	32.441	55.617	14.900	9.197
	5	71.023	125.400	31.983	20.372
19	6	49.828	90.167	22.283	14.835
	7	55.967	105.633	24.350	17.571
	8	58.187	111.400	24.700	18.962
	9	60.133	116.717	25.400	20.131
	10	60.969	118.417	25.233	20.604

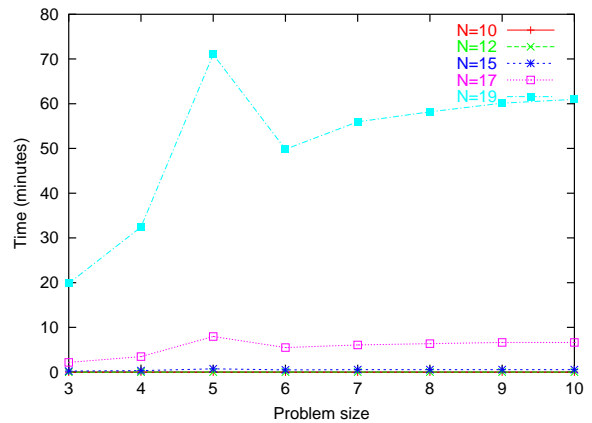
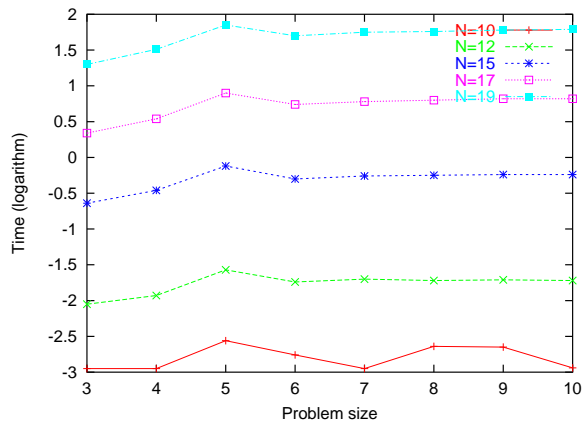
Table 3: Computational performance for problems with cost functions of type G3.



a) Problems with cost functions of type G1.



b) Problems with cost functions of type G2.



c) Problems with cost functions of type G3.

Figure 2: The effect of the arcs limit value on computational time.

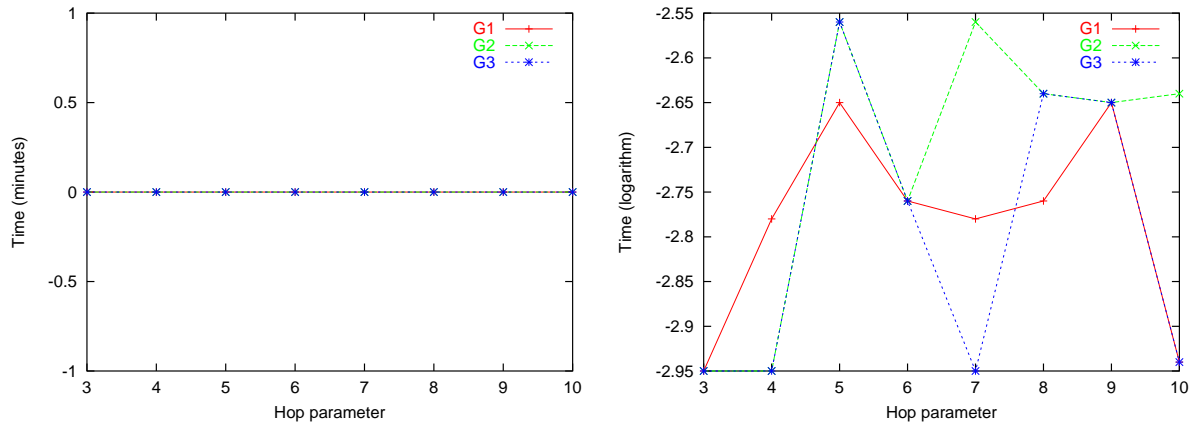


Figure 3: The effect of cost function type on computational time, for problems with 10 vertices.

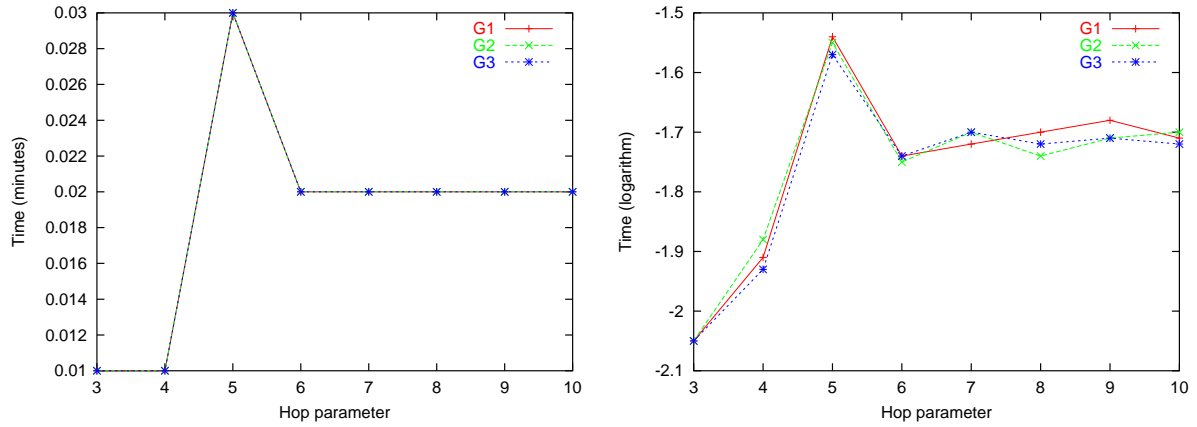


Figure 4: The effect of cost function type on computational time, for problems with 12 vertices.

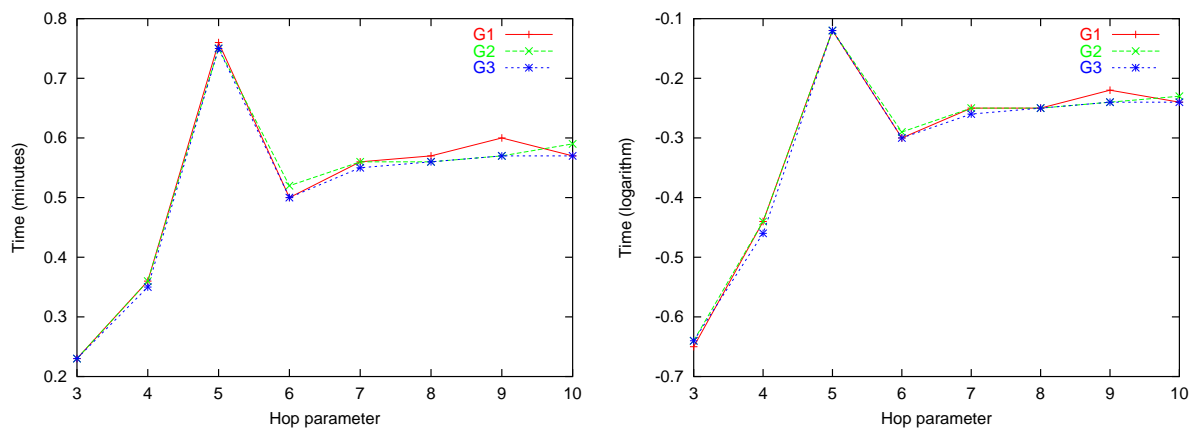


Figure 5: The effect of cost function type on computational time, for problems with 15 vertices.

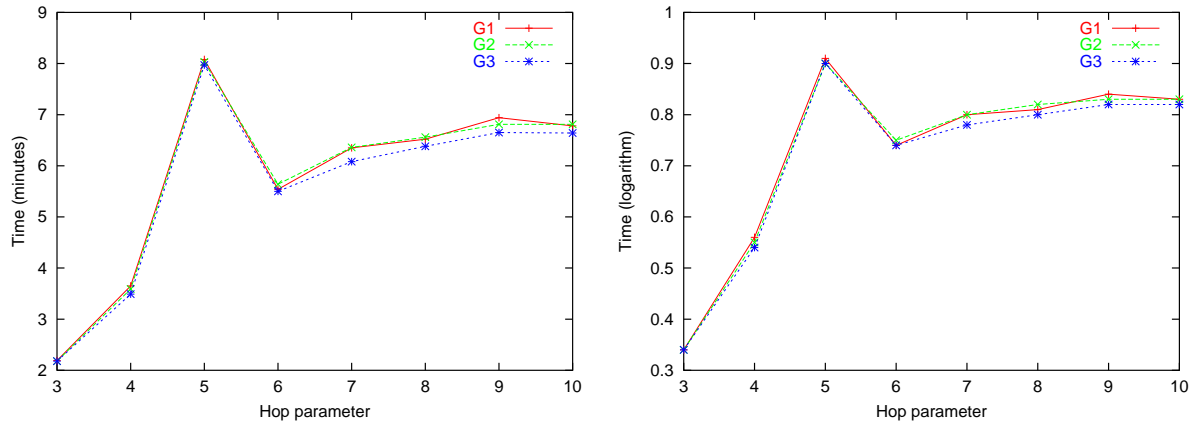


Figure 6: The effect of cost function type on computational time, for problems with 17 vertices.

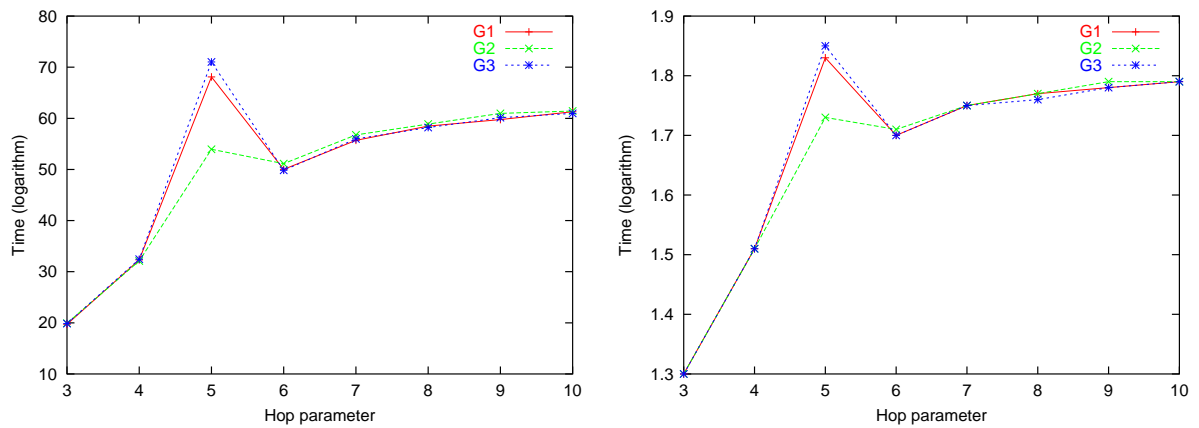


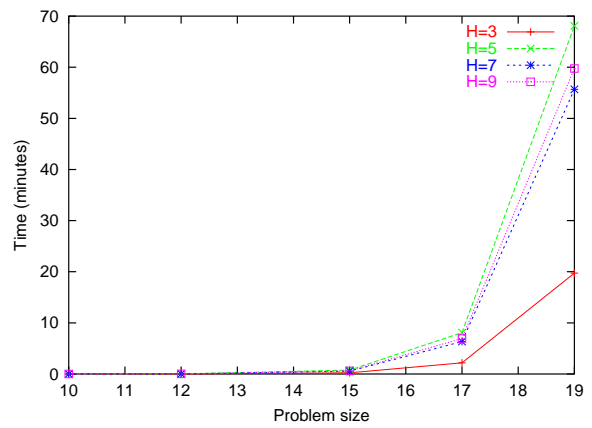
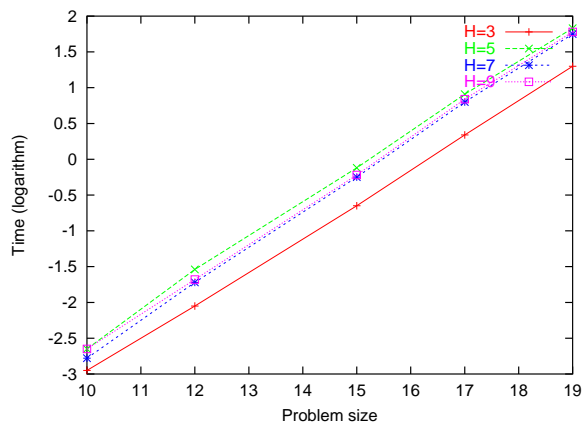
Figure 7: The effect of cost function type on computational time, for problems with 19 vertices.

This can be seen in Figure 8 where we have plotted the computational time for all problem sizes considering constraint values to be a) odd, b) even, and c) the best and worst performance for even and odd values.

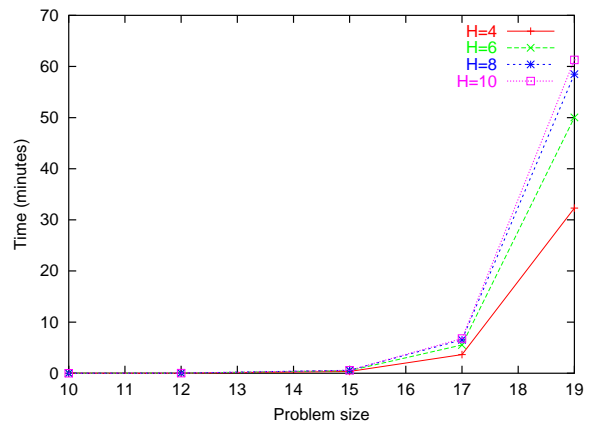
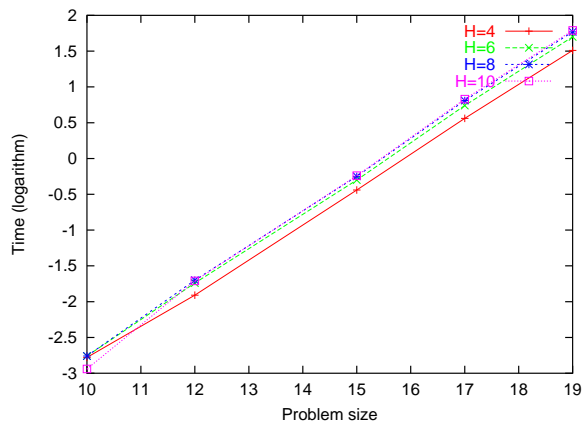
## 5 CONCLUSIONS

In this paper we have presented a DP methodology for finding path constrained trees that satisfy customer demands at minimum cost. The constraints considered, force the paths in the trees to have no more than a predefined number  $P$  of arcs. The cost functions considered may be neither differentiable nor continuous. Also, they might be neither convex nor concave having only to be separable and additive.

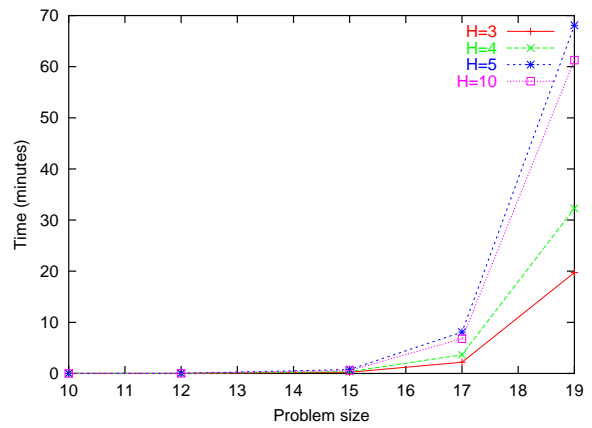
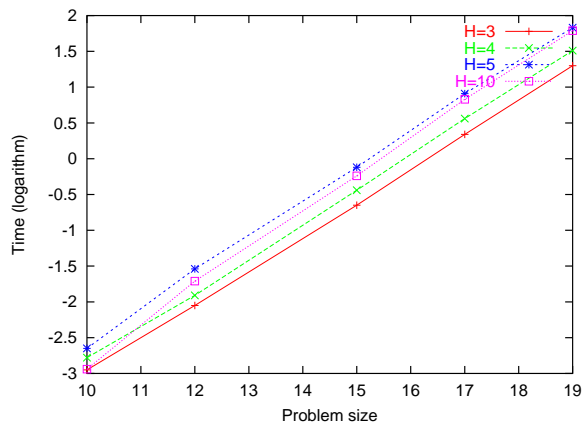
No other works have been founded in the literature for path constrained trees that involve gen-



a) Odd values.

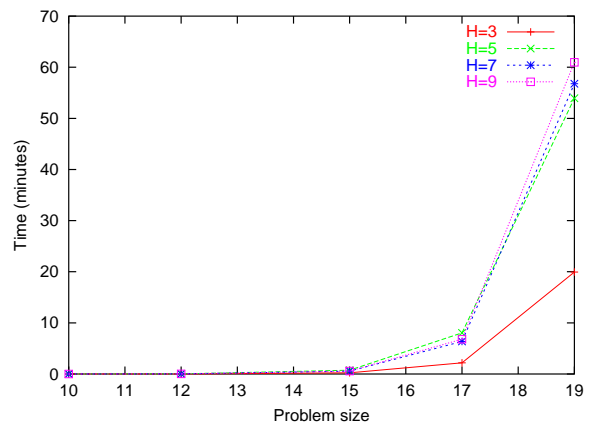
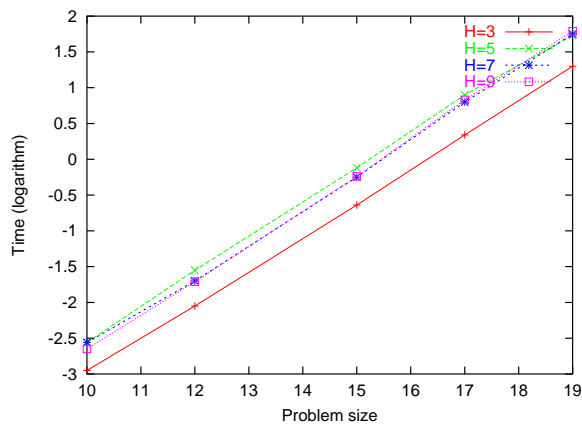


b) Even values.

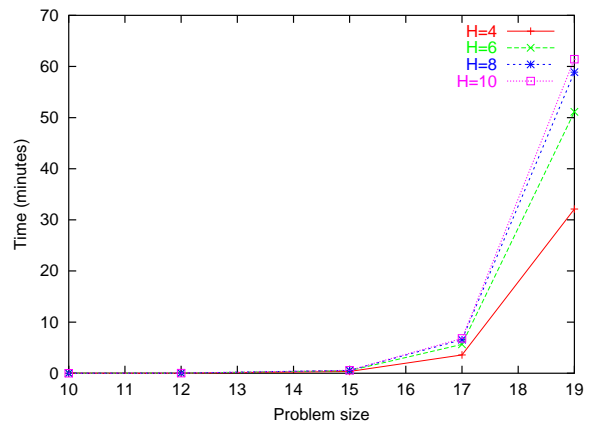
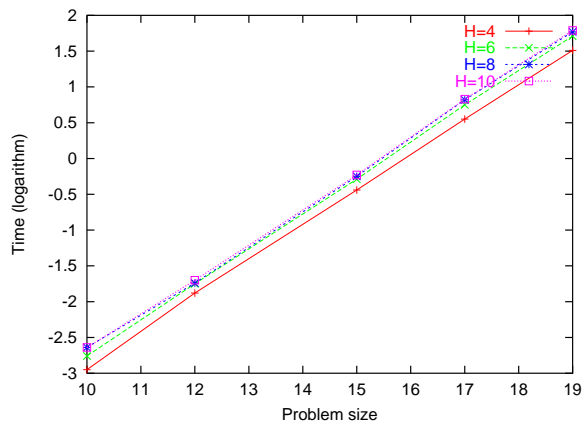


c) Best and worst, odd and even values.

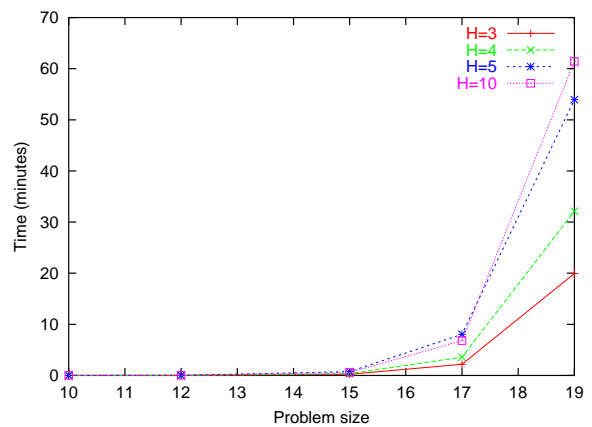
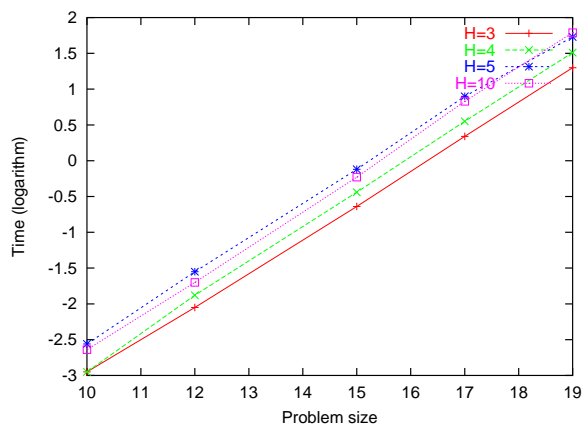
Figure 8: The effect of having odd and even constraint values on computational time, for problems type G1.



a) Odd values.

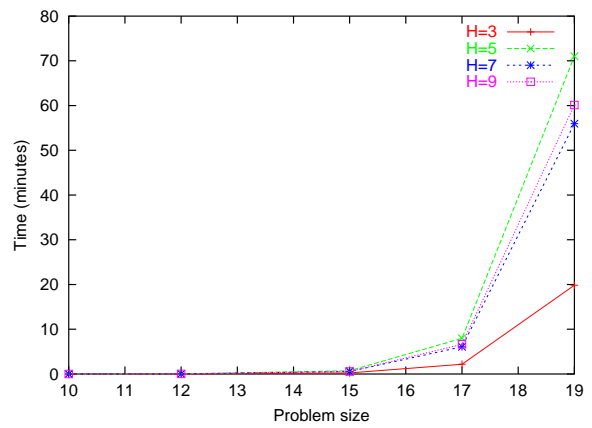
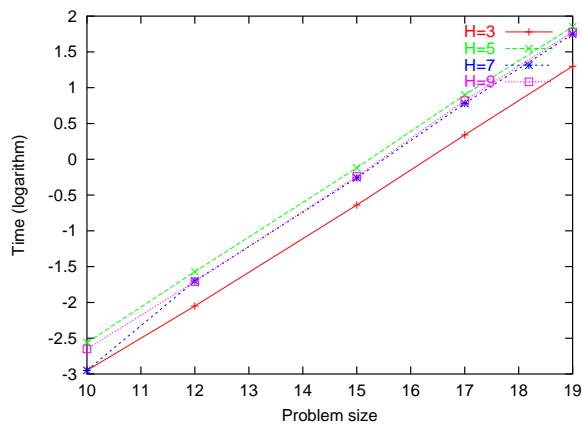


b) Even values.

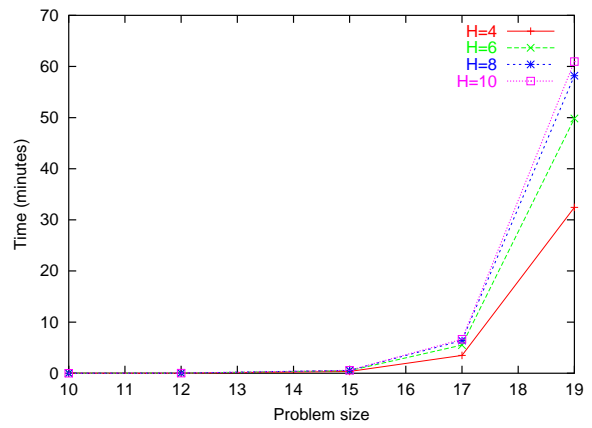
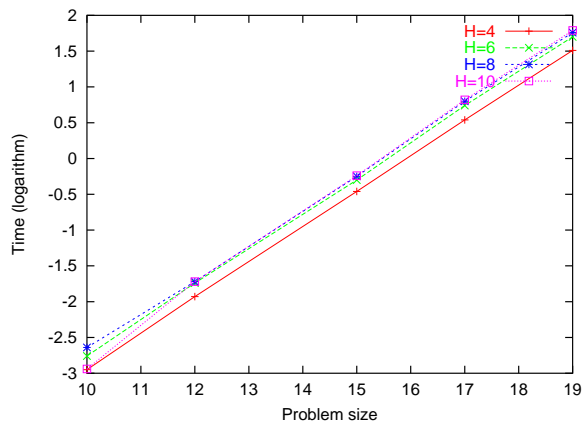


c) Best and worst, odd and even values.

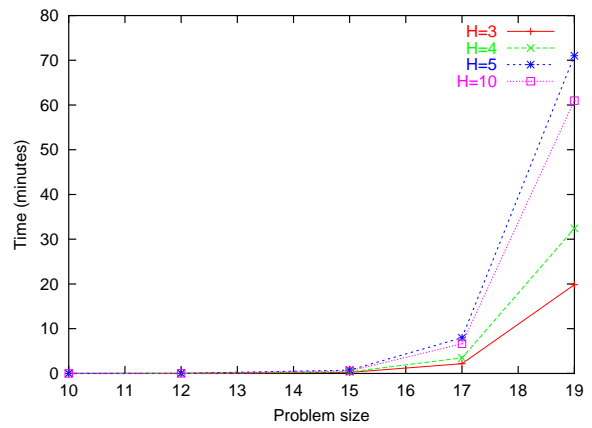
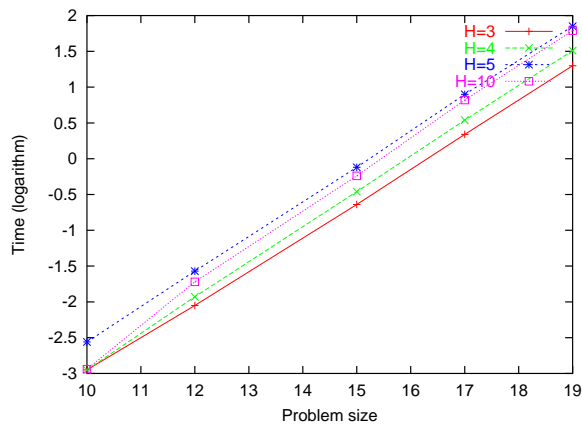
Figure 9: The effect of having odd and even constraint values on computational time, for problems type G2.



a) Odd values.



b) Even values.



c) Best and worst, odd and even values.

Figure 10: The effect of having odd and even constraint values on computational time, for problems type G3.



eral nonlinear arc costs that are neither convex nor concave. Furthermore, the works found are on graphs and do not involve flow routing on the arcs and flow supplying to the customers.

A large number of randomly generated test problems of varying size and complexity was used to evaluate the algorithms performance. Overall, computational experiments were carried out on 450 problem instances for each of the eight limit values for number of arcs considered. The results have shown the DP algorithm to be effective at solving such a problem for any type of cost function. Furthermore, the algorithm is also efficient, although only for small and medium size problem instances, since computational requirements grow rapidly with problem size.

A major advantage of the methodology proposed in this work, is that it is independent of the cost function type as well as of the number of nonlinear arcs (which have been shown to be the major factors defining problem complexity, see (Burkard et al. 2001, Hochbaum & Segev 1989)): Furthermore, the proposed methodology can address the path constraints and, actually take advantage of them, specially if they are very tight.

## References

- Beasley, J. E. (n.d.), 'Or-Library', <http://www.brunel.ac.uk/depts/ma/research/jeb/info.html>.
- Burkard, R. E., Dollani, H. & Thach, P. H. (2001), 'Linear approximations in a dynamic programming approach for the uncapacitated single-source minimum concave cost network flow problem in acyclic networks', *Journal of Global Optimization* **19**, 121–139.
- Chepoi, V. & Vaxes, Y. (2002), 'Augmenting trees to meet biconnectivity and diameter constraints', *Algorithmica* **33**, 243–262.
- Fontes, D. B. M. M. (2007), Diameter-constrained trees for general nonlinear cost flow networks, in B. Fortz, ed., 'INOC - International Conference on Network Optimization'.
- Fontes, D. B. M. M., Hadjiconstantinou, E. & Christofides, N. (2003), 'Upper bounds for single source uncapacitated minimum concave-cost network flow problems', *Networks* **41**, 221–228.

- Fontes, D. B. M. M., Hadjiconstantinou, E. & Christofides, N. (2006), 'A branch-and-bound algorithm for concave network flow problems', *Journal of Global Optimization* **34**, 127–155.
- Gouveia, L., Magnanti, T. L. & Requejo, C. (2004), 'A 2-path approach for odd-diameter-constrained minimum spanning and steiner trees', *NETWORKS* **44**(4), 254–265.
- Guisewite, G. M. (1994), Network problems, *in* R. Horst & P. M. Pardalos, eds, 'Handbook in Global Optimization', Kluwer Academic, pp. 506–648.
- Guisewite, G. M. & Pardalos, P. M. (1991), 'Algorithms for the single-source uncapacitated minimum concave-cost network flow problem', *Journal of Global Optimization* **3**, 245–265.
- Hochbaum, D. S. & Segev, A. (1989), 'Analysis of a flow problem with fixed charges', *Networks* **19**, 291–312.
- Kim, D. (2003), Piecewise linear network flow problems, *in* C. A. Floudas & P. M. Pardalos, eds, 'Encyclopedia of Optimization', Kluwer Academic Publisher.
- Woolston, K. & Albin, S. (1988), 'The design of centralized networks with reliability and availability constraints', *Computers & Operations Research* **15**, 207–217.

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