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João Correia-da-Silva

CEMPRE and
Faculdade de Economia, Universidade do Porto

U.PORTO
FEP FACULDADE DE ECONOMIA UNIVERSIDADE DO PORTO

Agreeing to disagree in a countable space of equiprobable states*

João Correia-da-Silva

CEMPRE and Faculdade de Economia. Universidade do Porto. PORTUGAL.

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Abstract. An example is given in which agents *agree to disagree*, showing that Aumann's (1976) Agreement Theorem does not extend to a countable space of equiprobable states of nature. Even in this unorthodox setting, if the sets of the information partitions are intervals, an agreement theorem holds. A result that describes the margin for disagreement is also obtained.

Keywords: Agreeing to disagree, Interactive epistemology, Bounded rationality.

JEL Classification Numbers: D82, D84.

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1 Introduction

The axiom of countable additivity states that given a countable set of disjoint events, the probability of occurrence of one of them is equal to the sum of the probabilities of the individual events. It is central to measure-theoretic probability theory, in spite of some controversy on whether it should be regarded as a normative principle or as a technical convenience.

As explained by Stinchcombe (1997), the framework for decision making under uncertainty built by Savage (1954) does not guarantee that subjective probabilities satisfy countable additivity, and this opens the way to paradoxes such as money pumps and indifference between an act and another that pointwise dominates it. A related paradox is presented here.

De Finetti (1974) claimed that probability theory should allow for the case of a uniform distribution over a countably infinite set of possibilities. This setting, which violates countable additivity, is the one that is considered in this paper.[1]

Aumann (1976) established the Agreement Theorem: if two agents have the same priors, and if their posteriors for an event are commonly known by both, then they must be equal. The insight provided by this beautiful result is that if agents have different expectations, then they must revise them taking into account the opinion of the others. This process can only stop when expectations coincide. An important consequence is that agents never engage in speculative trade (Milgrom & Stokey, 1982).

This result holds in the general setup considered in economic theory. The common

[1]Suppose that countable additivity held. If the probability of an individual possibility were strictly positive, then, the probability of the whole would be infinite. If the probability of an individual possibility were null, then, the probability of the whole would also be null. In any case, there is a contradiction with the notion that the probability of the whole universe is 1.

prior of the agents is a compact probability measure space, and the agent's private information is described by a partition of the state space (after receiving his private information, an agent knows which set of his private information partition includes the actual state of nature).

In this paper, an example is given showing that Aumann's (1976) Agreement Theorem does not extend to a countable space of equiprobable states of nature. In such a setting, the common prior assumption does not guarantee agreement. Agents may agree to disagree.

This equiprobability among a countable number of states constitutes an *improper common prior*: a σ-finite measure that assigns the same positive probability to each state of nature. Under this improper prior, the probabilities that an agent attributes to events are ill-defined, but the posterior probabilities may be well-defined (Heifetz, 2006).[2]

With the state space identified with the natural numbers, it makes sense to define the probability of an event as the limit of its relative frequency, which is known as the frequency probability. It may happen that we have to settle for lower and upper probabilities, defined as the infimum limit and the supremum limit of the relative frequency. This may be seen as reflecting a sort of bounded rationality.

In order to get positive results, a regularity assumption on the information structures is imposed: the sets of the partitions of information are assumed to be intervals. Under this assumption, the ability of agents to distinguish different states of nature is related to the natural ordering. If an agent believes that the true state of nature can be state 7 or state 9, then the agent cannot rule out the possibility that the true state is 8.

[2] For a detailed technical treatment, see the exposition on conditional probabilities and desintegration of measures by Chang and Pollard (1997).

This assumption brings some discomfort, lessened by the fact that, for an agreement theorem to hold, some assumption on the partitions involving the order on the naturals is necessary. Assuming that the sets of the partitions are intervals is more acceptable if the states of nature can be identified with imperfectly observable physical quantities like temperature or speed.

An agreement theorem is shown for this setting: if the agent's posterior probabilities are common information, then they are equal.

If only lower and upper prior probabilities exist, then there is a margin for disagreement. If common information is not significantly informative, it can only be guaranteed that the commonly known posteriors are between the lower and the upper prior probabilities.

The paper continues with the presentation of the model, in section 2, that also includes an agreement result and a result that describes the margin for disagreement. In section 3, we give an example that illustrates our results.

2 Agreement and Margin for Disagreement

In this section, we set up the model and obtain two results. One gives sufficient conditions for agreement (Theorem 1), and the other describes the possibilities of disagreement (Theorem 2).

Let the set of equally probable possible states of nature be $\mathbb{N}$.

An event (set of states), $E \subseteq \mathbb{N}$, is described by its characteristic function:

$x^E = \{x_s^E\}_{s \in \mathbb{N}}$, with $x_s^E = 1$ if $s \in E$, and $x_s^E = 0$ if $s \notin E$.

The probability of an event is the limit of the ratio of favorable cases over possible cases (relative frequency). The probability of an event is well defined if and only if the upper and lower probabilities coincide.

Definition 1 (PROBABILITIES)

Let $p_n(E) = \dfrac{1}{n}\sum_{s=1}^{n} x_s^E$. *Define:*

$$\underline{p}(E) = \liminf_{n \to +\infty} p_n(E);$$

$$\overline{p}(E) = \limsup_{n \to +\infty} p_n(E);$$

$$p(E) = \lim_{n \to +\infty} p_n(E).$$

The probability of E conditional on A is defined in a similar way.

Definition 2 (CONDITIONAL PROBABILITIES)

Given two sets E *and* A, *let* $p_n(E|A) = \sum_{s=1}^{n} x_s^E x_s^A / \sum_{s=1}^{n} x_s^A$, *Define:*

$$\underline{p}(E|A) = \liminf_{n \to +\infty} p_n(E|A);$$

$$\overline{p}(E|A) = \limsup_{n \to +\infty} p_n(E|A);$$

$$p(E|A) = \lim_{n \to +\infty} p_n(E|A).$$

The information of agent A is described by a partition of $\mathbb{N}$, such that the agent does not distinguish the true state of nature, t, from those that belong to the same set of the partition, $P_A(t)$.

After receiving its information, agent A knows that the true state of nature, t, belongs to $P_A(t)$. The characteristic function of $P_A(t)$ is $x^{A(t)} = \{x_s^{A(t)}\}_{s\in\mathbb{N}}$, with $x_s^{A(t)} = 1$ if $s \in P_A(t)$, and $x_s^{A(t)} = 0$ if $s \notin P_A(t)$.

The posterior probability that agent A attributes to the event E in state t is defined as follows.

Definition 3 (POSTERIOR PROBABILITY)

Let $p_n(E|P_A(t)) = \sum_{s=1}^{n} x_s^E x_s^{A(t)} / \sum_{s=1}^{n} x_s^{A(t)}$. *Define:*

$$\underline{p}(E|P_A(t)) = \liminf_{n\to+\infty} p_n(E|P_A(t)).$$

$$\overline{p}(E|P_A(t)) = \limsup_{n\to+\infty} p_n(E|P_A(t)).$$

$$p(E|P_A(t)) = \lim_{n\to+\infty} p_n(E|P_A(t)).$$

We restrict the analysis to the case in which the sets in the agent's information structures (partitions of $\mathbb{N}$) are intervals.

Assumption 1 (PARTITIONS COMPOSED BY INTERVALS)

$\forall s \in \mathbb{N}:$ $P_A(s)$ *and* $P_B(s)$ *are intervals.*

This assumption implies that the partitions are either composed by an infinite number of finite sets or a finite number of finite sets and one single infinite interval. We are mostly concerned on the former case, in which it is guaranteed that the posterior probabilities are well defined.

Following Aumann (1976), the common information can be described by a partition: the meet of the agents individual partitions, that is, the finest common coarsening of P_A and P_B.

Definition 4 (COMMON INFORMATION STRUCTURE)

The common information of agents A *and* B *is described by* $P = P_A \wedge P_B$.

Proposition 1

If P_A *and* P_B *are composed by intervals, then* $P = P_A \wedge P_B$ *is also composed by intervals.*

Proof.

By definition of common knowledge, we know that $P(s)$ is both a union of sets in P_A and a union of sets in P_B.

$$P(s) = \bigcup_{j \in J_A} P_A^j = \bigcup_{j \in J_B} P_B^j.$$

Suppose that the set $P(s)$ is not an interval, and decompose it into the coarsest intervals that compose it.

$$P(s) = \bigcup_{c \in J_C} P^c(s).$$

A set in J_A cannot intersect two sets in J_C, because, being an interval, it would also contain the states between these two sets in J_C. This would contradict the fact that P_A^j is contained in $P(s)$. Each $P^c(s)$ is, therefore, also a union of sets in P_A and a union of sets in P_B.

$$P^c(s) = \bigcup_{j \in J_A^c} P_A^j = \bigcup_{j \in J_B^c} P_B^j.$$

This means that $P^c(s)$ is an element of $P = P_A \wedge P_B$, and, therefore, $P^c(s) = P(s)$. Contradiction.

QED

In the cases where disagreement is possible, the posterior conditional on common information coincides with the prior.

Proposition 2

If $P(t)$ is an infinite interval, then $\underline{p}(x|P(t)) = \underline{p}(x)$ and $\overline{p}(x|P(t)) = \overline{p}(x)$

Proof.

Since $P(t)$ is an interval and is infinite, it is equal to the set of natural numbers that are greater than some number, N.

$$\underline{p}(x|P(t)) = \liminf_{n\to+\infty} \frac{\sum_{s=1}^{n} a_s x_s}{\sum_{s=1}^{n} a_s} =$$
$$= \liminf_{n\to+\infty} \frac{\sum_{s=1}^{N} a_s x_s + \sum_{s=N+1}^{n} a_s x_s}{\sum_{s=1}^{N} a_s + \sum_{s=N+1}^{n} a_s} =$$
$$= \liminf_{n\to+\infty} \frac{0 + \sum_{s=N+1}^{n} x_s}{0 + n - N} = \liminf_{n\to+\infty} \frac{\sum_{s=N+1}^{n} x_s}{n - N}.$$

It is easy to verify that this coincides with $\underline{p}(x)$, as a finite number of terms is always negligible among an infinite sample.

$$\liminf_{n\to+\infty} \frac{\sum_{s=N+1}^{n} x_s}{n - N} = \liminf_{n\to+\infty} \frac{\sum_{s=1}^{n} x_s}{n} = \underline{p}(x).$$

QED

The main results of this paper are based on the following lemma.

Lemma 1

Let $t \in \mathbb{N}$ be a state of nature, P_A and P_B be partitions composed by intervals, $P = P_A \wedge P_B$ represent the common information, and $x \subseteq \mathbb{N}$ be an event. Then:

i) If it is common knowledge at t that $\underline{p}(x|P_A(t)) = \underline{q}_A$, then $\underline{q}_A \geq \underline{p}(x|P(t))$.

ii) If it is common knowledge at t that $\overline{p}(x|P_A(t)) = \overline{q}_A$, then $\overline{q}_A \leq \overline{p}(x|P(t))$.

Proof.

Consider the partition describing the common information, $P = P_A \wedge P_B$. The set of this partition that contains t is $P(t)$. It is both a union of sets in P_A and a union of sets in P_B:

$$P(t) = \bigcup_{j \in J_A(t)} P_A^j = \bigcup_{j \in J_B(t)} P_B^j.$$

Since the posterior lower probability, $\underline{p}(x|P_A(t)) = \underline{q}_A$, is common information:

$$\forall s \in P(t): \;\; \underline{p}(x|P_A(s)) = \underline{q}_A.$$

Let $a^j = \{a_s^j\}_{s \in \mathbb{N}}$ and $c = \{c_s\}_{s \in \mathbb{N}}$ represent the events P_A^j and $P(t)$, respectively.

$$\forall s \in \mathbb{N}: \sum_{j \in J_A(t)} a_s^j = c_s.$$

<u>If $P(t)$ is finite</u>, all P_A^j are also finite, and $J_A(t)$ is finite (suppose that the number of sets in $J_A(t)$ is T).

$$\exists N_c \in \mathbb{N}: \;\; s > N_c \Rightarrow c_s = a_s^j = 0, \;\; \forall j = 1, ..., T.$$

In this finite case, the actual probability is surely well defined. With some manipulation, we obtain that $p(x|P(t)) = q_A$.

$$p(x|P(t)) = \frac{\sum_{s=1}^{N_c} x_s c_s}{\sum_{s=1}^{N_c} c_s} = \frac{\sum_{s=1}^{N_c} \sum_{j=1}^{T} x_s a_s^j}{\sum_{s=1}^{N_c} c_s} = \sum_{j=1}^{T} \left(\frac{\sum_{s=1}^{N_c} x_s a_s^j}{\sum_{s=1}^{N_c} a_s^j} \, \frac{\sum_{s=1}^{N_c} a_s^j}{\sum_{s=1}^{N_c} c_s} \right) =$$

$$= \sum_{j=1}^{T} q_A \frac{\sum_{s=1}^{N_c} a_s^j}{\sum_{s=1}^{N_c} c_s} = q_A \frac{\sum_{j=1}^{T} \sum_{s=1}^{N_c} a_s^j}{\sum_{s=1}^{N_c} c_s} = q_A \frac{\sum_{s=1}^{N_c} \sum_{j=1}^{T} a_s^j}{\sum_{s=1}^{N_c} c_s} = q_A \frac{\sum_{s=1}^{N_c} c_s}{\sum_{s=1}^{N_c} c_s} = q_A.$$

Of course that lower and upper probabilities necessarily coincide with the actual probability.

$$p(x|P(t)) = \underline{p}(x|P(t)) = \overline{p}(x|P(t)) = q_A = \underline{q}_A = \overline{q}_A.$$

The interesting case is when $P(t)$ infinite.

<u>If $P(t)$ is infinite</u>, then either $J_A(t)$ is finite with the last set being infinite, or

$J_A(t)$ is infinite and made up by finite sets.

Let's start with the case in which $J_A(t)$ is finite, with the last set, P_A^T having an infinite number of states. Denote by N_a the last state that belongs to P_A^{T-1}.

$$s > N_a \Rightarrow a_s^j = 0 \wedge a_s^T = c_s, \forall j = 1, ..., T-1.$$

Below, we manipulate the sequence of truncated conditional probabilities of the event x conditional on $P(t)$. Recall that $\underline{p}(x|P(t))$ is the liminf of this sequence.

$$\frac{\sum_{s=1}^n x_s c_s}{\sum_{s=1}^n c_s} = \frac{\sum_{j=1}^T \sum_{s=1}^n x_s a_s^j}{\sum_{s=1}^n c_s} =$$

$$= \sum_{j=1}^{T-1} \left(\frac{\sum_{s=1}^n x_s a_s^j}{\sum_{s=1}^n a_s^j} \frac{\sum_{s=1}^n a_s^j}{\sum_{s=1}^n c_s} \right) + \frac{\sum_{s=1}^n x_s a_s^T}{\sum_{s=1}^n a_s^T} \frac{\sum_{s=1}^n a_s^T}{\sum_{s=1}^n c_s} =$$

$$= \sum_{j=1}^{T-1} \left(\frac{\sum_{s=1}^{N_A} x_s a_s^j}{\sum_{s=1}^{N_A} a_s^j} \frac{\sum_{s=1}^{N_A} a_s^j}{\sum_{s=1}^n c_s} \right) + \frac{\sum_{s=1}^n x_s a_s^T}{\sum_{s=1}^n a_s^T} \frac{\sum_{s=1}^n a_s^T}{\sum_{s=1}^n c_s} =$$

$$= \sum_{j=1}^{T-1} q_A \frac{\sum_{s=1}^{N_A} a_s^j}{\sum_{s=1}^n c_s} + \frac{\sum_{s=1}^n x_s a_s^T}{\sum_{s=1}^n a_s^T} \frac{\sum_{s=1}^n a_s^T}{\sum_{s=1}^n c_s} =$$

$$= q_A \frac{\sum_{s=1}^{N_A} \sum_{j=1}^{T-1} a_s^j}{\sum_{s=1}^n c_s} + \frac{\sum_{s=1}^n x_s a_s^T}{\sum_{s=1}^n a_s^T} \frac{\sum_{s=1}^n a_s^T}{\sum_{s=1}^n c_s} =$$

$$= q_A \frac{\sum_{s=1}^{N_A} c_s}{\sum_{s=1}^n c_s} + \frac{\sum_{s=1}^n x_s a_s^T}{\sum_{s=1}^n a_s^T} \frac{\sum_{s=1}^n a_s^T}{\sum_{s=1}^n c_s}.$$

With $P(t)$ infinite, $\sum_{s=1}^n c_s$ grows to infinity, thus the first term is negligible. We proceed to show that, in this case, $\underline{p}(x|P(t)) = \underline{q}_A$

$$\underline{p}(x|P(t)) = \liminf_{n\to+\infty} \frac{\sum_{s=1}^n x_s c_s}{\sum_{s=1}^n c_s} = \liminf_{n\to+\infty} q_A \frac{\sum_{s=1}^{N_A} c_s}{\sum_{s=1}^n c_s} + \frac{\sum_{s=1}^n x_s a_s^T}{\sum_{s=1}^n a_s^T} \frac{\sum_{s=1}^n a_s^T}{\sum_{s=1}^n c_s} =$$

$$= \liminf_{n\to+\infty} \frac{\sum_{s=1}^n x_s a_s^T}{\sum_{s=1}^n a_s^T} \frac{\sum_{s=1}^n a_s^T}{\sum_{s=1}^n c_s} = \liminf_{n\to+\infty} \frac{\sum_{s=1}^n x_s a_s^T}{\sum_{s=1}^n a_s^T} \frac{\sum_{s=N_a}^n c_s}{\sum_{s=1}^n c_s} =$$

$$= \liminf_{n\to+\infty} \frac{\sum_{s=1}^n x_s a_s^T}{\sum_{s=1}^n a_s^T} = \underline{q}_A.$$

It remains to be considered the case in which J_A is infinite and made up by finite elements. Denote by $n(j)$ the last state included in the set P_A^j. It is obvious that

$j \leq n(j)$ and that $s > n(j) \Rightarrow a_s^j = 0$.

As before, the probability of x conditional on $P(t)$ is a limit of a sequence of truncated probability calculations.

$$\underline{p}(x|P(t)) = \liminf_{n\to+\infty} \frac{\sum_{s=1}^n x_s c_s}{\sum_{s=1}^n c_s} = \liminf_{n\to+\infty} \frac{\sum_{j\in J_A}\sum_{s=1}^n x_s a_s^j}{\sum_{s=1}^n c_s}.$$

The liminf of the following subsequence cannot, by definition, be higher than the liminf of the original sequence.

$$\underline{p}(x|P(t)) \leq \liminf_{k\to+\infty} \frac{\sum_{j=1}^k \sum_{s=1}^{n(k)} x_s a_s^j}{\sum_{s=1}^{n(k)} c_s} = \liminf_{k\to+\infty} \sum_{j=1}^k \left(\frac{\sum_{s=1}^{n(k)} x_s a_s^j}{\sum_{s=1}^{n(k)} a_s^j} \frac{\sum_{s=1}^{n(k)} a_s^j}{\sum_{s=1}^{n(k)} c_s} \right) =$$

$$= \liminf_{k\to+\infty} \sum_{j=1}^k \left(\underline{q}_A \frac{\sum_{s=1}^{n(k)} a_s^j}{\sum_{s=1}^{n(k)} c_s} \right) = \liminf_{k\to+\infty} \underline{q}_A \frac{\sum_{j=1}^k \sum_{s=1}^{n(k)} a_s^j}{\sum_{s=1}^{n(k)} c_s} =$$

$$= \lim_{k\to+\infty} \underline{q}_A \frac{\sum_{s=1}^{n(k)} c_s}{\sum_{s=1}^{n(k)} c_s} = \underline{q}_A.$$

In this remaining case, $\underline{p}(x|P(t)) \leq \underline{q}_A$. The proof of i) is complete. The same reasoning applies to prove ii).

QED

The following agreement theorem is a corollary of Lemma 1.

Theorem 1 (AGREEMENT THEOREM)

Let $t \in \mathbb{N}$ be a state of nature, $x \subseteq \mathbb{N}$ be an event, P_A and P_B be partitions composed by intervals, and $P = P_A \wedge P_B$. If it is common knowledge at t that $p(x|P_A(t)) = q_A$ and that $p(x|P_B(t)) = q_B$, then $q_A = q_B = p(x|P(t))$.

Proof.

This result is a corollary of Lemma 1.

Since it is common knowledge at t that $p(x|P_A(t)) = q_A$, then it is common knowledge at t that $\underline{p}(x|P_A(t)) = q_A$ and that $\overline{p}(x|P_A(t)) = q_A$.

By Lemma 1, $\underline{q} \leq q_A \leq \overline{q}$.

But the fact that $p(x|P(t)) = q$ means that $\underline{q} = \overline{q} = q$. Therefore: $q_A = q$.

The same argument shows that $q_B = q$.

QED

The following result, establishing the margin for disagreement, is a straightforward corollary of Lemma 1. The proof is omitted.

Theorem 2 (MARGIN FOR DISAGREEMENT THEOREM)

Let $t \in \mathbb{N}$ be a state of nature, $x \subseteq \mathbb{N}$ be an event, P_A and P_B be partitions composed by intervals, and $P = P_A \wedge P_B$. Let $\underline{p}(x|P(t)) = \underline{q}$ and $\overline{p}(x|P(t)) = \overline{q}$. If it is common knowledge at t that $p(x|P_A(t)) = q_A$ and that $p(x|P_B(t)) = q_B$, then $\underline{q} \leq q_A \leq \overline{q}$ and $\underline{q} \leq q_B \leq \overline{q}$.

Notice that if $P(t)$ is finite, then there is no margin for disagreement. If $P(t)$ is infinite, then (by Proposition 2), the lower and upper posteriors conditional on common information coincide with the lower and upper priors. Therefore, in the cases where disagreement is possible, $\underline{p}(x) \leq q_A \leq \overline{p}(x)$ and $\underline{p}(x) \leq q_B \leq \overline{p}(x)$.

Assuming that the sets of the partitions are intervals may be criticized on the grounds that it involves the order on $\mathbb{N}$. This is more acceptable if states of nature refer to imperfectly observable physical quantities like mass, temperature or luminous intensity. In any case, an assumption of this kind is needed for an agreement result to hold.

Suppose that no assumption on the partitions is made, and consider an event E with $\lim_{n \to +\infty} \frac{1}{n} \sum_{s=1}^{n} x_s^E \in (0,1)$. For any rational numbers q_A and q_B, we can construct partitions such that the posteriors q_A and q_B are common knowledge. For example, to construct P_A with a posterior $q_A = 1/2$ that is uniform (and, therefore, common knowledge), simply let P_A^1 contain only the first number in E and the first number not in E, let P_A^2 contain only the second number in E and the second number not in E, etc.[3]

Nevertheless, it is clear that the assumption that the elements of the partitions are intervals may be relaxed. It would be of interest to find necessary and sufficient conditions for the agreement theorem to hold.

[3] I thank for this interesting remark.

3 An Example of Agreeing to Disagree

Ann and Bob face a countable set of equally probable states of nature, that we denote by $\Omega = \{1, 2, ...\}$.

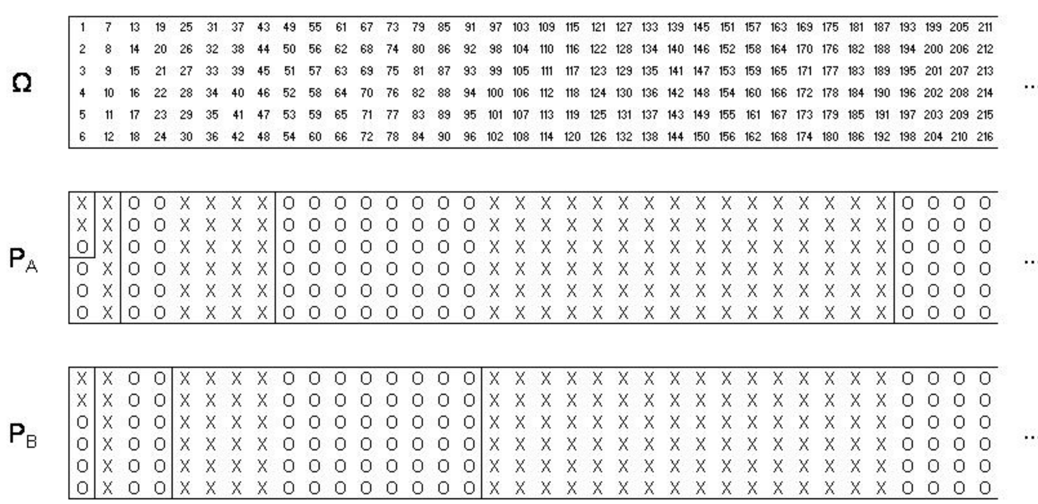
Ann and Bob have different observation capabilities, described by partitions of Ω. These are such that what is observed is the element of the partition that contains the actual state of nature. Their private information partitions are, respectively:

$$P_A = \{\{1, 2, 3\}, \{3 + 1, ..., 2^2 \cdot 3\}, ..., \{2^{2k} \cdot 3 + 1, ..., 2^{2k+2} \cdot 3\}, ...\};$$

$$P_B = \{\{1, ..., 2 \cdot 3\}, \{2 \cdot 3 + 1, ..., 2^3 \cdot 3\}, ..., \{2^{2k-1} \cdot 3 + 1, ..., 2^{2k+1} \cdot 3\}, ...\}.$$

Ann and Bob are curious about whether the event described as follows occurred:

$$X = \{1, 2\} \cup \{2 \cdot 3 + 1, 2^2 \cdot 3\} \cup ... \cup \{2^{(2k+1)} \cdot 3 + 1, 2^{(2k+2)} \cdot 3\} \cup ... \ .$$



All this is common information. The space of states, the information structures, and the event are described in the picture below. Notice that in every set of

Ann's information structure, the event X occurs in two thirds of the states of nature; while in every set of Bob's information structure, X occurs in only one third of the states.

Having received their private information (that is, knowing in which set of their partition of information is the actual state of nature), Ann will estimate that the probability of occurrence of X is $2/3$, and Bob will estimate that it is $1/3$. This is common knowledge, since it is independent of the state of nature that occurs. For the same reason, truthful exchange of information about their estimates does not lead to any update of their posteriors.

Pooling their information, Ann and Bob are always able to find out whether X occurred or not, except if the state of nature belongs to $\{1, 2, 3\}$. Being restricted to truthful communication of their probability beliefs, Ann and Bob always (agree to) disagree on the probability of occurrence of X.

After receiving their private information, Ann and Bob focus on a finite number of states and thus have well defined subjective posterior probabilities. But the prior probability of X is undetermined. As we advance in the naturals, the proportion of states that belong to X oscillates between $1/3$ and $2/3$.

As we have shown in the previous section, this lack of convergence is crucial to the posterior disagreement. We can only be sure that the commonly known posteriors are between $1/3$ and $2/3$. This is the margin for disagreement. In fact, Ann's posterior is $2/3$ and Bob's posterior is $1/3$. They agree to disagree.

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FACULDADE DE ECONOMIA DA UNIVERSIDADE DO PORTO
Rua Dr. Roberto Frias, 4200-464 Porto | Tel. 225 571 100
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