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Research Work in Progress

n. 287, August 2008

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August 4th, 2008

Abstract. We generalize the model of Krugman (1991) to allow for asymmetric trade costs between regions and for (asymmetric) trade costs that are internal to the regions. We find that industrial activity, in a region, is enhanced by higher costs of importing and lower costs of exporting (more precisely, by a higher ratio between the two trade costs). This suggests that countries may impose tariffs on imported goods and seek to remove the import tariffs in other countries (unilateral protectionism) in order to foster industrial activity. Industrial activity is also promoted by lower domestic internal trade costs and higher foreign internal trade costs (more precisely, by a lower ratio between the two trade costs).

Keywords: New Economic Geography, Core-Periphery, Trade costs, Unilateral protectionism.

JEL Classification Numbers: F12, F15, F21, R12, R13.

* We thank Diego Puga, José María Chamorro Rivas, and participants at the II Economic Theory Workshop in Vigo and the 2nd Meeting of the Portuguese Economic Journal. Vasco Leite (040421008@fep.up.pt) thanks Fundação para a Ciência e Tecnologia (FCT) for financial support (Ph.D. scholarship). Sofia B.S.D. Castro (sdcastro@fep.up.pt) acknowledges support from Centro de Matemática da Universidade do Porto, financed by FCT through the programmes POCTI and POSI. João Correia-da-Silva (joao@fep.up.pt) acknowledges support from CEMPRE and research grant PTDC/ECO/66186/2006 from FCT and FEDER.

1 Introduction

What is the impact of asymmetric internal and external trade costs on the spatial distribution of the industrial activity and on the welfare of the different interest groups in an economy?

Trade costs, broadly defined by Anderson and Wincoop (2004):

"*include all costs incurred in getting a good to a final user other than the marginal cost of producing the good itself: transportation costs (both freight costs and time costs), policy barriers (tariffs and nontariff barriers), information costs, contract enforcement costs, costs associated with the use of different currencies, legal and regulatory costs, and local distribution costs (wholesale and retail).*"

It is clear that trade costs are highly variable across countries. They are higher in landlocked countries than in coastal countries (Limão and Venables, 2001) and higher in developing countries than in industrialized countries (Anderson and Wincoop, 2004). Differences in trade costs, particularly those associated with the distance to the larger markets, explain some of the income inequality across countries (Redding and Venables, 2004).

The economic relevance of trade costs is beyond doubt, being equivalent (in industrialized countries) to a 170 % *ad valorem* tax equivalent, that can be decomposed into a 55% domestic trade cost, associated with local distribution, and a 74 % international trade cost (Anderson and Wincoop, 2004).

A monotonic relationship between trade costs and location of the economic activity is one of the main theoretical findings of the 'New Economic Geography' literature. If trade costs are high, economic activity is dispersed across regions, while if trade costs are low, then economic activity becomes concentrated in one region.[1]

This recent literature has allowed an understanding of a variety of matters in its relation with the location of economic activity such as trade policy (Baldwin *et al.*, 2003), economic development (Murata, 2002), qualification (Mori and Turrini, 2005; Toulemonde, 2006), quality of the infrastructure (Martin and Rogers, 1995) and the structure of the transport network (Fujita and Mori, 1996; Mun, 2004). A welfare

[1] The concept of "region" may refer to locations ranging from small geographical regions like cities (Fujita and Krugman, 1995; Mori, 1997) to larger areas such as countries or even continents (Krugman and Venables, 1995).

analysis of the agglomeration process was carried out by Ottaviano, Tabuchi and Thisse (2002).

In spite of the empirical evidence, most of the theoretical work has neglected the differences in trade costs across regions and the trade costs that are internal to a region, focusing on the case of symmetric trade costs associated with trade across regions.[2]

In this paper, we extend the model introduced by Krugman (1991) to allow for: (i) the existence of intra-regional (internal) trade costs, possibly different between regions; and (ii) the existence of asymmetric inter-regional (external) trade costs.

By asymmetric external trade costs we mean that the cost of trading from region 1 to region 2 is different from the cost of trading from region 2 to region 1. The assumption that trade costs from region 1 to region 2 are identical to those from region 2 to region 1 is pervasive in the existing literature. Nevertheless, it is clear that some trade barriers like tariffs and import quotas are unilateral (at least asymmetric) and that there may be different degrees of trade liberalization.[3] This asymmetry was illustrated by Krugman and Venables (1995, Section V) in the form of a unilateral import tariff.

There are papers that consider internal and external trade costs in new economic geography models, but none addressing the problem of extending the base model of Krugman (1991).

Martin and Rogers (1995) have considered asymmetric internal and external transportation costs in an economy with two countries, but in their model workers are immobile between regions. Therefore, it does not capture agglomeration as a self-reinforcing process generated by demand-linkage and cost-linkage circular causality.[4]

A kind of internal and external transportation costs also appears in Mansori (2003), who considers a country composed by two identical regions which trade with the

[2]See, for example, the works of Krugman (1991), Fujita and Krugman (1995), Mori (1997), Ottaviano and Puga (1998), Puga (1999), Fujita, Krugman and Venables (2001), Baldwin *et al.* (2003), Ottaviano and Thisse (2004) and the references therein.

[3]See Baldwin *et al.* (2003) for an overview on trade policy and economic geography.

[4]When workers migrate to a region, the size of the market increases, fostering economic activity in this region (demand-linkage). When economic activity is transfered to a region, trade costs in this region decrease, attracting workers (cost-linkage). See Baldwin *et al.* (2003) for a detailed explanation of the difference between the basic core-periphery model (Krugman, 1991) and the footloose captal model (Martin and Rogers, 1995).

rest of the world. Each region has a different cost of trading with the rest of the world, and there is also trade between the two regions.

Behrens et al. (2007) proposed a model in which there are two identical countries formed by two regions between which labor is mobile, while there is no international labor mobility. Goods can be traded both nationally and internationally at different costs. Particularly, they assume that countries have different internal trade costs, but still an identical external trade cost.

In sum, we study an economy in which there are agglomeration forces generated by the mobility of the industrial population, allowing for asymmetric internal and external trade costs. Technically, we extend Krugman's (1991) model to accommodate four different trade costs: external trade costs from region 1 to region 2 and from 2 to 1, and internal trade costs within region 1 and within region 2.[5]

We show that in the case of symmetric trade costs, the model is equivalent to the original model of Krugman (1991) with trade cost equal to the ratio between the external and the internal trade cost. The trade cost considered in the existing literature can, thus, be interpreted as the ratio between external and internal trade costs. Recall that this ratio was measured as a 74% tax by Anderson and Wincoop (2004).[6] The measures of the total trade cost as a 170% tax and of the domestic component as a 55% tax are irrelevant for the agglomeration process.

Not surprisingly, we find that industrial activity tends to shift to the region with lower internal trade costs, and to the region with higher cost of importing (lower cost of exporting).

Considering a decrease of the internal trade cost of a region, we find that, in this region, the real wages of the workers increase in the short-run, and economic activity increases in the long-run (workers migrate to the region). In terms of welfare, we observe a "win-lose" situation. In the short-run, the welfare of workers improves in this region but worsens in the other region, while the welfare of farmers improves in both regions (because prices go down).

In the case of a unilateral decrease in the cost of importing, there are different possible effects. If the size of the industrial sector is small, the real wages of the

[5]We do not address the issue of assigning trade costs to agricultural goods. See Fujita, Krugman and Venables (2001, chapter 7).

[6]A tax of 74% corresponds to an iceberg cost parameter of $0.57 = 1/1.74$. To receive 1 unit, the customer pays 1.74.

workers decrease in the short-run and economic activity decreases in the long-run. If the industrial sector is large, we arrive at the opposite conclusion. The real wages of the other regions' workers always increases. The welfare of farmers always improves in the region, and worsens in the other region.

In the next section we present an extension of the model of Krugman (1991) which accommodates different internal and external trade costs. Section 3 includes the main results, being divided into three subsections which corresponds to the cases of symmetric trade costs, asymmetric internal trade costs, and asymmetric external trade costs. Section 4 concludes the paper with some remarks. Proofs of the analytical results can be found in the appendixes.

2 The model

2.1 Setup

The economy comprises two sectors (agriculture and industry) and two regions (1 and 2). Regions are symmetric in terms of technology and preferences, but may have different internal and external trade costs.

The agricultural sector is perfectly competitive and produces a homogeneous good under constant returns to scale, using only labor supplied by farmers, who are immobile between regions.

The manufacturing sector is monopolistic competitive and produces a continuum of varieties of a horizontally differentiated product using only labor supplied by workers, who are mobile between regions.

Preferences

Both farmers and workers share a utility function of the form

$$U = C_M^{\mu} C_A^{1-\mu}, \tag{1}$$

where C_A is consumption of the agricultural goods and C_M is consumption of a manufactures aggregate. This functional form implies that $0 < \mu < 1$ is the share of spending on manufactured goods.

The manufacturing firms produce horizontally differentiated products, with the manufactures aggregate being defined as:

$$C_M = \left[\int_0^N c_i^{(\sigma-1)/\sigma} di \right]^{\sigma/(\sigma-1)},$$

where N is the quantity of horizontally differentiated products, c_i is the consumption of the i differentiated product and $\sigma > 1$ is the elasticity of substitution among the products. A low σ means that the products have a high degree of differentiation (or that the consumers have a high preference for variety).

Supply of factors

A fraction $0 < \mu < 1$ of the population works in the manufacturing sector, while the remaining, $1 - \mu$, works in the agricultural sector.[7] Farmers are evenly distributed between the regions, thus the agricultural population in each region is fixed and equal to $\frac{1-\mu}{2}$.

The industrial population in regions 1 and 2 is L_1 and L_2, with $L_1 + L_2 = \mu$. We denote the share of workers in region 1 by $f = \frac{L_1}{L_1+L_2}$ (the share of workers in region 2 is, obviously, $1 - f$).

Manufacturing sector

Production of each variety requires a fixed input involving $\alpha > 0$ units of labor and a variable input involving $\beta > 0$ units of labor, supplied by the industrial workers. The cost function, in region j, is:

$$CT_j = W_j(\alpha + \beta x_i),$$

where CT_j is the cost to produce one unit of some variety, W_j is the nominal wage of the workers in region j, and x_i is the output produced by the firm.

Given the profit-maximization pricing behavior of manufacturing firms that operate in a monopolistic competitive sector, the price of any manufactured product in region j is:

$$p_j = \frac{\sigma}{\sigma - 1} \beta W_j.$$

[7] The coincidence between the share of population in each sector and the share of spending on each sector only implies the equality between wages in both sectors, in equilibrium.

Free entry of firms into the manufacturing sector drives profits to zero, which implies that all firms produce the same output, given by:

$$x_i = \frac{\alpha(\sigma - 1)}{\beta}.$$

Each firm employs the same number of workers, therefore:

$$\frac{L_1}{L_2} = \frac{n_1}{n_2},$$

where n_j is the number of firms in region j.

Agricultural sector

The agricultural sector is perfectly competitive and has constant returns to scale. One unit of labor supplied by farmers is used to produce one unit of the agricultural good. Trade costs in this sector are neglected, therefore, the price of the agricultural good is the same in both regions, and chosen as the numeraire.

$$p_A = W_A = 1,$$

where p_A is the price of the agricultural good and W_A is the nominal wage of the farmers in both regions.

Trade costs

So far, we have presented the model of Krugman (1991). We now extend it by allowing for the existence of asymmetric trade costs between regions as well as different internal trade costs.

The trade of manufactures involves an *iceberg* trade cost. Of each unit of manufactures shipped from region i to region j, only a fraction $0 < \tau_{ij} < 1$ arrives. Thus, a high τ_{ij} corresponds to a low trade cost. The trade of agricultural products is assumed to be costless.[8]

Figure 1 represents the configuration of the economy with two regions and four different trade costs. The parameter τ_{12} represents the cost of shipping the manufactured goods from region 1 to region 2, while τ_{21} represents the cost necessary

[8]This assumption was relaxed by Adrian (1996). Agricultural transport costs were shown to render agglomeration of industry less likely.

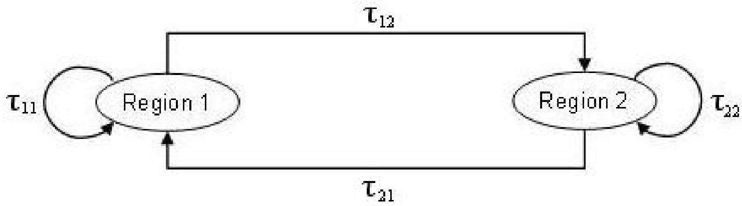

Figure 1: **Asymmetric internal and external trade costs**

to ship the manufactured goods from region 2 to region 1. We designate these as external trade costs. We also consider trade costs for goods that are produced and consumed in the same region, τ_{11} and τ_{22}, and designate them as internal trade costs. We assume that the internal trade costs are lower than the external trade costs.

2.2 Short-Run equilibrium

In the short-run, the spatial distribution of workers is taken as given (migrations do not occur). We start by computing output and nominal wages. After we compute prices and check whether there are incentives for workers to migrate by comparing the real wages in each region.

Demand

We denote consumption in region i of a representative region j product by Cji.

In region 1, the price of a local product is p_1/τ_{11}, with $\tau_{11} < 1$, while the price of an imported product is p_2/τ_{21}. Consumption is given by:

$$C_{11} = \left(\frac{p_1}{\tau_{11}}\right)^{-\sigma} \text{ and } C_{21} = \left(\frac{p_2}{\tau_{21}}\right)^{-\sigma}.$$

The expenditure on local manufactures, E_{11}, and on foreign manufactures, E_{21}, is:

$$E_{11} = \left(\frac{p_1}{\tau_{11}}\right)^{1-\sigma} n_1 \quad \text{and} \quad E_{21} = \left(\frac{p_2}{\tau_{21}}\right)^{1-\sigma} n_2$$

Given E_{11} and E_{21}, we define Z_{11} as the ratio between region 1's expenditure on local manufactures and region 1's expenditure on manufactures imported from region 2:

$$Z_{11} = \frac{E_{11}}{E_{21}} = \left(\frac{W_1\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma} \frac{n_1}{n_2} = \left(\frac{W_1\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma} \frac{L_1}{L_2}. \tag{2}$$

With a similar procedure we obtain Z_{12}, the ratio between region 2's spending on region 1 products and local products:

$$Z_{12} = \frac{E_{12}}{E_{22}} = \left(\frac{W_1\tau_{22}}{W_2\tau_{12}}\right)^{1-\sigma} \frac{L_1}{L_2}. \tag{3}$$

Nominal wages

Let Y_1 and Y_2 denote the nominal regional income, which is equal to the sum of the incomes in the agricultural and the manufacturing sectors:

$$Y_1 = \frac{1-\mu}{2} + W_1 L_1 \text{ and } Y_2 = \frac{1-\mu}{2} + W_2 L_2. \tag{4}$$

The nominal wage of workers in region 1 is equal to the spending on region 1's manufactures:

$$L_1 W_1 = \frac{Z_{11}}{1+Z_{11}}\mu Y_1 + \frac{Z_{12}}{1+Z_{12}}\mu Y_2 \Leftrightarrow W_1 = \frac{\mu}{L_1}\left[\frac{Z_{11}}{1+Z_{11}}Y_1 + \frac{Z_{12}}{1+Z_{12}}Y_2\right]. \tag{5}$$

Similarly, the nominal wage of workers in region 2 is:

$$W_2 = \frac{\mu}{L_2}\left[\frac{1}{1+Z_{11}}Y_1 + \frac{1}{1+Z_{12}}Y_2\right]. \tag{6}$$

Expressions (4)-(6) imply that the sum of the nominal wages across all agents is invariant:

$$L_1 W_1 + L_2 W_2 = \mu. \tag{7}$$

Equations (2)-(4) allow us to write (5) and (6) in the following way:

$$W_1 = \frac{\mu}{L_1}\left[\frac{\left(\frac{W_1\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma}\frac{L_1}{L_2}(\frac{1-\mu}{2}+W_1L_1)}{1+\left(\frac{W_1\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma}\frac{L_1}{L_2}} + \frac{\left(\frac{W_1\tau_{22}}{W_2\tau_{12}}\right)^{1-\sigma}\frac{L_1}{L_2}(\frac{1-\mu}{2}+W_2L_2)}{1+\left(\frac{W_1\tau_{22}}{W_2\tau_{12}}\right)^{1-\sigma}\frac{L_1}{L_2}}\right]; \tag{8}$$

$$W_2 = \frac{\mu}{L_2}\left[\frac{\frac{1-\mu}{2}+W_1L_1}{1+\left(\frac{W_1\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma}\frac{L_1}{L_2}} + \frac{\frac{1-\mu}{2}+W_2L_2}{1+\left(\frac{W_1\tau_{22}}{W_2\tau_{12}}\right)^{1-\sigma}\frac{L_1}{L_2}}\right]. \tag{9}$$

Equations (8) and (9) constitute a system that determines W_1 and W_2 for a given distribution of workers between regions 1 and 2. Using (7), equation (8) can be written as a function of W_1 and equation (9) can be written as a function of W_2.

Price index and real wage

Workers are interested not in nominal wages but in real wages, and these depend on the cost of living in each region.

The price indices, P_1 and P_2, reflect the relationship between expenditure and utility for individuals in region 1 and region 2, respectively. These depend on the price of the agricultural products (normalized to 1) as well as on the price indices of manufactured goods, P_{M1} and P_{M2}.

$$P_{M1} = \gamma\left[f\left(\frac{W_1}{\tau_{11}}\right)^{1-\sigma} + (1-f)\left(\frac{W_2}{\tau_{21}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}};$$
$$P_{M2} = \gamma\left[f\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + (1-f)\left(\frac{W_2}{\tau_{22}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

where $f = \frac{L_1}{L_1+L_2}$ and $\gamma = \frac{\sigma\beta}{\sigma-1}\left[\frac{\mu}{\frac{\alpha}{\beta}(\sigma-1)}\right]^{\frac{1}{1-\sigma}}$.

We have $P_1 = P_A^{1-\mu} P_{M1}^{\mu} = P_{M1}^{\mu}$ and $P_2 = P_A^{1-\mu} P_{M2}^{\mu} = P_{M2}^{\mu}$. Therefore:

$$P_1 = \gamma \left[f \left(\frac{W_1}{\tau_{11}} \right)^{1-\sigma} + (1-f) \left(\frac{W_2}{\tau_{21}} \right)^{1-\sigma} \right]^{-\frac{\mu}{\sigma-1}} ; \tag{10}$$

$$P_2 = \gamma \left[f \left(\frac{W_1}{\tau_{12}} \right)^{1-\sigma} + (1-f) \left(\frac{W_2}{\tau_{22}} \right)^{1-\sigma} \right]^{-\frac{\mu}{\sigma-1}} . \tag{11}$$

The maximization of (1) yields consumption of each product (manufacture aggregate and agricultural good), and given the price index of each of them, we obtain the utility in region i for workers, U_i^M, and farmers, U_i^A:

$$U_i^M = \mu^{\mu}(1-\mu)^{1-\mu} \frac{W_i}{P_i} \quad \text{and} \quad U_i^A = \mu^{\mu}(1-\mu)^{1-\mu} \frac{1}{P_i}. \tag{12}$$

Workers seek the region with the highest utility or, equivalently, the highest real wage. From equations (8), (9), (10) and (11), we obtain the relative real wage, ω_1/ω_2:

$$\frac{\omega_1}{\omega_2} = \frac{\frac{W_1}{P_1}}{\frac{W_2}{P_2}} = \frac{W_1 \left[f \left(\frac{W_1}{\tau_{12}} \right)^{1-\sigma} + (1-f) \left(\frac{W_2}{\tau_{22}} \right)^{1-\sigma} \right]^{\frac{\mu}{1-\sigma}}}{W_2 \left[f \left(\frac{W_1}{\tau_{11}} \right)^{1-\sigma} + (1-f) \left(\frac{W_2}{\tau_{21}} \right)^{1-\sigma} \right]^{\frac{\mu}{1-\sigma}}}.$$

2.3 Long-Run Equilibrium

The short-run equilibrium variables are determined taking as given the amount of industrial workers in each region, f. The long-run equilibrium is a situation where migration does not occur. We say that it is stable if it is robust to small perturbations of the distribution of workers across regions.

Dispersion is a long-run equilibrium configuration if regions have the same real wage. It is stable if a small migration to region 1 decreases the real wage in region 1, implying that the initial configuration is reestablished. Precisely:

$$\frac{\omega_1}{\omega_2}|_{f=f^*} = 1 \;\; \text{and} \;\; \left[\partial \left(\frac{w_1}{w_2} \right) / \partial f \right] |_{f=f^*} < 0.$$

If the equilibrium share of population in region 1 is 0.5 ($f^* = 0.5$), we say that dispersion is symmetric (otherwise, it is asymmetric).

Concentration is a long-run equilibrium configuration if all workers are concentrated in the region that has the highest real wage. Unless real wages exactly coincide, it is stable.

$$f^* = 1 \text{ and } \frac{\omega_1}{\omega_2}|_{f=f^*} \geq 1 \quad \text{(concentration in region 1).}$$

$$f^* = 0 \text{ and } \frac{\omega_1}{\omega_2}|_{f=f^*} \leq 1 \quad \text{(concentration in region 2).}$$

3 Results

In this section, we consider three different cases.

1. Symmetric internal and external trade costs: ($\tau_{12} = \tau_{21} = \tau_e$ and $\tau_{11} = \tau_{22} = \tau_i$).

2. Asymmetric internal trade costs: ($\tau_{12} = \tau_{21} = \tau_e$ and $\tau_{11} \neq \tau_{22}$).

3. Asymmetric external trade costs: ($\tau_{12} \neq \tau_{21}$ and $\tau_{11} = \tau_{22} = \tau_i$).

We provide analytical results for short-run equilibria and simulations describing the long-run behavior.

3.1 Symmetric internal and external trade costs

Suppose that regions have equal internal trade costs, $\tau_{11} = \tau_{22} = \tau_i$, and symmetric external trade costs, $\tau_{12} = \tau_{21} = \tau_e$.

In a short-run equilibrium, the relative real wage, $\frac{\omega_1}{\omega_2}$, is the same as in the model of Krugman (1991), with $\tau = \frac{\tau_e}{\tau_i}$.[9] In this sense, we can reinterpret the trade cost in the model of Krugman (1991) as a ratio between external and internal trade costs.

[9]Recall that the model of Krugman (1991) only considers the external trade cost.

Proposition 3.1. *When regions have equal internal trade costs,* $\tau_{11} = \tau_{22} = \tau_i$*, and equal external trade costs,* $\tau_{21} = \tau_{12} = \tau_e$*, the ratio between* ω_1 *and* ω_2 *is the same as in the case in which regions have only an external trade cost equal to* $\tau = \frac{\tau_e}{\tau_i}$*.*

The proof of this result is provided in the appendix.

Figure 2 illustrates the effect of a decrease of internal trade costs on the relative real wage (external trade costs are kept constant).

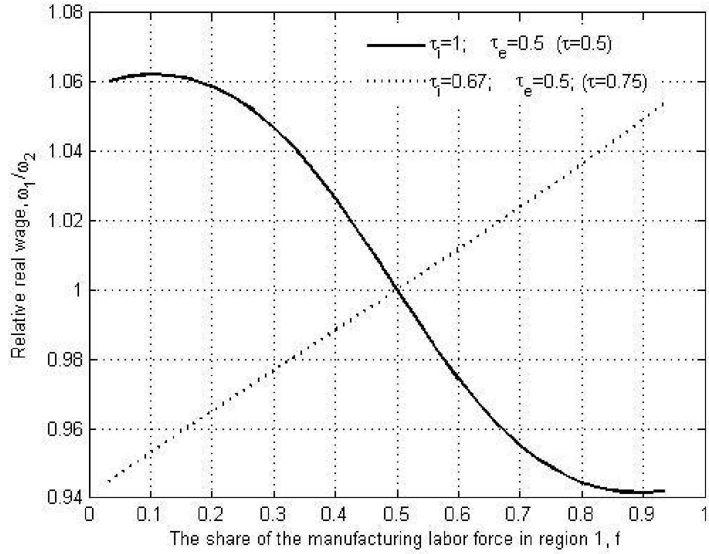

Figure 2: **The spatial distribution of the economic activity with symmetric internal and external trade costs** ($\tau_e = 0.5$, $\mu = 0.3$ and $\sigma = 4$). An increase in τ_i from 2/3 to 1 (a decrease in the internal trade costs) corresponds to a decrease in τ from 0.75 to 0.5. This figure is the same as the figure in Krugman (1991).

The bold line in figure 2 corresponds to short-run equilibria $(f, \frac{\omega_1}{\omega_2})$ in which there are no internal trade costs ($\tau_i = 1$) while the external trade costs are equal to 0.5 (the ratio between external and internal trade costs is $\tau = \frac{\tau_e}{\tau_i} = 0.5$). Any combination between internal and external trade costs such that $\frac{\tau_e}{\tau_i} = 0.5$ leads to the same relative real wage (Proposition 3.1), and therefore to the same spatial distribution of economic activity.

The dotted line corresponds to short-run equilibria in which the internal trade costs are 2/3 and the external trade costs are 0.5. In this case, the ratio between the external an internal trade costs is $\tau = \frac{\tau_e}{\tau_i} = 0.75$.

When the ratio between the internal and external trade costs changes, there is a change in the short-run equilibria. In this example, the decrease of internal trade costs (from $\tau_i = 2/3$ to $\tau_i = 1$) changes the equilibrium configuration from agglomeration to symmetric dispersion.

3.2 Asymmetric internal trade costs

What is the impact of a unilateral decrease in the internal trade costs:

- On the relative real wage, in the short-run?
- On the welfare of each interest group, in the short-run?
- On the distribution of industrial activity, in the long-run?

We provide analytical results for the first two questions, and use simulation to characterize the distribution of economic activity in the long-run. Proofs can be found in the appendix.

For the analytical results, we have chosen parameter values such that the initial long-run equilibrium is characterized by symmetric dispersion of economic activity. We start out with regions that have symmetric trade costs ($\tau_{12} = \tau_{21} = \tau_e$ and $\tau_{11} = \tau_{22} = \tau_i$) and we consider a marginal decrease of the internal trade cost of region 2 (an increase in τ_{22}).[10]

We study the two types of long-run equilibrium: dispersion and concentration.

Short-run effect on the relative real wage

Figure 3 compares short-run equilibria in the symmetric case ($\tau_{22} = \tau_{11}$) with short-run equilibria when region 2 has lower internal trade costs (i.e. $\tau_{22} > \tau_{11}$). Of course that, starting from a symmetric setting, analogous results are obtained for $\tau_{11} > \tau_{22}$, by interchanging the regions.

The curve which depicts the short-run equilibria moves downwards, which means that workers in region 2 will have a higher real wage than workers in region 1, in

[10] Recall that, for each unit produced and sold in region 2, a fraction τ_{22} is consumed, while $1 - \tau_{22}$ is dissipated as trade costs.

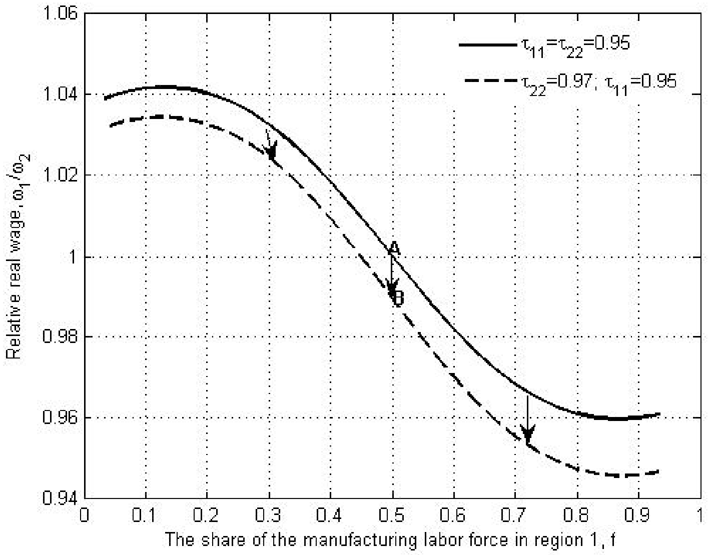

Figure 3: **Short-run equilibria and asymmetric internal trade costs** ($\tau_e = 0.5$, $\mu = 0.3$ and $\sigma = 4$). Point A is an initial asymmetric dispersion equilibrium in which $\tau_{11} = \tau_{22} = 0.95$. A decrease of the internal trade costs in region 2 (an increase in τ_{22} from 0.95 to 0.97) leads the economy from A to B, in the short-run.

the short-run. In particular, for $f = 0.5$, the relative real wage in region 1, $\frac{\omega_1}{\omega_2}$, is a decreasing function of τ_{22}.

According to proposition 3.2, and considering as a starting point a symmetric dispersion equilibrium in which $\tau_{21} = \tau_{12} = \tau_e$ and $\tau_{22} = \tau_{11} = \tau_i$, this outcome occurs for any value of σ and μ.

Proposition 3.2. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{22} = \tau_{11} = \tau_i$ and $L_1 = L_2$. The relative real wage, $\frac{\omega_1}{\omega_2}$, is a decreasing function of τ_{22}, for any $0 < \tau_e < \tau_i < 1$, $\mu \in (0, 1)$ and $\sigma > 1$.*

Short-run effect on the welfare of the four interest groups

There are four interest groups in this economy, namely, the workers and the farmers in each region. Here we analyze the effect of variations of τ_{22} on the utility, (12), of each interest group.

Notice that the utility of the workers coincides with the real wage, except for the constant term, $\mu^\mu(1-\mu)^{1-\mu}$.

The decrease of the internal trade costs in region 2, (increase in τ_{22}) influences the nominal wages and the price index in both regions, causing a "win-lose" outcome in which the workers in region 2 are the winners (Lemma 3.2), while the workers in region 1 are the losers (Lemma 3.2).

Lemma 3.1. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$. The real wage in region 1, ω_1, is a decreasing function of τ_{22}, for any $0 < \tau_e < \tau_i < 1$, $\mu \in (0,1)$ and $\sigma > 1$.*

Lemma 3.2. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$. The real wage in region 2, ω_2, is an increasing function of τ_{22}, for $0 < \tau_e < \tau_i < 1$, $\mu \in (0,1)$ and $\sigma > 1$.*

As the nominal wages of the farmers are always equal to 1, their real wages only depend on the price index. Therefore, all welfare effects stem from the cost-of-living effect. We show in Lemma 3.3 and Lemma 3.4 that the price indices in both regions are decreasing functions of τ_{22}.

Lemma 3.3. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$. The price index in region 1, P_1, is a decreasing function of τ_{22} for $0 < \tau_e < \tau_i < 1$, $\mu \in (0,1)$ and $\sigma > 1$.*

Lemma 3.4. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$, the price index in region 2, P_2, is a decreasing function of τ_{22} for $0 < \tau_e < \tau_i < 1$, $\mu \in (0,1)$ and $\sigma > 1$.*

In the short-run, a decrease in the internal trade costs of one region benefits the farmers in both regions.

Long-run effect on the distribution of industrial activity

Figures 4 and 5 illustrate the possible effects of a decrease in the internal trade costs of region 2. In the long-run, the economic activity can be asymmetrically dispersed between regions (figure 4) or fully concentrated in region 2 (figure 5).

Point A represents the initial long-run equilibrium (in which regions have symmetric internal and external trade costs). Industrial activity is equally divided between the regions ($L_1 = L_2 \Leftrightarrow f = 0.5$). The real wages are obviously identical in both regions (otherwise there would exist incentives for the workers to move to the region with higher real wages). However, when $\tau_{22} > \tau_{11}$, the economy finds a new short-run equilibrium, point B, in which $\omega_2 > \omega_1$.

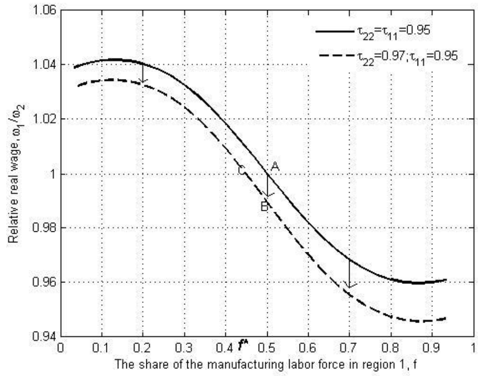

Figure 4: **Asymmetric dispersion** ($\tau_e = 0.5$, $\mu = 0.3$ and $\sigma = 4$). Point A represents an initial symmetric dispersion equilibrium, with $\tau_{22} = \tau_{11} = 0.95$. A small increase in τ_{22} from 0.95 to 0.97 raises the relative real wage in region 2 (to point B), attracting workers to that region. The long-run equilibrium (point C) is characterized by asymmetric dispersion.

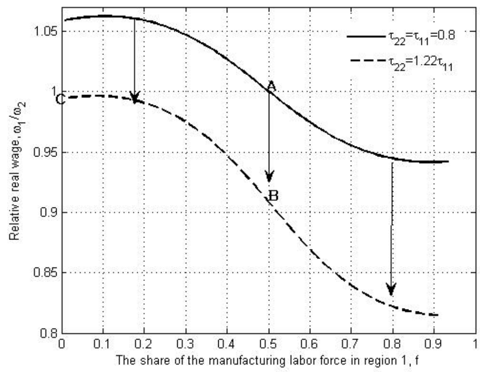

Figure 5: **Concentration** ($\tau_e = 0.4$, $\mu = 0.3$ and $\sigma = 4$). Point A corresponds to the initial symmetric dispersion equilibrium, with $\tau_{22} = \tau_{11} = 0.8$. The variation to $\tau_{22} = 1.22\tau_{11}$ raises the relative real wage in region 2 (to point B). In the long-run, there will be full concentration of the industrial population in region 2 (point C).

This attracts workers from region 1. There is migration to region 2 until the real wages coincide (figure 4) or until all workers have migrated to region 2 (figure 5).

Figure 4 shows an asymmetric dispersion equilibrium (point C) with $f^* < 0.5$ and $\omega_1 = \omega_2$, while figure 5 shows a concentration equilibrium (point C) with $f^* = 0$ and $\omega_2 > \omega_1$.

If economic activity is initially concentrated in a region, then a variation of internal trade costs may preserve this configuration (figure 6), or may imply that concentration can only occur in the region with the lower internal trade costs (figure 7).

An increase in τ_{22} favors concentration in region 2, as the basin of attraction is enlarged. Indeed, for $\tau_{22} = 0.97$ and $\tau_{11} = 0.95$, we observe that any $f < 0.6$ (point C), is sufficient to induce concentration in region 2.

In figure 7, we illustrate a case in which all industrial activity is initially concentrated in region 1, with $f = 1$ and $\omega_1 > \omega_2$ (point A) and consider a decrease in the internal trade costs in region 2 (an increase in τ_{22}). If the new relative real wage is above 1, the decrease in the internal trade costs in region 2 will have no effect on the spatial distribution of the industrial activity. But if the new relative real wage is below 1 (point B), then the impact on the location of the industry is drastic. In

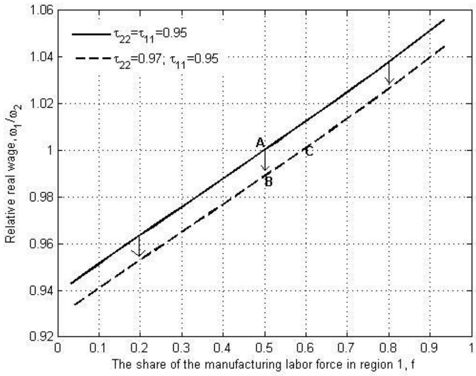

Figure 6: **Agglomeration in any region** ($\tau_e = 0.75$, $\mu = 0.3$ and $\sigma = 4$). A decrease in the internal trade cost of region 2 (increase in τ_{22} from 0.95 to 0.97) leads the economy to point B. Economic activity concentrates in region 2 if the initial distribution is such that $f < 0.6$.

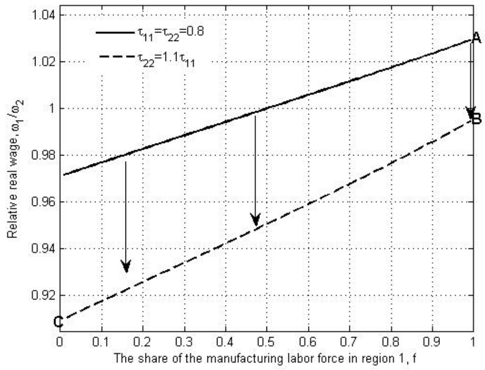

Figure 7: **Agglomeration in only one region** ($\tau_e = 0.75$, $\mu = 0.3$ and $\sigma = 4$). Suppose that all economic activity is concentrated in region 1 (point A). An increase in τ_{22} implies that all the industrial activity is transfered from region 1 to region 2 (point C).

our illustration, for a $\tau_{22} = 1.1\tau_{11}$, all industrial activity relocates from region 1 to region 2 (point C with $f = 0$ and $\omega_2 > \omega_1$).

3.3 Asymmetric external trade costs

What is the impact of an asymmetry between the cost of exporting and the cost of importing:

- On the relative real wage, in the short-run?
- On the welfare of each interest group, in the short-run?
- On the distribution of industrial activity, in the long-run?

We provide analytical results for the first two questions, and we use numerical methods to characterize the distribution of economic activity in the long-run. The proofs may be found on the appendix.

For the analytical results, we focus on the case in which the initial long-run equilibrium is characterized by symmetric dispersion of economic activity and we suppose that regions have initially symmetric trade costs, $\tau_{12} = \tau_{21} = \tau_e$ and $\tau_{11} = \tau_{22} = \tau_i$. Then, we consider a marginal decrease of the cost of trading goods from region 1 to region 2 ($\tau_{12} > \tau_{21}$), keeping constant the cost of trading goods from region 2 to region 1 (τ_{21}). We may interpret $\tau_{12} > \tau_{21}$ as the case in which region 2 has a lower cost of importing.[11]

We study the two types of long-run equilibrium: dispersion and concentration.

Short-run effect on the relative real wage

Figures 8 and 9 show the impact of a decrease in cost of trade from region 1 to region 2 (increase in τ_{12}) on the relative real wage, in the short-run (starting from a initial dispersion that is unstable and stable, respectively).

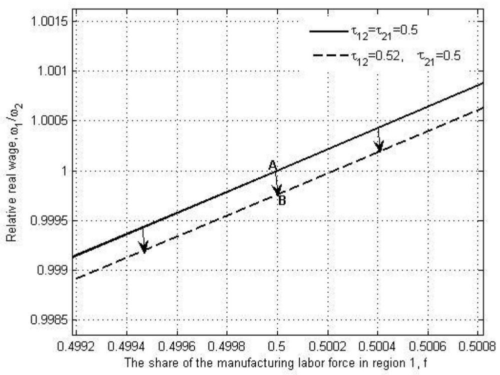

Figure 8: **Liberalization is good** ($\tau_i = 0.95$, $\sigma = 4$, and $\mu = 0.96$). For high μ, a increase in τ_{12} increases the relative real wage in region 2.

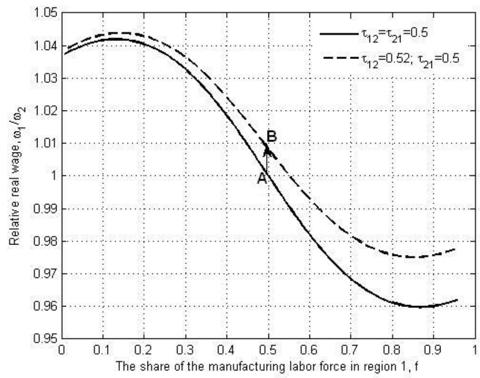

Figure 9: **Liberalization is bad** ($\tau_i = 0.95$, $\sigma = 4$ and $\mu = 0.3$). An increase in τ_{12} from 0.5 to 0.52 decreases the relative real wage in region 2, in the short-run (the economy moves from point A to B).

In the case illustrated in figure 8, the relative real wage of region 1 decreases (from point A to B), while figure 9 shows the opposite effect. The direction of the effect depends on the weight of the industrial sector in the economy.

Proposition 3.3. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{22} = \tau_{11} = \tau_i$ and $L_1 = L_2$. There is a $\mu^*(\sigma, \tau) \in (0, 1)$ such that: $\frac{d(\omega_1/\omega_2)}{d\tau_{12}} > 0$ for $\mu \in (0, \mu^*)$ and $\frac{d(\omega_1/\omega_2)}{d\tau_{12}} < 0$ for $\mu \in (\mu^*, 1)$*

[11] The trade costs from region 1 to region 2, τ_{12}, are supported by consumers in region 2. As p_1 is the price of the manufactured products produced in region 1 (determined in the market), then $\frac{p_1}{\tau_{12}}$ is the total price supported by consumers in region 2 when they purchase a manufacture produced in region 1. Then, if τ_{12} increases, the price paid by consumers in region 2 decreases.

Short-run effect on the welfare of each interest group

The four interest groups are the workers and the farmers in each of the regions. The utilities (12) coincide with the real wages, except for a constant. Therefore, we study the impact of τ_{12} on the real wages of each group.

We show that a decrease in τ_{12} increases the welfare of workers in region 1 (Proposition 3.4), whereas the effect on the welfare of workers in region 2 can be positive or negative (Proposition 3.5).

Proposition 3.4. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$. The real wage in region 1, ω_1, is an increasing function of τ_{12}, for any $0 < \tau_e < \tau_i < 1$, $\mu \in (0, 1)$ and $\sigma > 1$.*

Proposition 3.5. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{22} = \tau_{11} = \tau_i$ and $L_1 = L_2$. For any $0 < \tau_e < \tau_i < 1$, there is a $\mu^*(\sigma, \tau) \in (0, 1)$ such that: $\frac{d\omega_2}{d\tau_{12}} < 0$ for $\mu \in (0, \mu^*)$ and $\frac{d\omega_2}{d\tau_{12}} > 0$ for $\mu \in (\mu^*, 1)$.*

As the nominal wages of the farmers are always equal to 1, their real wages only depend on the price index. We show that the price index in region 1, P_1, is an increasing function of τ_{12} (Lemma 3.5) and that the price index in region 2, P_2, is a decreasing function of τ_{12} (Lemma 3.6). This means that a decrease in the cost of trading manufactured goods from region 1 to region 2 (an increase in τ_{12}) benefits the farmers of region 2 and penalizes the farmers of region 1.

Lemma 3.5. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$. Then, P_1 is an increasing function of τ_{12} for any $0 < \tau_e < \tau_i < 1$, $\mu \in (0, 1)$ and $\sigma > 1$.*

Lemma 3.6. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$. Then, P_2 is a decreasing function of τ_{12} for any $0 < \tau_e < \tau_i < 1$, $\mu \in (0, 1)$ and $\sigma > 1$.*

Long-run effect on the distribution of industrial activity

Figures 10 and 11 depict the long-run distribution of industrial activity. Depending on the extent of the asymmetry of external trade costs, the economic activity can be asymmetrically distributed between regions or fully concentrated in one region.

In figure 10, we present an asymmetric dispersion equilibrium. An increase in τ_{12} increases the relative real wage in region 1 attracting new workers to the region. The migration to region 1 leads to a decrease in ω_1/ω_2. This process continues until a new long-run equilibrium is reached, point C, with $\omega_1 = \omega_2$.

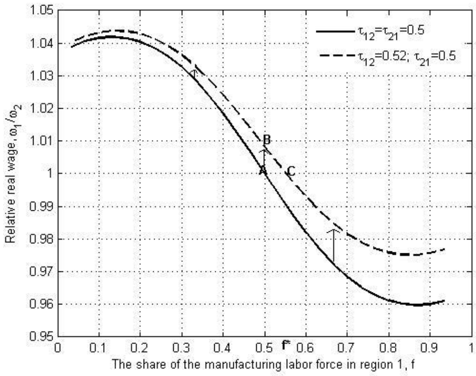

Figure 10: **Asymmetric dispersion** ($\tau_i = 0.95$, $\sigma = 4$ and $\mu = 0.3$). With $\tau_{12} = \tau_{21} = 0.5$, there is symmetric dispersion (point A). However, an increase in τ_{12} from 0.5 to 0.52 raises the relative real wage in region 1, in the short-run (point B). In the long-run, there will be asymmetric dispersion, as region 1 will have more workers than region 2 (point C).

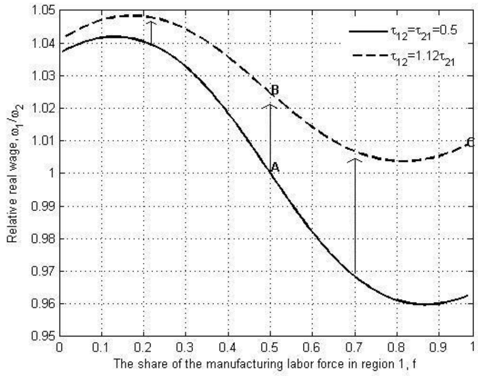

Figure 11: **Concentration** ($\tau_i = 0.95$, $\sigma = 4$ and $\mu = 0.3$). With $\tau_{12} = \tau_{21} = 0.5$, there is symmetric dispersion (point A). A strong decrease in the cost of trading from region 1 to region 2 ($\tau_{12} = 1.12\tau_{21}$) raises the real wage, in the short-run (point B). In the long-run, there is full concentration of the industrial population in region 1 (point C).

Figure 11 illustrates how a strong increase in τ_{12} may generate catastrophic agglomeration in region 1, producing a core-periphery structure.

Figures 12 and 13 describe the case in which there is an initial concentration of economic activity. Figure 12 deals with a case in which concentration may occur in any of the regions, while figure 13 shows a case in which all economic activity becomes concentrated in region 2.

Figure 12 shows that an increase in τ_{12} enlarges the basin of attraction of the equilibrium in which all economic activity is concentrated in region 1.

Figure 13 illustrates an environment in which region 2 initially concentrates all industrial activity (point A, with $f = 0$ and $\omega_2 > \omega_1$). An increase in τ_{12} raises the relative real wage in region 1, in the short-run (point B). As the real wage is higher in region 1, workers migrate from region 2 to region 1. In the long-run, there is full agglomeration in region 1 (point C).

There is a threshold degree of asymmetric trade liberalization between regions that generates the relocation of all industrial activity from one region to the other. However, there will be no migration of workers until the the asymmetry in trade liberalization reaches this threshold.

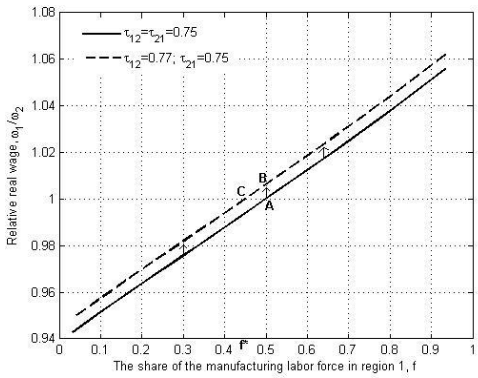

Figure 12: **Agglomeration in any region** ($\tau_i = 0.95$, $\sigma = 4$ and $\mu = 0.3$). When $\tau_{12} = \tau_{21} = 0.75$, an increase in τ_{12} from 0.75 to 0.77 changes the critical f^* that determines in which region all economic activity concentrates to $f^* \approx 0.42$ (point C). For any initial $f > 0.42$ all workers end up concentrating in region 1, in the long-run.

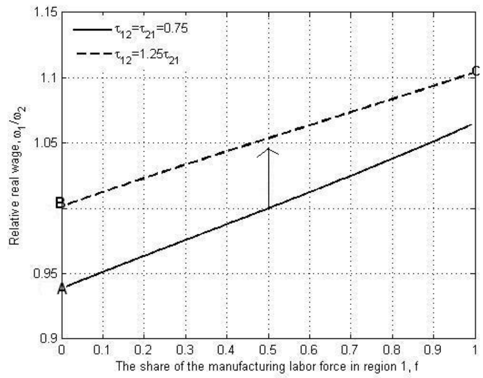

Figure 13: **Agglomeration in only one region** ($\tau_i = 0.95$, $\sigma = 4$ and $\mu = 0.3$). Consider a initial core-periphery structure in which all economic activity is concentrated in region 2 (point A). An increase in τ_{12} will transfer all industrial activity from region 2 to region 1 (point C).

4 Concluding Remarks

We have extended the model of Krugman (1991) in order to study the effects of internal and external trade costs on the location of industrial activity as well as on the welfare of the agents. The existence of an asymmetric dispersion equilibrium is not surprising, given that under distinct trade costs, the regions no longer are identical.

We find that industrial activity in a region is enhanced, *ceteris paribus*, by lower internal trade costs and by higher costs of importing (lower costs of exporting). The fact that asymmetries in the external trade costs lead to relocation of economic activity is a natural result. However, it was not present in the work of Martin and Rogers (1995). In their model, differentials in external trade costs only increase the sensitivity of industrial location to differentials in the internal trade costs.[12]

From the point of view of welfare, a decrease of the internal trade cost of a region benefits the workers of this region and the farmers of both regions, while the workers of the other region become worse off.

[12] If internal trade costs are equal, then differentials in external trade costs have no effect.

A decrease in the cost of trading from region 1 to region 2 benefits the workers of region 1 and the farmers of region 2, while the farmers of region 1 become worse off. The effect on the welfare of the workers of region 2 is positive if the weight of the industrial sector is large, and negative otherwise.

The analytical results obtained for short-run equilibria support the numerical evidence obtained for the long-run. In the long-run, numerical results indicate a new feature, namely that of sudden agglomeration in some instances.

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5 Appendix

Proposition 3.1. *When regions have equal internal trade costs, $\tau_{11} = \tau_{22} = \tau_i$, and equal external trade costs, $\tau_{21} = \tau_{12} = \tau_e$, the ratio between ω_1 and ω_2 is the same as in the case in which regions have only an external trade cost equal to $\tau = \frac{\tau_e}{\tau_i}$.*

Proof. Let Z_{11} and Z_{11}^K be the ratio between region 1's expenditure on local manufactures and that on manufactures from the other region in the model with equal internal and external trade costs and in the model without internal trade costs, respectively. Then, from (2), and for $\tau_{11} = \tau_{22} = \tau_i$, $\tau_{21} = \tau_{12} = \tau_e$ and $\tau = \dfrac{\tau_e}{\tau_i}$, we have:

$$Z_{11} = \frac{E_{11}}{E_{12}} = \left(\frac{W_1 \tau_e}{W_2 \tau_i}\right)^{1-\sigma} \frac{L_1}{L_2} = \left(\frac{W_1 \tau}{W_2}\right)^{1-\sigma} \frac{L_1}{L_2} = {Z_{11}}^K.$$

Analogously, from (3):

$$Z_{12} = \frac{E_{21}}{E_{22}} = \left(\frac{W_1 \tau_i}{W_2 \tau_e}\right)^{1-\sigma} \frac{L_1}{L_2} = \left(\frac{W_1}{W_2 \tau}\right)^{1-\sigma} \frac{L_1}{L_2} = {Z_{12}}^K$$

Next, we show that the nominal wages, W_1 and W_2, when regions have equal internal trade costs and equal external trades costs are the same as in the case in which regions have only an external trade costs, $\tau = \frac{\tau_e}{\tau_i}$.

From (8) and (9) with $\tau_i = \tau_{22} = \tau_{11}$ and $\tau_e = \tau_{12} = \tau_{21}$, the nominal wages satisfy:

$$W_1 = \frac{\mu}{L_1} \left[\frac{\left(\frac{W_1 \tau_e}{W_2 \tau_i}\right)^{1-\sigma} \frac{L_1}{L_2} (\frac{1-\mu}{2} + W_1 L_1)}{1 + \left(\frac{W_1 \tau_e}{W_2 \tau_i}\right)^{1-\sigma} \frac{L_1}{L_2}} + \frac{\left(\frac{W_1 \tau_i}{W_2 \tau_e}\right)^{1-\sigma} \frac{L_1}{L_2} (\frac{1-\mu}{2} + W_2 L_2)}{1 + \left(\frac{W_1 \tau_i}{W_2 \tau_e}\right)^{1-\sigma} \frac{L_1}{L_2}} \right]; \tag{13}$$

$$W_2 = \frac{\mu}{L_2} \left[\frac{\frac{1-\mu}{2} + W_1 L_1}{1 + \left(\frac{W_1 \tau_e}{W_2 \tau_i}\right)^{1-\sigma} \frac{L_1}{L_2}} + \frac{\frac{1-\mu}{2} + W_2 L_2}{1 + \left(\frac{W_1 \tau_i}{W_2 \tau_e}\right)^{1-\sigma} \frac{L_1}{L_2}} \right]. \tag{14}$$

Since $\tau = \frac{\tau_e}{\tau_i}$, (13) and (14) become:

$$W_1 = \frac{\mu}{L_1}\left[\frac{\left(\frac{W_1\tau}{W_2}\right)^{1-\sigma}\frac{L_1}{L_2}(\frac{1-\mu}{2}+W_1L_1)}{1+\left(\frac{W_1\tau}{W_2}\right)^{1-\sigma}\frac{L_1}{L_2}} + \frac{\left(\frac{W_1}{W_2\tau}\right)^{1-\sigma}\frac{L_1}{L_2}(\frac{1-\mu}{2}+W_2L_2)}{1+\left(\frac{W_1}{W_2\tau}\right)^{1-\sigma}\frac{L_1}{L_2}}\right];$$

$$W_2 = \frac{\mu}{L_2}\left[\frac{\frac{1-\mu}{2}+W_1L_1}{1+\left(\frac{W_1\tau}{W_2}\right)^{1-\sigma}\frac{L_1}{L_2}} + \frac{\frac{1-\mu}{2}+W_2L_2}{1+\left(\frac{W_1}{W_2\tau}\right)^{1-\sigma}\frac{L_1}{L_2}}\right].$$

These expressions coincide with those of the classical model of Krugman. There is a single equilibrium, as shown by Mossay (2006). Therefore, the nominal wages coincide:

$$W_1 = W_1^K \qquad \text{and} \qquad W_2 = W_2^K.$$

The inclusion of internal trade costs changes the price index of manufactured goods. Let P_{M1} be the price index of manufactured goods in the model with equal internal and external trade costs, and let P_{M1}^K be the price index of manufactured goods in the model without internal trade costs. For $\tau_{11} = \tau_{22} = \tau_i$, $\tau_{12} = \tau_{21} = \tau_e$, and $\tau = \frac{\tau_e}{\tau_i}$, we have:

$$\begin{aligned} P_{M1} &= \gamma\left[f\left(\frac{W_1}{\tau_i}\right)^{1-\sigma} + (1-f)\left(\frac{W_2}{\tau_e}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \Leftrightarrow \\ \Leftrightarrow P_{M1}\tau_i &= \gamma\left[fW_1{}^{1-\sigma} + (1-f)\left(\frac{W_2}{\tau}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = P_{M1}{}^K \end{aligned}$$

Doing the same for P_{M2}, we verify that the manufacturing price indexes increase to compensate for the internal "iceberg" trade costs:

$$P_{M1} = \frac{P_{M1}^K}{\tau_i} \qquad \text{and} \qquad P_{M2} = \frac{P_{M2}^K}{\tau_i}.$$

This implies that:

$$\omega_1 = \frac{W_1}{P_{M1}^\mu} = \frac{W_1^K}{(P_{M1}^K/\tau_i)^\mu} = \omega_1^K\tau_i^\mu \qquad \text{and} \qquad \omega_2 = \omega_2^K\tau_i^\mu.$$

The internal trade costs decrease the real wages in the same proportion, therefore, the relative real wage remains unaltered:

$$\frac{\omega_1}{\omega_2} = \frac{\omega_1^K \tau_i^\mu}{\omega_2^K \tau_i^\mu} = \frac{\omega_1^K}{\omega_2^K}$$

□

Claim 5.1. *If $L_1 = L_2$, then $W_1 + W_2 = 2$.*
When $L_1 = L_2$, $\tau_{21} = \tau_{12} = \tau_e$ and $\tau_{22} = \tau_{11} = \tau_i$, then $W_1 = W_2 = 1$.

Proof. Substituting $L_1 = L_2 = \mu$ in (7), we obtain $W_1 + W_2 = 2$.

If the trade costs are symmetric ($\tau_{21} = \tau_{12} = \tau_e$ and $\tau_{22} = \tau_{11} = \tau_i$), then $W_1 = W_2 = 1$ is a short-run equilibrium, as we verify below.

The nominal wage in region 1 is:

$$W_1 = \frac{\mu}{L_1}\left[\frac{\left(\frac{W_1\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma}\frac{L_1}{L_2}(\frac{1-\mu}{2}+W_1L_1)}{1+\left(\frac{W_1\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma}\frac{L_1}{L_2}} + \frac{\left(\frac{W_1\tau_{22}}{W_2\tau_{12}}\right)^{1-\sigma}\frac{L_1}{L_2}(\frac{1-\mu}{2}+W_2L_2)}{1+\left(\frac{W_1\tau_{22}}{W_2\tau_{12}}\right)^{1-\sigma}\frac{L_1}{L_2}}\right].$$

Substituting $W_1 = W_2 = 1$, $L_1 = L_2 = \frac{\mu}{2}$, $\tau_e = \tau_{12} = \tau_{21}$ and $\tau_i = \tau_{11} = \tau_{22}$, we obtain:

$$\begin{aligned}
1 &= 2\left[\frac{\left(\frac{\tau_e}{\tau_i}\right)^{1-\sigma}\frac{1-\mu+\mu}{2}}{1+\left(\frac{\tau_e}{\tau_i}\right)^{1-\sigma}} + \frac{\left(\frac{\tau_e}{\tau_i}\right)^{\sigma-1}\frac{1-\mu+\mu}{2}}{1+\left(\frac{\tau_e}{\tau_i}\right)^{\sigma-1}}\right] \Leftrightarrow \\
1 &= \left[\frac{\left(\frac{\tau_e}{\tau_i}\right)^{1-\sigma}}{1+\left(\frac{\tau_e}{\tau_i}\right)^{1-\sigma}} + \frac{\left(\frac{\tau_e}{\tau_i}\right)^{\sigma-1}}{1+\left(\frac{\tau_e}{\tau_i}\right)^{\sigma-1}}\right] \Leftrightarrow \\
1 &= \frac{2+\left(\frac{\tau_e}{\tau_i}\right)^{1-\sigma}+\left(\frac{\tau_e}{\tau_i}\right)^{\sigma-1}}{2+\left(\frac{\tau_e}{\tau_i}\right)^{1-\sigma}+\left(\frac{\tau_e}{\tau_i}\right)^{\sigma-1}} \Leftrightarrow \\
1 &= 1
\end{aligned}$$

This short-run equilibrium is unique (Mossay, 2006).

□

Claim 5.2. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{22} = \tau_{11} = \tau_i$, and $L_1 = L_2$. Then, $P_1 = P_2 \geq 1$.*

Proof. Substituting $W_1 = W_2 = 1$, $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{22} = \tau_{11} = \tau_i$ and $f = \frac{1}{2}$ into (10) and (11), we obtain:

$$P_1 = \left(\frac{\tau_i^{\sigma-1} + \tau_e^{\sigma-1}}{2} \right)^{\mu/(1-\sigma)},$$

and

$$P_2 = \left(\frac{\tau_e^{\sigma-1} + \tau_i^{\sigma-1}}{2} \right)^{\mu/(1-\sigma)}$$

We verify that $P_1 = P_2$, and since τ_i and τ_e are greater or equal than 1, $P_1 = P_2 \geq 1$.

□

Claim 5.3. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{22} = \tau_{11} = \tau_i$ and $L_1 = L_2$. Then, $0 < \omega_1 = \omega_2 \leq 1$.*

Proof. The real wage is simply given by the ratio between nominal wage ($W_1 = W_2 = 1$) and the price index ($P_1 = P_2 \geq 1$).

$$\omega_1 = \omega_2 = \frac{1}{P_1} = \frac{1}{P_2} = \left(\frac{\tau_i^{\sigma-1} + \tau_e^{\sigma-1}}{2} \right)^{\mu/(\sigma-1)}$$

We clearly have $0 < \omega_1 = \omega_2 \leq 1$.

□

Claim 5.4. *Let $L_1 = L_2$. In the short-run:*

$$\frac{dW_1}{d\tau_i} = -\frac{dW_2}{d\tau_i}, \quad \frac{dW_1}{d\tau_e} = -\frac{dW_2}{d\tau_e} \quad \textit{and} \quad \frac{dW_1}{d\tau_{22}} = -\frac{dW_2}{d\tau_{22}}$$

Proof. This is a straightforward consequence of $W_1 + W_2 = 2$.

□

Claim 5.5. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i = 1$ and $L_1 = L_2$. An increase in τ_{22} decreases W_1, for any $0 < \tau_e < \tau_i < 1$, $\mu \in (0,1)$ and $\sigma > 1$.*

Proof. We want to prove that $\left.\frac{dW_2}{d\tau_{22}}\right|_{L_1=L_2} > 0$.

By Claim 5.1, we can substitute $W_1 = 2 - W_2$ in equation (9).

$$\frac{W_2}{2} = \frac{\frac{1-\mu}{2} + (2-W_2)\frac{\mu}{2}}{1 + \left(\frac{(2-W_2)\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma}} + \frac{\frac{1-\mu}{2} + \frac{W_2\mu}{2}}{1 + \left(\frac{(2-W_2)\tau_{22}}{W_2\tau_{12}}\right)^{1-\sigma}} = \frac{A}{B} + \frac{C}{D}. \tag{15}$$

We compute $\frac{dW_2}{d\tau_{22}}$ (denoted, below, as W_2') by implicit differentiation, substituting $W_2 = 1$, $\tau_{21} = \tau_{12} = \tau_e$ and $\tau_{11} = \tau_{22} = \tau_i = 1$.

$$\frac{W_2'}{2} = \frac{A'}{B} - \frac{B'A}{B^2} + \frac{C'}{D} - \frac{D'C}{D^2},$$

where

$$A = C = \frac{1}{2}, \quad A' = -\frac{\mu}{2}W_2', \quad C' = \frac{\mu}{2}W_2', \quad B = 1 + \tau^{1-\sigma}, \quad D = 1 + \tau^{\sigma-1},$$
$$B' = 2(\sigma-1)\tau^{1-\sigma}W_2', \quad D' = 2(\sigma-1)\tau^{\sigma-1}\left(W_2' - \frac{1}{2\tau_i}\right).$$

With some manipulation:

$$\frac{W_2'}{2} = -\frac{\frac{\mu}{2}W_2'}{1+\tau^{1-\sigma}} + \frac{\frac{\mu}{2}W_2'}{1+\tau^{\sigma-1}} - \frac{(\sigma-1)\tau^{1-\sigma}W_2'}{(1+\tau^{1-\sigma})^2} - \frac{(\sigma-1)\tau^{\sigma-1}\left(W_2' - \frac{1}{2\tau_i}\right)}{(1+\tau^{\sigma-1})^2} \Leftrightarrow$$
$$\Leftrightarrow \quad W_2' \left[1 + \frac{\mu}{1+\tau^{1-\sigma}} - \frac{\mu}{1+\tau^{\sigma-1}} + \frac{(\sigma-1)\tau^{1-\sigma}}{(1+\tau^{1-\sigma})^2} + \frac{(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}\right] = \frac{(\sigma-1)\tau^{\sigma-1}}{2\tau_i\left(1+\tau^{\sigma-1}\right)^2}.$$

Using the fact that $\frac{\tau^{1-\sigma}}{(1+\tau^{1-\sigma})^2} = \frac{\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}$:

$$W_2' = \frac{\frac{(\sigma-1)\tau^{\sigma-1}}{2\tau_i(1+\tau^{\sigma-1})^2}}{1 + \frac{\mu}{1+\tau^{1-\sigma}} - \frac{\mu}{1+\tau^{\sigma-1}} + \frac{2(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}} \tag{16}$$

It should be clear that $W_2' = \frac{dW_2}{d\tau_{22}}$ is positive. Then, from Claim (5.4), we know that $\frac{dW_1}{d\tau_{22}}$ is negative.

□

Claim 5.6. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i =$ and $L_1 = L_2$. An increase in τ_{12} increases W_1, for any $\tau_i \in (0,1)$ $\tau_e \in (0,\tau_i)$, $\mu \in (0,1)$ and $\sigma > 1$.*

Proof. From Claim 5.4, we know that $\frac{dW_2}{d\tau_{12}} = -\frac{dW_1}{d\tau_{12}}$. Denote $\frac{dW_2}{d\tau_{12}}$ by W_2'.

Recalling (15):

$$\frac{W_2}{2} = \frac{\frac{1-\mu}{2} + \frac{\mu}{2}(2 - W_2)}{1 + \left(\frac{2-W_2}{W_2}\frac{\tau_{21}}{\tau_{11}}\right)^{1-\sigma}} + \frac{\frac{1-\mu}{2} + \frac{\mu}{2}W_2}{1 + \left(\frac{2-W_2}{W_2}\frac{\tau_{22}}{\tau_{12}}\right)^{1-\sigma}} = \frac{A}{B} + \frac{C}{D}.$$

Then:

$$\begin{aligned}
\frac{W_2'}{2} &= \frac{-\frac{\mu}{2}W_2'B - (1-\sigma)\left(\frac{2-W_2}{W_2}\frac{\tau_{21}}{\tau_{11}}\right)^{-\sigma}\frac{\tau_{21}}{\tau_{11}}\frac{-W_2'W_2 - W_2'(2-W_2)}{W_2^2}A}{B^2} + \\
&+ \frac{\frac{\mu}{2}W_2'D - (1-\sigma)\left(\frac{2-W_2}{W_2}\frac{\tau_{22}}{\tau_{12}}\right)^{-\sigma}\left[\frac{-W_2'W_2 - W_2'(2-W_2)}{W_2^2}\frac{\tau_{22}}{\tau_{12}} + \frac{2-W_2}{W_2}\left(-\frac{\tau_{22}}{\tau_{12}^2}\right)\right]C}{D^2} = \\
&= \frac{-\frac{\mu}{2}W_2'(1+\tau^{1-\sigma}) + (1-\sigma)\tau^{-\sigma}\tau W_2'}{(1+\tau^{1-\sigma})^2} + \\
&+ \frac{\frac{\mu}{2}W_2'(1+\tau^{\sigma-1}) + (1-\sigma)\tau^{\sigma}\left[W_2'\tau^{-1} + \frac{\tau_i}{2\tau_e^2}\right]}{(1+\tau^{\sigma-1})^2}.
\end{aligned}$$

Equivalently (using $W_1' = -W_2'$):

$$\begin{aligned}
\frac{W_1'}{2} &= \frac{W_1'\left[(1-\sigma)\tau^{1-\sigma} - \frac{\mu}{2}(1+\tau^{1-\sigma})\right]}{(1+\tau^{1-\sigma})^2} + \\
&+ \frac{W_1'\left[(1-\sigma)\tau^{\sigma-1} + \frac{\mu}{2}(1+\tau^{\sigma-1})\right]}{(1+\tau^{\sigma-1})^2} - \frac{\frac{1-\sigma}{2\tau_e}\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}
\end{aligned}$$

Solving for W_1':

$$W_1' = \frac{\frac{\sigma-1}{\tau_e}\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}\frac{1}{1-A-B}, \tag{17}$$

in which:

$$A = \frac{2(1-\sigma)\tau^{1-\sigma} - \mu(1+\tau^{1-\sigma})}{(1+\tau^{1-\sigma})^2}$$

$$B = \frac{2(1-\sigma)\tau^{\sigma-1} + \mu(1+\tau^{\sigma-1})}{(1+\tau^{\sigma-1})^2}$$

It is straightforward to see that $1 - A - B > 0$ (only the last term is negative):

$$1 - A - B = 1 + \frac{2(\sigma-1)\tau^{1-\sigma}}{(1+\tau^{1-\sigma})^2} + \frac{2(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2} + \frac{\mu}{1+\tau^{1-\sigma}} - \frac{\mu}{1+\tau^{\sigma-1}}.$$

□

Proposition 3.2. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{22} = \tau_{11} = \tau_i$ and $L_1 = L_2$. The relative real wage, $\frac{\omega_1}{\omega_2}$, is a decreasing function of τ_{22}, for $\tau_e \in (0,1)$, $\tau_i \in (\tau_e, 1)$, $\mu \in (0,1)$ and $\sigma > 1$.*

Proof. We want to show that $\left.\frac{d(\omega_1/\omega_2)}{d\tau_{22}}\right|_{L_1=L_2} < 0.$

We have:

$$\left.\frac{d(\omega_1/\omega_2)}{d\tau_{22}}\right|_{L_1=L_2} = \frac{\frac{d\omega_1}{d\tau_{22}}\omega_2 - \frac{d\omega_2}{d\tau_{22}}\omega_1}{\omega_2^2}. \tag{18}$$

From Claim 5.3, we know that $\omega_1 = \omega_2 > 0$, therefore, (18) can be written as:

$$\left.\frac{d(\omega_1/\omega_2)}{d\tau_{22}}\right|_{L_1=L_2} = \frac{\frac{d\omega_1}{d\tau_{22}} - \frac{d\omega_2}{d\tau_{22}}}{\omega_2}.$$

From Lemma 3.1 and Lemma 3.2, we know that $\frac{d\omega_1}{d\tau_{22}} < 0$ and $\frac{d\omega_2}{d\tau_{22}} > 0$ respectively, thus:

$$\left.\frac{d(\omega_1/\omega_2)}{d\tau_{22}}\right|_{L_1=L_2} < 0.$$

□

Lemma 3.1. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$. The real wage in region 1, ω_1, is a decreasing function of τ_{22}, for $\tau_e \in (0, 1)$, $\tau_i \in (\tau_e, 1)$, $\mu \in (0, 1)$ and $\sigma > 1$.*

Proof. We want to show that $\left.\dfrac{d\omega_1}{d\tau_{22}}\right|_{L_1=L_2} < 0$.

Using (7), we write W_2 as function of W_1, and differentiate the expression using the chain rule and the Implicit Function Theorem.

$$\frac{d\omega_1}{d\tau_{22}} = \frac{d\left(\frac{W_1(W_1,W_2)}{P_1(W_1,W_2)}\right)}{d\tau_{22}} = \frac{\frac{\partial W_1}{\partial \tau_{22}} P_1(W_1) - \frac{dP_1}{d\tau_{22}} W_1}{P_1(W_1)^2}, \tag{19}$$

where

$$\frac{dP_1}{d\tau_{22}} = \left[\frac{\partial P_1(W_1)}{\partial \tau_{22}} + \frac{\partial P_1(W_1)}{\partial W_1}\frac{\partial W_1}{\partial \tau_{22}}\right].$$

From Claim 5.1, when $L_1 = L_2$, $\tau_{11} = \tau_{22} = \tau_i$ and $\tau_{12} = \tau_{21} = \tau_e$, it is always the case that $W_1 = W_2 = 1$. Then (19) becomes:

$$\frac{d\omega_1}{d\tau_{22}} = \frac{\frac{\partial W_1}{\partial \tau_{22}} P_1(W_1) - \left[\frac{\partial P_1(W_1)}{\partial \tau_{22}} + \frac{\partial P_1(W_1)}{\partial W_1}\frac{\partial W_1}{\partial \tau_{22}}\right]}{P_1(W_1)^2}. \tag{20}$$

From Claims 5.5 and 5.2, we know that $\frac{\partial W_1}{\partial \tau_{22}} < 0$ and that $P_1(W_1) > 0$.

Since P_1 does not depend on the internal transportation costs of region 2, τ_{22}, from (10) we have that $\frac{\partial P_1(W_1)}{\partial \tau_{22}} = 0$. Therefore, the second term in the numerator of (20) becomes:

$$\begin{aligned}\frac{\partial P_1(W_1)}{\partial W_1}\frac{\partial W_1}{\partial \tau_{22}} = &-\frac{\gamma\mu}{\sigma - 1}\left\{\frac{1}{2}\left[\left(\frac{1}{\tau_{11}}\right)^{1-\sigma} + \left(\frac{1}{\tau_{21}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \\ &\left\{\frac{1-\sigma}{2}\left[\left(\frac{1}{\tau_{11}}\right)^{-\sigma}\tau_{11}^{-1} - \left(\frac{1}{\tau_{21}}\right)^{-\sigma}\tau_{21}^{-1}\right]\right\}\frac{\partial W_1}{\partial \tau_{22}}\end{aligned}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$:

$$\frac{\partial P_1(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_i^{\sigma-1} + \tau_e^{\sigma-1}\right)\right]^{\frac{-\mu}{\sigma-1}-1}\left(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}}. \tag{21}$$

With P_1 at equilibrium values, $P_1 = \gamma \left[\frac{1}{2}\left(\tau_i^{\sigma-1} + \tau_e^{\sigma-1}\right)\right]^{\frac{-\mu}{\sigma-1}}$, and using (21), (20) becomes:

$$\begin{aligned}
\frac{d\omega_1}{d\tau_{22}} &= \frac{\frac{dW_1}{d\tau_{22}}}{P_1} - \frac{\frac{\mu}{2} P_1 \left[\frac{1}{2}\left(\tau_i^{\sigma-1} + \tau_e^{\sigma-1}\right)\right]^{-1} \left(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}\right) \frac{dW_1}{d\tau_{22}}}{P_1^2} \\
&= \frac{\frac{dW_1}{d\tau_{22}}}{P_1} - \frac{\frac{\mu}{2} \left[\frac{1}{2}\left(\tau_i^{\sigma-1} + \tau_e^{\sigma-1}\right)\right]^{-1} \left(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}\right) \frac{dW_1}{d\tau_{22}}}{P_1}.
\end{aligned}$$

To prove Lemma 3.1 we note that:

$$\frac{dW_1}{d\tau_{22}} \left[1 - \mu \frac{\tau_i^{\sigma-1} - \tau_e^{\sigma-1}}{\tau_i^{\sigma-1} + \tau_e^{\sigma-1}}\right] < 0 \Rightarrow \frac{d\omega_1}{d\tau_{22}} < 0$$

Given that $\tau_i > \tau_e$, it is clear that $0 < \frac{\mu\left(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}\right)}{\tau_i^{\sigma-1} + \tau_e^{\sigma-1}} < 1$, finishing the proof.

□

Lemma 3.2. *Let* $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ *and* $L_1 = L_2$. *The real wage in region 2,* ω_2, *is a increasing function of* τ_{22}, *for any* $0 < \tau_e < \tau_i < 1$, $\mu \in (0,1)$ *and* $\sigma > 1$.

Proof. We want to show that $\left.\frac{d\omega_2}{d\tau_{22}}\right|_{L_1 = L_2} > 0$.

Using (7), we write W_2 as function of W_1, and differentiate the expression using the chain rule and the Implicit Function Theorem.

$$\frac{d\omega_2}{d\tau_{22}} = \frac{d\left(\frac{W_2(W_1,W_2)}{P_2(W_1,W_2)}\right)}{d\tau_{22}} = \frac{\frac{dW_2}{d\tau_{22}} P_2(W_1) - \frac{dP_2}{d\tau_{22}} W_2}{P_2(W_1)^2}, \tag{22}$$

where:

$$\frac{dP_2}{d\tau_{22}} = \left[\frac{\partial P_2(W_1)}{\partial \tau_{22}} + \frac{\partial P_2(W_1)}{\partial W_1} \frac{dW_1}{d\tau_{22}}\right]. \tag{23}$$

From (11), we compute the first term in (23):

$$\begin{aligned}
\frac{\partial P_2(W_1)}{\partial \tau_{22}} = -\frac{\gamma\mu}{\sigma-1} \left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \\
\frac{1-\sigma}{2}\left(\frac{2-W_1}{\tau_{22}}\right)^{-\sigma}\left(-\frac{2-W_1}{\tau_{22}^2}\right)
\end{aligned}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\frac{\partial P_2(W_1)}{\partial \tau_{22}} \quad = \quad -\frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1} \tau_i^{\sigma-2} \tag{24}$$

Secondly, we study the second term of (23):

$$\frac{\partial P_2(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}} = -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1}$$
$$\left\{\frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\tau_{12}^{-1} - \left(\frac{2-W_1}{\tau_{22}}\right)^{-\sigma}\tau_{22}^{-1}\right]\right\}\frac{dW_1}{d\tau_{22}}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\frac{\partial P_2(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_e^{\sigma-1} - \tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}} \tag{25}$$

With (24) and (25), equation (23) becomes:

$$\frac{dP_2}{d\tau_{22}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{\frac{-\mu}{\sigma-1}-1}\left[\left(\tau_e^{\sigma-1} - \tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}} - \tau_i^{\sigma-2}\right] \tag{26}$$

With P_2 at the equilibrium value, $P_2 = \gamma\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}}$, equation (22) becomes:

$$\begin{aligned}\frac{d\omega_2}{d\tau_{22}} &= \frac{\frac{dW_2}{d\tau_{22}}}{P_2} - \frac{\frac{\mu}{2}P_2\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-1}\left[\left(\tau_e^{\sigma-1} - \tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}} - \tau_i^{\sigma-2}\right]}{P_2^2} \\ &= \frac{\frac{dW_2}{d\tau_{22}}}{P_2} - \frac{\mu\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)^{-1}\left[\left(\tau_e^{\sigma-1} - \tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}} - \tau_i^{\sigma-2}\right]}{P_2}\end{aligned} \tag{27}$$

To prove that (27) is positive, we note that:

$$\frac{dW_2}{d\tau_{22}} - \mu\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)^{-1}\left(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}\right)\frac{dW_2}{d\tau_{22}} + \mu\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)^{-1}\tau_i^{\sigma-2} =$$
$$= \frac{dW_2}{d\tau_{22}}\left(1 - \mu\frac{\tau_i^{\sigma-1} - \tau_e^{\sigma-1}}{\tau_i^{\sigma-1} + \tau_e^{\sigma-1}}\right) + \frac{\mu\tau_i^{\sigma-2}}{\tau_e^{\sigma-1} + \tau_i^{\sigma-1}} > 0$$

From Claims 5.5 and 5.4, we know that $\frac{dW_2}{d\tau_{22}} > 0$, thus the proof is finished.

□

Lemma 3.3. *Let* $\tau_{21} = \tau_{12} = \tau_e$*,* $\tau_{11} = \tau_{22} = \tau_i$ *and* $L_1 = L_2$*. The price index in region 1,* P_1 *is a decreasing function of* τ_{22}*, for any* $0 < \tau_e < \tau_i < 1$*,* $\mu \in (0,1)$ *and* $\sigma > 1$*.*

Proof. By the chain rule:

$$\frac{dP_1}{d\tau_{22}} = \frac{\partial P_1(W_1)}{\partial \tau_{22}} + \frac{\partial P_1(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}} \tag{28}$$

From (10), we know that $\frac{\partial P_1(W_1)}{\partial \tau_{22}} = 0$, and thus (28) becomes:

$$\begin{aligned}\frac{dP_1}{d\tau_{22}} &= -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{11}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{21}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \\ & \quad \frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{11}}\right)^{-\sigma}\tau_{11}^{-1} - \left(\frac{2-W_1}{\tau_{21}}\right)^{-\sigma}\tau_{21}^{-1}\right]\frac{dW_1}{d\tau_{22}}\end{aligned}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\frac{dP_1}{d\tau_{22}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_i^{\sigma-1} + \tau_e^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}}.$$

As $\gamma > 0$, $\tau_i > \tau_e$ and (from Claim 5.5) $\frac{dW_1}{d\tau_{22}} < 0$, the proof is finished.

□

Lemma 3.4. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$. The price index in region 2, P_2, is a decreasing function of τ_{22}, for any $0 < \tau_e < \tau_i < 1$, $\mu \in (0,1)$ and $\sigma > 1$.*

Proof. By the chain rule:

$$\frac{dP_2}{d\tau_{22}} = \frac{\partial P_2(W_1)}{\partial \tau_{22}} + \frac{\partial P_2(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}}. \tag{29}$$

From (11), we compute the first term of (29):

$$\frac{\partial P_2(W_1)}{\partial \tau_{22}} = -\frac{\gamma\mu}{\sigma - 1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \frac{1-\sigma}{2}\left(\frac{2-W_1}{\tau_{22}}\right)^{-\sigma}\left(-\frac{2-W_1}{\tau_{22}^2}\right)$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$ above, we obtain:

$$\frac{\partial P_2(W_1)}{\partial \tau_{22}} = -\frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1} \tau_i^{\sigma-2} \tag{30}$$

Secondly, we study the second term of (29):

$$\frac{\partial P_2(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}} = -\frac{\gamma\mu}{\sigma - 1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \left\{\frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\tau_{12}^{-1} - \left(\frac{2-W_1}{\tau_{22}}\right)^{-\sigma}\tau_{22}^{-1}\right]\right\}\frac{dW_1}{d\tau_{22}}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\frac{\partial P_2(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_e^{\sigma-1} - \tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}} \tag{31}$$

With (30) and (31), we compute $\frac{dP_2}{d\tau_{22}}$:

$$\frac{dP_2}{d\tau_{22}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{\frac{-\mu}{\sigma-1}-1}\left[\left(\tau_e^{\sigma-1} - \tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}} - \tau_i^{\sigma-2}\right] \tag{32}$$

Given that, in (32), $\frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{\frac{-\mu}{\sigma-1}-1}$ is positive, to finish the proof we need to show the following inequality:

$$\left(\tau_e^{\sigma-1} - \tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}} < \tau_i^{\sigma-2} \Leftrightarrow \tau_i(1-\tau^{\sigma-1})\frac{dW_2}{d\tau_{22}} < 1$$

Importing the expression (16):

$$\tau_i(1-\tau^{\sigma-1})\frac{\frac{(\sigma-1)\tau^{\sigma-1}}{2\tau_i(1+\tau^{\sigma-1})^2}}{1+\frac{\mu}{1+\tau^{1-\sigma}}-\frac{\mu}{1+\tau^{\sigma-1}}+\frac{2(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}} < 1 \Leftrightarrow$$

$$\Leftrightarrow \frac{1-\tau^{\sigma-1}}{2}\frac{(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2} < 1+\frac{\mu}{1+\tau^{1-\sigma}}-\frac{\mu}{1+\tau^{\sigma-1}}+\frac{2(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}.$$

Since the expression on the left is lower than the last term of the expression on the right, the proof is finished.

□

Proposition 3.3. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{22} = \tau_{11} = \tau_i$ and $L_1 = L_2$. There is a $\mu^*(\sigma,\tau) \in (0,1)$ such that: $\frac{d(\omega_1/\omega_2)}{d\tau_{12}} > 0$ for $\mu \in (0,\mu^*)$ and $\frac{d(\omega_1/\omega_2)}{d\tau_{12}} < 0$ for $\mu \in (\mu^*,1)$.*

Proof. We want to know the sign of:

$$\left.\frac{d\left(\omega_1/\omega_2\right)}{d\tau_{12}}\right|_{L_1=L_2} = \frac{\frac{d\omega_1}{d\tau_{12}}\omega_2 - \frac{d\omega_2}{d\tau_{12}}\omega_1}{\omega_2^2},$$

where

$$\frac{d\omega_1}{d\tau_{12}} = \frac{d\left(\frac{W_1}{P_1}\right)}{d\tau_{12}} = \frac{\frac{dW_1}{d\tau_{12}}P_1 - \left(\frac{\partial P_1}{\partial \tau_{12}} + \frac{\partial P_1}{\partial W_1}\frac{dW_1}{d\tau_{12}}\right)W_1}{P_1^2}$$

and

$$\frac{d\omega_2}{d\tau_{12}} = \frac{d\left(\frac{W_2}{P_2}\right)}{d\tau_{12}} = \frac{\frac{dW_2}{d\tau_{12}}P_2 - \left(\frac{\partial P_2}{\partial \tau_{12}} + \frac{\partial P_2}{\partial W_1}\frac{dW_1}{d\tau_{12}}\right)W_2}{P_2^2}.$$

When $L_1 = L_2$, $\tau_{22} = \tau_{11} = \tau_i$ and $\tau_{12} = \tau_{21} = \tau_e$, from Claims 5.1, 5.2 and 5.3, we know that $W_1 = W_2 = 1$, $P_1 = P_2$ and $\omega_1 = \omega_2$. Simplifying:

$$\left.\frac{d\left(\omega_1/\omega_2\right)}{d\tau_{12}}\right|_{L_1=L_2} = \frac{\frac{d\omega_1}{d\tau_{12}} - \frac{d\omega_2}{d\tau_{12}}}{\omega_2},$$

with

$$\frac{d\omega_1}{d\tau_{12}} - \frac{d\omega_2}{d\tau_{12}} = \frac{\left(\frac{dW_1}{d\tau_{12}} - \frac{dW_2}{d\tau_{12}}\right) P_1 + \frac{\partial P_2}{\partial \tau_{12}} - \frac{\partial P_1}{\partial \tau_{12}} + \frac{\partial P_2}{\partial W_1}\frac{dW_1}{d\tau_{12}} - \frac{\partial P_1}{\partial W_1}\frac{dW_1}{d\tau_{12}}}{P_1^2}.$$

From Claim 5.4 and (10), we know that $\frac{dW_1}{d\tau_{12}} = -\frac{dW_2}{d\tau_{12}}$ and that $\frac{\partial P_1}{\partial \tau_{12}} = 0$. Then:

$$\frac{d\omega_1}{d\tau_{12}} - \frac{d\omega_2}{d\tau_{12}} = \frac{1}{P_1^2}\left[\frac{dW_1}{d\tau_{12}}\left(2P_1 + \frac{\partial P_2}{\partial W_1} - \frac{\partial P_1}{\partial W_1}\right) + \frac{\partial P_2}{\partial \tau_{12}}\right]$$

The next stage is to compute $\frac{\partial P_2}{\partial W_1}$, $\frac{\partial P_1}{\partial W_1}$ and $\frac{\partial P_2}{\partial \tau_{12}}$.

Using (11), we find:

$$\begin{aligned}\frac{\partial P_2}{\partial W_1} &= -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \\ &\quad \frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\tau_{12}^{-1} - \left(\frac{2-W_1}{\tau_{22}}\right)^{-\sigma}\tau_{22}^{-1}\right].\end{aligned}$$

Replacing $W_1 = 1$, $\tau_{11} = \tau_{22} = \tau_i$, $\tau_{12} = \tau_{21} = \tau_e$ and $P_1 = \gamma\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}}$:

$$\begin{aligned}\frac{\partial P_2}{\partial W_1} &= \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_e^{\sigma-1} - \tau_i^{\sigma-1}\right) = \\ &= \mu P_1 \frac{\tau_e^{\sigma-1} - \tau_i^{\sigma-1}}{\tau_e^{\sigma-1} + \tau_i^{\sigma-1}} = -\mu P_1 \frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}.\end{aligned}$$

Using (10) we find that $\frac{\partial P_1}{\partial W_1} = -\frac{\partial P_2}{\partial W_1}$:

$$\begin{aligned}\frac{\partial P_1}{\partial W_1} &= -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{11}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{21}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \\ &\quad \frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{11}}\right)^{-\sigma}\tau_{11}^{-1} - \left(\frac{2-W_1}{\tau_{21}}\right)^{-\sigma}\tau_{21}^{-1}\right] = \\ &= \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_i^{\sigma-1} + \tau_e^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}\right)\end{aligned}$$

From (11), we compute $\frac{\partial P_2}{\partial \tau_{12}}$:

$$\begin{aligned}\frac{\partial P_2}{\partial \tau_{12}} &= -\frac{\mu}{\sigma-1}\gamma\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma}+\left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \\ & \quad \frac{1-\sigma}{2}\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\left(-\frac{W_1}{\tau_{12}^2}\right) = -\frac{\gamma\mu}{2}\tau_e^{\sigma-2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1} \\ &= -\mu P_1\frac{\tau_e^{\sigma-2}}{\tau_e^{\sigma-1}+\tau_i^{\sigma-1}} = -\mu P_1\frac{\tau^{\sigma-1}\tau_e^{-1}}{1+\tau^{\sigma-1}}.\end{aligned}$$

Then, replacing:

$$P_1\left(\frac{d\omega_1}{d\tau_{12}}-\frac{d\omega_2}{d\tau_{12}}\right) = 2\frac{dW_1}{d\tau_{12}}\left(1-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}\right)-\frac{\mu\tau_e^{-1}\tau^{\sigma-1}}{1+\tau^{\sigma-1}}.$$

The sign of $\frac{d(\omega_1/\omega_2)}{d\tau_{12}}$ is positive whenever:

$$\frac{dW_1}{d\tau_{12}} > \frac{\frac{\mu\tau_e^{-1}}{2}\frac{\tau^{\sigma}-1}{1+\tau^{\sigma-1}}}{1-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}} = \frac{\frac{\mu\tau_e^{-1}}{2}\tau^{\sigma-1}}{1+\tau^{\sigma-1}-\mu(1-\tau^{\sigma-1})}.$$

Importing the expression (17) for $W_1' > 0$ obtained in Claim 5.6:

$$W_1' = \frac{\frac{\sigma-1}{\tau_e}\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}\frac{1}{1-A-B},$$

where

$$A = -\frac{2(\sigma-1)\tau^{1-\sigma}}{(1+\tau^{1-\sigma})^2}-\frac{\mu}{1+\tau^{1-\sigma}}, \quad \text{and} \quad B = -\frac{2(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}+\frac{\mu}{1+\tau^{\sigma-1}}.$$

Recalling that $\frac{\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2} = \frac{\tau^{1-\sigma}}{(1+\tau^{1-\sigma})^2}$, we know that $\frac{d(\omega_1/\omega_2)}{d\tau_{12}} > 0$ if and only if:

$$\frac{\sigma-1}{1+4(\sigma-1)\frac{\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}+\frac{\mu}{1+\tau^{1-\sigma}}-\frac{\mu}{1+\tau^{\sigma-1}}} > \frac{\frac{\mu}{2}(1+\tau^{\sigma-1})^2}{1+\tau^{\sigma-1}-\mu(1-\tau^{\sigma-1})}.$$

With some manipulation:

$$\frac{2(\sigma-1)}{(1+\tau^{\sigma-1})^2} > \frac{\mu+4\mu(\sigma-1)\frac{\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}-\mu^2\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}}{1+\tau^{\sigma-1}-\mu(1-\tau^{\sigma-1})}.$$

In is clear that this is true when $\mu \to 0$. When $\mu \to 1$, the expression becomes:

$$\frac{4(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2} > 1 + \frac{4(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2} - \frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}} 2\tau^{\sigma-1},$$

which is clearly false.

To show that the expression is true if and only if μ is lower or equal than some $\mu^* \in (0,1)$, notice that it is equivalent to a U-shaped parabola.

$$\frac{d\left(\omega_1/\omega_2\right)}{d\tau_{12}} > 0 \Leftrightarrow a\mu^2 + b\mu + c > 0, \quad \text{where} \quad a = \frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}.$$

□

Proposition 3.4. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$. Then, ω_1, is an increasing function of τ_{12}, for any $\tau_e \in (0,1)$, $\tau_i \in (\tau_e, 1)$, $\mu \in (0,1)$ and $\sigma > 1$.*

Proof. We want to show that $\left.\frac{d\omega_1}{d\tau_{12}}\right|_{L_1=L_2} > 0$.

$$\frac{d\omega_1}{d\tau_{12}} = \frac{d\left(\frac{W_1}{P_1}\right)}{d\tau_{12}} = \frac{\frac{dW_1}{d\tau_{12}} P_1 - \left[\frac{\partial P_1}{\partial \tau_{12}} + \frac{\partial P_1}{\partial W_1}\frac{dW_1}{d\tau_{12}}\right] W_1}{P_1^2} \tag{33}$$

When $L_1 = L_2$, $\tau_{11} = \tau_{22} = \tau_i$ and $\tau_{12} = \tau_{21} = \tau_e$, from Claim 5.1 we know that $W_1 = 1$ and from (10) we find that $\frac{\partial P_1}{\partial \tau_{12}} = 0$. Then, simplifying (33):

$$\left.\frac{d\omega_1}{d\tau_{12}}\right|_{L_1=L_2} = \frac{\frac{dW_1}{d\tau_{12}} P_1 - \frac{\partial P_1}{\partial W_1}\frac{dW_1}{d\tau_{12}}}{P_1^2} \tag{34}$$

From (10) we evaluate:

$$\frac{\partial P_1}{\partial W_1} = -\frac{\gamma\mu}{\sigma-1}\left[\frac{1}{2}(\tau_i^{\sigma-1} + \tau_e^{\sigma-1})\right]^{-\frac{\mu}{\sigma-1}-1} \frac{1-\sigma}{2}(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}) = \mu P_1 \frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}} < P_1.$$

□

Proposition 3.5. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{22} = \tau_{11} = \tau_i$ and $L_1 = L_2$. For any $0 < \tau_e < \tau_i < 1$, there exists a $\mu^*(\sigma, \tau) \in (0,1)$ such that:*
$\frac{d\omega_2}{d\tau_{12}} < 0$ for $\mu \in (0, \mu^*)$ and $\frac{d\omega_2}{d\tau_{12}} > 0$ for $\mu \in (\mu^*, 1)$.

Proof. We want to show that $\left.\dfrac{d\omega_2}{d\tau_{12}}\right|_{L_1=L_2} < 0$.

$$\frac{d\omega_2}{d\tau_{12}} = \frac{d\left(\frac{W_2}{P_2}\right)}{d\tau_{12}} = \frac{\frac{dW_2}{d\tau_{12}}P_2 - \left(\frac{\partial P_2}{\partial \tau_{12}} + \frac{\partial P_2}{\partial W_1}\frac{dW_1}{d\tau_{12}}\right)W_2}{P_2^2} \tag{35}$$

From Claim 5.1 we can rewrite (35) in the following way:

$$\left.\frac{d\omega_2}{d\tau_{12}}\right|_{L_1=L_2} = \frac{\frac{dW_2}{d\tau_{12}}P_2 - \frac{\partial P_2}{\partial \tau_{12}} - \frac{\partial P_2}{\partial W_1}\frac{dW_1}{d\tau_{12}}}{P_2^2} \tag{36}$$

From Claim 5.6 we know that $\frac{dW_1}{d\tau_{12}} > 0$ and from Claim 5.4 we know that $\frac{dW_2}{d\tau_{12}} = -\frac{dW_1}{d\tau_{12}}$. Then, the sign of (36) is the same as the sign of:

$$-\frac{dW_1}{d\tau_{12}}\left(P_2 + \frac{\partial P_2}{\partial W_1}\right) - \frac{\partial P_2}{\partial \tau_{12}}.$$

From (11):

$$\begin{aligned}\frac{\partial P_2}{\partial \tau_{12}} &= -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \\ &\times \left[\frac{1-\sigma}{2}\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\left(\frac{-W_1}{\tau_{12}^2}\right)\right]\end{aligned}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\begin{aligned}\frac{\partial P_2}{\partial \tau_{12}} &= -\frac{\gamma\mu}{\sigma-1}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left[\frac{\sigma-1}{2}\tau_e^{\sigma-2}\right] = \\ &= -\frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\tau_e^{\sigma-2} = -\mu P_2 \tau_e^{-1}\frac{\tau^{\sigma-1}}{1+\tau^{\sigma-1}}\end{aligned} \tag{37}$$

Again from (11):

$$\frac{\partial P_2}{\partial W_1} = -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma}+\left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \times \left\{\frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\tau_{12}^{-1}-\left(\frac{2-W_1}{\tau_{22}}\right)^{-\sigma}\tau_{22}^{-1}\right]\right\}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\frac{\partial P_2}{\partial W_1} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_e^{\sigma-1}-\tau_i^{\sigma-1}\right) = -\mu P_2\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}. \quad (38)$$

With (37) and (38), we observe that:

$$\frac{d\omega_2}{d\tau_{12}} < 0 \Leftrightarrow \frac{dW_1}{d\tau_{12}}\left(1-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}\right)-\mu\tau_e^{-1}\frac{\tau^{\sigma-1}}{1+\tau^{\sigma-1}} > 0.$$

Importing the expression (17) obtained in Claim 5.6:

$$\frac{dW_1}{d\tau_{12}} = \frac{\frac{\sigma-1}{\tau_e}\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}\frac{1}{1+\frac{4(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}}.$$

Thus $\frac{d\omega_2}{d\tau_{12}} < 0$ if and only if:

$$\frac{\frac{\sigma-1}{\tau_e}\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}\frac{1-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}}{1+\frac{4(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}} > \mu\tau_e^{-1}\frac{\tau^{\sigma-1}}{1+\tau^{\sigma-1}} \Leftrightarrow$$

$$\Leftrightarrow \frac{\sigma-1}{1+\tau^{\sigma-1}}\frac{1-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}}{1+\frac{4(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}} > \mu \Leftrightarrow$$

$$\Leftrightarrow \frac{\sigma-1}{1+\tau^{\sigma-1}}\left(1-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}\right)-\mu\left[1+\frac{4(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}\right] > 0.$$

The expression on the left is a U-shaped parabola, therefore, all we need to check is that the inequality is true when $\mu \to 0$ and false when $\mu \to 1$. It is easy to see that when $\mu \to 0$, the inequality is true. When $\mu \to 1$, the inequality becomes:

$$\frac{\sigma-1}{1+\tau^{\sigma-1}}-\frac{(\sigma-1)(1-\tau^{\sigma-1})}{(1+\tau^{\sigma-1})^2}-1-\frac{4(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}+\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}} > 0 \Leftrightarrow$$

$$\Leftrightarrow (\sigma-1)\left[\frac{-2\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}\right]-1+\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}} > 0.$$

Which is clearly false.

□

Lemma 3.5. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$. Then, the price index in region 1, P_1, is an increasing function of τ_{12} for any $0 < \tau_e < \tau_i < 1$, $\mu \in (0,1)$ and $\sigma > 1$.*

Proof. Observe that:

$$\left.\frac{dP_1}{d\tau_{12}}\right|_{L_1=L_2} = \frac{\partial P_1}{\partial \tau_{12}} + \frac{\partial P_1}{\partial W_1}\frac{dW_1}{d\tau_{12}}. \tag{39}$$

As $\frac{\partial P_1}{\partial \tau_{12}} = 0$, (39) becomes:

$$\begin{aligned}\frac{dP_1}{d\tau_{12}} &= -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{11}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{21}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \\ &\times \left\{\frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{11}}\right)^{-\sigma}\tau_{11}^{-1} - \left(\frac{2-W_1}{\tau_{21}}\right)^{-\sigma}\tau_{21}^{-1}\right]\right\}\frac{dW_1}{d\tau_{12}}.\end{aligned}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\frac{dP_1}{d\tau_{12}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_i^{\sigma-1} + \tau_e^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}\right)\frac{dW_1}{d\tau_{12}}$$

From Claim 5.6, we know that $\frac{\partial W_1}{\partial \tau_{12}} > 0$.

□

Lemma 3.6. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$. Then, the price index in region 2, P_2, is a decreasing function of τ_{12} for any $0 < \tau_e < \tau_i < 1$, $\mu \in (0,1)$ and $\sigma > 1$.*

Proof. Observe that:

$$\frac{dP_2}{\tau_{12}} = \frac{\partial P_2}{\partial \tau_{12}} + \frac{\partial P_2}{\partial W_1}\frac{dW_1}{d\tau_{12}} \tag{40}$$

Firstly, we study the first term in (40):

$$\begin{aligned}\frac{\partial P_2}{\partial \tau_{12}} &= -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma}+\left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1}\\ &\times \left[\frac{1-\sigma}{2}\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\left(\frac{-W_1}{\tau_{12}^2}\right)\right]\end{aligned}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\begin{aligned}\frac{\partial P_2}{\partial \tau_{12}} &= -\frac{\gamma\mu}{\sigma-1}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{\frac{-\mu}{\sigma-1}-1}\left[\frac{\sigma-1}{2}\tau_e^{\sigma-2}\right]\\ &= -\frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\tau_e^{\sigma-2} \qquad (41)\end{aligned}$$

Secondly, we study the second term in (40), then:

$$\begin{aligned}\frac{\partial P_2}{\partial W_1}\frac{dW_1}{d\tau_{12}} &= -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma}+\left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1}\\ &\times \left\{\frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\tau_{12}^{-1}-\left(\frac{2-W_1}{\tau_{22}}\right)^{-\sigma}\tau_{22}^{-1}\right]\right\}\frac{dW_1}{d\tau_{12}}\end{aligned}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\frac{\partial P_2}{\partial W_1}\frac{dW_1}{d\tau_{12}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_e^{\sigma-1}-\tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{12}} \qquad (42)$$

Adding (41) and (42), we obtain:

$$\frac{dP_2}{d\tau_{12}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left[\left(\tau_e^{\sigma-1}-\tau_i^{\sigma-1}\right)\frac{\partial W_1}{\partial \tau_{12}}-\tau_e^{\sigma-2}\right]. \qquad (43)$$

From Claim 5.6, $\frac{dW_1}{d\tau_{12}} > 0$, which implies that $\frac{dP_2}{d\tau_{12}} < 0$.

□

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