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Restructuring Facility Networks under Economy of Scales*

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Abstract

In this work we address the facility network restructuring problem. This problem is closely related to location/allocation and set covering problems. However, none of the above includes all its complexity nor involves all the decision types. Therefore we are extending current literature by considering a new problem. Due to the presence of economies of scale, another type of complexity arises since we must minimize a concave cost function. For this problem a local search heuristic is proposed, where an initially feasible solution, obtained by solving a related linear problem, is improved by a slope scaling procedure and then by drop and swap operations. Computational results showing the effectiveness and efficiency of the solution procedure are reported.

Keywords: Location/Allocation, Covering, Concave Optimization, Heuristics.
JEL Classifications: C61, C44.

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1 Introduction

The need for restructuring facility networks may be due to business relations, i.e. situations like mergers and acquisitions, but it can also be a consequence of social dynamics. New ”good” areas can be created for instance due to the installation of new infrastructures (hospitals, schools, shopping malls, roads, and the like) or due to changes in urban legislation, while others may loose its status, for instance due infrastructures aging and lack of reconstruction.

In this work we address the Facility Network Restructuring Problem (FNRP), where given an existing network of facilities, a set of potential sites where to locate new facilities, and a set of clients we want to provide service to, we wish to determine the new network configuration, i.e. facilities location and sizes, such that clients needs and target service quality are met, at minimum total cost. In order to do so we may close or resize existing facilities and open new facilities. Costs are incurred with physical facilities (opening, closing or resizing), with human resources (firing, hiring, and training) and with providing the service (servicing client needs and penalties associated with not meeting the target service quality). Clients are assumed to get their service from the closest facility. Furthermore, we also consider that there is a maximum distance that clients are willing to travel to get their service, this distance is termed standard distance.

The facility restructuring problem we address here is a mixture and an extension of the location/allocation and set covering problems. In the Set Covering Problem (SCP) we also wish to determine in which of the potential locations we should place facilities at minimum installation cost. However, in the SCP we are only interested in guaranteeing that clients are covered, that is have at least one facility within a predefined distance. An extension of this problem which has been termed Multi Covering Problem (MCP), requiring a number of covers ($>$1) for clients has also been studied, see for example (Gonsalvez, Hall, Rhee, & Siferd, 1987; Hall & Hochbaum, 1992; Peleg, Schechtman, & Wool, 1997). In both the SCP and the MCP, facility covers are binary, that is the facility either covers the client or it does not. Another version, named Unbounded Multi Covering problem, where a facility can provide an integer number of covers ($>$1) has also been studied (Xiaoming & Slyke, 1984). Batta and Mannur (1990) have considered the case where it is possible to have more than one facility at each potential site. Some authors have also studied the capacitated version of the SCP, where a limit is imposed on the capacity a facility has to provide covers, see for example (Chuzhoy, 2006). However, none of the extensions studied so far include costs other than facility installation costs, facilities with different sizes, different number of covers provided by each facility to each client it covers, constraints on both the total number of covers a facility may provide, as well as, on the number of covers a facility may provide to a single client. In the Facility Network Restructuring Problem (FNRP) we consider all these issues and others that are to be presented in the following discussion.

Location problems are some of the most widely studied problems in combinatorial optimization (see (Mirchandani & Francis, 1990) for a detailed introduction). In the Facility Location Problem (FLP) it is required to locate a number of facilities (industrial plants, warehouses, etc.) and allocate clients to them so as to minimize the total cost of satisfying the demand for some commodity or service, which is the sum of the servicing costs and opening costs. Some extensions of the FLP have been studied, for example when more than one facility can serve a client (Sherali & Al-Loughani, 1999; Jaramillo, Bhadury, & Batta, 2002) and when demand can take an integer value (≥ 1), (Holmberg, 1999). In the classical FLP, it is assumed that the setup cost of a facility depends only on its location. An extension of the classical problem where the opening costs are considered to be dependent on the amount of demand satisfied by the facility has been considered for example by Averbakh et al. (1998) which later also have considered that clients are the ones doing the allocation choice (Averbakh, Berman, Drezner, & Wesolowsky, 2007). Another extension of the FLP can be considered when the capacity of a facility to serve clients is limited. These problems are called *capacitated* FLP and have been studied by (Christofides & Beasley, 1983), (Maniezzo, Mingozzi, & Baldacci, 1998) and (Klose & Grtz, 2007). Other type of facility location problems, less related to ours have also been studied, for example (Zhang & Melachrinoudis, 2001) study the location of obnoxious facilities. For a very recent and comprehensive survey see (ReVelle & Eiselt, 2005).

Among all the papers on FLP and its variants, only a few considered the problem of opening new facilities or closing existing facilities when some facilities already exist. Berman and Simchi-Levi (1990) and Drezner (1995) considered the problem of adding some new facilities, while Leorch et al. (1996) considered the problem of closing some existing facilities. Chhajed and Lowe (1992) studied the problem of adding m new facilities on a tree, given that there are n pre-existing facilities. Dell (1998) focused on the formulation of closing or realigning of US Army installations. The problem of relocating a facility can be viewed also a highly application specific situation, see for example the study by Min and Melachrinoudis (1999) in which they present a real-world case study for the relocation of a combined manufacturing and distribution (warehouse) facility and in Melachrinoudis and Min (2000) a mathematical programming model is built for the same problem. Even if some times it may make sense to decide on opening new facilities or closing existing ones in isolation, in many real applications it is more likely to have to consider both issues simultaneously. As far as the authors are aware of, only two such studies have been made (Wang, Batta, Bhadury, & Rump, 2003; Ghosh, 2003). The problem addressed by Wang et al. (2003) is probably the closest to our work in literature, however not as many issues (e.g. service capacity, service quality, employees, etc. are considered and the cost function considered is linear. The authors develop three heuristic algorithms (greedy interchange, tabu search and Lagrangian relaxation approximation) to solve the problem of opening and closing facilities with budget constraints. Ghosh (2003) addresses the uncapacitated FLP with two cost components: a fixed cost component of opening a facility at a given site and a service cost component of satisfying the client requirements. For this problem the author develops a local search procedure based on add and swap operations. This is then embedded into a Tabu search and also into a complete local search with memory algorithms, that closely follow (Rolland, Schilling, & Current, 1996) and (Ghosh & Sierksma, 2002), respectively.

The contribution of our work is to address a new problem, since it is far more complex than the related problems reported in current literature. For this problem we propose a mathematical model and a solution methodology to solve it. The solution procedure proposed, successfully adapts solution techniques from the literature for other related and also non-related problems.

2 Problem Description and Formulation

In the facility network restructuring problem we seek to find the sites of the existing facilities that are to be closed or resized and the sites of new facilities that are to be opened and their respective size, so that the total cost is minimized, subject to clients demand constraints and facility capacity constraints. Each facility, regardless of being an existent one or just a potential one, is characterized by its location, size, service capacity, human resources, and costs. There is a set of clients, each of which characterized by its location, service requirements, and service quality. The latter one being set by the service provider. Also there is a distance up to which clients are willing to travel, designated by standard distance S.

The choice of facility locations is not completely free since there is already a set of existing facilities that we denote by B. From this set of facilities, some can be closed CB, while some others are not closable NCB, with $B = CB \cup NCB$. The former may just be resized, that is their size may be upgraded or downgraded. We denote the set of facility sizes. by K. Let D represent all site locations, i.e. D includes the locations of the existing facilities, as well as, the potential facility locations. Since, in what concerns employees the locations can be grouped we also defined C as the set of counties. Each county $j \in C$ is made up of a set of districts D_j, such that $D = \cup_j D_j$ for all $j \in C$. Therefore, B, CB, and NCB are also partitioned accordingly. We must decide on the number of employees to be hired and fired, so that we have the requiring number ϵ_{ij}^k of employees for a facility with size $k \in K$ operating at district $i \in D_j$ of county $j \in C$. Since employees can be moved within a county at no cost we define he_j as the number of employees hired in county $j \in C$ and fe_j as the number of employees fired in county $j \in C$.

Since we must make decisions on where to locate facilities, that is in which sites are we opening, closing, or resizing we have defined the following decision variables.

$$y_{ij}^k = \begin{cases} 1, & \text{if a facility of size } k \in K \text{ is closed in district } i \in CB_j \text{ of county } j \in C, \\ 0, & \text{otherwise,} \end{cases}$$

$$z_{ij}^k = \begin{cases} 1, & \text{if a facility of size } k \text{ is opened in district } i \in D_j \backslash NCB_j \text{ of county } j \in C, \\ 0, & \text{otherwise,} \end{cases}$$

We also need variables that specify in which sites we are operating facilities since operating facilities incur in costs that are dependent of their size and location, regardless of being previously existent or newly opened.

$$x_{ij}^k = \begin{cases} 1, & \text{if a facility of size } k \in K \text{ is operating in district } i \in D_j \text{ of county } j \in C, \\ 0, & \text{otherwise,} \end{cases}$$

Given that we also must satisfy clients demand $\underline{W}_{lm}$ and that this can be done by more that one facility we also need to determine the service that is being provided by each facility to each client. (Recall that we also must try to meet service quality targets $\overline{W}_{lm}$.) Therefore, we define q_{ij}^{lm} to be the service provided by the facility located in district $i \in D_j$ of county $j \in C$ to the client in district $l \in D_m$ of county $m \in C$.

Let us define a_{ij}^{lm}=1 if the distance between the facility in district $i \in D_j$ of county $j \in C$ and the client in district $l \in D_m$ of county $m \in C$ is not larger than the standard distance S. Therefore the problem can be formulated as follows, where a 10 years operating time horizon is being considered:

$$\text{(P)} \qquad \min \qquad \sum_{j \in C} \sum_{i \in D_j} \sum_{k \in K} f(x_{ij}^k) + \sum_{j \in C} \sum_{i \in CB_j} \sum_{k \in K} g(y_{ij}^k) + \sum_{j \in C} \sum_{i \in D_j \backslash NCB_j} \sum_{k \in K} h(z_{ij}^k) \qquad (1)$$
$$+ \sum_{j \in C} T_j \times he_j + \sum_{j \in C} CMP_j \times fe_j +$$
$$+ \sum_{j \in C} \sum_{i \in D_j} \sum_{m \in C} \sum_{l \in D_m} q_{ij}^{lm} \times v_{ij}^{lm} +$$
$$+ \sum_{m \in C} \sum_{l \in D_m} P_{lm} \times (\overline{W}_{lm} - \sum_{j \in C} \sum_{i \in D_j} q_{ij}^{lm}).$$

subject to:

$$x_{ij}^{k_i} = 1, \quad \forall j \in C, \forall i \in NCB_j, k_i = k(i,j), \tag{2}$$

$$x_{ij}^{k \neq k_i} = 0, \quad \forall j \in C, \forall i \in NCB_j, \forall k \neq k_i \in K, \forall k_i = k(i,j), \tag{3}$$

$$x_{ij}^{k_i} = 1 - y_{ij}^{k_i}, \quad \forall j \in C, \forall i \in CB_j, k_i = k(i,j), \tag{4}$$

$$x_{ij}^{k \neq k_i} = z_{ij}^{k \neq k_i}, \quad \forall j \in C, \forall i \in CB_j, \forall k \neq k_i \in K, \forall k_i = k(i,j), \tag{5}$$

$$\sum_{k \in K} z_{ij}^k \leq 1, \quad \forall j \in C, \forall i \in D_j \backslash B_j, \tag{6}$$

$$x_{ij}^k = z_{ij}^k, \quad \forall j \in C, \forall i \in D_j \backslash B_j, \forall k \in K, \tag{7}$$

$$\underline{W}_{lm} \leq \sum_{j \in C} \sum_{i \in D_j} q_{ij}^{lm} \leq \overline{W}_{lm}, \quad \forall m \in C, \forall l \in D_m, \tag{8}$$

$$q_{ij}^{lm} \leq a_{ij}^{lm} \times \sum_{k \in K} k \times x_{ij}^k, \forall j, m \in C, \forall i \in D_j, \forall l \in D_m, \tag{9}$$

$$\sum_{m \in C} \sum_{l \in D_m} q_{ij}^{lm} \leq \alpha \times a_{ij}^{lm} \sum_{k \in K} k \times x_{ij}^k, \forall j \in C, \forall i \in D_j, \tag{10}$$

$$\sum_{j \in C} \sum_{i \in D_j} \sum_{k \in K} \epsilon_{ij}^k(x) \times x_{ij}^k = E + \sum_{j \in C} he_j - \sum_{j \in C} fe_j, \tag{11}$$

$$\sum_{i \in CB_j} \sum_{k \in K} \epsilon_{ij}^k(y) \times y_{ij}^k - fe_j \geq 0, \quad \forall j \in C, \tag{12}$$

$$\sum_{i \in D_j \backslash B_j} \sum_{k \in K} \epsilon_{ij}^k(z) \times z_{ij}^k - he_j \geq 0, \quad \forall j \in C, \tag{13}$$

$$he_j, fe_j, q_{ij}^{lm} \geq 0, \text{ integer}, \tag{14}$$

$$x_{ij}^k, y_{ij}^k, z_{ij}^k \in \{0, 1\} . \tag{15}$$

The objective function, which is given by equation (1), minimizes the total cost incurred with the restructure of the existing network of facilities. This cost is made up four main components: facility costs (operating f, closing g, and opening h costs); employee costs (hiring T_j, and firing CMP_j costs); service v_{ij}^{lm} costs; and costs incurred by not attaining the desired quality of service. This corresponds to penalty costs P_{lm}, since service quality targets are not seen as constraints, but rather as desirable goals, therefore the lack of service quality is understood as a cost. It should be noticed that the objective function is nonlinear and concave. And also that functions f_{ij}^k, g_{ij}^k, and h_{ij}^k, are non linearly dependent on size and location, see (Monteiro, 2005) for more details.

Constraints (2) and (3) ensure the operation of non closable facilities with the current size, while constraints (4) and (5) ensure that closed facilities are not operated unless they have been resized. We also must ensure that no more than one facility is opened at any potential location, equation (6) and if indeed it is opened then it must be operating, equation (7). Constraints (8) guarantee that client' needs are satisfied, while at the same time service is not wasted beyond service quality targets (set by the service provider).

Each facility has two kinds of capacity limits. On one hand, there is a limit on the service it can provide to a single client, which is given by equation (9), and on the other hand, it is also limited in the quantity of service it can overall provide, equation (10). Constraints (11), (12), and (13) set the boundaries for the number of employees needed, fired, and hired, respectively. Finally, constraints (14) and (15), state the integer and binary nature of the variables, respectively.

3 Solution Methodology

A The solution methodology proposed here consists of two phases. In the first phase we obtain an initial feasible solution from solving a related linear programming model. This solution is then improved by adapting the dynamic slope scaling procedure developed by (Kim & Pardalos, 1999). In the second phase, we use a local search procedure to improve upon this solution

3.1 A Dynamic Slope Scaling Based Procedure

Originally, the Dynamic Slope Scaling has being proposed for fixed charge network flow problems (Kim & Pardalos, 1999). The motivation of this approach is to find a linear factor that effectively reflects the variable costs and fixed costs simultaneously. Since then, many adaptations have been proposed to address other problems, see for example (Fontes, Hadjiconstantinou, & Christofides, 2003; Nahapetyan & Pardalos, 1982).

The linear programming model (P') is obtained from the original mathematical programming model (P) by disregarding the binary variables, as well as, all costs except for the service costs. The problem is then, given by:

$$\text{(P')} \qquad \min \quad \sum_{j\in C}\sum_{i\in D_j}\sum_{m\in C}\sum_{l\in D_m} \phi_{ij}^{lm} \tag{16}$$

subject to:

$$\sum_{j\in C}\sum_{i\in D_j} q_{ij}^{lm} \geq \underline{W}_{lm}, \quad \forall m \in C, \forall l \in D_m, \tag{17}$$

$$\sum_{j\in C}\sum_{i\in D_j} q_{ij}^{lm} \leq \overline{W}_{lm}, \quad \forall m \in C, \forall l \in D_m, \tag{18}$$

$$\sum_{m\in C}\sum_{l\in D_m} q_{ij}^{lm} \leq k_{\max} \times \alpha, \quad \forall j \in C, \forall i \in D_j \backslash NCB_j, \tag{19}$$

$$\sum_{m\in C}\sum_{l\in D_m} q_{ij}^{lm} \leq k_{ij}^{e} \times \alpha, \quad \forall j \in C, \forall i \in NCB_j, \tag{20}$$

$$q_{ij}^{lm} \leq k_{ij}^{e} \times a_{ij}^{lm}, \quad \forall j, m \in C, \forall l \in D_m, \forall i \in NCB_j, \tag{21}$$

$$q_{ij}^{lm} \leq k_{\max} \times a_{ij}^{lm}, \ \forall j, m \in C, \forall l \in D_m, \forall i \in D_j \backslash NCB_j, \tag{22}$$

$$q_{ij}^{lm} \geq 0 \quad integer\ . \tag{23}$$

It is clear that this initial solution, which is obtained by solving the LP problem (P') with a simple linear underestimation $\phi_{ij}^{lm} = q_{ij}^{lm} \times v_{ij}^{lm}$ of the original concave cost, provides an upper bound to the optimal solution once its cost is computed using the original cost function. Then, we iteratively solve the above LP model updating the cost function as follows:

$$(\phi_{ij}^{lm})^{T+1} = \begin{cases} (\overline{v}_{ij}^{lm})^{T} + \dfrac{h_{ij} + f_{ij}}{(q_{ij}^{lm})^{T} \times \varphi_{ij}^{T}}, & \text{if} \quad (q_{ij}^{lm})^{T} > 0, \\ (\overline{v}_{ij}^{lm})^{R}, & \text{otherwise}, \end{cases}$$

where φ_{ij}^{T} is the number of demand locations serviced by the facility located in district $i \in D_j$ of county $j \in C$ at iteration T and R is the index of the last iteration where $(q_{ij}^{lm})^{T} > 0$.

The update procedure stops either when we obtain the same solution in two consecutive iterations (meaning that no further improvements may be achieved) or when the pre-defined maximum number of iterations has been performed, whichever happens first.

3.2 The Local Search Procedure

Local search is perhaps the simplest among neighborhood search methods. It starts with a given initial solution as the current solution and then checks its neighborhood for a better solution and repeats the process. In case the neighborhood of the current solution does not contain any better solution, than the local search returns the current solution and terminates. This method does not guarantee globally optimal solutions to most combinatorial problems, but generally returns relatively good quality solutions. Of course, the effectiveness of the method depends on the neighborhood structure used. In this section we develop a local search on the FNRP using two neighborhood structures: the Drop neighborhood and the Swap neighborhood.

Drop neighborhood: In this type of neighborhood there is an attempt to decreases the number of opened facilities by attempting to drop facilities one at the time. We have defined 3 different ways of performing such a movement.

Step 1 - This step consists of dropping facilities that serve only one client, if no infeasibility results from this operation.

Step 2 - In step 2, we attempt to drop the remaining facilities servicing only one client, one at the time, by distributing the service units provided to their clients by other facilities still having available capacity.

Step 3 - In this step we try to eliminate facilities which are not using all service capacity, by redistributing their clients as in step 2.

Swap neighborhood: This neighborhood structure is defined using two kinds of swap moves. A swap move either removes a facility from one of the sites where it was located in the current solution and simultaneously opens a facility in a site that had none, or removes a facility from one of the sites where it was located in the current solution replacing it by another facility in the same site but with different size. This kind of move keeps the number of open facilities in the solution constant. Moves in this type of neighborhoods have been define in 3 different ways.

Step 4 - Here we analyze the possibility of downsizing for each facility that has remained opened. If the number of service units being given (to all the clients) beyond the minimum required is larger than the number of service units to be removed due to a downsize, then the facility is swapped by a smaller one.

Step 5 - In step 5 we attempt to downsize facilities by redistributing some of its service to other facilities with available capacity. It should be noticed that the difference between the operational costs is always larger than the difference between servicing costs, and in this case no service is lost.

Step 6 - In this step we try to swap facilities of different locations. The facility swap with the largest positive gain is selected to be performed. This step is repeated until no more cost improving swaps exist.

The procedure starts by dropping facilities in one of the locations on the instance, by using the steps defined above. It then enters a swap phase in which swap moves are executed until no more swap moves improve the solution. The drop and swap phases alternate until a local optimum is reached with respect to both add and swap moves.

4 Computational Experiments

The proposed heuristic has been implemented in Visual C++ 6.0. Computational experiments were carried out on a 1.8-GHz Pentium4 with 256 MB of RAM.

The mathematical programming model (P) given in Section 2 has been implemented and solved in CPLEX. The results obtained by the proposed heuristic are compared to the results obtained by CPLEX, since the later provides optimal solutions. Let $\overline{x}$ be the optimal solution value and x the heuristic solution value. Then the percentage optimality gap is given by,

$$Gap = \frac{(x - \overline{x})}{\overline{x}} \times 100.$$

In Table 1, we report on the variation of the number of employees E; the percentage ratio Q between service provided and service quality target; the number of operating facilities B; and the computational time required to solve the problem, in CPU seconds, both for CPLEX and for the Heuristic.

The results reported in each row of Table 1 are averages obtained from the fifteen problem instances considered, each of which having m districts and n counties.

Our work does not have a specific application behind it, as is for instance the case of Min and Melachrinoudis (2000; 1999). However, the intended application for our model is that of locating bank branches in an urban environment. For this reason, we have chosen to do the computational testing on problems whose parameters have been generated in accordance with this application setting. We have used data from the Portuguese Bank Association (2006) and decisions on parameters have been made by following other authors (Wang et al., 2003), whenever possible or otherwise by using the information collected from specialists, i.e. bank managers, accountants, real estate traders, amongst others.

Overall 165 problems have been solved and the average optimality gap has been found to be 4.07%, ranging from 0.42%, for a problem with 20 districts and 16 counties, to 12.84%, for a problem with 20 districts and 13 counties. The standard deviation is 3%.

Although the solution obtained by the heuristic is, usually, more expensive, it has more facilities operating. Therefore, it is better equipped to deal with market growth, since CPLEX solution would have to be changed even for a very small increase in service requirements.

In terms of computational time needed to solve the problems, on average the heuristic is faster,

Table 1: Comparison of the solutions provided by the CPLEX and the heuristic.

		CPLEX				Heuristic				
m	n	E	Q(%)	B	Time	E	Q(%)	B	Time	Gap(%)
15	13	-52	99	6	1	-45	98	7	1	5.24
20	13	-23	99	8	9	-16	98	8	1	6.33
25	16	-38	99	9	7	-30	99	10	3	4.99
30	20	-54	100	11	32	-46	99	12	2	3.64
35	23	-73	98	13	38	-63	100	14	3	2.81
40	27	-83	99	15	26	-72	99	16	4	3.56
45	37	-150	99	16	397	-138	100	18	5	2.42
50	27	-42	99	18	1949	-30	99	19	7	3.93
55	36	-104	99	20	688	-92	100	22	8	3.16
60	36	-89	99	22	1129	-75	99	23	10	3.51
65	16	55	97	25	1966	75	99	24	11	5.18
Aver.		**-59**	**99**	**15**	**568**	**-48**	**99**	**16**	**5**	**4.07**

even when the CPLEX is very quick. For large size problem instances the heuristic is much faster then CPLEX. As it can be seen from the results reported in Table 1, if medium to large size problems are to be solved the proposed heuristic is actually the only solution method that can be used.

We have also analyze the effectiveness of the steps in the developed local search heuristic. As it can be seen from Table 2 step 3 has never contributed for the decrease of the total cost, whereas step 1 and 6 always improve the current solution. In our study, we have concluded that step 2, 4 and 5 improve the current solution in about 50%, 83%, and 67% of the problem instances solved.

Table 2: Average cost improvement for each step, in % of the initial solution.

m	**Step 1**	**Step 2**	**Step 3**	**Step 4**	**Step 5**	**Step 6**
15	99.13	99.13	99.13	99.05	99.05	97.01
20	100.00	100.00	100.00	99.85	99.87	96.70
25	99.29	99.14	99.14	98.53	98.33	93.53
35	98.42	98.42	98.42	97.86	97.86	93.51
40	99.62	99.43	99.43	99.43	98.94	95.64
45	99.12	99.12	99.12	99.05	98.71	96.12

We have tested CPLEX in order to find out the largest problem that could be solved by it. CPLEX cannot solve problems with more than 65 to 70 districts, depending obviously on the number of counties. The problem size impact on the heuristic has also been tested, and it has been observed that it can solve problems up to 150 or 160 districts. Furthermore, the dimensionality problem we are faced with for the heuristic is due to the fact that we have chosen

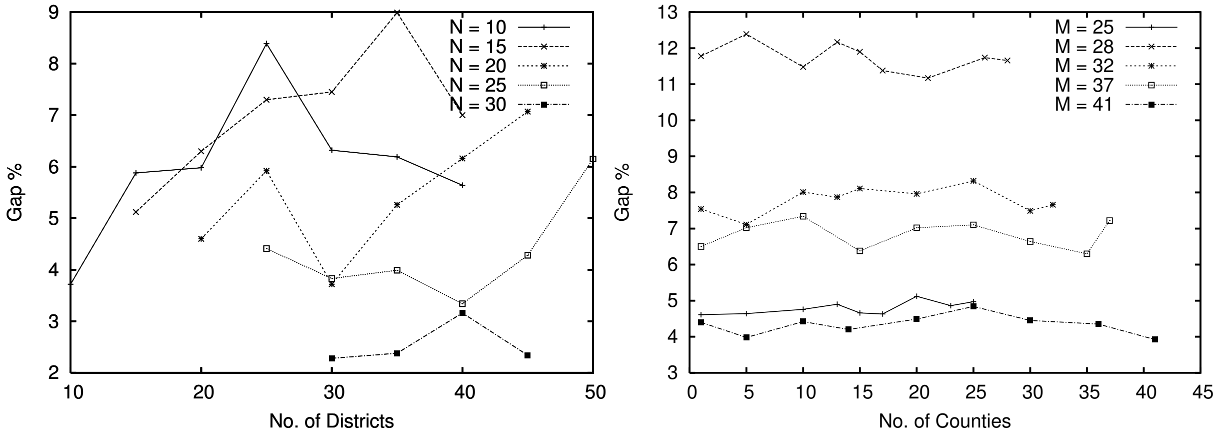

Figure 1: Solution quality versus number of districts and counties, respectively.

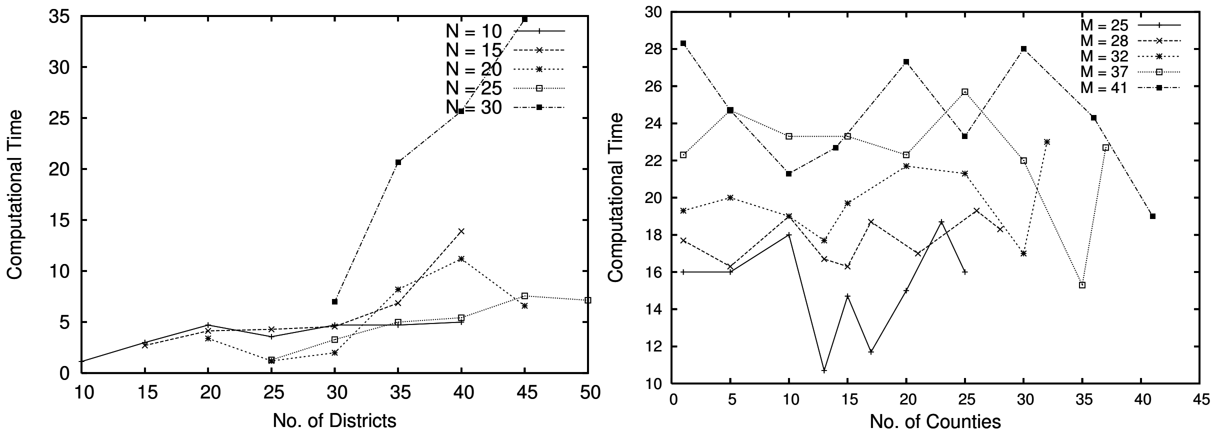

Figure 2: Average computational time versus number of districts and counties, respectively.

CPLEX to implement and solve the LP model (P')

We also did some computational testing to analyze the sensitivity of the solution to the number of districts and counties. As it can be seen in Figure 1 the increase in the number of districts, for a fixed number of counties, seems to increase the optimality gap, but the variation on the number of counties, for a fixed number of districts, seems to have no pattern at all. The average computational time needed to solve the problems is depicted in the graphs of Figure 2. As it can be seen, the computational time increases slightly with the number of districts. However, the increase in the number of counties seems not to affect the computational time.

In Figure 3, we have plotted the optimality gap and the computational time for different values of the distance standard. As it can be seen, the optimality gap decreases as the distance standard increases. This has to do with the possibility of choosing between many more facilities to provide the service. If we look closer, we can see that after a certain value the error seems to stabilize.

After a certain number of facilities is reached, the optimum set of facilities is identified, and the fact that more facilities can be added, will no longer have any influence in the results. An

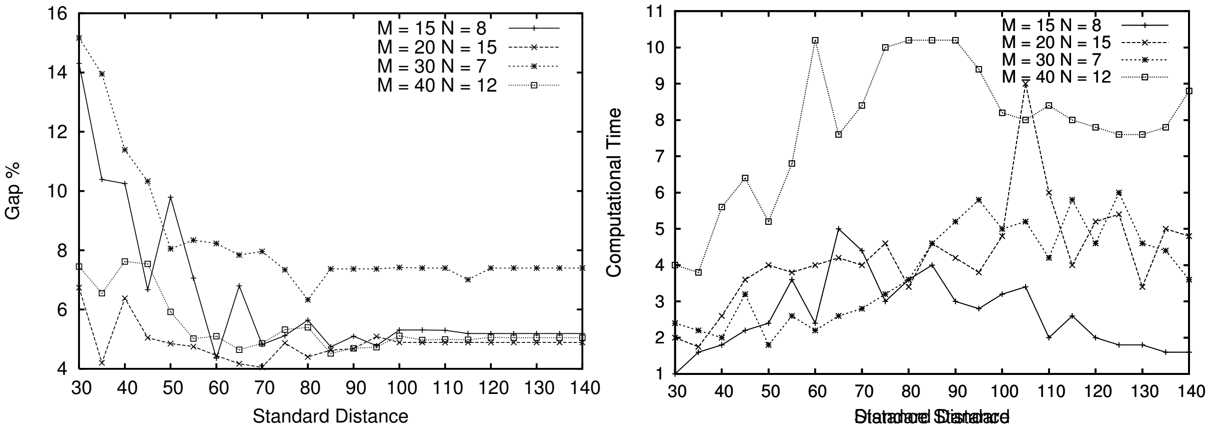

Figure 3: Solution quality and computational time versus Distance Standard, respectively.

increasing pattern, regarding the computational time can be observed, which is probably due to the larger number of branches that has to be tested in order to find the cheaper ones.

5 Conclusion

In this work, we propose a two stage heuristic. An initial good solution is obtained by a dynamic slope scaling procedure consisting on iteratively solving a linear programming model, obtained after relaxing the original problem, with updated cost functions. This procedure is a modified version of the dynamic slope scaling procedure developed by (Kim & Pardalos, 1999). The second stage consists of a local search algorithm using two neighborhood structures: the Drop neighborhood and the Swap neighborhood. The local search algorithm starts by dropping facilities in the locations on the instance and then it enters a swap phase in which swap moves are executed until no more swap moves improve the solution. The drop and swap phases alternate until a local optimum is reached with respect to both add and swap moves.

The computational experiments have shown that our heuristic is very fast and that for larger size problem instances it can be, on average, up to 100 times faster than CPLEX. Computational testing of these algorithms includes analysis of the sensitivity of the solution quality and computational time to the number of counties and districts and also to the standard distance. We have found out that the number of counties does not affect the solution quality, however regarding the variation of the number of districts and of the distance standard this is no longer the case. The computational time increases both with the number of districts and the distance standard, although it seems to be unaffected by varying the number of counties.

Regarding the steps used in the local search algorithm of the heuristic, we were able to conclude that: step 3 has never achieved any improvement; steps 1 and 6 improve the solution in 100% of the problem instances. And that, the other three steps although improving the solution could be restructured in order to increase their improvement rate. (Recall that steps 1 to 3 are from the drop neighborhood, while steps 4 to 6 are from the swap neighborhood).

The heuristic proposed is therefore, a good method of solving such problems. Although computational time grows quickly with problem size, this can be obviated by using a LP solver other than CPLEX. Due to the nature of the heuristic developed, it stops at the first local optimum that it reaches. In order to improve the solution quality further we intend to investigate its behavior when embedded in a genetic algorithm, tabu search or other method that gives it the opportunity to search for other local optima after reaching the first one.

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