ANIMAL SPIRITS AND THE COMPOSITION OF INNOVATION IN A LAB-EQUIPMENT R&D MODEL

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We revisit the issue of self-fulfilling "waves of enthusiasm" as stationary rational expectations equilibrium outcomes in endogenous-growth models that merge the quality-ladders with the expanding-variety mechanism. By considering a lab-equipment specification with vertical-innovation intertemporal spillovers but no intersectoral spillovers, we extend previous results of a negative impact of animal spirits on both horizontal aggregate R&D and number of firms to a framework where decreasing returns to horizontal entry are not a necessary condition. In contrast, our general-equilibrium setting allows us to predict an effect of animal spirits on R&D composition impacting neither on aggregate growth nor on aggregate vertical R&D, as reduced outlays in "mature" industries compensate for the increased R&D intensity in newly-born industries.

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1. Introduction

This paper studies the effect of animal spirits, or sunspots, on the composition of aggregate R&D, the industrial structure (number of firms and average firm size), consumption level and aggregate growth in an endogenous-growth model. The model merges the quality-ladders (vertical R&D) with the expanding variety (horizontal R&D) mechanism under a non-scale full lab-equipment specification without intersectoral spillovers. The model is shown to admit multiple deterministic stationary equilibria and is used to study the effect of animal spirits in a full endogenous-growth setting.

Since Azariadis (1981) and Cass and Shell (1983), among others, it is well-known that we can construct economic environments in which changes in economic agents' beliefs per se, if shared by everyone, influence current choices in such a way that the variations in beliefs are self-fulfilling, i.e., there are sunspot equilibria. More specifically, these are rational expectations equilibria that are perfectly correlated with stochastic factors that do not affect the preferences, endowment and production set of any individual; hence, only purely extrinsic uncertainty affects prices and allocations.

This paper is concerned with the existence of sunspot equilibria in a continuous-time endogenous-growth setting with multiple stationary equilibria, as highlighted by Cozzi (2005). We focus on the concept of Stationary Sunspot Equilibria (SSE),[1]built as a randomisation over multiple deterministic stationary equilibria, in the tradition of Cass and Shell (1983). Due to the existence of multiple equilibria, there exists a coordination problem between firms, which cannot be solved by referring solely to the fundamentals of the economic system. Following a common practice in the literature, we assume that firms tackle this coordination problem by referring to some extrinsic stochastic process, which "selects" a specific equilibrium over the others. Cozzi (2005) studies simultaneously the existence of multiple stationary deterministic equilibria and SSE, whereas we define the latter explicitly in line with Cass and Shell (1983), after the derivation of the multiple equilibria.

We adopt Cozzi (2005)'s approach in order to show that our lab-equipment model of vertical and horizontal R&D admits multiple deterministic stationary equilibria (balanced-growth paths, BGPs). Cozzi highlights the asymmetric BGPs admitted by R&D-driven endogenous growth models that merge the quality-ladders with the expanding variety mechanism, addressing the often observed waves of innovations characterized by a flood of quality improving R&D into newly introduced sectors. This is in accordance to some of the stylised facts on industry life cycle (e.g., Klepper, 1996).[2]

Cozzi assumes that, as soon as a new good is introduced, there will be a "wave of enthusiasm" for that sector, in the sense that the new product line attracts more vertical R&D than the older ones, thus implying a "supernormal" process of creative destruc-

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[1]If the extrinsic factors are subject to a stationary stochastic process, then the sunspot equilibria are called "stationary", that is, the effect of beliefs agents hold about their environment does not vanish asymptotically. As argued by, e.g., Azariadis and Guesnerie (1986), this property is relevant namely because stationary beliefs are likely to be the asymptotic outcome of many stable learning processes (e.g., Woodford, 1990; and, more recently, Evans, Honkapohja, and Marimon, 2007).

[2]For different references on this topic, see Cozzi (2005).

tion in the new sector. He also assumes that "enthusiasm" disappears after the first quality improvement, which implies that, along the BGP, horizontal innovation will be discouraged owing to an expected increase in the rate of creative destruction until the first quality jump. Therefore, the model predicts an asymmetric equilibrium (in fact, a continuum of asymmetric equilibria) with a larger vertical R&D intensity engaged in improving the quality of already improved products, and thus of aggregate vertical R&D intensity, at the expense of a lower horizontal R&D intensity.

Cozzi explores a version of Howitt (1999), who - similarly to e.g., Young (1998) and Dinopoulos and Thompson (1998) - presents a growth model that features the two types of R&D within a knowledge-driven specification. By focusing on the removal of scale effects of population, such models make the expanding-variety mechanism basically exogenous, i.e., predict a steady-state flow of new goods at the same rate as (or proportional to) exogenous population growth.

In contrast, we consider with a full lab-equipment specification, where the input to both R&D activities and to differentiated-goods production is measured in units of the homogeneous final good (e.g., Barro and Sala-i-Martin, 2004, ch. 6,7),[3]and which allows for a fully endogenous expanding-variety mechanism, such that the flow of new goods is independent of population growth. Moreover, we model the quality-ladders mechanism with intertemporal spillovers but no intersectoral spillovers, similarly to, e.g., Segerstrom and Zolnierek (1999) and Barro and Sala-i-Martin (2004, ch. 7), and in contrast to Howitt (1999) and Dinopoulos and Thompson (1998), among others.

Apart from a few studies (e.g., Rivera-Batiz and Romer, 1991; Acemoglu, 2002), the endogenous-growth literature has usually ignored the effect of the R&D specification on results. In the knowledge-driven R&D models, such as Howitt (1999), horizontal innovation competes away exogenously-driven scarce resources (labour) from manufacturing and vertical innovation activities. In our lab-equipment model, this effect is dampened as horizontal and vertical innovation re-inforce each other's impact on aggregate productivity, thus enlarging the pool of resources (the amount of final good) available to allocate as inputs to either manufacturing or R&D activities. On the other hand, models where vertical innovation exhibits no intersectoral spillovers, such as ours, display an adjustment mechanism that is absent from the models with intersectoral spillovers (such as Howitt, 1999), based on the response of the *relative average quality* of the differentiated goods to shocks that move the economy's steady-state equilibrium.

Consequently, we are able to extend Cozzi (2005)'s result of a negative impact of animal spirits on both horizontal aggregate R&D and the number of industries to a framework where decreasing returns to horizontal entry are not a necessary condition. In our model, the adjustment mechanism runs from the number of industries to the relative average quality and then to the level of horizontal R&D. Instead, in Cozzi (2005), it runs from horizontal R&D as characterised by decreasing marginal returns, to the number of industries. Similarly to Cozzi, we predict that the "waves of enthusiasm" have an effect

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[3]Using Rivera-Batiz and Romer (1991)'s terminology, the assumption that the final good is the R&D input means that one adopts the "lab-equipment" version of R&D, instead of the "knowledge-driven" specification, in which labour is the only input.

on the composition of R&D, but without impacting on aggregate vertical R&D: in our explicit general-equilibrium setting, reduced outlays in "mature" industries compensate for the increased R&D intensity in newly-born industries. Thus, we have an inter-R&D composition effect combined with an intra-(vertical)-R&D composition effect. Besides, the "waves of enthusiasm" have no impact on the aggregate growth rate but a positive impact on the level of per-capita consumption, which contrasts with the, respectively, positive and negative effect obtained by Cozzi.

The rest of the paper has the following structure. The next section briefly discusses the related literature on SSE. Section 3 outlines the symmetric equilibrium produced by the model of vertical and horizontal R&D within a full lab-equipment specification. Section 4 focus on the class of asymmetric general equilibria characterised by supernormal waves of creative destruction in the newly introduced intermediate goods. Section 5 is concerned with the existence of SSE obtained as a randomisation over the multiple deterministic equilibria. Section 6 gives the conclusion.

2. Related literature on SSE

Distinct forms of SSE have been explored by the literature, in the context of different classes of general-equilibrium rational-expectations models.

The seminal work by Cass and Shell (1983) describes SSE obtained as a lottery, or randomisation, over *multiple* deterministic equilibria within a discrete-time overlapping-generation model. Using the stationary transition probabilities (from sunspot to no-sunspot activity and vice versa) as weighting factors, SSE are derived as the convex combination of the multiple solutions to the deterministic optimisation problem. Azariadis and Guesnerie (1986) derive a sufficient condition for the existence of SSE around the *single* deterministic stationary equilibrium (or deterministic cycle) also within a discrete-time overlapping-generation model. The authors identify a subset of all stationary transition probability matrices for which SSE exist as the solution of a "stochastic deformation" of the deterministic optimisation problem. Woodford (1986) presents a method of constructing SSE generated by a stationary transition probability matrix in a discrete-time model that exhibits the *indeterminacy* of equilibrium near a steady state. Our paper is in line with the first approach, by studying a model with multiple stationary equilibria and constructing SSE as a randomisation over these equilibria. In this respect, the SSE we study have a simpler origin than the other forms of SSE extant in the literature.

Like us, more recently some authors have studied SSE in continuous-time infinitely-lived agents models of endogenous growth. Drugeon and Wigniolle (1996) develop an extension of Grossman and Helpman (1991), and establish sufficient conditions for the existence of a SSE around the single deterministic stationary equilibrium, which include restrictions from above on the stationary transition probabilities of a continuous-time Markov process. Nishimura and Shigoka (2006) show how to construct a SSE in multi-sector endogenous-growth models with local indeterminacy,[4] based on, e.g., Lucas (1988)

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[4]This paper extends the continuous-time analysis in Shigoka (1994).

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and Romer (1990).[5] Those papers exhibit a positive relationship between sunspots (optimistic expectations of firms) and the long-run aggregate growth (just like Cozzi, 2005), while in our model sunspots have no effect on long-run aggregate growth.

By focusing on the endogenous number of firms, we also relate to some papers in the business-cycle literature with endogenous firm entry and exit. Dos Santos Ferreira and Dufourt (2006) study a discrete-time real-business-cycle (RBC) model, while Chatterjee, Cooper, and Ravikumar (1993) present a discrete-time two-sector overlapping-generation model. In both models, there is Cournot competition and variations in the number of active firms are associated with the aggregate fluctuations due to sunspots, constructed as a randomisation between multiple deterministic steady states. Jaimovich (2007) studies an RBC model with firm entry and exit where the SSE are derived upon local indeterminacy of the deterministic steady state. Notably, in all these models, sunspots have a positive effect on the number of active firms, whereas in our growth model (similarly to Cozzi, 2005), sunspots have a negative effect on the number of firms.[6]

3. The benchmark model

In this section, we first present the full-endogenous growth model of quality ladders and expanding variety and then derive its symmetric equilibrium. This, in turn, will serve as a benchmark to the analysis carried out in Section 4, where "waves of enthusiasm" are considered.

We explore a dynamic general equilibrium model of a closed economy where there is a single competitively-produced final good that can be used in consumption, C, production of intermediate goods, X, and horizontal and vertical R&D activities, R_n and R_v, respectively. The final consumption good is produced by a (large) number of firms each using labour and a continuum of intermediate inputs indexed by ω on the interval $[0, N(t)]$.

The economy is populated by L identical dynastic families, each endowed with one unit of labour that is inelastically supplied to final-good firms. Thus, the total labour level is L, which, by assumption, is constant over time. In turn, families invest in firms' equity.

In the intermediate-good sector, firms can devote resources to R&D either to create a new product line (a new industry) or, within an existing industry ω, to improve the quality of its good. Quality is indexed by j, where higher values denote higher quality products. In particular, when a new quality rung is reached in ω, the jth innovator is the sole producer with the quality level $\lambda^{j(\omega)}$, where the parameter $\lambda > 1$ measures

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[5]Also within the endogenous-growth literature, Francois and Lloyd-Ellis (2003) study the effects of animal spirits on long-run growth by developing an extension of Grossman and Helpman (1991) where the realisation of innovations is separated in time from their implementation. However, in their model, expectations are deterministic.

[6]Note also that we study the particular class of SSE of order two, i.e. with two possible events or states of nature, as in Cass and Shell (1983), Azariadis and Guesnerie (1986), Drugeon and Wigniolle (1996), Nishimura and Shigoka (2006), among others, but in contrast to, e.g., Shigoka (1994) and Dos Santos Ferreira and Dufourt (2006).

the size of each quality upgrade. By improving on the current best quality index j, a successful R&D firm earns monopoly profits from selling the leading-edge $j(\omega)+1$ quality to final-good firms and, in equilibrium, lower qualities of ω are priced out of business. As each leader is driven out of business by further innovation supported by other firm, the duration of the monopoly is finite.

3.1. The consumer sector

The economy consists of L identical dynastic families who consume and collect income (dividends) from investments in financial assets (equity) and labour income. We assume consumers have perfect foresight concerning the aggregate rate of technological change over time,[7] and choose the path of final-good aggregate consumption $\{C(t), t \geq 0\}$ to maximise the discounted lifetime utility

$$U = \int_0^\infty \left(\frac{C(t)^{1-\Theta} - 1}{1 - \Theta} \right) e^{-\rho t} dt \tag{1}$$

where $\rho > 0$ is the subjective discount rate and $\Theta > 0$ is the constant elasticity of marginal utility with respect to consumption. Intertemporal utility is maximised subject to the flow budget constraint (henceforth, the dot denotes time derivative)

$$\dot{a}(t) = r(t)a(t) + w(t)L - C(t) \tag{2}$$

where a stands for households' financial assets (equity) holdings, measured in terms of final-good output Y. Households take the real rate of return on financial assets, r, and the real labour wage, w, as given. The initial level of wealth $a(0)$ is also given, whereas the condition $lim_{t\to\infty} e^{-\int_0^t r(s)ds} a(t) \geq 0$ is imposed in order to prevent Ponzi schemes. The optimal path of consumption satisfies the well-known differential Euler equation

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\Theta} \left(r(t) - \rho \right) \tag{3}$$

as well as the transversality condition $\underset{t\to\infty}{lim}\, e^{-\rho t} C(t)^{-\Theta} a(t) = 0$.

3.2. Production and price decisions

We consider that the final-good sector faces the following production function

$$Y(t) = L^{1-\alpha} \cdot \int_0^{N(t)} \left[\lambda^{j(\omega,t)} \cdot x(\omega, t) \right]^\alpha d\omega \tag{4}$$

where L is labour input; $(1-\alpha)$, $0 < \alpha < 1$, is the labour share in production; $x(\omega, t)$ is the amount used of the intermediate good ω, weighted by its quality level $\lambda^{j(\omega,t)}$, $\lambda > 1$.

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[7]As we will see below, the uncertainty associated with R&D at the industry level creates jumpiness in microeconomic outcomes. However, as the probabilities of successful R&D across industries are independent and there is a continuum of industries this jumpiness is not transmitted to macroeconomic variables.

It is implicit in (4) that only the highest grade of each $\omega \in [0, N(t)]$ are actually produced and used in equilibrium, meaning $x(j, \omega, t) = x(\omega, t)$; thus, $N(t) > 0$ is the measure of how many different intermediate goods (i.e., product lines) ω exist at time t.

Letting final output be the numeraire (that is, setting its price equal to unity), firms in the final-good sector seeks to maximise profit by choice of L and $x(\omega), \omega \in [0, N(t)]$. From the first-order condition with respect to x one derives the aggregate demand of ω

$$x(\omega, t) = L \cdot \left(\frac{\lambda^{j(\omega,t)\alpha} \cdot \alpha}{p(\omega, t)} \right)^{\frac{1}{1-\alpha}} \tag{5}$$

where $p(\omega, t)$ is the price of ω relative to the final-good price.

The intermediate good is nondurable and entails a unit marginal cost of production, measured in terms of final-good output Y. Since there is a continuum of intermediate goods, one can assume that firms are atomistic and take as given the price of final output (numeraire). Monopolistic competition, therefore, prevails and firms face isoelastic demand curves (5). The obtained intermediate-good profit maximising price is a constant markup over marginal costs $p(\omega, t) \equiv p = \frac{1}{\alpha},^8$ which implies the aggregate quantity produced of ω

$$x(\omega, t) = L \cdot \left(\lambda^{j(\omega,t)\alpha} \cdot \alpha^2 \right)^{\frac{1}{1-\alpha}} \tag{6}$$

Using the results above we get the profit accrued by the monopolist in ω

$$\pi(\omega, t) = \bar{\pi} \cdot L \cdot \lambda^{j(\omega,t)\left(\frac{\alpha}{1-\alpha}\right)} \tag{7}$$

where $\bar{\pi} \equiv \left(\frac{1-\alpha}{\alpha} \right) \cdot \alpha^{\frac{2}{1-\alpha}}$ and $\lambda^{\frac{\alpha}{1-\alpha}} > 1$.

Substituting (6) in (4) yields the aggregate output

$$Y(t) = \alpha^{\frac{2\alpha}{1-\alpha}} \cdot L \cdot Q(t) \tag{8}$$

where

$$Q(t) = \int_0^{N(t)} \lambda^{j(\omega,t)\left(\frac{\alpha}{1-\alpha}\right)} d\omega \tag{9}$$

is the intermediate-input aggregate quality index, which can also be interpreted as the *technological-knowledge stock* of the economy, since, by assumption, there are no intersectoral spillovers. Total resources devoted to intermediate input production at t are also proportional to $Q(t)$

$$X(t) = \int_0^{N(t)} x(\omega, t) d\omega = \alpha^{\frac{2}{1-\alpha}} \cdot L \cdot Q(t) \tag{10}$$

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[8]We assume that $\frac{1}{\alpha} < \lambda \Leftrightarrow \frac{1}{\alpha\lambda} < 1$, that is, if $\frac{1}{\alpha}$ is the price of the leading-edge good, the price of the next lowest grade, $\frac{1}{\alpha\lambda}$, is less than the unit marginal cost of production. Only in this case are the lower grades of ω unable to provide any effective competition for the leading-edge type, so that its producer can charge the unconstrained monopoly price.

as are total profits

$$\Pi(t) = \int_0^{N(t)} \pi(\omega, t) d\omega = \bar{\pi} \cdot L \cdot Q(t) \tag{11}$$

3.3. R&D decisions

As in the standard model of quality ladders, firms decide over their optimal vertical-R&D level, which constitutes the search for new designs (blueprints) that lead to a higher quality of existing intermediate goods. Each new design is granted a patent, meaning that a successful researcher retains exclusive rights over the use of his/her improved intermediate good. In each industry only (potential) entrants can do R&D and innovation arrival follows a Poisson process.[9]

Let $I_i(j, \omega, t)$ denote the instantaneous probability of R&D success by potential entrant i in industry ω when the highest quality is j (I is also interpreted as the vertical innovation rate). This probability is independently distributed across firms, industries and over time, and depends on the flow of resources $R_{vi}(j, \omega, t)$ devoted to vertical R&D by entrants in each ω at t, measured in units of the final good. We assume perfect competition among entrants and that each entrant's instantaneous probability of R&D success is given by a relation exhibiting constant returns in R&D expenditures, $I_i(j, \omega, t) = R_{vi}(j, \omega, t) \cdot \Phi(j, \omega, t)$, where the function Φ is the same for every firm in ω and captures the effect of the current technological-knowledge position j (e.g., Barro and Sala-i-Martin, 2004, ch. 7). Now, let

$$\Phi(j, \omega, t) = \frac{1}{\zeta \cdot L} \cdot \lambda^{-(j(\omega,t)+1)\left(\frac{\alpha}{1-\alpha}\right)} \tag{12}$$

where $\zeta > 0$ is a constant that stands for the (flow) fixed vertical-R&D cost. In order to eschew the usual scale-effect in endogenous growth models associated to the size of the labour force, in (12) we assume that an increase in market scale, measured as L, dilutes the effect of R&D outlays on innovation probability. Observe also that the R&D technology exhibits intertemporal spillovers but no intersectoral spillovers (e.g., Segerstrom and Zolnierek, 1999), in contrast with, e.g., Howitt (1999) and Dinopoulos and Thompson (1998). By aggregating across firms in ω, we get $R_v(j, \omega, t) = \sum_i R_{vi}(j, \omega, t)$ and $I(j, \omega, t) = \sum_i I_i(j, \omega, t)$, such that

$$I(j, \omega, t) = R_v(j, \omega, t) \cdot \frac{1}{\zeta \cdot L} \cdot \lambda^{-(j(\omega,t)+1)\left(\frac{\alpha}{1-\alpha}\right)} \tag{13}$$

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[9]Zero equilibrium R&D by incumbents is a well-known result claimed by the traditional quality-ladders models (e.g., Aghion and Howitt, 1992). However, as shown by Cozzi (2007), the assumption of R&D firms (potential entrants and the incumbent) operating under perfect competition and constant returns at the firm level, taken rigorously, yields an indeterminate investment for the incumbent, which is consistent with the latter doing any amount of R&D, from zero to a very large number. Our assumption of zero equilibrium R&D by incumbents is only for the sake of simplicity in what regards the microstructure of our model.

As the terminal date of each monopoly arrives with probability $I(j, \omega, t)$ per (infinitesimal) increment of time, the present value of a monopolist's profits is a random variable. Let $V(j, \omega, t)$ denote the expected value of a successful R&D firm,[10] such that $V(j, \omega, t) = \int_t^\infty \pi(j, \omega, t) e^{-\int_t^s (r(v) + I(j, \omega, v)) dv} ds$, where r is the equilibrium market real interest rate and $\pi(j, \omega, t)$ is given by (7). Along the BGP, r and I are constant; hence, we can further write

$$V(j) = \frac{\pi(j)}{r + I(j)} \tag{14}$$

On the other hand, with free-entry into the vertical R&D business, we have the free-entry condition

$$I(j) \cdot V(j+1) = R_v(j) \tag{15}$$

By substituting (14) into (15) and using (7) and (13) to simplify, we get

$$r = \frac{\bar{\pi}}{\zeta} - I \tag{16}$$

According to (16), the relationship between r and I is independent of t, ω, and j. Along the BGP, where we expect I to be constant, r is also constant.

Variety expansion results from R&D aimed at creating a new intermediate-good line, corresponding to a new firm, at a cost of η units of final output. In particular, we view the creation of new product lines as a product development activity without positive spillovers and allow for entry as well as exit of product lines from the market. After a new product is launched, an initial quality level is observed, drawn at random from the distribution of quality indexes matching the existing product lines (e.g., Dinopoulos and Thompson, 1998; Howitt, 1999). Let $q(j, \omega, t) \equiv \lambda^{j(\omega, t)\left(\frac{\alpha}{1-\alpha}\right)}$ be an alternative measure of product quality. Then, from (9), we have

$$Q(t) = \int_0^{N(t)} q(j, \omega, t) d\omega = q(j, \bar{\omega}, t) \cdot N(t) \tag{17}$$

where $q(j, \bar{\omega}, t) \equiv E_\omega(q)$ is the average of q over industries and $\bar{\omega}$ denotes the average intermediate-good sector for a given $N(t)$.

We assume perfect competition among R&D firms and *static* constant returns combined with *dynamic* decreasing returns to scale to horizontal R&D. That is, $\dot{N}_e(t) = \frac{1}{\eta} R_{ne}(t)$, where $\dot{N}_e(t)$ is the contribution to the instantaneous flow of new product line by R&D firm e at a unit cost of η and $R_{ne}(t)$ is the flow of resources devoted to horizontal R&D by e at t. Cost η is the same for every firm doing horizontal R&D, with $\eta \equiv \eta(N) = N^\sigma$, $\sigma > 0$ (e.g., Barro and Sala-i-Martin, 2004, ch. 6).[11] By aggregating across firms, we have $R_n = \sum_e R_{ne}$ and $\dot{N}(t) = \sum_e \dot{N}_e(t)$, which implies

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[10] We assume that entrants are risk-neutral and, thus, only care about the expected value of the firm.

[11] The positive dependence of η on N is necessary to eschew the explosive growth that would occur in our model, e.g., if η were constant over time. This is not the case in Barro and Sala-i-Martin (2004, ch. 6)'s basic model of pure expanding variety.

$$R_n(t) = \eta \dot{N}(t) \tag{18}$$

Since entry generates value $V(q(j,\bar{\omega},t)) \equiv V(j,\bar{\omega},t)$, a free-entry equilibrium requires that new product lines are created (or destroyed) at a rate $\dot{N}$ necessary to ensure the free-entry condition $\dot{N}(t) \cdot V(j,\bar{\omega},t) = R_n(t)$, which simplifies to

$$V(j,\bar{\omega},t) = \eta(N) \tag{19}$$

Finally, a consistency condition between vertical and horizontal arbitrage conditions is needed. Having in mind that horizontal entry occurs at the average product quality, $q(j,\bar{\omega},t)$, firstwe find an expression for $R_v(j-1,\bar{\omega},t)$, by solving (13) in order to R_v, considering the average intermediate-good sector, $\bar{\omega}$, and applying to $j-1$, for a given $N(t)$,

$$R_v(j-1,\bar{\omega},t) = I(t) \cdot \zeta \cdot L \cdot q(j,\bar{\omega},t) \tag{20}$$

where we used $I(t) \equiv I(j-1,\bar{\omega},t)$. Then, from the vertical free-entry condition, (15), solved in order to V, we get $V(j,\bar{\omega},t) = \frac{R_v(j-1,\bar{\omega},t)}{I(j-1,\bar{\omega},t)}$. Together with (20), we have

$$V(j,\bar{\omega},t) = \zeta \cdot L \cdot q(j,\bar{\omega},t) \tag{21}$$

Last, equating (21) and the horizontal free-entry condition, (19), together with (17), yields

$$q(j,\bar{\omega},t) = \frac{Q(t)}{N(t)} = \frac{\eta(N)}{\zeta \cdot L} \tag{22}$$

3.4. The symmetric general-equilibrium BGP

The dynamic general equilibrium is defined by the paths of $\{N(t), C(t), Q(t), I(t), r(t), t \geq 0\}$, such that: (i) consumers, final-good firms and intermediate-good firms solve their problems; (ii) consistency conditions are met; and (iii) markets clear.

The final-product market equilibrium condition is

$$Y(t) = C(t) + X(t) + R_v(t) + R_n(t) \tag{23}$$

which defines the aggregate resource constraint. At the aggregate level, the households' equilibrium condition (balance sheet) is

$$a(t) = V(j,\bar{\omega},t) \cdot N(t) = \eta(t) \cdot N(t) \tag{24}$$

which we can prove, by substituting in (2), that is equivalent to (23) (see Gil, Brito, and Afonso, 2008).

We now derive and characterise the interior BGP. Let $g_y \equiv \frac{\dot{y}}{y}$, the growth rate of y. Along the BGP, the aggregate resource constraint (23)is satisfied with Y, X, C, R_v and R_n growing at the same constant rate. By considering (8) and by time-differentiating

(22) with $\eta(N) = N^\sigma$, the following necessary conditions for the existence of a BGP are derived: (i) $g_C = g_Q = g$; (ii) $g_I = 0$; and (iii) $\frac{g_Q}{g_N} = (\sigma + 1)$, $g_N \neq 0$. Observe that g is the long-run aggregate growth rate and that g_Q and g_N are monotonically related.

By assuming that the number of sectors, N, is large enough to treat Q as time-differentiable and the time interval dt is small enough to have $\dot{Q}$ non-stochastic, we time-differentiate (17) in order to get $\dot{Q}(t) = \int_0^{N(t)} \dot{q}(j, \omega, t) d\omega + q(N, t) \dot{N}(t)$. After some algebraic manipulation of the latter, we can write

$$g_Q = I \cdot \left(\lambda^{\frac{\alpha}{1-\alpha}} - 1 \right) + g_N \tag{25}$$

Next, solve (3) with respect to r and note that, along the BGP, $g_C = g$, to get $r = \rho + \Theta g$. The latter, combined with $g = (1 + \sigma) \cdot g_N$, (25) and (16), and solved with respect to g, yields

$$g = \frac{1}{\Theta} \left(\frac{\bar{\pi}}{\zeta} - \rho \right) \frac{\left(\lambda^{\frac{\alpha}{1-\alpha}} - 1 \right) \cdot (\sigma + 1)}{\left(\lambda^{\frac{\alpha}{1-\alpha}} - 1 \right) \cdot (\sigma + 1) + \frac{1}{\Theta} \sigma} \tag{26}$$

Observe that $\lim_{\sigma \to \infty} g = g_{no-entry}$ and that $g > 0$ requires $\mu > 0$. Since, from (3), $g = g_C = \frac{1}{\theta} (r - \rho)$, then $r > \rho$ must occur; this condition also guarantees $g_N > 0$.[12] Thus, under a sufficiently productive technology, our model predicts a BGP with constant positive g and g_N, where the former exceeds the latter by an amount corresponding to the growth of intermediate-good quality, driven by vertical innovation; to verify this, just check (25) and solve to get $\frac{\dot{Q}}{Q} - \frac{\dot{N}}{N} = I \cdot \left(\lambda^{\frac{1-\alpha}{\alpha}} - 1 \right)$, which is positive if $I > 0$. This implies that the consumption growth rate equals the growth rate of the number of varieties plus the growth rate of intermediate-good quality, in line with the view that industrial growth proceeds both along an intensive and an extensive margin. A similar result can be found, e.g., in Arnold (1998), Peretto (1998) and Howitt (1999).[13]

But differently from Dinopoulos and Thompson (1998), Howitt (1999) and other quality-ladders models with expanding variety, g_N is not linked to the (exogenous) population growth rate.[14] The dynamic decreasing returns due to $\eta(N) = N^\sigma$, per se, determine a constant N along BGP (see Barro and Sala-i-Martin, 2004, ch. 6); however, the N expansion is sustained by technological-knowledge accumulation, as the expected

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[12]Also, having in mind (24) and (22), we re-write the transversality condition as

$$\lim_{t \to \infty} e^{-\rho t} C(t)^{-\theta} \zeta \cdot L \cdot Q(t) = \lim_{t \to \infty} e^{-\rho t} \left(\frac{C(t)}{Q(t)} \right)^{-\theta} \zeta \cdot L \cdot \left(\hat{Q} e^{gt} \right)^{1-\theta} = 0 \tag{27}$$

where $Q = \hat{Q} e^{gt}$ and $\hat{Q}$ denotes detrended Q. Thus, the transversality condition implies $\rho > (1-\theta)g$; i.e., $r > g$, since $g = \frac{1}{\theta}(r - \rho)$. This condition also guarantees that attainable utility is bounded, i.e., the integral (1) converges to infinity.

[13]In Peretto (1998)'s endogenous growth model with cost-reducing R&D, the intensive margin is due to productivity growth, whilst in Arnold (1998) reflects human-capital accumulation.

[14]The link between the expanding variety and the exogenous population growth can also be found in the class of endogenous-growth model where incumbents do in-house cost-reducing R&D, while entrants bring new products to the market, such as Peretto and Connolly (2007).

growth of intermediate-good quality due to vertical R&D makes it attractive, in terms of intertemporal profits, for potential entrants to always put up an entry cost, in spite of its increase with N. In this sense, it is not necessarily the larger economy, measured by population size, that produces the greater number of varieties, but that with the larger technological-knowledge stock, which thus emerges as the relevant endogenous economic size measure.

4. The model with asymmetric equilibria

We now adopt Cozzi (2005)'s approach in order to show that the lab-equipment model of vertical and horizontal R&D admits (at least) a continuum of asymmetric stationary deterministic equilibria, besides the symmetric equilibrium derived in the previous section.

4.1. R&D decisions

We focus on the class of asymmetric general equilibria characterised by supernormal waves of creative destruction in the newly introduced intermediate goods, and which satisfy the following assumption:

Assumption 1. Consider $\theta \in (1, \infty)$, constant across industries and over time. When a new intermediate good ω is introduced, vertical R&D will be θ times higher in industry ω than in the product lines that have experienced at least one quality jump ("mature" industries). After the next quality jump occurs in ω, its vertical R&D level will become equal to the R&D carried out in the other "mature" industries.

This assumption is in line with some stylised facts on industry life cycle (e.g., Klepper, 1996), according to which frequently new industries are initially and quickly developed by new entrants. Moreover, it implies that the BGP expected value of the first monopolist in a newly introduced product line is

$$V^E(j) = \frac{\pi(j)}{r + \theta I(j)} = \Omega \cdot V(j) \tag{28}$$

where $V(j) = \frac{\pi(j)}{r + I(j)}$, π is given by (7) and $\Omega \equiv \left[1 - \frac{(\theta-1)I(j)}{r + \theta I(j)} \right] \in (0,1), \forall r, I, > 0, \theta \in (1, \infty)$ captures the negative effect of increased creative destruction on the value of the monopolist firm (i.e., $V^E < V$, due to $\theta > 1$). Provided that I and r are constant along the BGP, Ω is also constant. This implies that the horizontal R&D free-entry condition can be rewritten as

$$\eta(N) = \Omega \cdot V(j) \tag{29}$$

Now we turn to vertical innovation under Assumption 1. Let N^I denote the number of industries that have not experienced the first quality jump. Since all new industries start without having innovated in the vertical direction, N^I increases as far as N increases;

however, at any t, there is also a number $\theta \cdot I \cdot N^I$ that innovates and leaves the group of N^I. Thus, $\dot{N^I} = -\theta \cdot I \cdot N^I + \dot{N}$, from which, with $\frac{N^I}{N}$ constant in BGP, we have

$$\frac{N^I}{N} = \frac{g_N}{g_N + \theta I} \tag{30}$$

Under Assumption 1, a general BGP equilibrium with "waves of enthusiasm" requires that financial markets recognise the "enthusiasm" about the newly-introduced industries and thus are willing to channel savings to aggregate vertical innovation according to the arbitrage condition

$$r = \frac{\pi}{V} - A \cdot I \tag{31}$$

where $A \equiv \left[1 + (\theta - 1)\frac{g_N}{g_N + \theta I} \right] \in (1, \infty), \forall g_N, I, > 0, \theta \in (1, \infty)$. This implies that the average Poisson rate of vertical innovation at the aggregate level, $A \cdot I$, exceeds the average Poisson rate in the "mature" industries, I. Thus, investors require that the real interest rate, r, equals the dividend rate, $\frac{\pi}{V}$, plus the rate of capital gain $-A \cdot I$. The latter term incorporates the fact that all industries are undertaking vertical R&D every period but some are still waiting for their first innovation in the vertical direction. Provided that I and g_N are constant along the BGP, A is also constant. Substituting (15) solved in order to V, together with (7) and (13), in (31), yields

$$r + A \cdot I = \frac{\bar{\pi}}{\zeta} \tag{32}$$

Finally, given the free-entry conditions (15) and (47), we rewrite the consistency condition (22) as

$$\frac{Q(t)}{N(t)} = \frac{\eta(N)}{\zeta \cdot L} \cdot \Omega^{-1} \tag{33}$$

This is our model's version of arbitrage equation (H´) in Cozzi (2005) (itself a generalisation of equation (H) in Howitt, 1999).[15]Having in mind that $0 < \Omega < 1$, it is clear from (33) that "waves of enthusiasm" imply a smaller N vis-à-vis the symmetric equilibrium implicit in (22): a lower N enhances average quality, $\frac{Q}{N}$, for a given Q (i.e., relative average quality $\frac{1}{N}$) received by a newly-born industry, in order to compensate for the higher subsequent creative destruction rate ($\Omega^{-1} > 1$) due to $\theta > 1$. On the right-hand side of (33), we get $\Omega^{-1} \rightarrow 1$ with $\theta \rightarrow 1$, and $\Omega^{-1} \rightarrow +\infty$ with $\theta \rightarrow +\infty$. In the latter case, N must approach zero in order to elevate relative average quality to infinity.

Notice that the described mechanism: (i) stems from our assumption of no intersectoral spillovers in vertical innovation, in as much as the latter implies that the BGP relative average quality is given by $\frac{1}{N}$ (see (17)); (ii) is only weakened by $\eta \equiv \eta(N)$, $\eta' > 0$, in (33); (iii) does not depend qualitatively on the presence of *static* decreasing returns to horizontal R&D.

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[15]To see this, consider Cozzi's arbitrage equation with "waves of enthusiasm" as a certain event, i.e., $p_E = 1$.

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In contrast, Howitt (1999)'s model features both intersectoral spillovers in vertical innovation and *static* decreasing returns to horizontal R&D; the former assumption implies that relative average quality is independent of the number of industries in BGP (see Howitt's equation (13)), and thus the latter assumption is necessary to ensure that the consistency condition between vertical and horizontal innovation has a finite and determined solution (see Howitt's equation (H)). "Waves of enthusiasm", as analysed by Cozzi (2005), imply a lower number of industries than in the symmetric equilibrium in Howitt (1999)'s model, too, but the mechanism runs from horizontal R&D - which decreases as a direct effect of decreasing marginal returns to horizontal R&D in Cozzi's equation (H´) - to the number of industries.[16]

4.2. The asymmetric general equilibrium BGP

In order to derive the asymmetric equilibrium characterised by "waves of enthusiasm" in an explicit general-equilibrium setting, we must take into account that the final-product market equilibrium condition is now given by

$$Y(t) = X(t) + C(t) + R_n(t) + A \cdot R_v(t) \tag{34}$$

where $A \cdot R_v$ is the aggregate vertical R&D and R_v is the vertical R&D conducted by the "mature" industries. At the aggregate level, the households' balance sheet, equity being taken at its market value, is now

$$a(t) = V(j, \bar{\omega}, t) \cdot N(t) = \eta(t) \cdot \Omega^{-1} \cdot N(t) \tag{35}$$

while the households' flow budget constraint is

$$\dot{a}(t) = r(t)a(t) + w(t)L - C(t) + \Lambda(t) \tag{36}$$

where Λ is a real pure profit. We can prove, by substituting (35) in (36), that the former is equivalent to (34) (see Appendix A).

The necessary conditions for the existence of a BGP continue to be those presented in Subsection 3.4.

Again, by time-differentiating (17), and by taking into account that the asymmetric BGP is characterised by (30), we can write (see (25))

$$g_Q = A \cdot I \cdot \left(\lambda^{\frac{\alpha}{1-\alpha}} - 1 \right) + g_N \tag{37}$$

which is our version of vertical innovation equation (14) in Cozzi (2005).[17] Then, solve (3) with respect to r, to get $r = \rho + \Theta g$. The latter, combined with (32) and (37), yields

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[16]We can easily add static decreasing returns to horizontal R&D to our model, as in Howitt (1999) and Cozzi (2005), such that $\dot{N} = \frac{1}{\eta}\phi(R_n)$, $\phi' > 0$, $\phi'' < 0$, and $\frac{Q(t)}{N(t)} = \frac{\eta(\cdot)}{\zeta \cdot \phi'(R_n)} \cdot \Omega_0^{-1}$. In this case, the impact of $\Omega_0^{-1} > 1$ on the right-hand side of (33) will be matched by a decrease in both N and R_n. See Appendix B for details.

[17]To see this, use $p_E = 1$ in Cozzi's equation.

$$g = \left(\frac{\sigma+1}{\sigma}\right) \cdot A \cdot I \cdot \left(\lambda^{\frac{\alpha}{1-\alpha}} - 1\right) = \left(\frac{\sigma+1}{\sigma}\right) \cdot \left(\frac{\bar{\pi}}{\zeta} - \rho - \Theta g\right) \cdot \left(\lambda^{\frac{\alpha}{1-\alpha}} - 1\right) \quad (38)$$

By solving (38) with respect to g, we find

$$g = \frac{1}{\Theta}\left(\frac{\bar{\pi}}{\zeta} - \rho\right) \frac{\left(\lambda^{\frac{\alpha}{1-\alpha}} - 1\right) \cdot (\sigma+1)}{\left(\lambda^{\frac{\alpha}{1-\alpha}} - 1\right) \cdot (\sigma+1) + \frac{1}{\Theta}\sigma} \quad (39)$$

which is the BGP aggregate growth rate under asymmetric equilibrium satisfying Assumption 1. However, (26) is also the expression for the BGP aggregate growth rate under symmetric equilibrium (i.e., $\theta = 1 \Rightarrow A = \Omega = 1$), as found in (26). Given $g = (1+\sigma) \cdot g_N$, the same is true for g_N.

Now, let s denote a given variable value along the symmetric BGP, as derived for the model in Section 3. Then, from (39) and (32), we have, respectively

$$g = g^s \Leftrightarrow g_N = g_N^s \quad (40)$$

$$A \cdot I = \frac{\bar{\pi}}{\zeta} - \rho - \Theta g = I^s \Leftrightarrow I = \frac{I^s}{A} < I^s \quad (41)$$

plus, given (33) and our assumption of $\eta(N) = N^\sigma$, for a given Q,

$$N = (\Omega \cdot \zeta \cdot L \cdot Q)^{\frac{1}{\sigma+1}} < N^s \quad (42)$$

Also, by solving (33) with respect to η and substituting in (18), we get, for a given Q,

$$R_n = \eta \cdot \dot{N} = \Omega \cdot g_N \cdot \zeta \cdot L \cdot Q < R_n^s \quad (43)$$

and, by solving (13) with respect to R_v and aggregating across ω having in mind the asymmetric BGP is characterised by an aggregate rate of creative destruction $A \cdot I$,

$$A \cdot R_v = A \cdot I \cdot \zeta \cdot L \cdot \lambda^{\frac{\alpha}{1-\alpha}} \cdot Q = R_v^s \Leftrightarrow R_v = \frac{R_v^s}{A_0} < R_v^s \quad (44)$$

Finally, substitute (8), (10), (43) and (44) in (34), to get, for a given Q,

$$C = \left[\left(\alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}\right) - \Omega \cdot g_N \cdot \zeta - A \cdot I \cdot \zeta \cdot \lambda^{\frac{\alpha}{1-\alpha}}\right] \cdot L \cdot Q > C^s \quad (45)$$

Observe that, given Q and L, the latter can be interpreted as the productivity-adjusted level of per-capita consumption.

In Appendix C, we prove the existence of a finite and unique $A > 1$, given the set $(\theta; g_N^s, I^s)$, that solves for the asymmetric BGP (26)-(45). Moreover, because the number $\theta \in (1, \infty)$ is arbitrary, it can then be shown that there is a continuum of asymmetric BGPs (see Section 5, below, for more detail).

Thus, in our model, the "waves of enthusiasm" have no impact on aggregate vertical R&D intensity (that is, for a given Q, or a given Y - see (8)), as reduced outlays in

"mature" industries compensate for the increased R&D intensity in newly-born industries. This intra-R&D composition effect is "forced" by the fact that financial markets link the *effective* return to vertical R&D (the real interest rate plus the average rate of creative destruction) to the fundamentals, which must be the same whether we consider the symmetric or the asymmetric equilibrium.[18] The latter effect, together with the fact that, as a consequence of our lab-equipment specification, vertical innovation is the ultimate growth engine, in the sense that it sustains both variety expansion and aggregate growth, implies that the "waves of enthusiasm" have no impact on either the growth rate of the number of varieties or the aggregate growth rate. This result contrasts with the positive effect obtained by Cozzi (2005).

Our results coincide with Cozzi's in what concerns the negative impact of "waves of enthusiasm" on horizontal R&D intensity. However, as already alluded above with respect to the effect of "waves of enthusiasm" on N, the mechanism at work is subtly different in our case: it predicts a lower horizontal R&D in response to a lower N vis-à-vis the symmetric equilibrium, since the latter requires a smaller $\dot{N}$ to sustain a given BGP growth rate, g_N, independently of the type of returns to horizontal R&D we postulate.[19] Given the fact that, in our model, "waves of enthusiasm" have no impact on the productivity-adjusted level of per-capita final-good production, $\frac{Y}{QL}$ (see (8)), then the lower horizontal R&D intensity implies that more resources become available to per-capita consumption, also in productivity-adjusted terms, $\frac{C}{QL}$. This contrasts with the negative effect on the latter found by Cozzi.

5. Stationary sunspot equilibria

In the previous sections, we described a set of multiple stationary deterministic equilibria (BGPs) admitted by our model: one symmetric equilibrium and a continuum of asymmetric equilibria characterised by "waves of enthusiasm". This section is concerned with the existence of stationary equilibria in which all firms expect a "wave of enthusiasm" for the new product, such that "waves of enthusiasm" constitute self-fulfilling stationary rational expectations equilibrium outcomes.

Because of the existence of multiple equilibria, there exists a coordination problem between firms,[20] which cannot be solved by referring solely to the fundamentals of the economic system. Following a common practice in the literature, we explicitly assume that firms tackle this coordination problem by referring to some extrinsic stochastic process, as described below:

Assumption 2. At each t, all R&D firms observe an exogenous variable $z(t) \in \{0, 1\}$ and establish a link between this observation and the existence of a "wave of enthusi-

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[18]Notice term $\frac{\bar{\pi}}{\zeta}$ in (32) and (16).

[19]Observe that the assumption of $\eta \equiv \eta(N), \eta' > 0$, has no qualitative effects on the described mechanism.

[20]As noted by Cozzi (2005), since after the second quality jump all sectors experience the same degree of vertical innovation, before that jump R&D firms are indifferent among sectors, which opens the door namely to an asymmetric allocation of R&D.

asm" towards a newly created intermediate good ω at t: "no wave of enthusiasm" ($z(t) = 0$) or "wave of enthusiasm" ($z(t) = 1$). Consider $p \in (0, 1)$, constant across industries and over t. With probability p, $z(t) = 1$, i.e., vertical R&D will be $\theta > 1$ times higher in the new industry than in the "mature" industries. With probability $1 - p$, $z(t) = 0$, i.e., the new industry is immediately as R&D intensive as the "mature" industries.

Observe that, like in Cozzi (2005), the nature of extrinsic uncertainty is industry specific. As the probabilities of "wave of enthusiasm" across industries are independent and there is a continuum of industries, uncertainty is not transmitted to macroeconomic variables. Therefore, the SSE studied herein do not generate aggregate uncertainty.[21]

Having Assumptions 1 and 2 in mind, SSE are then built as a randomisation over the multiple deterministic stationary equilibria (e.g., Cass and Shell, 1983, Drugeon and Wigniolle, 1996 and Dos Santos Ferreira and Dufourt, 2006). First, consider the BGP expected value of the first monopolist in a newly introduced product line, which is now

$$V(j) = \frac{\pi(j)}{r + I} \qquad \text{with probability } 1 - p \tag{46a}$$

$$V^E(j) = \frac{\pi(j)}{r + \theta I} = \left[1 - \frac{(\theta - 1)I(j)}{r + \theta I(j)}\right] \cdot V(j) \quad \text{with probability } p \tag{46b}$$

where π is given by (7).

Next, recall from Subsection 3.3 that the contribution to the instantaneous flow of new goods by R&D firm e in the horizontal-R&D sector is $\dot{N}_e = \frac{1}{\eta(N)} R_{ne}$. Given perfect competition among innovator firms, each of them takes as given the marginal value of entry, V. Under Assumption 2, R&D firms solve the lottery[22]

$$R_{ne} = argmax \left[p \cdot \left(\dot{N}_e \cdot V^E - R_{ne}\right) + (1 - p) \cdot \left(\dot{N}_e \cdot V - R_{ne}\right)\right]$$

The single-valuedness and continuity of the latter, for a given p, is guaranteed by the strict concavity and continuity of the maximand with respect to R_{ne}. After aggregation, the associated first-order condition implies the horizontal R&D free-entry condition

$$\eta(N) = (1 - p) \cdot V(j) + p \cdot V^E(j) = \Omega_p \cdot V(j) \tag{47}$$

where $\Omega_p \equiv \left[1 - \frac{(\theta - 1)I}{r + \theta I} \cdot p\right] \in (0, 1), \forall r, I, > 0, p \in (0, 1), \theta > 1$ and $p = 1 \Rightarrow \Omega_p = \Omega$. By using Ω_p in (33), we have $\Omega_p^{-1} \to 1$ with $p \to 0 \vee \theta \to 1$, and $\Omega_p^{-1} \to (1 - p)^{-1}$ with $\theta \to +\infty$. Thus, in the latter case, with $p \in (0, 1)$, N need not approach zero and hence elevate relative average quality to infinity.

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[21]Thus, the deterministic nature of the households' optimisation program is not affected by the consideration of sunspots (see Subsection 3.1, above).

[22]Without sunspots, R&D firms solve $R_{ne} = argmax \left(\dot{N}_e \cdot V - R_{ne}\right)$, with the associated first-order condition implying (19). Given constant returns to scale, this is equivalent to consider directly the free-entry condition $\dot{N}_e \cdot V = R_{ne}$, as in Subsection 3.3.

On the other hand, the vertical-innovation arbitrage condition and the aggregate growth rate are now given by

$$r + A_p \cdot I = \frac{\bar{\pi}}{\zeta} \tag{48}$$

$$g_Q = A_p \cdot I \cdot \left(\lambda^{\frac{\alpha}{1-\alpha}} - 1 \right) + g_N \tag{49}$$

where $A_p \equiv \left[1 + (\theta - 1) \frac{g_N}{g_N + \theta I} \cdot p \right] \in (1, \infty), \forall g_N, I, > 0, p \in (0, 1), \theta > 1$ and $p = 1 \Rightarrow A_p = A$. Then, the combination of (3), (48) and (49) yields (39), while the BGP relationships (40)-(45) apply now with Ω and A replaced by Ω_p and A_p, respectively. In Appendix C, we prove the existence of a finite and unique $A_p > 1$, given the set $(\theta, p; g_N^s, I^s)$, that solves for the asymmetric BGP under Assumption 2.

Let the variables in the state without "waves of enthusiasm" be indexed by 0, whereas the ones in the state with "waves of enthusiasm" be indexed by 1. We define formally a SSE as follows:

Definition. For a given $\theta \in (1, \infty)$, a SSE is a quintuple $(\Omega_z, A_z, p)_{z \in \{0,1\}}$, with $\Omega_0 = A_0 = 1$, $\Omega_1 = \Omega_p \in (0, 1)$, $A_1 = A_p \in (1, \infty)$ and $p \in (0, 1)$, that yields the equality parts in (40)-(45), with Ω and A replaced by Ω_z and A_z, $z \in \{0, 1\}$, respectively.

Finally, a continuous monotonic relationship between the BGP values of the endogenous variables under Assumption 1 and 2 and, respectively, θ and p is needed, such that a continuum of SSE exists. By using the implicit function theorem to compute $\frac{dA_p}{d\theta}$ and $\frac{d\Omega_p}{d\theta}$, it is easy to verify that A_p and Ω_p relate monotonically with θ. In concrete, A_p *increases* monotonically with θ at a decreasing rate ($lim_{\theta \to \infty} \frac{dA_p}{d\theta} = 0$), whereas Ω_p *decreases* monotonically with θ also at a decreasing rate. With respect to the latter, we add that, on one hand, the direct negative effect of θ on Ω_p overweights the positive effect of θ through A_p on Ω_p and, on the other hand, the decreasing marginal direct effect of θ on Ω_p overweights the impact of $lim_{\theta \to \infty} \frac{dA_p}{d\theta} = 0$. Re-iterating the same steps as above, one can give a proof of the monotonic relationship between p and, respectively, A_p and Ω_p. Because sunspots are the random factors that, with probability p, choose one specific realisation from the underlying multiple deterministic equilibria, and because the numbers θ and p are arbitrary, then there is a continuum of SSE.

With these ingredients, we are able to rewrite Cozzi (2005)'s Proposition 1 as

Proposition 1. A continuum of BGPs exist parametrised by "wave of enthusiasm" probabilities $p \in (0, 1)$ and amplitudes $\theta \in (1, \infty)$. Comparing such BGPs, the *larger* p and θ, the *smaller* the number of firms, aggregate horizontal R&D and vertical R&D in "mature" industries, and the *larger* vertical R&D in newly-born industries and the level of per-capita consumption. Aggregate vertical R&D, the growth rate of the number of varieties and of intermediate-good quality, and thus the aggregate growth rate, do not depend on either p or θ.

In what follows, we briefly relate our main results to recent work on SSE. With respect to the impact of sunspots (optimistic expectations of firms) on the number of firms, our

growth model predicts a negative relationship similarly to Cozzi (2005). The opposite result obtains in the business-cycle models with endogenous firm entry by, e.g., Chatterjee, Cooper, and Ravikumar (1993), Dos Santos Ferreira and Dufourt (2006) and Jaimovich (2007). In our study, sunspots have no effect on long-run aggregate growth, in contrast to Cozzi (2005), but also to, e.g., the endogenous-growth models analysed in Drugeon and Wigniolle (1996) and Nishimura and Shigoka (2006). These papers all exhibit a positive relation between sunspots and the aggregate growth rate. On the other hand, the positive impact we find of sunspots on per-capita consumption is in line with, e.g., Chatterjee, Cooper, and Ravikumar (1993) and Dos Santos Ferreira and Dufourt (2006), but opposes to the negative effect described by Cozzi (2005).

6. Concluding remarks

In this paper, we revisit the issue of self-fulfilling "waves of enthusiasm" as stationary rational expectations equilibrium outcomes in endogenous-growth models that feature both the quality-ladders and the expanding-variety mechanism. For that purpose, we develop a model that merges the two mechanisms under a non-scale full lab-equipment specification without intersectoral spillovers.

The model predicts, under a sufficiently productive technology, a stationary BGP with constant positive growth rates, and where the consumption growth rate equals the growth rate of the number of varieties plus the growth rate of intermediate-good quality, in line with the general view that industrial growth proceeds both along an intensive and an extensive margin. Nevertheless, different from Dinopoulos and Thompson (1998) and Howitt (1999), among others, the growth of the number of varieties is not linked to the exogenous population growth rate. It is sustained by endogenous technological-knowledge accumulation, as the expected growth of intermediate-good quality makes it attractive for potential entrants to always put up an entry cost, in spite of its upward trend.

In line with Cozzi (2005), this paper focuses on asymmetric equilibria derived from self-fulfilling prophecies a la Cass and Shell (1983), instead of any asymmetry in market fundamentals, such as cost structures or technologies. It shows that the BGP composition of aggregate R&D, the industrial structure (number and average firm size) and the per-capita consumption level can be affected in a relevant manner by animal spirits.

However, by considering a vertical and horizontal R&D model in some aspects distinct from Howitt (1999), our results coincide only partially with those presented by Cozzi (2005). Both models predict a smaller number of firms in the asymmetric equilibrium with sunspots, in order to enhance the returns to horizontal entry, given subsequent higher obsolescence (creative destruction) rate. In our model, returns are increased because relative average quality is higher when the number of varieties is lower, whereas in Cozzi (2005), returns are increased because marginal returns to horizontal R&D are higher when R&D outlays are lower, which in turn imply a smaller number of industries in equilibrium.

More importantly, balanced growth dynamics are distinct. In our model, long-run aggregate growth is not affected by the "waves of enthusiasm", while Cozzi's model pre-

dicts a higher aggregate growth rate, due to a higher aggregate rate of vertical innovation. The mechanism behind our result is twofold. One one hand, we consider an explicit general BGP equilibrium where financial markets link the *effective* return to vertical R&D (the real interest rate plus the average rate of creative destruction) to the fundamentals, which must be the same whether we consider the symmetric equilibrium or the asymmetric equilibrium with sunspots. This "forces" an intra-R&D composition effect between "mature" and newly-born industries, thus dampening the impact of animal spirits on aggregate vertical R&D intensity. The latter, together with the fact that in our model vertical innovation is the ultimate growth engine, in the sense that it sustains both variety expansion and aggregate growth, implies that the "waves of enthusiasm" have no impact on the growth rate of the number of varieties and on the aggregate growth rate. However, our model predicts a positive impact on the level of per-capita consumption, in contrast to Cozzi's negative effect.

This set of results suggests that the risk that policy intervention, by acting itself as a potential source of extrinsic uncertainty, sees its effectiveness reduced - as explained by Cozzi (2005) - may ultimately be more relevant to the impact of public policy on the industrial structure and the level of consumption per capita than on the long-run aggregate growth rate.

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A. Derivation of the aggregate resource constraint with "waves of enthusiasm"

Consider the households' balance sheet (35), in the text. Hence, we can characterise the change in the value of equity as

$$\dot{a}(t) = \eta(t) \cdot \Omega^{-1} \cdot \dot{N}(t) + \dot{\eta}(t) \cdot \Omega^{-1} \cdot N(t) \tag{50}$$

Substitute (36) in the left-hand side of (50) and $\frac{\dot{\eta}}{\eta} = \frac{\dot{Q}}{Q} - \frac{\dot{N}}{N} = AI\left(\lambda^{\frac{\alpha}{1-\alpha}} - 1\right)$ - derived from (33) and (37) - in the right-hand side, to get

$$(r(t) + A \cdot I) \cdot a(t) - A \cdot I \cdot a(t) + w(t) \cdot L - C(t) + \Lambda(t) =$$

$$= A \cdot I \cdot \left(\lambda^{\frac{\alpha}{1-\alpha}} - 1\right) \cdot \eta(t) \cdot \Omega^{-1} \cdot N(t) + \eta(t) \cdot \Omega^{-1} \cdot \dot{N}(t) \tag{51}$$

Then, using (31) solved in order to π, together with $Y - X = wL + \pi N$,[23] in (51), we find

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[23] Having in mind the price markup $p = \frac{1}{\alpha}$, equations (4), (8), (10) and (11), and that, in equilibrium, w and p are equated to the marginal product of labour and the marginal product of intermediate goods, respectively, it is easily shown that $wL = (1-\alpha)Y$, $X = \alpha^2 Y$, $pX = \alpha Y$ and total profits $\Pi = X \cdot (p-1) = \alpha Y - \alpha^2 Y$. Also, have in mind that, by definition of $q(j, \bar{\omega}, t)$, total profits can be represented as $\Pi = \pi(j, \bar{\omega}, t) \cdot N$.

$$Y(t) - X(t) - C(t) + \Lambda(t) = A \cdot I \cdot \lambda^{\frac{\alpha}{1-\alpha}} \cdot a(t) + \eta(t) \cdot \Omega^{-1} \cdot \dot{N}(t) \tag{52}$$

Finally, recall that $R_n = \eta \dot{N}$ and $R_v = I\lambda^{\frac{1-\alpha}{\alpha}}a$,[24] and let

$$\Lambda = (\Omega^{-1} - 1)\eta\dot{N} \tag{53}$$

such that (52) reads

$$Y(t) = X(t) + C(t) + R_n(t) + A \cdot R_v(t)$$

which is (23), in the text. Observe that (53) means that the "real pure profit" term, Λ, in (36) must capture the net increase in the households' balance sheet induced by horizontal entry, in turn explained by the fact that the average value from entry exceeds the average cost, $V > \eta$, when $\theta > 1$ (thus implying $\Omega^{-1} > 1$). Notice that Λ is zero in the symmetric equilibrium, $\theta = 1$ (in that case, $\Omega^{-1} = 1$), which means that in fact (36) generalises (2).

B. Model with static decreasing returns to horizontal R&D

We consider first the symmetric equilibrium. Let horizontal R&D technology (18), in the text, be replaced by[25]

$$\dot{N} = \frac{1}{\eta(N)}\phi(R_n) \tag{54}$$

where $\phi' > 0$, $\phi'' < 0$ and $\eta(\cdot)$ as defined in the text. Term $\phi(R_n)$ introduces *static* decreasing returns to horizontal R&D in the model. We keep term $\frac{1}{\eta(N)}$ in (54), as the *dynamic* decreasing returns to horizontal R&D implied by $\eta(N)$, $\eta' > 0$, are necessary to eschew the explosive growth that would occur, e.g., if η were constant over time.[26] More specifically, in line with Segerstrom (2000), let $\dot{N}_e(t)$ be the contribution to the instantaneous flow of new goods by R&D firm e in the horizontal-R&D sector and $R_{ne}(t)$ the flow of resources devoted to horizontal R&D by e at t (measured in units of final-good output Y), such that $\dot{N}_e = \frac{1}{\eta(N)}\phi(R_{ne})$. Given perfect competition among R&D firms, each of them takes as given the marginal value of entry, V. R&D firms solve $max_{R_{ne}} \dot{N}_e \cdot V - R_{ne}$, with the associated first-order condition implying, under a convenient aggregation procedure, $V = \eta(N)\frac{1}{\phi'(R_n)}$. Combining the latter with (21), in the text, we get the consistency condition

$$\frac{Q}{N} = \frac{\eta(N)}{\zeta \cdot L \cdot \phi'(R_n)} \tag{55}$$

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[24]By solving (13) with respect to R_v and aggregating across ω, we obtain $R_v = \int_0^N \Phi(\omega)^{-1} I(\omega) d\omega = I\zeta L\lambda^{\frac{\alpha}{1-\alpha}}Q$ (see (44)). From (33) and (35), we then get $R_v = I\zeta L\lambda^{\frac{\alpha}{1-\alpha}}Q = I\lambda^{\frac{\alpha}{1-\alpha}}a$.

[25]In this appendix we omit time subscripts for sake of simplicity.

[26]More generally, we need some type of dynamic friction in horizontal entry in order to eschew the explosive growth that would otherwise occur given the feedback between horizontal and vertical innovation in a lab-equipment setup. It can be shown that the specification $\eta \equiv \eta(Q)$, $\eta' > 0$, $\eta'' < 0$, produces a similar result.

For concreteness, assume the specification $\phi(R_n) = R_n^{\sigma_1}$ and $\eta(N) = N^{\sigma_2}$, with $0 < \sigma_1 < 1$ and $\sigma_2 > 0$.[27] By considering the latter and that, along the BGP, $\frac{\dot{R}_n}{R_n} = \frac{\dot{Q}}{Q} = g$, time-differentiation of (59) yields the BGP relationship

$$g = \left(\frac{1 + \sigma_2}{\sigma_1}\right) \cdot g_N \tag{56}$$

where $\left(\frac{1+\sigma_2}{\sigma_1}\right) > 1$. Solving (54) in order to R_n, together with (55), leads to

$$R_n = g_N \cdot \sigma_1 \cdot \zeta \cdot L \cdot Q \tag{57}$$

$$N = \left[\zeta \cdot L \cdot \phi'(R_n) \cdot Q\right]^{\frac{1}{\sigma_2+1}} = \left[(\zeta \cdot L \cdot \sigma_1 \cdot Q)^{\sigma_1} g_N^{1-\sigma_1}\right]^{\frac{1}{\sigma_2+1}} \tag{58}$$

Now, we focus on the asymmetric equilibrium under Assumption 1. Re-iterating the same steps as in Section 4, we find that the consistency condition (55) is replaced by

$$\frac{Q}{N} = \frac{\eta(N)}{\zeta \cdot L \cdot \phi'(R_n)} \cdot \Omega^{-1} \tag{59}$$

where $\Omega \equiv \left[1 - \frac{(\theta-1)I}{r+\theta I}\right] \in (0,1), \forall r, I, > 0, \theta \in (1, \infty)$. Given (59), we get

$$R_n = g_N \cdot \sigma_1 \cdot \zeta \cdot L \cdot \Omega \cdot Q \tag{60}$$

$$N = \left[\zeta \cdot L \cdot \Omega \cdot \phi'(R_n) \cdot Q\right]^{\frac{1}{\sigma_2+1}} = \left[(\zeta \cdot L \cdot \Omega \cdot \sigma_1 \cdot Q)^{\sigma_1} g_N^{\sigma_1-1}\right]^{\frac{1}{\sigma_2+1}} \tag{61}$$

It is straightforward to show that the presence of $\phi(R_n)$ in (54) reduces the (negative) impact of "waves of enthusiasm" on R_n vis-á-vis the symmetric equilibrium without sunspots (in absolute terms), because now Ω multiplies by $\sigma_1 < 1$ in (60). The effect of $\phi(R_n)$ on the (negative) impact of "waves of enthusiasm" on N is ambiguous: since R_n feeds back on N through $\phi'(R_n)$ - Ω multiplies by $\phi'(R_n)$ in (61) - the exact magnitude depends also on the level of $\phi'(R_n)$, in particular whether it is below or above unity.

C. Existence and uniqueness of $A_p > 1$

We give a sketch of the formal proof for the existence and uniqueness of the value of $A_p > 1$, given the set $(\theta, p; g_N^s, I^s)$, that solves for the asymmetric BGP both under Assumption 1 ($p = 1 \Rightarrow A_p = A$) and Assumption 2 ($p \in (0,1)$). Recall from (41) that $I \equiv I(A_p) = \frac{1}{A_p} I^s$, where $I^s = \frac{\bar{\pi}}{\zeta} - \rho - \Theta g$. Thus, we wish to show that the equation

---

[27]Given this specification for $\phi(R_n)$, the aggregation procedure that allows one to derive (54), and thus (55), can be achieved by postulating $\dot{N}_e = \frac{1}{\eta(N)} R_{ne}^{\sigma_1} M^{\sigma_1-1}$, where M is the total number of firms in the horizontal-R&D sector (e.g., Segerstrom and Zolnierek, 1999). With all R&D firms choosing the same amount of R&D, this technology allows for convenient aggregation, such that $\dot{N} = \frac{1}{\eta(N)} R_n^{\sigma_1}$, where $\dot{N} = M \cdot \dot{N}_e$ and $R_n = M \cdot R_{ne}$. Observe that, as $M \to \infty$, the aggregate flow of new goods does not change, but the individual contribution of any firm e becomes negligible.

$$A_p = 1 + (\theta - 1) \cdot \frac{g_N \cdot A_p}{g_N \cdot A_p + \theta \cdot I^s} \cdot p \equiv \chi(A_p) \tag{62}$$

has a unique and finite solution, i.e., that the locus of A_p (i.e., the 45º line) and $\chi(A_p)$ coincide for a unique and finite value of $A_p > 1$. Firstly, see that $\chi(A_p)$ is a continuous increasing concave function, that is, for A_p finite,

$$\frac{d\chi(A_p)}{dA_p} = (\theta - 1)\frac{g_N \cdot p}{g_N \cdot A_p + \theta \cdot I^s}\left(1 - \frac{g_N \cdot A_p}{g_N \cdot A_p + \theta \cdot I^s}\right) > 0$$

and

$$\frac{d^2\chi(A_p)}{dA_p^2} = 2(\theta - 1)\frac{g_N^2 \cdot p}{(g_N \cdot A_p + \theta \cdot I^s)^2}\left(-1 + \frac{g_N \cdot A_p}{g_N \cdot A_p + \theta \cdot I^s}\right) < 0$$

since $1 - \frac{g_N A_p}{g_N A_p + \theta I^s} > 0$. Secondly, note that $lim_{A_p \to \infty} \frac{d\chi}{dA_p} = 0$. Thus, $\chi(A_p)$ is monotonically increasing such that $\exists A_p > 1, \frac{d\chi}{dA_p} < 1$. The latter, combined with $\chi(0) = 1$, implies that the curve $\chi(A_p)$ and the 45º line cross only once, at a given $A_p > 1$ finite.

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