

**CONFORMITY BASED BEHAVIOR AND  
THE DYNAMICS OF PRICE  
COMPETITION: A NEW RATIONAL FOR  
FASHION SHIFTS**

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# Conformity based behavior and the dynamics of price competition: a new rational for fashion shifts.

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## Abstract

This paper deals with dynamic price competition in markets in which the perception of consumers regarding the value of goods depends on the choices of other consumers in the market. In particular, we consider the case in which consumers tend to imitate their peers, generating a conformity effect.

In the context of a finite horizon model, we show that conformity based behavior creates new channels of dynamic interaction between firms, changing the nature of price competition. As time evolves, both price strategic complementarity and substitutability may arise along the equilibrium trajectory. This leads to *V-shaped* equilibrium price paths and oscillating trajectories of market shares. We provide also a new rational for the inversion of fashion trends.

**Keywords:** dynamic price competition, consumer behavior, conformity, fashion shift.

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# 1 Introduction

The emergence of trends, fads and fashion is a very common phenomenon. Often, agents view some of these trends as a path to social interaction and adopt them as means of social recognition by their peers. Agents who are not willing to stand out imitate their peers in their consumption choices, generating *conformity based behavior*.<sup>1</sup> There is a wide range of goods for which this behavior can arise, for example: alcoholic drinks, beverages, entertainment goods, garments, cars, restaurants, clubs, touristic destinations, etc. In general, these are goods whose value has two components: the intrinsic value of the good and its social value. For example, the intrinsic utility of being a costumer of a certain restaurant is related to the meals served in that restaurant, whereas its social utility is related to the social status attributed to the people who frequent that particular place.

Conformity based behavior has implications on firms' strategies both in a static environment, as studied by Grilo et al. (2001) and in a dynamic framework, as the one we study here. We consider the dynamic competition of two firms that interact on prices during three periods. Each firm produces a differentiated variant of the good and there exist conformity effects in consumption. We assume that agents take one period of time to observe the consumption trend and to conform with it.<sup>2</sup> Accordingly, the number of past consumers is one of the main determinants of consumption patterns. In addition, we assume that the conformity effect is non-cumulative in time, in the sense that the current value of a good only depends on the number of consumers using a variant in the preceding period. In a context of fast shifting trends, consumers will have no interest in imitating the choices made by their peers in a distant past.

The hypothesis of non-cumulative and delayed conformity behavior is particularly suitable for some types of goods, as it is the case of fashion goods (garments, restaurants, dancing clubs, touristic destinations). On the one hand, the conformity effect in the consumption of these goods tends to be delayed as a consequence of some learning or word-of-mouth process (very often there is a time lag to get information on the most recent fashion tendency).<sup>3</sup> On the other hand, the conformity effect in the consumption of fashion goods tends to be non-cumulative since the number of consumers who was using the good far-off in the past can hardly influence its current value.

In this paper, we study to which extent the existence of conformity behavior in consumption creates new (dynamic) channels of interaction between firms, affecting the nature of price competition. From the point of view of firms, the conformity based behavior of consumers generates inter-dependent demands across periods: the higher the firm's demand in the preceding period, the higher the social value provided by its good. This amounts to saying that, in each period, firms' demands depend on the size of the base of users created in the preceding period. Hence, by lowering their prices, firms enhance both present and future consumption. In this context, we expect

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<sup>1</sup>To better understand this type of behavior see, for example, Veblen (1922), Bikhchandani et al. (1992), Bernheim (1994), Corneo and Jeanne (1997, 1999) and Grilo et al. (2001).

<sup>2</sup>Bikhchandani et al (1992) also refer to delayed conformity effects, considering the case of informational cascades in which "it is optimal for an individual, having observed the actions of those ahead of him, to follow the behavior of the preceding individual without regard to his own information".

<sup>3</sup>See for example Corneo and Jeanne (1999).

conformity based behavior to stimulate tougher competition between firms in the first periods of interaction.

However, this paper reveals that there are conditions in which this initial competition boost does not arise in equilibrium. Our analysis unveils interesting results in what concerns the behavior of the duopolists. First, we observe that, for the levels of conformity considered in this paper, both firms adopt *V-shaped* equilibrium price trajectories: prices decrease from period one to period two, increasing afterwards.

The fact that firms increase their prices as the game moves towards the end is not surprising. In the last period of interaction, firms relax price competition in order to profit from the higher social value of their goods. On the contrary, initial price reductions are not so intuitive. One could be led to think that, in the beginning of the game, firms have maximal incentives to adopt price-cutting strategies. However, this result does not hold in this paper due to the fierceness of competition in the intermediary period. Indeed, we observe that, in this period, both firms quote negative prices, privileging a strategy of enlarging the base of consumers for the last period. The higher the firms' demands in the intermediary period, the higher the cost of the former strategy because firms would charge a negative price on a larger number of consumers. As a consequence, firms refrain from adopting aggressive demand-enhancing strategies in the first period.

We also analyse the evolution of the goods' social value along the equilibrium path. For the level of conformity effects addressed in this paper, the firm holding the good that initially yields a higher social value ends up reverting its position. In other words, the good *à la mode* becomes the unfashionable good! The reasons for this fashion reversion are twofold. First, the initially fashionable firm has some slack and it is able to increase the price of the good *à la mode*, maintaining a high level of sales. Second, for the reasons explained above, in the intermediary period, this firm has incentives to refrain from adopting demand-enhancing strategies, even if this makes its good less fashionable.

The previous results are related the specific nature of dynamic price competition in the presence of conformity based behavior. In each period of time, we have investigated the existence of strategic complementarity or substitutability on prices. The actions of two or more players are called strategic complements if they mutually reinforce one another, i.e. if an increase in the action of one player increases the marginal payoff of the other player, leading him to also increase his strategy. Likewise, they are called strategic substitutes if they mutually offset one another. In other words, in the case of strategic complements the reaction functions of the firms are increasing in the rival's strategy, whereas in the case of strategic substitutes, they are decreasing. These terms were originally coined by Bulow et al. (1985).<sup>4</sup> Our results can be summarized as follows. When the conformity effect is relatively less important than differentiation between products, firms' prices are strategic substitutes in period one and two. Only in the last period of interaction, there is a switch to strategic complementarity. For higher intensity of the conformity effect there is a change in the

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<sup>4</sup>Topkis (1979), Vives (1990) and Milgrom and Roberts (1990) have developed further the study of games with strategic complements and substitutes. See Vives (2005) for a survey.

nature of strategic interaction: prices are strategic complements in periods one and three, and strategic substitutes in the second period. These results follow from the dynamic nature of our model together with the delayed non-cumulative conformity effect that is being analyzed here. In a static setting (as well as in the last period of interaction), the only incentives that firms have to compete for consumers stem from the instantaneous profits that they get. This necessarily leads to price strategic complementarity. However, in a dynamic duopoly with non-cumulative conformity effects, the inter-temporal dependence of demands creates additional incentives to compete for consumers. When quoting their prices, firms will take into account that their attractiveness in the forthcoming periods depends on goods' social value, which, in turn depends on the size of the user base of the preceding period.

This paper belongs to the literature on conformity based behavior and conspicuous consumption (Veblen (1922), Bikhchandani et al. (1992), Bernheim (1994), Corneo and Jeanne (1997, 1999) and Grilo et al. (2001)). As we deal with price competition when consumer behavior is characterized by conformity, the paper that is more closely related to ours is Grilo et al. (2001). However, while the latter brings a static perspective on this issue, we shed light on the dynamic aspects of firms interaction.

Our paper also adds to the literature on dynamic price competition in network industries.<sup>5</sup> A number of recent works have studied the problem of dynamic competition in network industries (Doganoglu (2003), Mitchell and Skrzypacz (2006), Laussel et al. (2004), Markovich (2008), Markovich and Moenius (2009), Doraszelski, Chen, and Harrington (2009) and Cabral (2007), among others). While some models predict monotonic price trajectories (e.g. Doganoglu (2003) and Laussel et al. (2004)), Cabral (2007) proposes a computational framework, in which prices do not evolve linearly with market shares, a prediction which is in line with our results.

The remainder of the paper is organized as follows. Section 2 presents the basic ingredients of the model. Section 3 characterizes the equilibrium in which both firms survive in every period and provides conditions under which this equilibrium exists and it is unique. Section 4 analyzes strategic interaction along the equilibrium path and, finally, Section 5 concludes.

## 2 The model

We consider a model in which two firms, indexed by 1 and 2, produce differentiated variants of a good, whose lifetime is equal to one period. Variants are differentiated *à la* Hotelling and firms are assumed to be located in the extrema of the Hotelling line  $[0, 1]$ . Firms interact for three periods choosing noncooperatively their prices.

The consumers (with mass equal to 1) are uniformly distributed in the Hotelling line. Every period  $t$ , they buy one of the two existing variants. Notice that, in our setup, consumers are considered to be atomistic and, therefore, there is no such thing as trendsetters.

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<sup>5</sup>The literature on competition in network industries has flourished after the seminal paper by Katz and Shapiro (1985).

Consumers' utility when buying good  $i$  is

$$u_{i,t}(x, p_{i,t}, D_{i,t-1}) = V - \tau(x - x_i)^2 + \alpha D_{i,t-1} - p_{i,t}. \quad (1)$$

where (i)  $V$  represents the intrinsic value of consumers' ideal variant of the good; (ii)  $\tau > 0$ , the unit travel cost; (iii)  $x \in [0, 1]$ , consumers' location along the Hotelling line; and (iv)  $x_i$ , the location of firm  $i$ , with  $x_1 = 0$  and  $x_2 = 1$ .<sup>6</sup> The price of good  $i$  in period  $t$  is denoted by  $p_{i,t}$ .

The utility function specified in (1) also takes into consideration the social value of goods stemming from consumers' desire for conformity. For the variant  $i$ , the social value is determined by the installed base of users for this variant in the preceding period ( $D_{i,t-1}$ ). The intensity of the conformity effect is measured by the parameter  $\alpha > 0$ .

To better understand the spirit of our model, consider the following example regarding the choice of restaurants. Each period, consumers may opt for one of two restaurants: a sushi bar and a pizzeria. Each consumer has a preference for the type of food offered by each restaurant. In our model, such preference is measured by  $V - \tau(x - x_i)^2$ . However, when choosing which restaurant to go to, consumers also take into consideration which is the restaurant *à la mode*, i.e., the restaurant with a larger number of patrons in the preceding period.<sup>7</sup> In our model, this corresponds to the conformity effect and it is measured by  $\alpha D_{i,t-1}$ . This type of conformity effect is applicable to fashion goods, capturing the idea that consumers' choices do not depend on trends established in a distant past. In the restaurant example, this amounts to saying that consumers' perception of which restaurant is *à la mode* in the last period does not depend on consumers' choices in period one.

According to (1), in period  $t$ , the consumer who is indifferent between the two existing variants of the good is located at  $\tilde{x}_t(p_{1,t}, p_{2,t}, D_{1,t-1}, D_{2,t-1})$  such that

$$\tilde{x}_t = \frac{1}{2} + \frac{\alpha}{2\tau} \Delta D_{t-1} - \frac{1}{2\tau} (p_{1,t} - p_{2,t}),$$

where  $\Delta D_{t-1} = D_{1,t-1} - D_{2,t-1}$ .

The demand for good 1 in period  $t$  as a function of the prices quoted by firms at  $t$  (resp.  $p_{1,t}$  and  $p_{2,t}$ ) and firms' market shares at the preceding period (resp.  $D_{1,t-1}$  and  $D_{2,t-1}$ ) is given by:

$$D_{1,t}(p_{1,t}, p_{2,t}, \Delta D_{t-1}) = \frac{1}{2} + \frac{\alpha}{2\tau} \Delta D_{t-1} - \frac{p_{1,t} - p_{2,t}}{2\tau}, \quad (2)$$

with  $|p_{2,t} - p_{1,t} + \alpha \Delta D_{t-1}| < \tau$  to guarantee that both firms have strictly positive market shares. The demand for good 1 in period  $t$  is simply  $D_{2,t} = 1 - D_{1,t}$ .

Evaluating (2) in period  $t - 1$ , one obtains  $D_{1,t-1}(p_{1,t-1}, p_{2,t-1}, \Delta D_{t-2})$ . Replacing this expression in (2) it is possible to define market shares of period  $t$  conditional on the size of the installed

<sup>6</sup>We consider that  $V$  is sufficiently large to guarantee that, at equilibrium, consumers always buy one of the goods. Also, following d'Aspremont *et al.* (1979), we consider quadratic travel costs.

<sup>7</sup>Obviously, depending on the interplay of intrinsic and social values, a consumer may opt differently in different periods.

base of firms in period  $t - 2$  and the prices quoted by firms at periods  $t$  and  $t - 1$ . Repeating this exercise sequentially, we obtain the demand of firm  $i$  in period  $t$  conditional on firms' initial market shares (i.e.  $\Delta D_0 = D_{1,0} - D_{2,0}$ ) and the sequence of prices charged by firms from period 1 to period  $t$  (i.e.  $(\mathbf{p}_1, \dots, \mathbf{p}_t)$ , where  $\mathbf{p}_t = (p_{1,t}, p_{2,t})$  and  $t = 1, 2, 3$  :

$$D_{1,t}(\Delta D_0, \mathbf{p}_1, \dots, \mathbf{p}_t) = \frac{1}{2} + \frac{1}{2} \left(\frac{\alpha}{\tau}\right)^t \Delta D_0 - \frac{1}{2\tau} \sum_{k=1}^t \left(\frac{\alpha}{\tau}\right)^{k-i} (p_{1,k} - p_{2,k}).$$

Assuming that firms produce goods at a zero cost, the instantaneous profits of firm  $i = 1, 2$  in period  $t$  are equal to

$$\pi_{i,t}(p_{1,t}, p_{2,t}, \Delta D_{t-1}) = D_{i,t}(p_{1,t}, p_{2,t}, \Delta D_{t-1}) p_{i,t}.$$

In our dynamic game, the players are the two firms and the strategies correspond to the three dimensional vectors of prices  $\mathbf{P}_i \in \mathbb{R}^3$ , with  $\mathbf{P}_i = \{p_{i,t}\}_{t=1}^3$  and  $i = 1, 2$ . In each period  $t$ , firms' payoffs are given by the value of the discounted profits accumulated from  $t$  to  $T = 3$ .

To characterize optimal price strategies in the context of this dynamic game, we rely on the equilibrium notion of *Subgame Perfect Nash Equilibrium* (SPNE): at equilibrium, for each period, each firm quotes the price  $p_{i,t}^*$  that maximizes the discounted profits accumulated from  $t$  to  $T = 3$  when firm  $i$  takes as given the price of the rival firm in period  $t$  ( $p_{j,t}$ ) as well as the differential on the size of the past demands faced by firms in the preceding period ( $\Delta D_{t-1}^i = D_{i,t-1} - D_{j,t-1}$ ).

Formally, at each period each firm solves the following optimization problem:

$$\max_{p_{i,t}} \left\{ \pi_{i,t}(\cdot) + \sum_{k=t+1}^T \pi_{i,k}^*(p_{i,k}^*(\cdot), p_{j,k}^*(\cdot), \Delta D_{k-1}^*) \right\}, i = 1, 2 \quad (3)$$

where, for the sake of simplicity, we denote the vector  $(p_{1,t}, p_{2,t}, \Delta D_{t-1})$  by  $(\cdot)$ . Note that, for given  $p_{j,t}$  and  $\Delta D_{t-1}$ , the price  $p_{i,t}^*(p_{j,t}, \Delta D_{t-1})$  that solves problem (3) takes into consideration the impact of  $p_{i,t}$  on contemporaneous profits of firm  $i$  as well as the impact of this strategic decision on the future profits of this firm (dynamic effect). This dynamic effect arises because the choice of  $p_{i,t}$  has an effect on the social value of the good offered by firm  $i$  in period  $t + 1$ . Accordingly, price decisions in period  $t$  will influence the attractiveness of each variant for future consumers, affecting firms' future profits through the conformity effect.

In the following section, we focus on the domain of parameters  $(\alpha, \tau)$  and initial conditions  $(\Delta D_0)$ , for which the SPNE corresponds to interior solutions with both firms having strictly positive market shares in every period.<sup>8</sup> Under this restriction, we use backward induction techniques to derive the explicit path of  $\mathbf{P} = (\mathbf{P}_1^*, \mathbf{P}_2^*)$ , corresponding to a SPNE of the multi-stage game.

<sup>8</sup>For example, there could be exit barriers, which are sufficiently strong so that predatory strategies aiming to evict the rival firm are unprofitable.

### 3 Equilibrium path

In this section, we derive the candidate SPNE in which both firms are active in every period of interaction. Afterwards, we provide conditions under which the duopolistic candidate SPNE corresponds to an effective equilibrium.

We start by investigating strategic interaction in the last period. At  $t = 3$ , the vector of equilibrium prices conditional on firms' previous market shares  $(p_{1,3}^*(\Delta D_2), p_{2,3}^*(\Delta D_2))$  solves

$$\max_{p_{i,3}} p_{i,3} D_{i,3}(p_{i,3}, p_{j,3}, \Delta D_2), \quad (4)$$

with  $i = 1, 2$  and  $D_{i,3}(p_{i,3}, p_{j,3}, \Delta D_2)$  obtained from (2) when  $t = 3$ .

For the domain of parameters  $(\alpha, \tau)$  and initial conditions  $(\Delta D_0)$  that leads to duopolistic equilibrium outcomes in period  $t = 3$ , the candidate equilibrium price strategies conditional on  $\Delta D_2$  are given by the solution to the system of first order conditions, namely:

$$p_{1,3}^*(\Delta D_2) = \tau + \frac{1}{3}\alpha\Delta D_2, \quad (5)$$

$$p_{2,3}^*(\Delta D_2) = \tau - \frac{1}{3}\alpha\Delta D_2. \quad (6)$$

In the light of (5) and (6), when both firms are active in the market, equilibrium market shares conditional on  $\Delta D_2$  are equal to:

$$\begin{aligned} D_{1,3}^*(\Delta D_2) &= \frac{1}{2} + \frac{\alpha}{6\tau}\Delta D_2, \\ D_{2,3}^*(\Delta D_2) &= 1 - D_{1,3}^*(\Delta D_2). \end{aligned}$$

Equilibrium profits at  $T = 3$  conditional on the differential  $\Delta D_2$  are given by:

$$\pi_{i,3}^*(\Delta D_2) = D_{i,3}^*(\Delta D_2) p_{i,3}^*(\Delta D_2). \quad (7)$$

In period 2, firms choose  $(p_{1,2}^*(\Delta D_1), p_{2,2}^*(\Delta D_1))$  that simultaneously solve the following optimization problem:

$$\max_{p_{i,2}} \pi_{i,2}(p_{i,2}, p_{j,2}, \Delta D_1) + \pi_{i,3}^*(\Delta D_2), \quad i = 1, 2, \quad (8)$$

where, each firm  $i$  takes as given the price quoted by the rival at  $t = 2$  as well as  $\Delta D_1$ . For the sake of simplicity, the discount rate is assumed to be equal to 1.

The previous optimization problem can be re-written as follows:

$$\max_{p_{i,2}} p_{i,2} D_{i,2}(p_{i,2}, p_{j,2}, \Delta D_1) + \pi_{i,3}^*(p_{i,2}, p_{j,2}, \Delta D_1), \quad i = 1, 2,$$

since  $\Delta D_2$  is a function of  $(p_{i,2}, p_{j,2}, \Delta D_1)$ . The candidate price equilibrium can be obtained from



the system of first order conditions, yielding:

$$p_{1,2}^*(\Delta D_1) = \tau - \frac{2}{3}\alpha + \alpha \frac{2\Omega + 9\tau^2}{4\Omega + 9\tau^2} \Delta D_1, \quad (9)$$

$$p_{2,2}^*(\Delta D_1) = \tau - \frac{2}{3}\alpha - \alpha \frac{2\Omega + 9\tau^2}{4\Omega + 9\tau^2} \Delta D_1, \quad (10)$$

where  $\Omega = \alpha^2 - 9\tau^2$ . The equilibrium market shares in period 2 conditional on  $\Delta D_1$  are equal to

$$D_{1,2}^*(\Delta D_1) = \frac{1}{2} - \frac{\tau}{2} \frac{9\alpha}{4\Omega + 9\tau^2} \Delta D_1, \quad (11)$$

$$D_{2,2}^*(\Delta D_1) = 1 - D_{1,2}^*(\Delta D_1), \quad (12)$$

yielding:

$$\Delta D_2^*(\Delta D_1) = -\frac{9\alpha\tau}{4\Omega + 9\tau^2} \Delta D_1.$$

Equilibrium instantaneous profits are given by:

$$\pi_{1,2}^*(\Delta D_1) = p_{1,2}^*(\Delta D_1) D_{1,2}^*(\Delta D_1).$$

Introducing the equilibrium differential  $\Delta D_2^*(\Delta D_1)$  in the expressions of equilibrium price strategies at  $t = 3$  conditional on  $\Delta D_2$  (respectively given by (5) and (6)), we obtain:

$$p_{1,3}^*(\Delta D_1) = \tau - \frac{3\alpha^2\tau}{4\Omega + 9\tau^2} \Delta D_1, \quad (13)$$

$$p_{2,3}^*(\Delta D_1) = \tau + \frac{3\alpha^2\tau}{4\Omega + 9\tau^2} \Delta D_1. \quad (14)$$

To get a full description of equilibrium price paths, we move now to the analysis of firms' price decisions at  $t = 1$ . Considering that firms take as given the price quoted by the rival at  $t = 1$  as well as initial market shares ( $\Delta D_0$ ), the duopolistic candidate equilibrium at  $t = 1$  corresponds to the vector of prices  $(p_{1,1}^*(\Delta D_0), p_{2,1}^*(\Delta D_0))$  that simultaneously solves the following optimization problem:

$$\max_{p_{i,1}} p_{i,1} D_{i,1}(p_{i,1}, p_{j,1}, \Delta D_0) + \sum_{t=2}^{T=3} \pi_{i,t}^*(p_{i,1}, p_{j,1}, \Delta D_0), \quad i = 1, 2. \quad (15)$$

The candidate equilibrium prices at  $t = 1$  are given by:

$$p_{1,1}^*(\Delta D_0) = \tau - \frac{2\alpha\Omega}{4\Omega + 9\tau^2} + \frac{\alpha}{3} \frac{(4\Omega + 9\tau^2)^2 + 2(3\alpha)^2\Omega}{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2\Omega} \Delta D_0, \quad (16)$$

$$p_{2,1}^*(\Delta D_0) = \tau - \frac{2\alpha\Omega}{4\Omega + 9\tau^2} - \frac{\alpha}{3} \frac{(4\Omega + 9\tau^2)^2 + 2(3\alpha)^2\Omega}{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2\Omega} \Delta D_0. \quad (17)$$

and firms' market shares at  $t = 1$  conditional on  $\Delta D_0$  are equal to:

$$\begin{aligned} D_{1,1}^* (\Delta D_0) &= \frac{1}{2} + \frac{\alpha}{6\tau} \frac{(4\Omega + 9\tau^2)^2}{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega} \Delta D_0, \\ D_{2,1}^* (\Delta D_0) &= 1 - D_{1,1}^* (\Delta D_0), \end{aligned}$$

implying:

$$\Delta D_1^* (\Delta D_0) = \frac{\alpha}{3\tau} \frac{(4\Omega + 9\tau^2)^2}{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega} \Delta D_0. \quad (18)$$

Introducing (18) in (9); (10); (13) and (14), it is possible to describe the vector  $\mathbf{P}_i^*$ , corresponding to the path of prices that constitutes the candidate SPNE as a function of the initial market shares and the parameters of the model:

$$\begin{cases} p_{i,1}^* (\Delta D_0^i) = \tau - \frac{2\alpha\Omega}{4\Omega + 9\tau^2} + \frac{\alpha}{3} \frac{(4\Omega + 9\tau^2)^2 + 2(3\alpha)^2 \Omega}{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega} \Delta D_0^i, \\ p_{i,2}^* (\Delta D_0^i) = \tau - \frac{2}{3}\alpha + \frac{\alpha^2}{3\tau} \frac{(2\Omega + 9\tau^2)(4\Omega + 9\tau^2)}{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega} \Delta D_0^i, \\ p_{i,3}^* (\Delta D_0^i) = \tau - \frac{\alpha^3(4\Omega + 9\tau^2)}{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega} \Delta D_0^i, \end{cases} \quad (19)$$

where  $\Delta D_0^i = D_{i,0} - D_{j,0}$ ,  $i, j = 1, 2$  and  $i \neq j$ .

From (19) follows that the vector of equilibrium prices is non monotonic in  $\Delta D_0^i$ . For  $\hat{\alpha}(\tau) < \alpha < 3\tau$ , the sign of  $\frac{\partial p_{i,2}^*(\Delta D_0)}{\partial \Delta D_0}$  is always positive, while the sign of  $\frac{\partial p_{i,3}^*(\Delta D_0)}{\partial \Delta D_0}$  is always negative. The sign of  $\frac{\partial p_{i,1}^*(\Delta D_0)}{\partial \Delta D_0}$  depends on the parameters of the model, being negative for  $\hat{\alpha}(\tau) < \alpha < \hat{\hat{\alpha}}(\tau)$  and positive for  $\hat{\hat{\alpha}}(\tau) < \alpha < 3\tau$ , with  $\hat{\alpha}(\tau) = \frac{\tau}{2} \left( \frac{54}{7} (3 + \sqrt{2}) \right)^{\frac{1}{2}}$  and  $\hat{\hat{\alpha}}(\tau) = \frac{1}{2}\tau \left( \frac{54}{17} (\sqrt{15} + 7) \right)^{\frac{1}{2}}$ .<sup>9</sup>

At the candidate SPNE, market shares evolve according to the following trajectory:

$$\begin{cases} D_{i,1}^* (\Delta D_0) = \frac{1}{2} + \frac{\alpha}{6\tau} \frac{(4\Omega + 9\tau^2)^2}{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega} \Delta D_0^i \\ D_{i,2}^* (\Delta D_0) = \frac{1}{2} - \frac{3\alpha^2}{2} \frac{(4\Omega + 9\tau^2)}{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega} \Delta D_0^i \\ D_{i,3}^* (\Delta D_0) = \frac{1}{2} - \frac{\alpha^3}{2\tau} \frac{(4\Omega + 9\tau^2)}{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega} \Delta D_0^i. \end{cases} \quad (20)$$

We proceed to investigate under which conditions the candidate equilibrium corresponds to an effective SPNE of the multi-stage game. These conditions are identified in Proposition 1 below:

**Proposition 1 (*Existence and uniqueness*)** *When the intensity of the conformity effect is such that  $\alpha \in (\hat{\alpha}(\tau), 3\tau)$ , the vectors of prices  $\mathbf{P}_i^*$  constitute a unique SPNE of the multi-stage game if and only if the differential of firms' initial installed base of customers is not too large:*

$$|\Delta D_0^i| < \frac{1}{3\alpha^2} \frac{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega}{(4\Omega + 9\tau^2)}. \quad (21)$$

<sup>9</sup>The thresholds  $\hat{\alpha}(\tau)$  and  $\hat{\hat{\alpha}}(\tau)$  are respectively the roots of the following polynomials:  $(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega$  and  $(4\Omega + 9\tau^2)^2 + 2(3\alpha)^2 \Omega$ .

**Proof.** See the Appendix. ■

## 4 Equilibrium Analysis

### 4.1 Strategic substitutability and complementarity

Concentrating on the case of intermediate conformity effects,  $\alpha \in (\widehat{\alpha}(\tau), 3\tau)$ , and assuming that the initial degree of asymmetry between the size of firms' installed base of consumers is not too large (condition (21) holds), we investigate the properties of  $\mathbf{P}_i^*$  with respect to the strategic complementarity and substitutability between firms' contemporaneous price strategies. As already mentioned in the Introduction, the actions of two or more players are called strategic complements if they mutually reinforce one another, i.e. if the best reply of a firm to an increase in the action of the rival is to increase its action. Likewise, they are called strategic substitutes if the reverse holds.

For the sake of exposition, within  $\alpha \in (\widehat{\alpha}(\tau), 3\tau)$ , we distinguish a range of strong conformity effects,  $\widehat{\alpha}(\tau) < \alpha < 3\tau$ , and a range of weak conformity effects,  $\widehat{\alpha}(\tau) < \alpha < \widehat{\alpha}(\tau)$ . Different results are obtained for these two ranges of parameters.

**Lemma 1** *Strategic price complementarity and substitutability*

*In period 1, for weak conformity effects, prices are strategic substitutes, whereas for strong conformity effects, prices are strategic complements. In period 2, prices are strategic substitutes while in period 3, prices are strategic complements.*

**Proof.** See the appendix. ■

In a context of price competition, it is well established in the literature that prices are often strategic complements: a price-cutting strategy tends to induce price-cutting strategies by the rival firms. However, according to the previous Lemma, along the equilibrium path, both price strategic complementarity and price strategic substitutability arise. To clarify to which extent the existence of conformity effects may affect the nature of price competition, we investigate the characteristics of firms' best reply functions in each period of interaction, starting with  $T = 3$ . In the last period, firms' best reply functions are given by:

$$\begin{aligned} p_{1,3}(p_{2,3}, \Delta D_2) &= \frac{1}{2}\tau + \frac{1}{2}(p_{2,3} + \alpha\Delta D_2), \\ p_{2,3}(p_{1,3}, \Delta D_2) &= \frac{1}{2}\tau + \frac{1}{2}(p_{1,3} - \alpha\Delta D_2). \end{aligned}$$

We observe that,  $\frac{\partial p_{i,3}(p_{j,3}, \Delta D_2)}{\partial p_{j,3}} = \frac{1}{2}$ , which yields strategic complementarity. This result is not surprising since, in the last period of interaction, firms do not have any incentives to adopt strategies seeking to increase the social value of their goods in subsequent periods. For a given  $\Delta D_2$ , an increase in  $p_{j,3}$  leads to an increase in  $p_{i,3}$ .

In period  $t = 2$ , the best reply functions are

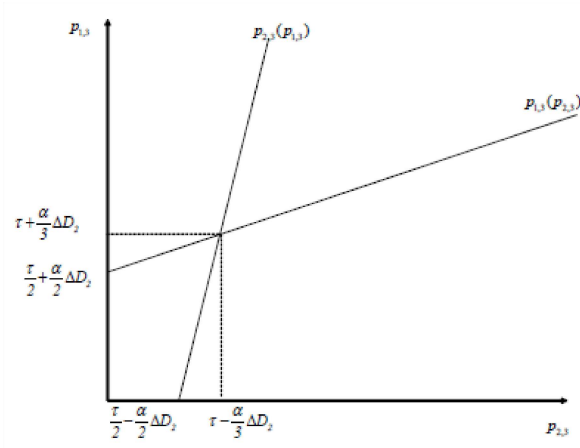


Figure 1: Strategic complementarity,  $t = 3$ .

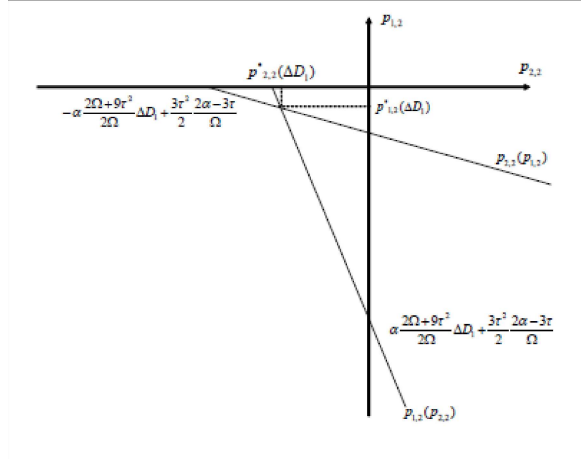


Figure 2: Strategic substitutability,  $t = 2$ .

$$\begin{aligned}
 p_{1,2}(p_{2,2}, \Delta D_1) &= \frac{2\Omega + 9\tau^2}{2\Omega} (p_{2,2} + \alpha \Delta D_1) + \frac{3}{2} \tau^2 \frac{2\alpha - 3\tau}{\Omega}, \\
 p_{2,2}(p_{1,2}, \Delta D_1) &= \frac{2\Omega + 9\tau^2}{2\Omega} (p_{1,2} - \alpha \Delta D_1) + \frac{3}{2} \tau^2 \frac{2\alpha - 3\tau}{\Omega}.
 \end{aligned}$$

It is easy to observe that  $\frac{\partial p_{i,2}}{\partial p_{j,2}} < 0$ , resulting in price strategic substitutability that sharply contrasts with the strategic complementarity result obtained for period  $t = 3$ .

The fact that prices are strategic substitutes in period  $t = 2$  results from the dynamic effects created by conformity based behavior. If firms only considered the impact of  $p_{j,2}$  on instantaneous profits  $\pi_{i,2}(p_{i,2}, p_{j,2}, \Delta D_1)$ , we would get  $\frac{\partial^2 \pi_{i,2}(p_{i,2}, p_{j,2}, \Delta D_1)}{\partial p_{j,2} \partial p_{i,2}} = \frac{1}{2\tau} > 0$  (price strategic complementarity). However, along the SPNE firms take into account the impact of their current

decisions on future profits. This dynamic effect more than offsets the direct (static effect), yielding strategic substitutability between  $p_{i,2}$  and  $p_{j,2}$ . In period  $t = 2$ , firms have strong incentives to invest in price-cutting strategies in order to induce favorable variations in  $\Delta D_2$ , which increase the willingness to pay of consumers in the following period. In the light of these incentives, at  $t = 2$ , competition is quite tough and both firms charge negative prices at equilibrium (see Figure 2).

Finally, in period  $t = 1$ , depending on the intensity of conformity effects, prices can be either strategic substitutes or strategic complements. The best reply functions are given by:

$$\begin{aligned} p_{1,1}(p_{2,1}, \Delta D_0) &= \frac{1}{2} \frac{(4\Omega + 9\tau^2)^2 + 2(3\alpha)^2\Omega}{(4\Omega + 9\tau^2)^2 + (3\alpha)^2\Omega} (p_{2,1} + \alpha\Delta D_0) + \frac{4\alpha^2 - 27\tau^2}{2} \frac{\tau(4\Omega + 9\tau^2) - 2\alpha\Omega}{(4\Omega + 9\tau^2)^2 + (3\alpha)^2\Omega}, \\ p_{2,1}(p_{1,1}, \Delta D_0) &= \frac{1}{2} \frac{(4\Omega + 9\tau^2)^2 + 2(3\alpha)^2\Omega}{(4\Omega + 9\tau^2)^2 + (3\alpha)^2\Omega} (p_{1,1} - \alpha\Delta D_0) + \frac{4\alpha^2 - 27\tau^2}{2} \frac{\tau(4\Omega + 9\tau^2) - 2\alpha\Omega}{(4\Omega + 9\tau^2)^2 + (3\alpha)^2\Omega}, \end{aligned}$$

with  $\frac{\partial p_{i,1}}{\partial p_{i,j}} < 0$  when  $\hat{\alpha}(\tau) < \alpha < \hat{\alpha}(\tau)$  and  $\frac{\partial p_{i,1}}{\partial p_{i,j}} > 0$  when  $\hat{\alpha}(\tau) < \alpha < 3\tau$ . In this period, prices are not necessarily strategic complements because the static effect coexists with dynamic effects caused by the intertemporal linkages of demands. However, notice that, in period  $t = 1$ , the dynamic effect is more complex than in period  $t = 2$ . The former can be decomposed in two distinct components: (i) the effect of  $(p_{i,1}, p_{j,1})$  on  $\pi_{i,2}^*$  via  $\Delta D_1$ ; and (ii) the effect of  $(p_{i,1}, p_{j,1})$  on  $\pi_{i,3}^*$  via  $\Delta D_2^*$  ( $\Delta D_1$ ). To be more precise, the static and the two dynamic effects are given by:

$$\begin{aligned} \frac{\partial^2 \pi_{i,1}(p_{i,1}, p_{j,1}, \Delta D_0)}{\partial p_{j,1} \partial p_{i,1}} &= \frac{1}{2\tau} > 0; \\ \frac{\partial^2 \pi_{i,2}^*(\Delta D_1(p_{i,1}, p_{j,1}, \Delta D_0))}{\partial p_{j,1} \partial p_{i,1}} &= 9\alpha^2 \frac{2\Omega + 9\tau^2}{\tau(4\Omega + 9\tau^2)^2} > 0; \\ \frac{\partial^2 \pi_{i,3}^*(\Delta D_2^*(\Delta D_1(p_{i,1}, p_{j,1}, \Delta D_0)))}{\partial p_{j,1} \partial p_{i,1}} &= -9 \frac{\alpha^4}{\tau(4\Omega + 9\tau^2)^2} < 0. \end{aligned}$$

Again, the static effect favors price strategic complementarity, while the dynamic effects have opposite signs. The full dynamic effect is equal to  $9\alpha^2\Omega\tau^{-1}(4\Omega + 9\tau^2)^{-2} < 0$ , favoring strategic substitutability. It is also worth noting that the sign of the total effect (static and dynamic) depends on the intensity of the conformity effect.<sup>10</sup> In the case of a weak conformity effect, the negative dynamic effect dominates, inducing tougher price competition. In contrast, in the case of a strong conformity effect, the positive effect dominates and prices are strategic complements.

## 4.2 Non-monotonic equilibrium trajectories and fashion shifts

In the context of a dynamic duopoly with conformity based behavior, we observe that the nature of price strategic interaction changes substantially as time evolves. For this reason, along the equilibrium path, firms' prices (as well as the corresponding market shares) are non-monotonic.

### Proposition 2 (*Non monotonic price path*)

*Along the equilibrium price trajectory  $\mathbf{P}_i^*$ , prices evolve non-monotonically: decreasing between period  $t = 1$  and  $t = 2$  but increasing afterwards.*

<sup>10</sup>The intensity of the conformity effect does not influence the static effect ( $\frac{1}{2\tau}$ ) but it positively influences the magnitude of the negative full dynamic effect ( $9\alpha^2\Omega\tau^{-1}(4\Omega + 9\tau^2)^{-2}$ ).

**Proof.** See the appendix. ■

From Proposition 2, equilibrium price trajectories are not monotonic, corresponding instead to a *V-shaped* trajectory:

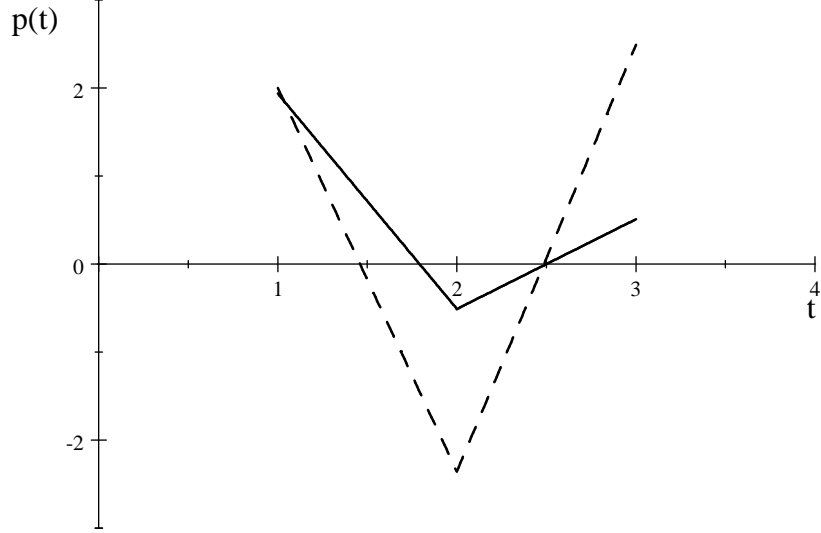


Figure 3: Equilibrium price trajectories ( $\tau = 1.5, \alpha = 4.4, \Delta D_0 = 0.05$ )

The fact that  $p_{i,3}^*(\Delta D_0^i) > p_{i,2}^*(\Delta D_0^i)$ , is associated with the lack of dynamic incentives to adopt price-cutting strategies in period  $t = 3$ . In period  $t = 2$ , differently from period  $t = 3$ , firms enjoy dynamic benefits from the enlargement of their base of users. For this reason, in period  $t = 2$ , price competition tends to be very tough. This is not the case in period  $t = 3$ , in which the user base expansion does not yield any dynamic benefit, relaxing competition between firms and inducing an increase of prices. The fact that  $p_{i,2}^*(\Delta D_0^i) < p_{i,1}^*(\Delta D_0^i)$  is *a priori* less intuitive, since one could think that the dynamic interplay of the conformity effects could lead firms to quote lower prices in period 1 (as it is the case in a monopoly setting, see Gabszewicz and Garcia (2007, 2008)). However, in our setting, competition in period 2 is very intense and both firms charge negative prices.<sup>11</sup> Clearly, the larger are the firms' market shares in period 2, the larger is the cost of adopting demand enhancing strategies (since firms are charging a negative price on a larger base of customers). As such, the fierceness of competition in period 2 ends up creating a dynamic effect that dampens competition in period 1.<sup>12</sup>

In line with the equilibrium price paths, the equilibrium trajectories of market shares are not monotonic and, in addition, there is a phenomenon of fashion shift in period  $t = 2$ .

<sup>11</sup>For the values of the parameters considered in this paper,  $\alpha \in (\hat{\alpha}(\tau), 3\tau)$ , it follows that accumulated profits from period  $t = 2$  and  $t = 3$  are positive for both firms, despite the profit losses incurred in the second period.

<sup>12</sup>Indeed, the firm enjoying the larger social value for its good in period 1 becomes, *ceteris paribus*, more attractive in period 2. In a context of negative prices, this increases the cost of adopting demand-enhancing strategies in  $t = 2$ .

**Proposition 3 (*Fashion shift*)**

When existence conditions are met, the firm benefiting from a larger initial base of customers (the firm *à la mode*) loses this dominance after one period of interaction, becoming *démodé*.

**Proof.** See the appendix. ■

The phenomenon of fashion shift is caused by the fierceness of competition in period 2. As mentioned before, in period 2, both firms have strong incentives to invest in price cutting strategies in order to boost the social values of their goods for the last period. In fact prices are actually negative. Nevertheless, the magnitude of such incentives is not the same for both firms, with the firm inheriting a smaller social value from the first period (the former *démodé* firm) having more powerful incentives to invest in price cutting strategies.<sup>13</sup> As a consequence, the price charged by the previously unfashionable firm is substantially more negative than the price charged by its rival, causing the phenomenon of fashion shift.

In the Figure 3, we depict the equilibrium trajectory of the firms' market shares that exhibit an oscillating behavior.

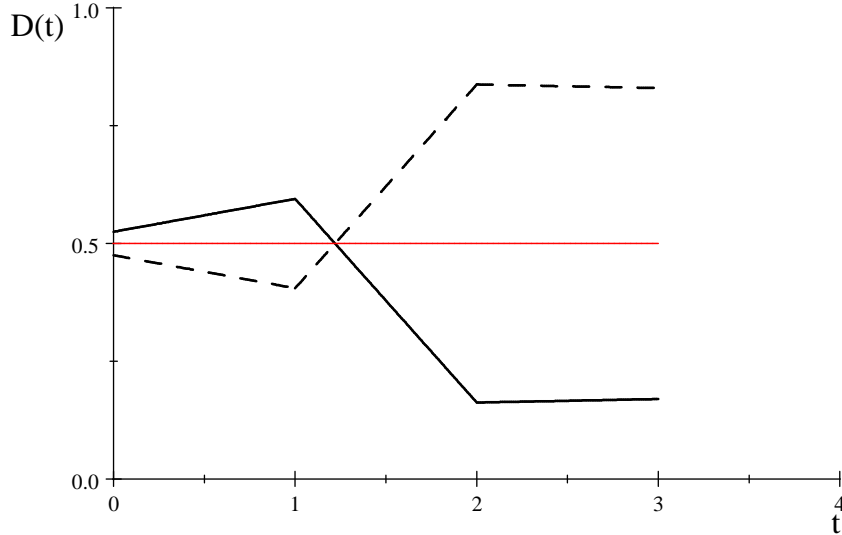


Figure 4: Equilibrium market shares path ( $\tau = 1.5, \alpha = 4.4, \Delta D_0 = 0.05$ )

This figure reflects the evolution of the differential of equilibrium market shares inherent to (20):  $\Delta D_1^i > \Delta D_0^i > \Delta D_3^i > \Delta D_2^i$ , where  $i = 1, 2$  denotes the firm *à la mode* in the initial period. (the reverse inequalities hold in the case of the *démodé* firm).

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<sup>13</sup>Remind that the initially unfashionable firm has a lower initial base of consumers and therefore the cost of adopting negative prices tends to be less significant than the cost incurred by the firm *à la mode*.

## 5 Conclusion

In this paper, we investigate dynamic price competition in a duopoly where consumers are conformist. We consider the conformity effect to be delayed and non cumulative, in the sense that it takes one period of time to observe the consumption trends and to follow them and that distant past trends become irrelevant. To our knowledge, previous papers investigating dynamic price competition in the presence of consumption externalities have been mostly focused on dynamic models of infinite time horizon, relying on equilibrium concepts of Markov Perfect Equilibrium, in certain cases involving linear strategies. Also, most results are obtained through numeric simulation. Our paper departs from this literature in two ways: first, we consider a finite time horizon model and, second, we privilege SPNE in detriment of the Markov Perfect Equilibrium. Under these two assumptions, we show that equilibrium outcomes may substantially differ from the predictions of the existing models dealing with dynamic price competition in industries with positive externalities in consumption.

We have characterized the properties of the SPNE in which both firms have positive market shares, identifying a range of parameters in which such equilibrium exists and it is unique.

We conclude that firms change their pricing behavior as time evolves, with both price strategic complementarity and price strategic substitutability arising along the equilibrium path. We observe that, for the range of parameters in which the interior SPNE exists, the equilibrium path of prices is V-shaped, with firms decreasing their prices in initial periods of interaction and increasing them afterwards. In the intermediary period (corresponding to the valley of the V trajectory), competition is quite tough, with both firms investing in price-cutting strategies that lead to negative equilibrium prices.

With respect to the evolution of market shares, we observe that equilibrium paths are oscillating. Also, we unveil a phenomenon of fashion shift, with the firm initially *à la mode* losing its dominance after one period of interaction. In the context of our model, this fashion shift phenomenon is explained by the fierceness of competition arising in the intermediary period.

Several questions naturally arise from our work. In our future research, we aim to investigate the possibility of eviction outcomes, studying under which conditions firms find it profitable to evict the rival. We also aim to investigate how the length of the interaction period affects our results. To this end, we intend to develop a dynamic duopoly model with  $T$  periods. In the context of this model, we intend to investigate whether the properties of fashion shift and V-shaped equilibrium path prices still hold when the time horizon is extended.

## 6 Appendix

### Proof of Proposition 1

First, one must take into consideration that the candidate SPNE can only constitute an interior SPNE in which both firms are active in the market, if the price strategies that compose  $\mathbf{P}_i^*$ ,  $i = 1, 2$ ,



actually lead to duopolistic equilibrium outcomes. This amounts to say that the following conditions must hold:

$$|p_{2,t}^* (\Delta D_0) - p_{1,t}^* (\Delta D_0) + \alpha \Delta D_{t-1}^* (\Delta D_0)| < \tau, \quad (22)$$

with  $t = 1, 2, 3$ . Substituting  $p_{1,1}^* (\Delta D_0)$  and  $p_{2,1}^* (\Delta D_0)$  in (22), we obtain, that in period  $t = 1$ :

$$|\Delta D_0^i| < \frac{3\tau}{\alpha(4\Omega + 9\tau^2)} \frac{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega}{(4\Omega + 9\tau^2)}.$$

Similarly, in period  $t = 2$ :

$$|\Delta D_0^i| < \frac{1}{3\alpha^2} \frac{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega}{(4\Omega + 9\tau^2)} \quad (23)$$

and, finally, in period  $t = 3$ :

$$|\Delta D_0^i| < \frac{\tau}{\alpha^3} \frac{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega}{(4\Omega + 9\tau^2)}.$$

For  $\alpha \in (\hat{\alpha}(\tau), 3\tau)$ , it follows that condition (23) is more restrictive than the other consistency conditions and therefore this guarantees that  $\mathbf{P}_i^*$  is consistent with an interior equilibrium in which both firms have strictly positive market shares. Under the assumption that there are exit barriers or significant costs associated to no-production (i.e. we impose that both firms are active in the market in every period), to demonstrate that the SPNE exists and is unique, it is sufficient to show that the objective functions in every period of interaction are concave. In period  $t = 3$ , we have

$$\frac{\partial^2 (\pi_{i,3}(p_{i,3}, p_{j,3}, \Delta D_2))}{\partial^2 p_{i,3}} = -\frac{1}{\tau} < 0,$$

therefore the objective function is concave. In period  $t = 2$ , we have

$$\frac{\partial^2 \left( \pi_{i,2}(p_{i,2}, p_{j,2}, \Delta D_1) + \pi_{i,3}^*(p_{i,2}, p_{j,2}, \Delta D_1) \right)}{\partial^2 p_{i,2}} = \frac{1}{9} \frac{\Omega}{\tau^3},$$

which is negative for  $\alpha < 3\tau$ . Finally, in period  $t = 1$ , we obtain

$$\frac{\partial^2 \left( \pi_{i,1}(\circ) + \pi_{i,2}^*(\circ) + \pi_{i,3}^*(\circ) \right)}{\partial^2 p_{i,1}} = -\frac{(4\Omega + 9\tau^2)^2 + (3\alpha)^2 \Omega}{\tau(4\Omega + 9\tau^2)^2},$$

which is negative when  $\alpha > \hat{\alpha}(\tau)$ . In the previous expression, for the sake of simplicity, the vector  $(p_{i,1}, p_{j,1}, \Delta D_0)$  is denoted by  $(\circ)$ . Accordingly, the objective functions are concave in every period and, therefore, the equilibrium exists and it is unique. ■

### Proof of Lemma 1

Firms' contemporaneous prices are said to be strategic complements when

$$\frac{\partial^2 \Pi_{i,t}(p_{i,t}, p_{j,t}, \Delta D_{t-1})}{\partial p_{i,t} \partial p_{j,t}} > 0,$$

or equivalently

$$\frac{\partial p_{i,t}^*(p_{j,t}, \Delta D_{t-1})}{\partial p_{j,t}} > 0,$$

with  $\Pi_{i,t}(p_i, p_j, \Delta D_{t-1})$  standing for the objective function of firm  $i$  in period  $t$  and  $i, j = 1, 2$ ,  $i \neq j$ . Conversely, when the previous inequalities hold with the reverse sign, i.e.

$$\frac{\partial^2 \Pi_{i,t}(p_i, p_j, \Delta D_{t-1})}{\partial p_{i,t} \partial p_{j,t}} < 0,$$

or equivalently,

$$\frac{\partial p_{i,t}^*(p_{i,t}, \Delta D_{t-1})}{\partial p_{j,t}} < 0,$$

firms' contemporaneous prices are said to be strategic substitutes. These derivatives can be directly obtained from (4), (8) and (15).

In period  $t = 3$ , we obtain

$$\frac{\partial^2 \Pi_{1,3}(p_{1,3}, p_{2,3}, \Delta D_2)}{\partial p_{2,3} \partial p_{1,3}} = \frac{\partial^2 \Pi_{2,3}(p_{1,3}, p_{2,3}, \Delta D_2)}{\partial p_{1,3} \partial p_{2,3}} = \frac{1}{2\tau},$$

which is always positive, yielding strategic complementarity on prices. In period  $t = 2$ , we get

$$\frac{\partial^2 \Pi_{1,2}(p_{1,2}, p_{2,2}, \Delta D_1)}{\partial p_{2,2} \partial p_{1,2}} = \frac{\partial^2 \Pi_{2,2}(p_{1,2}, p_{2,2}, \Delta D_1)}{\partial p_{1,2} \partial p_{2,2}} = -\frac{1}{18} \frac{2\Omega + 9\tau^2}{\tau^3},$$

which is negative for any  $\alpha$  such that  $\hat{\alpha}(\tau) < \alpha < 3\tau$ . Accordingly, in period  $t = 2$ , prices are strategic substitutes. Finally, in period  $t = 1$ , we have

$$\frac{\partial^2 \Pi_{1,1}(p_{1,1}, p_{2,1}, \Delta D_0)}{\partial p_{2,1} \partial p_{1,1}} = \frac{\partial^2 \Pi_{2,1}((p_{1,1}, p_{2,1}, \Delta D_0))}{\partial p_{1,1} \partial p_{2,1}} = \frac{(4\Omega + 9\tau^2)^2 + 2(3\alpha)^2 \Omega}{2\tau(4\Omega + 9\tau^2)^2},$$

which is negative when  $\hat{\alpha}(\tau) < \alpha < \hat{\hat{\alpha}}(\tau)$ , and positive otherwise. Hence, when  $\hat{\alpha}(\tau) < \alpha < \hat{\hat{\alpha}}(\tau)$ , prices in period  $t = 1$  are strategic substitutes. In contrast, when  $\hat{\hat{\alpha}}(\tau) < \alpha < 3\tau$ , prices in period  $t = 1$  are strategic complements. ■

### Proof of Proposition 2

First we show that  $p_{i,2}^*(\Delta D_0^i) - p_{i,1}^*(\Delta D_0^i)$  is negative, being equal to

$$\frac{2}{3} \frac{\alpha^3}{(27\tau^2 - 4\alpha^2)} + \frac{1}{9\tau\alpha} \frac{(2\alpha^3 + 27\tau^3 - 18\alpha\tau^2 - 4\alpha^2\tau)(9\alpha\tau - 4\alpha^2 + 27\tau^2)}{(27\tau^2 - 4\alpha^2)} \Delta D_0^i$$

The first term is negative when  $\alpha \in (\hat{\alpha}, 3\tau)$ . Likewise, the second term is positive, hence, the

price difference is increasing in  $\Delta D_0^i$ . If it is negative for the highest possible  $\Delta D_0^i$ , it is negative everywhere. We evaluate the difference upper limit of the relevant domain for  $\Delta D_0^i$ .

$$p_{i,2}^* (\Delta D_0^i) - p_{i,1}^* (\Delta D_0^i) = \frac{1}{9\tau\alpha} (2\alpha^3 + 27\tau^3 - 9\alpha\tau^2 - 10\alpha^2\tau) < 0$$

Likewise  $p_{i,3}^* (\Delta D_0^i) - p_{i,2}^* (\Delta D_0^i)$  is positive, being equal to:

$$\frac{2}{3}\alpha - \frac{1}{3} \frac{(27\tau^2 - 4\alpha^2) (3\tau - 2\alpha) (\alpha + 3\tau) \alpha^2}{(28\alpha^4 + 729\tau^4 - 324\alpha^2\tau^2) \tau} \Delta D_0^i$$

The difference is decreasing in  $\Delta D_0^i$  and hence, if it is positive for the highest value of  $\Delta D_0^i$ , it is positive everywhere.

$$p_{i,3}^* (\Delta D_0^i) - p_{i,2}^* (\Delta D_0^i) = \frac{2}{3}\alpha - \frac{1}{9\tau} (2\alpha - 3\tau) (\alpha + 3\tau) > 0. \blacksquare$$

### Proof of Proposition 3

The trajectory of market shares corresponding to the SPNE  $\mathbf{P}_i^*$  is described in (19), yielding:

$$\begin{aligned} \frac{\partial D_{i,1}^* (\Delta D_0)}{\partial \Delta D_0^i} &= \frac{\alpha}{6\tau} \frac{(4\Omega + 9\tau^2)^2}{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega} > 0, \\ \frac{\partial D_{i,2}^* (\Delta D_0)}{\partial \Delta D_0^i} &= -\frac{3\alpha^2}{2} \frac{(4\Omega + 9\tau^2)}{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega} < 0, \\ \frac{\partial D_{i,3}^* (\Delta D_0)}{\partial \Delta D_0^i} &= -\frac{\alpha^3}{2\tau} \frac{(4\Omega + 9\tau^2)}{(4\Omega + 9\tau^2)^2 + 3(2\alpha)^2 \Omega} < 0. \end{aligned}$$

The negative sign of  $\frac{\partial D_{i,2}^* (\Delta D_0)}{\partial \Delta D_0^i}$  implies that, the firm that accumulates a larger user base in period  $t = 2$  corresponds to the firm endowed with a narrower initial base of customers. Furthermore, it is also the case that  $\frac{\partial D_{i,3}^* (\Delta D_0)}{\partial \Delta D_0^i} < \frac{\partial D_{i,2}^* (\Delta D_0)}{\partial \Delta D_0^i} < 0$ . Accordingly, in period  $t = 3$ , the initially *démodé* firm has a larger market share than the firm that was initially *à la mode*.  $\blacksquare$

## References

- [1] Bernheim, B. D. 1994. A Theory of Conformity, *The Journal of Political Economy*, 102(5), 841-877.
- [2] Bikhchandani, S., Hirshleifer, D., and Welch, I. 1992. A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades, *Journal of Political Economy*, 100(5), 992-1026.
- [3] Bulow, J., Geanakoplos, J. and Klemperer, P. 1985. Multimarket oligopoly: strategic substitutes and strategic complements. *Journal of Political Economy*, 93, 488-511.

- [4] Cabral, L. 2007. Dynamic Price Competition with Network Effects, *Working Paper 22*, Portuguese Competition Authority.
- [5] Corneo, G. and Jeanne, O. 1997. Conspicuous consumption, snobbism and conformism, *Journal of Public Economics*, 66, 55-71
- [6] Corneo, G. and Jeanne, O. 1998. Segmented communication and fashionable behavior, *Journal of Economic Behavior & Organization*, 39, 371-385.
- [7] d'Aspremont, C., Gabszewicz, J.J. and Thisse, J. 1979. On Hotelling's Stability in Competition, *Econometrica*, 17, 1145-1151.
- [8] Doganoglu, T. 2003. Dynamic Competition with Consumption Externalities, *Netnomics*, 5, 116-35.
- [9] Doraszelski, U., Chen, J. and Harrington, J. 2009. Avoiding Market Dominance: Product Compatibility in Markets with Network Effects, *RAND Journal of Economics*, 40(3), 455-485.
- [10] Gabszewicz, J. J. and Garcia, F. 2008. A note on Expanding Networks and Monopoly Pricing, *Economics Letters*, 98, 19-15.
- [11] Gabszewicz, J.J. and Garcia, F. 2007. Optimal Monopoly Price Paths with Expanding Networks, *Review of Network Economics*, 6, 142-49.
- [12] Grilo, I., O. Shy, and J. Thisse. 2001. "Price Competition When Consumer Behavior is Characterized by Conformity or Vanity", *Journal of Public Economics*, 80, 385-408.
- [13] Katz, M. and Shapiro, C. 1985. "Network Effect, Competition, and Compatibility, *American Economic Review*, 75(3), 424-440.
- [14] Laussel, D., Montmarin, M. and Van Long, N. 2004. Dynamic Duopoly with Congestion Effects, *International Journal of Industrial Organization*, 22(5), 655-677.
- [15] Markovich, S. 2008. Snowball: a Dynamic Oligopoly Model with Indirect Network Effects, *Journal of Economic Dynamics & Control*, 32, 909-938.
- [16] Markovich, S. and Moenius, J. 2009. Winning While Losing: Competition Dynamics in the Presence of Indirect Network Effects, *International Journal of Industrial Organization*, 27(3), 346-357.
- [17] Milgrom, P. and Roberts, J. 1990. Rationalizability, learning, and equilibrium in games with strategic complementarities, *Econometrica*, 58, 1255-1277.
- [18] Mitchell, M. and Skrzypacz, A. 2006. Network Externalities and Long-run Market Shares, *Economic Theory*, 29(3), 621-648.

- [19] Topkis, D. M. 1979. Equilibrium points in nonzero-sum n-person submodular games. *SIAM Journal of Control and Optimization*, 17, 773-787.
- [20] Vives, X. 1990. Nash equilibrium with strategic complementarities. *Journal of Mathematical Economics*, 19, 305-321.
- [21] Vives, X. 2005. Complementarities and games: New developments. *Journal of Economic Literature*, 43, 437-479.
- [22] Veblen, T., 1922. *The Theory of the Leisure Class. An Economic Study of Institutions*, George Allen Unwin, London. (First published, 1899).

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