

**TWO-PERIOD ECONOMIES
WITH PRIVATE STATE
VERIFICATION**

JOÃO CORREIA-DA-SILVA ¹
CARLOS HERVÉS-BELOSÓ ²

¹ CEF.UP, FACULDADE DE ECONOMIA, UNIV. DO PORTO

² RGEA, FACULTAD DE ECONÓMICAS, UNIVERSIDAD DE VIGO

U. PORTO

FEP FACULDADE DE ECONOMIA
UNIVERSIDADE DO PORTO

Two-period economies with private state verification^{*}

João Correia-da-Silva

CEF.UP and Faculdade de Economia, Universidade do Porto.

Carlos Hervés-Beloso

RGEA, Facultad de Económicas. Universidad de Vigo.

April 23rd, 2010

Abstract. Private state verification is introduced in a two-period economy with spot markets in both periods and complete futures markets for contingent delivery in the second period. Existence of equilibrium is established, under standard assumptions. An example is presented in which a complete set of contingent markets allows agents to arrive at the optimal allocation of risk-bearing, while securities are not sufficient.

Keywords: General equilibrium, Asymmetric information, Private state verification, Two-period economies.

JEL Classification Numbers: C62, C72, D51, D82.

^{*} We thank Alberto Bennardo, Marco LiCalzi and the participants in the CSEF Workshop on Equilibrium Analysis Under Uncertainty at U. Napoli Federico II for useful comments and suggestions. João Correia-da-Silva (joao@fep.up.pt) acknowledges support from CEF.UP, Fundação para a Ciência e Tecnologia and FEDER (PTDC/ECO/66186/2006). Carlos Hervés-Beloso (cherves@uvigo.es) acknowledges support from Xunta de Galicia (PGIDIT07PXIB300095PR) and Ministerio de Educación y Ciencia (ECO2009-14457-C0401ECON).

1 Introduction

Agents frequently wish to make contracts that are contingent on a future event (like any insurance contract), but the enforcement of such contracts may be problematic if only one party is able to observe the event. Even if both parties observe the event, this may not be sufficient to enforce a contract. It may be necessary to prove to a third party that the event has occurred.

The first attempt to incorporate this kind of information asymmetries in general equilibrium theory was made by Radner (1968), who restricted agents to make contracts that are contingent on events that they can observe. This is too restrictive, as the other party may find it to be in his interest to honor the contract, even if a violation of the contract could be concealed. Such contracts are said to be incentive compatible (Hurwicz, 1972).

Allowing agents to make any incentive compatible contract, Prescott and Townsend (1984a, 1984b) showed the existence of optimal allocations and sought to decentralize them through a price system. However, to induce agents to self-select incentive compatible lotteries, such decentralization requires non-linear prices (Jerez, 2005; Rustichini and Siconolfi, 2008).

Our purpose is to investigate the economic effects of asymmetries in the ability to verify the occurrence of events, in the context of competitive markets (with linear price systems). Our framework may be described as a model of general equilibrium with private and incomplete state verification. While Townsend (1979) studied the effects of costly state verification, we assume that to verify the occurrence of an event is either free or impossible. State verification is incomplete, and this incompleteness varies across agents.

We consider a two-period economy with spot markets in both periods, present and future, and complete futures markets (in the first period) for contingent delivery (in the second period). In the present, being uncertain about the future state of nature, agents trade in the spot markets and in the futures markets. The trade in the spot markets determines present consumption (goods are assumed to be non-durable). The contracts

made in the futures markets determine the bundle that the agents have the right to receive in each of the possible future states of nature. In the second-period spot markets, agents sell the bundle that is delivered to them, together with their second period endowments, to acquire their second-period consumption bundle. It is assumed that agents trade in the present anticipating the future spot prices and, therefore, the bundle that they will be able to consume in the future.¹

This market structure coincides with that of Arrow (1953) and Debreu (1959). The difference here is that agents are assumed to have incomplete and asymmetric abilities to verify the future state of nature. Each agent has an exogenously given information structure, which is a partition of the set of possible states of nature. In the future, with the objective of enforcing the contracts made in the present, all that agents can verify is that the state of nature belongs to a certain element of their information partition.

We assume that trade in the futures markets is mediated by profit-maximizing firms.² In the first period, each agent makes a contract with one of these firms, stipulating a net trade for each of the possible future states of nature. In the second period, given the agent's incomplete ability to verify the states of nature, the firm may have the opportunity of delivering a less valuable net trade. We assume that, in case of litigation between the agent and the firm, it is the agent that bears the burden of the proof, and that her ability to prove that a certain state of nature has occurred or not is exogenous and described by the agent's information partition. The firm may choose, therefore, among the net trades that corresponds to states of nature that the agent cannot distinguish from the true state of nature. As a result, the agent always receives, in each state of nature, the less valuable of those possible net trades (according to the spot prices in that state of nature). In the spirit of the revelation principle (Myerson, 1979), we restrict agents to make trades which induce truthful deliveries.

¹This modifies the model of an economy with uncertain delivery (Correia-da-Silva and Hervés-Beloso, 2008, 2009a, 2009b) by opening spot markets in the second period.

²This was also assumed by Prescott and Townsend (1984a, 1984b), Jerez (2005), Bisin and Gottardi (2006) and Rustichini and Siconolfi (2008).

It is assumed that agents cannot use prices to prove to a third party that a certain state has occurred. This contrasts with what is assumed in the works of Radner (1979) and Allen (1981). We rule out revelation through prices because it eliminates the information asymmetries, thus rendering the model useless to explain their economic effects. Furthermore, allowing revelation through prices would lead to an implicit assumption that every contract can be enforced. We assume that, even if prices allow an agent to infer the true state of nature, this is useless as a means of enforcing contracts.

In our framework, there are contracts which cannot be enforced because agents have incomplete information. Markets are, therefore, incomplete. But in a fundamentally different way from that considered by Radner (1972) and Magill and Quinzii (1996). Here, each agent faces different trade possibilities, which are, in addition, endogenous.

The “Hidden Information Economy” of Bisin and Gottardi (1999) is closely related to ours. The main differences are the following: (i) they consider an aggregate shock that is publicly observed and an idiosyncratic shock that is privately observed, while we consider the more general case of uncertainty described by a set of future states of nature and private verification described by agent-specific information partitions; (ii) we allow for state-dependent preferences and consider concave utility functions instead of strictly concave; (iii) we consider a single agent of each type, instead of countably many; (iv) we consider a complete set of markets for contingent delivery, while they only consider securities that are payable in a numeraire good; (v) we suppose that each agent can only use her information to enforce contracts, while they allow the outcome of trade for one agent to depend on messages that are sent by others.

Our main result is the proof of existence of equilibrium, under standard assumptions. The usual techniques are not sufficient because, as a consequence of restricting agents to make trades that induce truthful delivery, the choice set is not lower hemicontinuous with respect to prices. Adapting a technique used in a related contribution (Correia-da-Silva and Hervés-Beloso, 2009b), we start by constructing a sequence of economies in which a violation of the truthful delivery restrictions is possible, but implies utility penalties that

are increasingly harsh along the sequence. After obtaining the corresponding sequence of equilibria, we prove that an accumulation point of this sequence is an equilibrium of the economy under study.

We illustrate our equilibrium concept by way of a simple insurance example. In this example, it turns out that agents are able to attain the optimal allocation of risk-bearing (as in the case of complete information). However, if we restrict the deliveries contracted in the futures markets to be payable in a numeraire good, then the optimal allocation of risk-bearing is not attainable. This contrasts with the result obtained by Arrow (1953) for economies with public state verification.

The paper is organized as follows. In Section 2, we present the model of a two-period economy with private state verification and establish existence of equilibrium. In Section 3, we present and discuss an illustrative example. In Section 4, we conclude the paper with some remarks.

2 The model

We consider an economy that extends over two time periods, the present ($\tau = 0$) and the future ($\tau = 1$), in which a finite number of agents, $\mathcal{I} = \{1, \dots, I\}$, trade a finite number of commodities, $\mathcal{L} = \{1, \dots, L\}$.

In the present, there is uncertainty about the state of the environment that will prevail in the future. There is a finite set of possible states of nature, $\mathcal{S} = \{1, \dots, S\}$, and agents agree that the probabilities of occurrence of each state are given by $\mu \in \Delta^S$.

Each agent's private information is described by a partition of \mathcal{S} . Agent i knows that if state s occurs, she will only be able to prove that the state of nature belongs to the element of her information partition that contains s , which is denoted by $P^i(s)$.

The initial endowments of agent i are $e_0^i \in \mathbb{R}_+^L$ and $e_1^i \in \mathbb{R}_+^{SL}$.

Assumption 1 (Endowments).

The endowments of each agent are strictly positive: $e_0^i \gg 0$ and $e_1^i \gg 0$, $\forall i \in \mathcal{I}$.

The agent's preferences about consumption in both periods, (x_0^i, x_1^i) , are described by an utility function, $U^i : \mathbb{R}_+^L \times \mathbb{R}_+^{SL} \rightarrow \mathbb{R}$.

Assumption 2 (Preferences).

*The utility functions of the agents are continuous, concave and strictly increasing.*³

There are spot markets at $\tau = 0$ and at $\tau = 1$, and futures markets at $\tau = 0$ for contingent delivery at $\tau = 1$. The deliveries contracted in the futures markets may be conditional on the occurrence of any event (set of states of nature), thus each agent i chooses a plan of net deliveries, specifying what she should receive in each state of nature, $y^i = (y^i(1), \dots, y^i(s), \dots, y^i(S)) \in \mathbb{R}^{SL}$.

The prices in the spot markets are denoted by p_0 and p_1 , and prices in the futures markets are denoted by q . We normalize prices by imposing that $(p_0, q) \in \Delta^{L+SL}$ and that $p_1(s) \in \Delta^L$, for each $s \in \mathcal{S}$.

At $\tau = 0$, agent i trades her endowments, e_0^i , for a consumption bundle, $x_0^i \in \mathbb{R}_+^L$, and a plan of future net deliveries, $y^i \in \mathbb{R}^{SL}$. The corresponding budget restriction is:

$$(x_0^i, y^i) \in B^i(p_0, q) = \{(z_0, w) \in \mathbb{R}_+^L \times \mathbb{R}^{SL} : p_0 \cdot z_0 + q \cdot w \leq p_0 \cdot e_0^i\}.$$

Trade in the futures markets is mediated by profit-maximizing intermediaries, who are also price takers. The relationship between agents and financial intermediaries is asymmetric, as it is the agent that bears the burden of proof. At $\tau = 1$, if state s occurs, each agent i can only prove that the state of nature belongs to $P^i(s)$, therefore, the

³By strictly increasing, it is meant that an increase in consumption of any of the goods is strictly desired by the agents: $(x_0^i, x_1^i) \geq (z_0^i, z_1^i)$ and $(x_0^i, x_1^i) \neq (z_0^i, z_1^i)$ implies that $U^i(x_0^i, x_1^i) > U^i(z_0^i, z_1^i)$.

financial intermediaries decide which of the alternatives among $\{y^i(t)\}_{t \in P^i(s)}$ is delivered to each agent i .⁴ Profit maximization implies that only the cheapest alternatives, according to $p_1(s)$, may be delivered.

Hence, agents receive, in each state s , one of the cheapest bundles among those that they cannot prove, using only $P^i(s)$, that are not the truthful delivery.⁵ Accordingly, we can restrict (without loss of generality) the choice of agent i to satisfy the following restrictions, which induce truthful delivery:⁶

$$y^i \in D^i(p_1) = \{w \in \mathbb{R}^{SL} : p_1(s) \cdot w(s) \leq p_1(s) \cdot w(t), \forall t \in P^i(s), \forall s \in \mathcal{S}\}.$$

At $\tau = 1$, in state s , agent i receives $y^i(s)$ (truthful delivery), which she trades, together with her endowments, $e_1^i(s)$, for a consumption bundle, $x_1^i(s) \in \mathbb{R}_+^L$. The corresponding budget restriction is:

$$x_1^i(s) \in B_s^i(p_1(s), y^i(s)) = \{z_1(s) \in \mathbb{R}_+^L : p_1(s) \cdot z_1(s) \leq p_1(s) \cdot [y^i(s) + e_1^i(s)]\}.$$

The budget set for future consumption in all states, $B_1^i(p_1, y^i)$, is defined as follows:

$$x_1^i \in B_1^i(p_1, y^i) \Leftrightarrow x_1^i(s) \in B_s^i(p_1(s), y^i(s)), \forall s \in \mathcal{S}.$$

Let $x^i = (x_0^i, y^i, x_1^i)$, $e^i = (e_0^i, 0, e_1^i)$ and $p = (p_0, q, p_1)$. We write $x^i \in B^i(p)$ whenever $(x_0^i, y^i) \in B_0^i(p_0, q)$ and $x_1^i \in B_1^i(p_1, y^i)$.

The choice set of agent i is, therefore:

$$C^i(p) = \{x^i = (x_0^i, y^i, x_1^i) \in \mathbb{R}_+^L \times \mathbb{R}^{SL} \times \mathbb{R}_+^{SL} : x^i \in B^i(p) \wedge y^i \in D^i(p_1)\}.$$

⁴It is assumed that the information conveyed by prices cannot be used to enforce contracts.

⁵See Correia-da-Silva and Hervés-Beloso (2008, 2009a, 2009b) for a more detailed justification.

⁶The choice of $y^i \notin D^i(p_1)$ would never be optimal, as it would lead to the delivery of some $w^i \in D^i(p_1)$, cheaper than y^i . The agent would be better off by choosing w^i instead of y^i .

In sum, the problem of agent i can be written as:

$$\begin{aligned}
& \max U^i(x_0^i, x_1^i) \\
& \text{s.t. } p_0 \cdot x_0^i + q \cdot y^i \leq p_0 \cdot e_0^i, \\
& \quad p_1(s) \cdot x_1^i(s) \leq p_1(s) \cdot y^i(s) + p_1(s) \cdot e_1^i(s), \quad \forall s \in \mathcal{S}, \\
& \quad p_1(s) \cdot y^i(s) \leq p_1(s) \cdot y^i(t), \quad \forall t \in P^i(s), \quad \forall s \in \mathcal{S}.
\end{aligned}$$

Or, equivalently, as:

$$\max U^i(x_0^i, x_1^i) \quad \text{s.t. } x^i \in C^i(p).$$

The choice of the financial intermediaries is denoted $x^f = (x_0^f, y^f, x_1^f) \in B^f(p)$, where the choice set, $B^f(p)$, is defined as $B^i(p)$ but with null endowments.⁷ We assume that they wish to maximize an objective function that is strictly increasing:⁸

$$\max U^f(x_0^f, x_1^f) \quad \text{s.t. } x^f \in B^f(p).$$

The demand of the financial intermediaries becomes unbounded whenever, for some state s , the relative prices in the spot markets at $\tau = 1$ are different from the relative prices in the futures markets for contingent delivery in this state. That is, there are arbitrage opportunities unless we have $q(s)$ parallel to $p_1(s)$, for all $s \in \mathcal{S}$. If, for every state of nature, the prices in the futures markets and the prices in the spot markets are parallel, the financial intermediaries cannot obtain any positive consumption plan, $(x_0^f, x_1^f) \neq 0$, and are, therefore, indifferent among any alternative in their choice set:

$$q(s) \parallel p_1(s), \quad \forall s \in \mathcal{S} \Rightarrow q \cdot y^f = 0, \quad \forall x^f \in B^f(p).$$

⁷As long as free entry is allowed, the number of financial intermediaries is irrelevant. We can assume that they behave as a single intermediary.

⁸It should be clear that any weighted function of their present and future profits is a particular case.

Hence, from now on, we will restrict our search for equilibrium prices to the following set of *no arbitrage* price systems:

$$\mathcal{P} = \{(p_0, q, p_1) \in \Delta^{L+SL} \times \Delta^L : \forall s \in \mathcal{S}, q(s) \parallel p_1(s)\},$$

and suppose that the financial intermediaries clear the futures markets by choosing:

$$x^f = (0, y^f, 0), \quad \text{with} \quad y^f = - \sum_{i \in \mathcal{I}} y^i,$$

which is an optimal choice that belongs to their choice set.

If agents make optimal choices and markets clear, the economy is in equilibrium.

Definition 1 (Equilibrium).

An equilibrium of an economy, $\mathcal{E} = \{e^i, U^i, P^i\}_{i \in \mathcal{I}}$, is a pair (x^*, p^*) , where x^* is a vector of individual choices, $x^* = \{x^{i*}\}_{i \in \mathcal{I}}$, and $p^* \in \mathcal{P}$ is a price system, satisfying:

$$(i) \quad x^{i*} \in \operatorname{argmax}_{z \in C^i(p^*)} U^i(z_0, z_1), \quad \forall i \in \mathcal{I} \text{ [individual optimality];}$$

$$(ii) \quad \sum_{i \in \mathcal{I}} (x_0^{i*}, x_1^{i*}) = \sum_{i \in \mathcal{I}} (e_0^i, e_1^i) \text{ [feasibility].}$$

To establish existence of equilibrium, we construct a sequence of economies in which the choice set of each agent i is $B_i(p)$ instead of $C_i(p)$. However, the choice of an $x_i \notin C_i(p)$ implies a utility penalty that is increasingly harsher along the sequence of economies. After obtaining a corresponding sequence of equilibria, we prove that an accumulation point (which exists) is an equilibrium of the original economy.⁹ Under Assumptions 1 and 2, existence of equilibrium is guaranteed.

⁹If the correspondences from prices to the choice sets, $C_i(p)$, were continuous, it would be straightforward to establish existence of equilibrium (Debreu, 1952). But the choice correspondences are not lower hemicontinuous. This property fails when prices in some state are null ($\exists s : p^1(s) = 0$) or when prices in two indistinguished states are collinear ($\exists s, t \in P_i(s), k \in \mathbb{R}_{++} : p^1(s) = kp^1(t)$).

Theorem 1 (Existence).

There exists an equilibrium of the economy $\mathcal{E} = \{e_i, U_i, P_i\}_{i \in \mathcal{I}}$.

The welfare theorems do not necessarily hold. The informational asymmetry may generate an inefficient allocation of risk-bearing, because the uninformed agents may not be able to make the desired wealth transfers across states and time.

Interestingly, the existence of markets for the future delivery of various goods (as opposed to contingent claims that are only payable in the numeraire good) generates further possibilities for the transference of wealth across states and time. We show this by way of an example, presented below, in which a complete set of contingent markets allows agents to arrive at the optimal allocation of risk-bearing (the same as in the case of complete information), while securities are not sufficient. This contrasts with the general result obtained by Arrow (1953) for the case of public state verification.

3 An example

Consider an economy with two agents who have the same preferences and the same present endowment. They differ in their future endowments: while agent A has a certain future endowment, agent B has a risky one. This is a classic setup, in which agent A should provide insurance to agent B in exchange for a premium. But here we introduce an obstacle to trade, by assuming that agent A will not be able to verify the realization of the endowment of agent B .

We denote by x_{sl}^i the quantity of good $l \in \{1, 2\}$ that is consumed by agent $i \in \{1, 2\}$ in state $s \in \{0\} \cup \{1, 2\}$, where 0 stands for the present and $\{1, 2\}$ is the set of possible future states of nature. The corresponding spot price is denoted by p_{sl} . In the futures markets, quantities and prices are denoted by y_{sl}^i and q_{sl} . For compactness, a pair (z_{s1}, z_{s2}) is denoted by z_s .

The preferences of the agents are described by:

$$U^i = x_{01}^{i \frac{1}{4}} x_{02}^{i \frac{1}{4}} + \frac{1}{2} x_{11}^{i \frac{1}{4}} x_{12}^{i \frac{1}{4}} + \frac{1}{2} x_{21}^{i \frac{1}{4}} x_{22}^{i \frac{1}{4}},$$

and their endowments are:

$$\begin{aligned} [(e_{01}^A, e_{02}^A); (e_{11}^A, e_{12}^A); (e_{21}^A, e_{22}^A)] &= [(2, 2); (2, 2); (2, 2)], \\ [(e_{01}^B, e_{02}^B); (e_{11}^B, e_{12}^B); (e_{21}^B, e_{22}^B)] &= [(2, 2); (1, 1); (4, 1)]. \end{aligned}$$

The problem of agent A , who cannot verify the state of nature, is the following:

$$\begin{aligned} \max \quad & \left\{ x_{01}^A \frac{1}{4} x_{02}^A \frac{1}{4} + \frac{1}{2} x_{11}^A \frac{1}{4} x_{12}^A \frac{1}{4} + \frac{1}{2} x_{21}^A \frac{1}{4} x_{22}^A \frac{1}{4} \right\} \\ \text{s.t.} \quad & \\ & p_0 \cdot (x_0^A - e_0^A) + q_1 \cdot y_1^A + q_2 \cdot y_2^A \leq 0, \\ & p_1 \cdot (x_1^A - y_1^A - e_1^A) \leq 0, \\ & p_2 \cdot (x_2^A - y_2^A - e_2^A) \leq 0, \\ & p_1 \cdot (y_1^A - y_2^A) \leq 0, \\ & p_2 \cdot (y_2^A - y_1^A) \leq 0. \end{aligned}$$

While the problem of the fully informed agent B is:

$$\begin{aligned} \max \quad & \left\{ x_{01}^B \frac{1}{4} x_{02}^B \frac{1}{4} + \frac{1}{2} x_{11}^B \frac{1}{4} x_{12}^B \frac{1}{4} + \frac{1}{2} x_{21}^B \frac{1}{4} x_{22}^B \frac{1}{4} \right\} \\ \text{s.t.} \quad & \\ & p_0 \cdot (x_0^B - e_0^B) + q_1 \cdot y_1^B + q_2 \cdot y_2^B \leq 0, \\ & p_1 \cdot (x_1^B - y_1^B - e_1^B) \leq 0, \\ & p_2 \cdot (x_2^B - y_2^B - e_2^B) \leq 0. \end{aligned}$$

For convenience of exposition, in this example we will use a different normalization

from that of the general model, choosing good 2 as the numeraire. We set $p_{s2} = 1$ for each $s \in 0, 1, 2$.

Since the instantaneous utility functions have equal elasticities with respect to goods 1 and 2, the agents find it optimal to spend equal shares of their incomes in each of the goods (in a given state). Therefore, in a given state, the relative prices of goods 1 and 2 are the inverse of the ratio between the aggregate endowments of each good:

$$p_{01} = p_{02} = 1, \quad p_{11} = p_{12} = 1, \quad \text{and} \quad 2p_{21} = p_{22} = 1.$$

And it is optimal for the agents to consume quantities of goods 1 and 2 that are inversely proportional to their prices:

$$x_{01} = x_{02}, \quad x_{11} = x_{12} \quad \text{and} \quad x_{21} = 2x_{22}.$$

The relative prices in the futures markets must be, as we have shown before, equal to the relative prices in the future spot markets:

$$q_{11} = q_{12} \quad \text{and} \quad 2q_{21} = q_{22}.$$

After some calculations, we find that the deliverability restrictions do not constrain the choice of agent A . The solution is the same as in the case of perfect information. This is somewhat surprising.

The equilibrium prices and quantities are the following:

$$\begin{aligned} (p_0, p_1, p_2, q_1, q_2) &= [(1, 1); (1, 1); (0.5, 1); (0.577, 0.577); (0.343, 0.687)], \\ (x_0^A, x_1^A, x_2^A) &= [(2.148, 2.148); (1.611, 1.611); (3.222, 1.611)], \\ (x_0^B, x_1^B, x_2^B) &= [(1.852, 1.852); (1.389, 1.389); (2.778, 1.389)], \\ (y_1^A, y_2^A) &= - (y_1^B, y_2^B) = [(-2.000, 1.222); (-2.000, 1.222)]. \end{aligned}$$

Agent A transfers wealth from state 1 to state 2 and to the present (while agent B does, necessarily, the opposite transfer, from the present and from state 2 to state 1).

The equality of the claims for delivery to agent A in states 1 and 2 is not essential. We remain in equilibrium if, for example, $(y_1^A, y_2^A) = [(-3.000, 2.222); (0.000, 0.222)]$. There is an indeterminacy of equilibrium related to the different possibilities of transferring wealth across time and states using the available assets. But the equilibrium values of x , p and q are unique.

With perfect information, it is clear that securities markets are enough for the desired wealth transfers to take place, as shown by Arrow (1953). But with asymmetric information, if there were only future markets for delivery of the numeraire good, then agent A would not be able to make the desired transfers. Suppose that $(y_{12}^A, y_{22}^A) = (-0.778, 0.222)$. If state 2 occurs, agent A has the right to receive 0.222 units of the numeraire. However, she cannot prove that the state of nature is not 1, in which she has the obligation to deliver 0.778 units of the numeraire. Under the assumptions of our economic scenario, she would be forced to pay 0.778 units instead of receiving 0.222 units.

Anticipating this enforcement issue, agent A would choose a pair (y_{12}^A, y_{22}^A) that satisfies the deliverability conditions. She would choose $y_{12}^A = y_{22}^A$, implying that $x_{12}^A = x_{22}^A + \frac{1}{2}$.

In this incomplete markets setup, the solution would be given by:

$$\begin{aligned} (p_0, p_1, p_2) &= [(1, 1); (1, 1); (0.5, 1)], \\ (q_{12}, q_{22}) &= (0.642, 0.635), \\ (x_0^A, x_1^A, x_2^A) &= [(2.153, 2.153); (1.881, 1.881); (2.761, 1.381)], \\ (x_0^B, x_1^B, x_2^B) &= [(1.847, 1.847); (1.119, 1.119); (3.239, 1.619)], \\ (y_{12}^A, y_{22}^A) &= - (y_{12}^B, y_{22}^B) = (-0.239, -0.239). \end{aligned}$$

With private state verification, the markets for future delivery of commodities other than the numeraire play, therefore, a relevant role. They expand the possibilities for

wealth transfers across states and time. Agents are able to induce truthful delivery by choosing, for delivery in each state, goods that are relatively cheap in this state but relatively expensive in the other states.

We remark that, in spite of the fact that the relative prices for future delivery in a given state coincide with the relative prices in the future spot markets in the same state, the agents do not buy, in the futures markets, the bundle that they desire to consume in the future (this would render the future spot markets irrelevant). In the futures markets, agents select a “bridge portfolio”, not intended for consumption, but to induce the desired wealth transfers in the absence of complete state verification.

4 Conclusion

In a seminal work, Arrow (1953) has shown that an optimal allocation of risk-bearing could be achieved by a system of securities and commodity markets, with securities being payable in money. This permits economizing on markets. Only $S + L$ markets (where S is the number of states of nature, and L is the number of commodities) are needed, instead of a complete set of markets for contingent claims on commodities, which totals a number of SL markets.

We have seen that if agents have incomplete abilities to verify the occurrence of relevant events, then this is not the case. What was a redundancy in the ways of transferring wealth across states, becomes useful as a means of enforcing truthful deliveries. It may be the case that a complete set of contingent markets allows agents to arrive at an optimal allocation of risk-bearing, while a system of securities and commodity markets does not.

5 Appendix

Proof of Theorem 1.

We start by constructing a sequence of economies without differential information, $\{\mathcal{E}_n\}_{n \in \mathbf{N}}$. In each economy of the sequence, agents have the same endowments as in the economy under study, but modified utility functions. The choice set of each agent i is $B^i(p)$ instead of $C^i(p)$, but agent i suffers a utility penalty if she chooses an $x^i \notin C^i(p)$. These penalties become harsher along the sequence.

In the economy $\mathcal{E}_n = \{e^i, U_n^i\}_{i \in \mathcal{I}}$, the utility functions of the agents are:¹⁰

$$U_n^i(x^i, p_1) = U^i(x_0^i, x_1^i) - n \sum_{s \in \mathcal{S}} \mu(s) \max_{t \in P^i(s)} \{p_1(s) \cdot y^i(s) - p_1(s) \cdot y^i(t)\}.$$

It is obvious that, for any $n \in \mathbf{N}$, the utility functions, U_n^i , are continuous. The maximum of linear functions is a convex function, and multiplying a convex function by a negative constant, $-n$, yields a concave function. Hence, the objective functions, $U_n^i(x^i, p_1)$, are concave in the first variable. Observe also that the utility penalty preserves *no satiation*. The plan $x^i + \epsilon \bar{1}$ is always preferred to x^i (the utility penalty is kept constant).

To show existence of competitive equilibrium in \mathcal{E}_n , consider, for now, the following convex and bounded choice space:

$$\bar{X} = \{z \in \mathbb{R}_+^L \times \mathbb{R}^{SL} \times \mathbb{R}_+^{SL} : (0, -2e_1^T, 0) \leq (z_0, w, z_1) \leq (2e_0^T, 2e_1^T, 2e_1^T)\}.$$

The budget correspondence of agent i , in this bounded economy, is:

$$\bar{B}^i(p) = B^i(p) \cap \bar{X}.$$

¹⁰Notice that, since $s \in P^i(s)$, penalties are never negative.

For each $i \in \mathcal{I}$, let $\psi_n^i(x, p) = \operatorname{argmax}_{z^i \in \bar{B}^i(p)} \{U_n^i(z^i, p_1)\}$.

By Lemma 1, the budget correspondences, $\bar{B}^i(p)$, are continuous with nonempty compact values. Hence, by Berge's Maximum Theorem, the demand correspondence, $\psi_n^i(x, p)$, is u.h.c. with nonempty compact values.¹¹ It is also convex-valued, because U_n^i is concave in the first variable.

An auctioneer chooses a price system with the objective of maximizing the value of excess demand. Since \mathcal{P} is not convex, let the auctioneer choose prices with $(p_0, p_1) \in \Delta^{L+SL}$ and $q = p_1$, and denote this space by $\hat{\mathcal{P}}$.

$$\text{Let } \psi_n^p(x, p) = \operatorname{argmax}_{p \in \hat{\mathcal{P}}} \left\{ p'_0 \cdot \sum_{i \in \mathcal{I}} (x_0^i - e_0^i) + p'_1 \cdot \sum_{i \in \mathcal{I}} (x_1^i - e_1^i) \right\}.$$

This correspondence is also u.h.c. with nonempty compact and convex values. Therefore, the product correspondence, $\psi_n = \prod_{i \in \mathcal{I}} \psi_n^i \times \psi_n^p$, also is. Applying the Theorem of Kakutani, we find that there exists a fixed point of ψ_n , that we denote by (x_n, p_n) . To prove that it is an equilibrium of \mathcal{E}_n , we must show that it satisfies feasibility.

Suppose that there is excess demand for some good. If another good does not have excess demand, its price must be zero, which, in turn, implies excess demand. Hence, there must be excess demand for all the goods in the spot markets (at $\tau = 0$ and at $\tau = 1$).

Aggregating the budget restrictions at $\tau = 0$, we obtain (recall that $q = p_1$):

$$\sum_{i \in \mathcal{I}} p_1 \cdot y^i \leq \sum_{i \in \mathcal{I}} p_0 \cdot (e_0^i - x_0^i) \leq 0.$$

On the other hand, aggregating the budget restrictions at $\tau = 1$, we obtain:

$$\sum_{i \in \mathcal{I}} p_1(s) \cdot y^i(s) \geq \sum_{i \in \mathcal{I}} p_1(s) \cdot [x_1^i(s) - e_1^i(s)] \geq 0, \quad \forall s \in \mathcal{S}.$$

¹¹See, for example, Aliprantis and Border (2006).

This implies that:

$$\sum_{i \in \mathcal{I}} p_1(s) \cdot y^i(s) = \sum_{i \in \mathcal{I}} p_1(s) \cdot [e_1^i(s) - x_1^i(s)] = 0, \quad \forall s \in \mathcal{S}.$$

Therefore, $p_0 = 0$ and $p_1 = 0$. Contradiction. There is no excess demand.

The usual extension to the unbounded choice set applies, therefore, (x_n, p_n) is an equilibrium of $\mathcal{E}_n = \{e^i, U_n^i\}_{i \in \mathcal{I}}$. Convert the price system from $\hat{\mathcal{P}}$ to \mathcal{P} , dividing each $p_{1n}(s)$ by $\|p_{1n}(s)\|_1$.

The resulting sequence of equilibria, $\{(x_n, p_n)\}_{n \in \mathbf{N}}$, which is contained in a compact set, has an accumulation point, denoted by (x^*, p^*) . This is our candidate for an equilibrium of the original economy.

It is straightforward to see that x^* is feasible, $\sum_{i \in \mathcal{I}} x^{i*} \leq \sum_{i \in \mathcal{I}} e^i$, and that it satisfies the budget restrictions, $x^{i*} \in B^i(p^*)$, $\forall i \in \mathcal{I}$.

Suppose that x^{i*} violated one of the delivery restrictions, $x^{i*} \notin D^i(p_1^*)$, by more than $\delta > 0$. Then, for sufficiently high n , x_n^i would also violate the corresponding restriction by more than δ . For $t \in P^i(s)$, $\exists n_0 \in \mathbf{N}$ such that, for all $n > n_0$:

$$p_1^*(s) \cdot y^{i*}(s) > p_1^*(s) \cdot y^{i*}(t) + \delta \Rightarrow p_{1n}(s) \cdot y_n^i(s) > p_{1n}(s) \cdot y_n^i(t) + \delta.$$

Utility among feasible allocations is bounded by $U^i(e^T)$, so we can consider a n_0 that is sufficiently high for $n_0\delta > U^i(e^T) - U^i(e^i)$. It would follow that $U_n^i(x_n^i, p_n) < U^i(x_n^i) - n_0\delta < U^i(x_n^i) - U^i(e^T) + U^i(e^i) < U^i(e^i) = U_n^i(e^i, p_n)$. Contradiction.

To establish that (x^*, p^*) is an equilibrium, we only need to prove that each x^{i*} is individually optimal at prices p^* .

Individual optimality of x^{i*} .

Assume (by way of contradiction) that there exists $x' \in C^i(p^*)$ such that $U^i(x') > U^i(x^{i*})$.

We will show that this implies that (x_n, p_n) is not an equilibrium of \mathcal{E}_n , for large n .

Observe that if $p_1(s) = p_1(t)$ with $t \in P^i(s)$, the deliverability conditions imply:

$$\begin{cases} p_1^*(s) \cdot [y'(s) - y'(t)] \leq 0 \\ p_1^*(t) \cdot [y'(t) - y'(s)] \leq 0 \end{cases} \Rightarrow \begin{cases} p_1^*(s) \cdot [y'(s) - y'(t)] = 0 \\ p_1^*(t) \cdot [y'(s) - y'(t)] = 0 \\ q^*(s) \cdot [y'(s) - y'(t)] = 0 \\ q^*(t) \cdot [y'(s) - y'(t)] = 0. \end{cases}$$

Therefore, the agent obtains the same utility by choosing $y''(s) = y''(t) = \frac{y'(s) + y'(t)}{2}$ instead of $y'(s)$ and $y'(t)$. Define $w \in C^i(p^*)$ by modifying y' in this way.

By continuity of U^i , there exists $\delta > 0$ such that $x'' = (1 - \delta)w$ is strictly preferred to x^{i*} , belongs to $C^i(p^*)$, is in the interior of $B^i(p^*)$, and is also in the interior of $B^i(p_n)$, for n greater than some n_0 .

Furthermore, there exists $\epsilon > 0$ such that $d(z, x'') < \epsilon$ implies that $U^i(z) > U^i(x^{i*})$, with z in the interior of $B^i(p^*)$. There also exists $n_1 > n_0$ such that $d(z, x'') < \epsilon$ implies that z is in the interior of $B^i(p_n)$ and that $U^i(z) > U_i(x_n^i)$ (notice that we are considering U^i and not U_n^i), for all $n > n_1$.

Let $n_2 > n_1$ be sufficiently large for $d(p_n, p^*) < \epsilon, \forall n > n_2$.

To finish the proof, we will construct $\hat{x} \in B(x'', \epsilon)$ that belongs to $C^i(p_n)$, contradicting the fact that x_n^i maximizes U_n^i at prices p_n .

Let $k^{(s,t)} = p_1^*(s) \cdot [y''(t) - y''(s)]$. Since $x'' \in C^i(p^*)$:

$$t \in P^i(s) \Rightarrow p_1^*(s) \cdot [y''(t) - y''(s)] = k^{(s,t)} \geq 0.$$

Let $d\hat{x} = \hat{x} - x''$ and $dp_n = p_n - p^*$. Manipulating a deliverability condition:

$$\begin{aligned}
p_1^*(s) \cdot [y''(t) - y''(s)] &= k^{(s,t)} \Leftrightarrow \\
\Leftrightarrow [p_{1n}(s) - dp_{1n}(s)] \cdot [\hat{y}(t) - d\hat{y}(t) - \hat{y}(s) + d\hat{y}(s)] &= k^{(s,t)} \Leftrightarrow \\
\Leftrightarrow p_{1n}(s) \cdot [\hat{y}(t) - \hat{y}(s)] = k^{(s,t)} + p_{1n}(s) \cdot [d\hat{y}(t) - d\hat{y}(s)] + dp_{1n}(s) \cdot [y''(t) - y''(s)] &\Leftrightarrow \\
\Leftrightarrow p_{1n}(s) \cdot [\hat{y}(t) - \hat{y}(s)] > k^{(s,t)} - 2\epsilon - 2\epsilon\|e_T\|. &
\end{aligned}$$

Define k^{min} as the minimum among the strictly positive $k^{(s,t)}$.

Choose a smaller $\epsilon > 0$, if necessary, to make $2\epsilon(\|e_T\| + 1) < k^{min}$. This guarantees that the strict inequalities for x'' and p_1^* remain strict for any $\hat{x} \in B(x'', \epsilon)$ and p_{1n} with $n > n_2$. If all $k^{(s,t)}$ were strictly positive, then \hat{x} would have no utility penalty. We would have $U_n^i(\hat{x}) > U_n^i(x_n^i)$, which would be a contradiction (the consumption plan in the equilibrium sequence, x_n^i , would not be a maximizer of U_n^i).

If some inequalities are not strict for x'' and p_1^* , we need to guarantee that they are still satisfied for some $\hat{x} \in B(x'', \epsilon)$ and some p_{1n} with $n > n_2$.

Select displacements from y'' to \hat{y} that are parallel to p_1^* , choosing:

$$d\hat{y}(s) = -\frac{\epsilon}{2} \frac{p_1^*(s)}{\|p_1^*(s)\|}.$$

Now define $\gamma^{(s,t)} = \left(1 - \frac{p_1^*(s) \cdot p_1^*(t)}{\|p_1^*(s)\| \|p_1^*(t)\|}\right) \|p_1^*(s)\|$. Notice that $\gamma^{(s,t)} = 0$ if and only if $p_1^*(s) = p_1^*(t)$. Let γ^{min} as the lowest of the strictly positive $\gamma^{(s,t)}$.

Let $\epsilon_2 = \frac{\epsilon\gamma^{min}}{4\|e^T\|}$, and consider some $n_3 > n_2$ that is large enough for:

$$d(p_n, p^*) < \min\{\epsilon_2, \epsilon\}, \forall n > n_3.$$

Consider an inequality that is not strict for p^* and w'' , i.e., some $k^{ab} = 0$. If $p_1(a) \neq p_1(b)$,

we have $\gamma^{ab} \geq \gamma^{min}$. This inequality still holds for p_n , with $n > n_3$, and \hat{y} :

$$\begin{aligned}
& p_{1n}(a) \cdot [\hat{y}(b) - \hat{y}(a)] = \\
& = p_1^*(a) \cdot [w''(b) + d\hat{y}(b) - w''(a) - d\hat{y}(a)] + dp_{1n}(a) \cdot [\hat{y}(b) - \hat{y}(a)] = \\
& = p_1^*(a) \cdot [d\hat{y}(b) - d\hat{y}(a)] + dp_{1n}(a) \cdot [\hat{y}(b) - \hat{y}(a)] > \\
& > p_1^*(a) \cdot [d\hat{y}(b) - d\hat{y}(a)] - 2\epsilon_2 \|e_T\| = \\
& = p_1^*(a) \cdot \frac{\epsilon}{2} \left[\frac{p_1^*(a)}{\|p_1^*(a)\|} - \frac{p_1^*(b)}{\|p_1^*(b)\|} \right] - 2\epsilon_2 \|e_T\| = \\
& = \frac{\epsilon}{2} \frac{p_1^*(a) \cdot p_1^*(a)}{\|p_1^*(a)\| \|p_1^*(a)\|} \|p_1^*(a)\| - \frac{\epsilon}{2} \frac{p_1^*(a) \cdot p_1^*(b)}{\|p_1^*(a)\| \|p_1^*(b)\|} \|p_1^*(a)\| - \frac{\epsilon}{2} \gamma^{min} = \\
& = \frac{\epsilon}{2} \gamma^{ab} - \frac{\epsilon}{2} \gamma^{min} \geq 0.
\end{aligned}$$

If $p^1(a) = p^1(b)$, then $x''^1(a) = x''^1(b)$ and $d\hat{y}(a) = d\hat{y}(b)$. In this case, $\hat{y}(a) = \hat{y}(b)$ and the deliverability condition is also satisfied.

Hence, $U_i^n(\hat{x}) > U_i^n(x_i^n)$. Contradiction. □

Lemma 1.

In the bounded economy, the budget correspondence, \bar{B}^i , is continuous.

Proof of Lemma 1:

It is easy to see that $\bar{B}^i(p)$ is upper hemicontinuous, as the inequalities which must be satisfied are not strict.

Let $x \in \bar{B}^i(p)$ and consider a ball centered at x with radius $\epsilon > 0$, denoted $B(x, \epsilon)$. To prove that \bar{B}^i is lower hemicontinuous, we need to show that $\exists \delta > 0$ such that, for a given $p' \in B(p, \delta)$, there exists $z \in B(x, \epsilon) \cap \bar{B}^i(p')$.

Observe that $(x'_0, y', x'_1) = (0, 0, 0)$ strictly satisfies all the budget restrictions. Therefore, any convex combination of x and x' also does. Let x'' be a convex combination of x and x' with enough weight on x so that it belongs to $B(x, \epsilon)$.

We have $p_0 \cdot x_0'' + q \cdot y'' - p_0 \cdot e_0^i < 0$ and $p_1(s) \cdot x_1''(s) - p_1(s) [y''(s) + e_1^i(s)] < 0, \forall s \in \mathcal{S}$. By continuity, for sufficiently small δ , any $p' \in B(p, \delta)$ preserves the inequalities. \square

References

Allen, B. (1981), “Generic Existence of Completely Revealing Equilibria for Economies with Uncertainty when Prices Convey Information”, *Econometrica*, 49 (5), 1173-1199.

Arrow, K.J. (1953), “The Role of Securities in the Optimal Allocation of Risk-Bearing”, *Econometrie*, translated and reprinted in 1964, *Review of Economic Studies*, 31 (2), 91-96.

Bisin, A. and P. Gottardi (1999), “Competitive Equilibria with Asymmetric Information”, *Journal of Economic Theory*, 87 (1), 1-48.

Bisin, A. and P. Gottardi (2006), “Efficient Competitive Equilibria with Adverse Selection”, *Journal of Political Economy*, 114 (3), 485-516.

Correia-da-Silva, J. and C. Hervés-Beloso (2008), “Subjective Expectations Equilibrium in Economies with Uncertain Delivery”, *Journal of Mathematical Economics*, 44 (7-8), 641-650.

Correia-da-Silva, J. and C. Hervés-Beloso (2009a), “Prudent Expectations Equilibrium in Economies with Uncertain Delivery”, *Economic Theory*, 39 (1), 67-92.

Correia-da-Silva, J. and C. Hervés-Beloso (2009b), “General Equilibrium with Uncertain Delivery”, mimeo.

Debreu, G. (1952), “A Social Equilibrium Existence Theorem”, *Proc. Natl. Acad. Sci. USA*, 38, 803-886.

Debreu, G. (1959), “Theory of Value”, Wiley, New York.

Hurwicz, L. (1972), “On Informationally Decentralized Systems”, in *Decision and Organization*, edited by M. McGuire and R. Radner, North Holland, Amsterdam.

- Jerez, B. (2005), "Incentive Compatibility and Pricing Under Moral Hazard", *Review of Economic Dynamics*, 8, 2847.
- Magill, M. and M. Quinzii (1996), "Theory of Incomplete Markets", MIT Press, Cambridge.
- Myerson, R.B. (1979), "Incentive Compatibility and the Bargaining Problem", *Econometrica*, 47 (1), 61-74.
- Prescott, E.C. and R.M. Townsend (1984a), "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard", *Econometrica*, 52 (1), 21-46.
- Prescott, E.C. and R.M. Townsend (1984b), "General Competitive Equilibria Analysis in an Economy with Private Information", *International Economic Review*, 25 (1), 1-20.
- Radner, R. (1968), "Competitive Equilibrium Under Uncertainty", *Econometrica*, 36 (1), 31-58.
- Radner, R. (1972), "Existence of equilibrium of plans, prices, and price expectations in a sequence of markets", *Econometrica*, 40 (2), 289-304.
- Radner, R. (1979), "Rational Expectations Equilibrium: Generic Existence and the Information Revealed by Prices", *Econometrica*, 47 (3), 655-678.
- Rustichini, A. and P. Siconolfi (2008), "General Equilibrium in Economies with Adverse Selection", *Economic Theory*, 37 (1), 1-29.
- Townsend, R.M. (1979), "Optimal Contracts and Competitive Markets with Costly State Verification", *Journal of Economic Theory*, 21 (2), 265-293.

Recent FEP Working Papers

Nº 373	Armando Silva, Óscar Afonso and Ana Paula Africano, " <i>Do Portuguese manufacturing firms learn by exporting?</i> ", April 2010
Nº 372	Ana Maria Bandeira and Óscar Afonso, " <i>Value of intangibles arising from R&D activities</i> ", April 2010
Nº 371	Armando Silva, Óscar Afonso and Ana Paula Africano, " <i>Do Portuguese manufacturing firms self select to exports?</i> ", April 2010
Nº 370	Óscar Afonso, Sara Monteiro and Maria Thompson, " <i>A Growth Model for the Quadruple Helix Innovation Theory</i> ", April 2010
Nº 369	Armando Silva, Óscar Afonso and Ana Paula Africano, " <i>Economic performance and international trade engagement: the case of Portuguese manufacturing firms</i> ", April 2010
Nº 368	Andrés Carvajal and João Correia-da-Silva, " <i>Agreeing to Disagree with Multiple Priors</i> ", April 2010
Nº 367	Pedro Gonzaga, " <i>Simulador de Mercados de Oligopólio</i> ", March 2010
Nº 366	Aurora A.C. Teixeira and Luís Pinheiro, " <i>The process of emergency, evolution, and sustainability of University-Firm relations in a context of open innovation</i> ", March 2010
Nº 365	Miguel Fonseca, António Mendonça and José Passos, " <i>Home Country Trade Effects of Outward FDI: an analysis of the Portuguese case, 1996-2007</i> ", March 2010
Nº 364	Armando Silva, Ana Paula Africano and Óscar Afonso, " <i>Learning-by-exporting: what we know and what we would like to know</i> ", March 2010
Nº 363	Pedro Cosme da Costa Vieira, " <i>O problema do crescente endividamento de Portugal à luz da New Macroeconomics</i> ", February 2010
Nº 362	Argentino Pessoa, " <i>Reviewing PPP Performance in Developing Economies</i> ", February 2010
Nº 361	Ana Paula Africano, Aurora A.C. Teixeira and André Caiado, " <i>The usefulness of State trade missions for the internationalization of firms: an econometric analysis</i> ", February 2010
Nº 360	Beatriz Casais and João F. Proença, " <i>Inhibitions and implications associated with celebrity participation in social marketing programs focusing on HIV prevention: an exploratory research</i> ", February 2010
Nº 359	Ana Maria Bandeira, " <i>Valorização de activos intangíveis resultantes de actividades de I&D</i> ", February 2010
Nº 358	Maria Antónia Rodrigues and João F. Proença, " <i>SST and the Consumer Behaviour in Portuguese Financial Services</i> ", January 2010
Nº 357	Carlos Brito and Ricardo Correia, " <i>Regions as Networks: Towards a Conceptual Framework of Territorial Dynamics</i> ", January 2010
Nº 356	Pedro Rui Mazedra Gil, Paulo Brito and Óscar Afonso, " <i>Growth and Firm Dynamics with Horizontal and Vertical R&D</i> ", January 2010
Nº 355	Aurora A.C. Teixeira and José Miguel Silva, " <i>Emergent and declining themes in the Economics and Management of Innovation scientific area over the past three decades</i> ", January 2010
Nº 354	José Miguel Silva and Aurora A.C. Teixeira, " <i>Identifying the intellectual scientific basis of the Economics and Management of Innovation Management area</i> ", January 2010
Nº 353	Paulo Guimarães, Octávio Figueiredo and Douglas Woodward, " <i>Accounting for Neighboring Effects in Measures of Spatial Concentration</i> ", December 2009
Nº 352	Vasco Leite, Sofia B.S.D. Castro and João Correia-da-Silva, " <i>A third sector in the core-periphery model: non-tradable goods</i> ", December 2009
Nº 351	João Correia-da-Silva and Joana Pinho, " <i>Costly horizontal differentiation</i> ", December 2009
Nº 350	João Correia-da-Silva and Joana Resende, " <i>Free daily newspapers: too many incentives to print?</i> ", December 2009
Nº 349	Ricardo Correia and Carlos Brito, " <i>Análise Conjunta da Dinâmica Territorial e Industrial: O Caso da IKEA – Swedwood</i> ", December 2009
Nº 348	Gonçalo Faria, João Correia-da-Silva and Cláudia Ribeiro, " <i>Dynamic Consumption and</i>

	<i>Portfolio Choice with Ambiguity about Stochastic Volatility</i> , December 2009
Nº 347	André Caiado, Ana Paula Africano and Aurora A.C. Teixeira, " <i>Firms' perceptions on the usefulness of State trade missions: an exploratory micro level empirical analysis</i> ", December 2009
Nº 346	Luís Pinheiro and Aurora A.C. Teixeira, " <i>Bridging University-Firm relationships and Open Innovation literature: a critical synthesis</i> ", November 2009
Nº 345	Cláudia Carvalho, Carlos Brito and José Sarsfield Cabral, " <i>Assessing the Quality of Public Services: A Conceptual Model</i> ", November 2009
Nº 344	Margarida Catarino and Aurora A.C. Teixeira, " <i>International R&D cooperation: the perceptions of SMEs and Intermediaries</i> ", November 2009
Nº 343	Nuno Torres, Óscar Afonso and Isabel Soares, " <i>Geographic oil concentration and economic growth – a panel data analysis</i> ", November 2009
Nº 342	Catarina Roseira and Carlos Brito, " <i>Value Co-Creation with Suppliers</i> ", November 2009
Nº 341	José Fernando Gonçalves and Paulo S. A. Sousa, " <i>A Genetic Algorithm for Lot Size and Scheduling under Capacity Constraints and Allowing Backorders</i> ", November 2009
Nº 340	Nuno Gonçalves and Ana Paula Africano, " <i>The Immigration and Trade Link in the European Union Integration Process</i> ", November 2009
Nº 339	Filomena Garcia and Joana Resende, " <i>Conformity based behavior and the dynamics of price competition: a new rational for fashion shifts</i> ", October 2009
Nº 338	Nuno Torres, Óscar Afonso and Isabel Soares, " <i>Natural resources, economic growth and institutions – a panel approach</i> ", October 2009
Nº 337	Ana Pinto Borges, João Correia-da-Silva and Didier Laussel, " <i>Regulating a monopolist with unknown bureaucratic tendencies</i> ", October 2009
Nº 336	Pedro Rui Mazedo Gil, " <i>Animal Spirits and the Composition of Innovation in a Lab-Equipment R&D Model</i> ", September 2009
Nº 335	Cristina Santos and Aurora A.C. Teixeira, " <i>The evolution of the literature on entrepreneurship. Uncovering some under researched themes</i> ", September 2009
Nº 334	Maria das Dores B. Moura Oliveira and Aurora A.C. Teixeira, " <i>Policy approaches regarding technology transfer: Portugal and Switzerland compared</i> ", September 2009
Nº 333	Ana Sofia Ferreira, Leonídio Fonseca and Lilian Santos, " <i>Serão os 'estudantes empreendedores' os empreendedores do futuro? O contributo das empresas juniores para o empreendedorismo</i> ", August 2009
Nº 332	Raquel Almeida, Marina Silva and Tiago Soares, " <i>Coesão Territorial - As relações de fronteira entre Portugal e Espanha</i> ", August 2009
Nº 331	Custódia Bastos, Suzi Ladeira and Sofia Silva, " <i>Empreendedorismo nas Artes ou Artes do Empreendedorismo? Um estudo empírico do 'Cluster' da Rua Miguel Bombarda</i> ", August 2009
Nº 330	Filipe A. Ribeiro, Ana N. Veloso and Artur V. Vieira, " <i>Empreendedorismo Social: Uma análise via associativismo juvenil</i> ", August 2009
Nº 329	Argentino Pessoa, " <i>Outsourcing And Public Sector Efficiency: How Effective Is Outsourcing In Dealing With Impure Public Goods?</i> ", July 2009
Nº 328	Joana Almodovar, Aurora A.C. Teixeira, " <i>Conceptualizing clusters through the lens of networks: a critical synthesis</i> ", July 2009
Nº 327	Pedro Mazedo Gil, Fernanda Figueiredo and Óscar Afonso, " <i>Equilibrium Price Distribution with Directed Technical Change</i> ", July 2009
Nº 326	Armando Silva, Ana Paula Africano and Óscar Afonso, " <i>Which Portuguese firms are more innovative? The importance of multinationals and exporters</i> ", June 2009

Editor: Sandra Silva (sandras@fep.up.pt)

Download available at:

<http://www.fep.up.pt/investigacao/workingpapers/>

also in <http://ideas.repec.org/PaperSeries.html>

www.fep.up.pt

FACULDADE DE ECONOMIA DA UNIVERSIDADE DO PORTO

Rua Dr. Roberto Frias, 4200-464 Porto | Tel. 225 571 100

Tel. 225571100 | www.fep.up.pt