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DIRECTED TECHNOLOGICAL CHANGE WITH COSTLY INVESTMENT AND COMPLEMENTARITIES, AND THE SKILL PREMIUM

ELENA SOCHIRCA [1]
ÓSCAR AFONSO [1 2 3]
PEDRO MAZEDA GIL [1 2]

[1] FACULDADE DE ECONOMIA, UNIVERSIDADE DO PORTO
[2] CEF.UP
[3] OBEGEF

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Directed technological change with costly investment and complementarities, and the skill premium

Elena Sochirca,*Oscar Afonso,† Pedro Mazeda Gil‡

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Abstract

We develop an extended directed technological change model with R&D driven growth to analyze the growth rate, technological-knowledge bias, skill premium and industrial structure, assuming: *(i)* complementarities between intermediate goods in production, and *(ii)* internal costly investment. We find that complementarities directly affect equilibrium technological-knowledge bias, both elements influence equilibrium growth rate and neither affects skill premium and industrial structure. We also find that equilibrium skill premium is independent of relative labour endowments, being determined solely by workers' productivities, suggesting that the persisting increase in wage inequality observed in several developed countries over the last decades may have been due to increases in productivity advantages of skilled workers favoured by technological development.

Keywords: technological-knowledge bias, skill premium, complementarities, costly investment, vertical and horizontal R&D.

JEL classification: J31, O31, O33, O41

*Faculty of Economics, University of Porto.

†Faculty of Economics, University of Porto, and CEF.UP. Corresponding author: please email to oafonso@fep.up.pt or address to Rua Dr Roberto Frias, 4200-464, Porto, Portugal.

‡Faculty of Economics, University of Porto, and CEF.UP.

1 Introduction

In this work, we develop an extended directed technological change model with R&D driven growth, complementarities between intermediate goods in production, and internal costly investment, in order to study the growth rate, technological-knowledge bias, industrial structure and intra-country skill premium behaviour.

Our baseline closed economy model builds on the original framework of Acemoglu and Zilibotti (2001) extended by the introduction of physical capital and: (*i*) vertical differentiation following Aghion and Howitt (1992), (*ii*) Hayashi's (1982) internal investment costs in both physical capital and R&D and (*iii*) complementarities between intermediate goods used in the production of final goods, as in Evans *et al.* (1998). In particular, the model assumes that final goods are produced through a combination of labour and quality-adjusted complementary intermediate goods and that two distinct production technologies, skilled and unskilled, are available. Then, to produce each final good, firms can employ either skilled labour and skilled-specific intermediate goods or unskilled labour and unskilled-specific intermediate goods.

The perfectly competitive final goods can then be used in consumption, production of intermediate goods or R&D. On their turn, intermediate good firms produce under monopolistic competition, using physical capital and choosing between investing into either vertical or horizontal R&D. That is, we model aggregate economic growth as occurring both along an extensive (horizontal R&D) and an intensive (vertical R&D) margin, i.e. both through expanding variety and increasing quality of intermediate goods, and we relate horizontal R&D to industrial structure and show that vertical R&D is the ultimate engine of growth. Given that a larger number of product lines (horizontal innovation) puts pressure on economy's resources due to the existence of positive fixed and recurrent operating costs, while greater productivity of existing product lines (vertical innovation) does not, we believe that using the two dimensions of technology (e.g., as in Howitt, 1999; Segerstrom, 2000; Peretto and Connolly, 2007) allows us to attain a more comprehensive reflection of the endogenous economic growth mechanism. We also assume that temporary monopoly position generates higher profits for the currently innovating firm, this Schumpeterian approach enabling us to endogenize economic processes designated as the causes of modern economic development and recurring structural change (e.g. Howitt, 1999; François and Lloyd-Ellis, 2009).

Our decision to introduce the assumption of costly investment is motivated by the argument that growth models should consider investments as a decision variable of the firm, implying that firms undergo expenses associated to investments both in capital accumulation and R&D, as part of total capital investments (Benavie *et al.*, 1996; Romer, 1996; Anagnostopoulou, 2008). The addition of this specific element enables us to analyse skill-biased technological development in an environment of internal capital investment costs, which include R&D expenses.

In order to reflect another relevant feature of modern industrialized economies in our baseline model, we introduce the element of complementarity between intermediate goods in final goods production. This assumption is primarily motivated by the argument that complementarities should be an essential feature in explaining economic growth, business cycles and underdevelopment (Matsuyama, 1995). We incorporate the important idea that an increase in the number of complementary goods leads to an increase in the production of a capital good, and that an increase in the production of a specific intermediate good raises the demand for its complementary intermediate goods. Introducing this specific element into the model contributes to enriching our analysis on the skill-biased technological development and economic growth.

Within the proposed framework, we analyze the effects of internal costly investment and complementarity degree on the economy's steady-state growth rate, technological-knowledge bias, and skill premium. Additionally, we also examine the same effects on industrial structure, measured by the number of intermediate good varieties for each technology type. We find that both elements – costly investment and complementarity – affect the economy's growth rate, the complementarity degree alone influences technological-knowledge bias through the price channel, while neither one of the two elements affects the skill premium and industrial structure in equilibrium. We consider that the absence of an explicit impact of costly investment and complementarity on the equilibrium skill premium results from their incorporated equitable effects on both available technologies, and consequently on the derived skilled and unskilled labour prices, thus canceling out in the relative factor price calculation.

We also examine the impact of an increase in the skilled-labour endowment on the key equilibrium variables of the model and, additionally, on industrial structure. We perform this analysis due to the increased importance of skilled labour in most developed (and developing) economies, and also because of the key role that this issue has played in the Skill Biased Technological Change ($SBTC$) literature. We find that an increase in the relative labour supply positively affects the economy's growth rate, technological-knowledge bias and industrial structure. As regards the skill premium, the relative labour supply variations have no explicit impact on its equilibrium value. We show that this is due in particular to our production function characteristics (namely, a constant elasticity of substitution between factors equal to 2) exactly offsetting the initial supply and the market-size and price-channel effects, which leaves the equilibrium skill premium being determined solely by the absolute productivities ratio. Intuitively, if technological development induced by changes in the relative supply of skills, i.e. $SBTC$, leads to an increase in the productivity of labour favoured by technological development (in particular skilled labour), this result suggests that the persisting increase in the wage inequality between skilled and unskilled workers observed in several developed countries throughout the past 30 years may have been due to such increases in the productive advantage of skilled workers.

The remainder of this work is organised as follows. After the Introduction, Section 2 sets up the model specifying the role of internal costly investment and complementarity degree, and presents the main results focusing on consumers, final-goods and intermediate-goods sectors, and R&D decisions. In Section 3 the steady-state equilibrium is defined and discussed. Section 4 provides a comparative analysis of the steady-state effects of costly investment, complementarity degree, and relative labour endowment. Conclusions are presented in Section 5.

2 Model specifications

2.1 Consumers

The economy consists of $L+H$ identical and infinitely-lived households and has a zero population growth. Indexed with $a \in [0,1]$ depending on their ability level, households consume final goods, own firms and supply unskilled or skilled labour, which we denote by L (indexed by $a \leq \overline{a}$) and H (indexed by $a > \overline{a}$) respectively. The amount of both types of labour supplied to the economy is fixed, implying $L+H=1$, with $L = \int_0^{\overline{a}} L_a da$ and $H = \int_{\overline{a}}^1 H_a da$. All households have identical preferences and in each period t they decide on the division of their income (from wages and interest) between consumption and saving, maximizing lifetime utility:

$$maxU(t) = \int_0^\infty e^{-\rho t} \cdot \frac{C_a(t)^{(1-\sigma)} - 1}{1-\sigma} dt$$

subject to their budget constraint:

$$\dot{E}_a(t) = r(t)E_a(t) + W_m(t)m_a(t) - C_a(t)$$

(with $m = L$ if $a \leq \overline{a}$ and $m = H$ if $a > \overline{a}$), and the transversality condition:

$$\lim_{t \to \infty} E_a(t)e^{-\rho t} = 0$$

where σ is the relative risk aversion coefficient,[1] ρ is the subjective discount rate, $C_a(t)$ is the consumption of the representative household a at time t and $\dot{E}_a(t)$ is the change in the asset stock, $E_a(t)$, given by the difference between income from interest and wages, $r(t)E_a(t)$ and $W_m(t)m_a(t)$ respectively, and consumption.

The optimal consumption path (independent from a) is the standard *Euler* equation:

$$\frac{\dot{C_a(t)}}{C_a(t)} = \frac{\dot{C(t)}}{C(t)} = \frac{r(t) - \rho}{\sigma} \tag{1}$$

Equation (1) implies that in a balanced-growth path (BGP) the interest rate, $r(t)$, must be constant.

2.2 Final-Goods Sector

Building on Acemoglu and Zilibotti (2001) and Afonso (2006), we assume that the composite final good, Y, is produced by competitive final-good firms continuously indexed by $n \in [0, 1]$. Firms produce employing a combination of labour and intermediate goods, using one of the two substitute production technologies available. In particular, firms can either use unskilled labour, L, and a continuum of L-specific intermediate goods indexed by $j_L \in [0, A_L(t)]$, or skilled labour, H, and a continuum of H-specific intermediate goods indexed by $j_H \in [0, A_H(t)]$. Thus, each set of intermediate goods only combines with its corresponding type of labour, and each firm produces with one production technology exclusively, which we denote by L- and H-technology respectively. Also, following Evans *et al.* (1998), we assume that intermediate goods enter complementarily the production function, which for a representative firm n is given by:

$$Y(n,t) = ((1-n)lL(n))^{(1-\alpha)} \left(\int_0^{A_L(t)} \lambda^{\gamma k_L(j_L,t)} \cdot x_L(n, j_L, t)^\gamma dj_L \right)^\phi +$$

$$+ (nhH(n))^{(1-\alpha)} \left(\int_0^{A_H(t)} \lambda^{\gamma k_H(j_H,t)} \cdot x_H(n, j_H, t)^\gamma dj_H \right)^\phi \tag{2}$$

where $x_m(n, j_m, t)$ represents the quantity of the intermediate good j_m, $m = L, H$, and variables $H(n)$ and $L(n)$ denote the amount of skilled and unskilled labour used to produce the final good n at time

[1] Determining the willingness to shift consumption between periods, a smaller σ indicates that marginal utility is slowly decreasing in $C_a(t)$, implying thus a higher degree of consumption variation over time.

t; variables $A_H(t)$ and $A_L(t)$ represent the measure of variety of H- and L-specific intermediate goods available at time t; $\lambda > 1$ reflects the improvement in the quality of an intermediate good j brought in by successful vertical innovation, and k indicates both the number of quality improvements and the top quality rung at time t; $h > l \geq 1$ reflects the *absolute* productivity advantage of skilled over unskilled labour, while the terms n and $(1-n)$ reflect the *relative* productivity advantage of each labour type, implying that skilled labour is relatively more productive for manufacturing final goods indexed by larger ns and vice versa; and the terms $(1-\alpha)$ and $\alpha = \gamma\phi$ denote the share of labour and intermediate goods in production (the integral terms summing up the contribution of intermediate goods to the final-goods production). The terms n and $(1-n)$ also imply that at each time t there exists an endogenous threshold $\overline{n}$, at which switching from one production technology to another becomes advantageous, and consequently, each final good n is produced with one technology – H or L – exclusively. Parameter $\phi > 1$ reflects our assumption of complementarity between intermediate goods used in production, implying that an increase in the quantity of one intermediate good increases the marginal productivity of the other intermediate goods. Restriction $\alpha = \gamma\phi$ is imposed to ensure constant returns to scale (with $0 < \alpha < 1$).

Normalizing the price of the composite final good n, $P(n,t)$, to 1 in each t such that $exp \int_0^1 \ln P(n)dn = 1$, the economy's aggregate output at time t, $Y(t)$, is given by the integral over the n final goods:

$$Y(t) = \int_0^1 P(n,t) \cdot Y(n,t)dn = exp \int_0^1 \ln Y(n,t)dn \tag{3}$$

Operating in a perfect competition environment, each final good firm n seeks to maximize its profits at time t, taking as given the prices of the complementary intermediate goods j and the wages of skilled or unskilled labour employed in production:

$$\max_{x_m(n,j_m,t)} \pi(n,t) = P(n,t) \cdot Y(n,t) - W_L(t)L(n) - W_H(t)H(n) -$$

$$- \int_0^{A_L(t)} p_L(j_L,t) \cdot x_L(n,j_L,t)dj_L - \int_0^{A_H(t)} p_H(j_H,t) \cdot x_H(n,j_H,t)dj_H$$

where $Y(n,t)$ is given by (2), $p_m(j_m,t)$ denotes the price of the m-type complementary intermediate good j_m, and $W_m(t)$ is the wage paid for each unit of $m-$type labour. Note that, since each final-good firm produces exclusively with one type of technology, profits at each t are maximized with respect to $x_m(n,j_m,t)$. Then, from the first order conditions (FOCs) of each final-producer type's maximization problem, we can derive the inverse demand functions for L- and H-specific intermediate goods used in final goods production:

$$\begin{aligned} p_L(j_L,t) &= P(n,t)\left((1-n)lL(n)\right)^{(1-\alpha)} \alpha\lambda^{\gamma k_L(j_L,t)} x_L(n,j_L,t)^{(\gamma-1)} \left(\int_0^{A_L(t)} \lambda^{\gamma k_L(j_L,t)} \cdot x_L(n,j_L,t)^{\gamma} dj_L\right)^{(\phi-1)} \\ p_H(j_H,t) &= P(n,t)\left(nhH(n)\right)^{(1-\alpha)} \alpha\lambda^{\gamma k_H(j_H,t)} x_H(n,j_H,t)^{(\gamma-1)} \left(\int_0^{A_H(t)} \lambda^{\gamma k_H(j_H,t)} \cdot x_H(n,j_H,t)^{\gamma} dj_H\right)^{(\phi-1)} \end{aligned} \tag{4}$$

Solving expressions (4) for $\int_0^{A_L(t)} \lambda^{\gamma k_L(j_L,t)} \cdot x_L(n,j_L,t)^{\gamma} dj_L$ and $\int_0^{A_H(t)} \lambda^{\gamma k_H(j_H,t)} \cdot x_H(n,j_H,t)^{\gamma} dj_H$ respectively, and substituting in equation (2), the economy's aggregate output can be re-written as:

$$Y(n,t) = \left(\frac{\alpha P(n,t)}{p_m(j_m,t)}\right)^{\frac{\alpha}{1-\alpha}} \cdot \left[(1-n)lL(n) \cdot Q_L(t)^{(\varepsilon+1)} + nhH(n) \cdot Q_H(t)^{(\varepsilon+1)}\right] \tag{5}$$

where, as we will further show, $p_L(j_L,t)$ and $p_H(j_H,t)$ are equal,[2] thus allowing us to place it in front of the brackets; the term $\varepsilon \equiv \frac{\phi-1}{1-\alpha}$ in (5) is a positive constant reflecting the effect of complementarity degree between intermediate goods on final goods production; and $Q_H(t)$ and $Q_L(t)$ are aggregate quality indexes denoting the technological-knowledge stock accumulated in t for the H- and L-technology group respectively, being defined by:

$$Q_m(t) \equiv \int_0^{A_m(t)} \left(\lambda^{k_m(j_m,t)}\right)^{\frac{\gamma}{1-\gamma}} dj_m, \; m = L, H \tag{6}$$

2.3 Intermediate-Goods Sector and Internal Costly Investment

The intermediate-goods sector fosters the production of specialized intermediate goods and the R&D activity, horizontal and vertical, meaning that intermediate-good firms both produce intermediate goods to supply the final-goods sector and decide on the amount of R&D investments. Operating under monopolistic competition, intermediate-good firms use both physical and R&D capital in their productive activity. That is, the production of intermediate goods requires the use of physical machines corresponding to each type of intermediate goods previously invented through R&D (with one of the same technologies available for the production of final and intermediate goods, i.e. H- or L-technology).

Assuming that producing one unit of any intermediate good j takes one unit of physical capital, we define the physical capital stock as the total amount of various types of intermediate goods produced in the economy, $K(t) = K_L(t) + K_H(t)$, with $K_L(t) = \int_0^{A_L(t)} X_L(j_L,t)dj_L$ and $K_H(t) = \int_0^{A_H(t)} X_H(j_H,t)dj_H$, where $X_L(j_L,t) = \int_0^{\bar{n}} x_L(n,j_L,t)dn$ and $X_H(j_H,t) = \int_{\bar{n}}^1 x_H(n,j_H,t)dn$ denote the demand faced by intermediate-good producers (obtained from the expressions (4)). Total investment, $\dot{Z}(t)$, in each period t, is given by the sum of investment in physical capital, $\dot{K}(t)$, and investment in vertical, $R_{v,m}(t)$, and horizontal, $R_{h,m}(t)$, R&D:

$$\dot{Z}(t) = \dot{K}(t) + R_{h,L}(t) + R_{h,H}(t) + R_{v,L}(t) + R_{v,H}(t)$$

Following Thompson (2008) we consider that investments (both in physical and R&D capital) involve an internal cost associated to the installation of new capital. Assuming zero capital depreciation, we specify that installing $I(t) = \dot{Z}(t)$ new units of total capital requires spending an amount given by $J(t) = I(t) + \frac{1}{2}\theta\frac{I(t)^2}{Z(t)}$, where $Z(t)$ is the total capital stock at t, $\frac{1}{2}\theta\frac{I(t)^2}{Z(t)}$ represents the Hayashi's (1982) internal installation cost, and θ denotes the adjustment cost parameter.

In every period t, firms choose their investment rate so as to maximize the present discounted value of cash flows, and the current-value Hamiltonian for our optimal control problem is:

$$H(t) = Y(t) - I(t) - \frac{1}{2}\theta\frac{I(t)^2}{Z(t)} + q(t)\left[I(t) - \dot{Z}(t)\right]$$

where $q(t)$ is the market value of capital. As we show later on, the output growth rate, $g \equiv g_Y$, is equal to the investment rate, that is $g = g_Z = \frac{I(t)}{Z(t)}$. Then, from the FOC for an optimum we have:

[2] This results from the intermediate-good firms profit maximization problem.

$$q(t) = 1 + \theta g \tag{7}$$

and the transversality condition $\lim_{t\to\infty} \exp(-rt)q(t)Z(t) = 0$.

Expression (7) defines the market value of capital, $q(t)$, in a BGP.

In each period t, an intermediate-good firm must decide on the optimal price and production quantity of intermediate good to supply to the final-good producers. Thus, in each t, a monopolistic H- or L-technology intermediate-good producer maximizes its profits taking as given the previously derived inverse demand curves (4) for its good:

$$\max_{X_m(j_m,t)} \pi_m(j_m,t) = p_m(j_m,t) \cdot X_m(j_m,t) - rq \cdot X_m(j_m,t),\ \ m = L, H,$$

where rq (the interest rate times the market value of capital) denotes the production cost of one unit of intermediate good j to supply to the final-good firm n.

As previously mentioned, the FOCs yield $p_L(j_L,t) = p_H(j_H,t) = p$, which leads to the mark-up rule:

$$p = \frac{rq}{\gamma} \tag{8}$$

Equation (8) implies that the price charged by intermediate-good producers for their differentiated goods is equal for all j and *H(L)*-technology sectors, i.e., it is *sector-invariant*. Moreover, in a BGP when $r(t)$ and $q(t)$ are constant, the mark-up $p = \frac{rq}{\gamma}$ (where γ is a parameter) is both *sector*- and *time-invariant*.

Next, using expressions (4) and (8), recalling that in any moment in time we have $L = \int_0^{\overline{n}} L(n)dn$ and $H = \int_{\overline{n}}^1 H(n)dn$, and normalizing prices such that:

$$P_L(t)^{\frac{1}{1-\alpha}} \equiv P(n,t)^{\frac{1}{1-\alpha}}(1-n), \quad P_H(t)^{\frac{1}{1-\alpha}} \equiv P(n,t)^{\frac{1}{1-\alpha}} n\ \ , \tag{9}$$

we derive the total quantity that each *H(L)*-technology intermediate-good firm produces and sells, accounting for the threshold final good $\overline{n}$, and resulting profits:

$$X_m(j_m,t) = M\left(\frac{\alpha P_m(t)}{p}\right)^{\frac{1}{1-\alpha}}\left(\lambda^{k_m(j_m,t)}\right)^{\frac{\gamma}{1-\gamma}} Q_m(t)^{\varepsilon}$$

$$\pi_m(t) = \pi_0 M P_m(t)^{\frac{1}{1-\alpha}}\left(\lambda^{k_m(j_m,t)}\right)^{\frac{\gamma}{1-\gamma}} Q_m(t)^{\varepsilon} \tag{10}$$

with $m = L$, H and $M = lL$, hH, and where $\pi_0 \equiv (1-\gamma)\left(\frac{rq}{\gamma}\right)^{-\frac{\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}}$.

We can now proceed to the economy's R&D sector, where we derive the free-entry and dynamic arbitrage conditions for horizontal and vertical R&D.

2.4 R&D Sector

We assume that intermediate-goods sector firms can choose between investing into either vertical or horizontal R&D. By devoting their resources to vertical R&D, intermediate-good firms target qualitative improvements of already existing intermediate-product lines, while in case of horizontal R&D they aim at variety expansion, i.e. creating a new product line. We also assume that, contrarily to vertical R&D, when the improved intermediate good will possess the highest quality in the given industry j, in case of horizontal R&D the newly-created product line will be characterized by the *average* quality level in the targeted intermediate-goods industry. We proceed by describing each R&D dimension's key features, and then derive the vertical and horizontal innovation arbitrage conditions, as well as the consistency and the inter-technology arbitrage conditions.

2.4.1 Vertical R&D free-entry and dynamic arbitrage conditions

In devoting resources to vertical R&D, each potential entrant seeks to improve the quality of an existent intermediate good. As in standard quality-ladders models, the R&D technology used for developing new designs is common for all intermediate-good firms, there is free entry into vertical R&D, and the innovation arrival rate follows a Poisson process. Each newly developed design is protected by a patent which grants the successful innovator exclusive rights over the use of improved quality. Since vertical R&D is a standard creative destruction process, this Schumpeterian monopoly is temporary. However, temporary exclusive rights over the top-quality intermediate good confer the successful innovator higher profits for the duration of the patent, thus motivating investment in vertical R&D. On its turn, the duration of the patent is determined by the instantaneous vertical-R&D arrival rate, which we denote by $I^i_m(k_m)$ – vertical innovation rate at time t, by firm i, in the intermediate-good industry j_m, when the top quality is at $k_m(j_m,t)$ level. Aggregated across firms in j_m, the instantaneous arrival rate of a new quality improvement is given by:

$$I_m(k_m) = R_{v,m}(k_m) \cdot \frac{1}{\varsigma} \cdot \left(\lambda^{-(k_m+1)}\right)^{\frac{\gamma}{1-\gamma}} \cdot Q_m^{-\varepsilon} \tag{11}$$

where $R_{v,m}(k_m)$ denotes resources devoted to vertical R&D in j_m; ς is a constant fixed vertical-R&D cost, assumed equal for both production technologies; and the term $\left(\lambda^{-(k_m+1)}\right)^{\frac{\gamma}{1-\gamma}} \cdot Q_m^{-\varepsilon}$ accounts for the adverse effect resulting from the increasing complexity of quality improvements, implying that research becomes progressively more difficult with each new vertical innovation.[3]

The expected present value of an intermediate-good firm's profits from developing the k_m+1 quality level depends on the expected monopoly profits, given by (10), the market real interest rate, $r(t)$, and the duration of the monopoly position determined by the instantaneous arrival rate, $I_m(k_m)$:

$$V_m(k_m+1) = \pi_0 M \left(\lambda^{(k_m+1)}\right)^{\frac{\gamma}{1-\gamma}} \int_t^{\infty} P_m(s)^{\frac{1}{1-\alpha}} \cdot Q_m(s)^{\epsilon} \cdot e^{-\int_t^s [r(v)+I_m(k_m(v))]dv} ds. \tag{12}$$

Assuming free-entry into vertical R&D, the dynamic zero-profit condition is:

[3] The assumption of complementarity between intermediate goods further adds to the progressive complexity of new quality improvements, captured by the term $Q_m^{-\varepsilon}$. Note that our complementarity specification implies that incentives for discovering new goods would grow rapidly over time (more complements raise the values of new intermediate goods), thus leading to explosive growth. Therefore, as in Evans *et al.* (1998), $\varepsilon \equiv \frac{\phi-1}{1-\alpha}$ reflects the offsetting effect induced by a higher cost for designing intermediate goods with a higher index, implying that the values for r and g are constant over time, and allowing us to solve the model for a BGP equilibrium.

$$I_m(k_m) \cdot V_m(k_m+1) = R_{v,m}(k_m) \quad (13)$$

To derive the vertical R&D arbitrage condition we time-differentiate expression (13) with $V_m(k_m+1)$ given by (12), bearing in mind Leibnitz's rule, which yields:

$$r(t) + I_m(t) = (1-\gamma)\frac{1}{\varsigma}M\left(\frac{rq}{\gamma}\right)^{-\frac{\alpha}{1-\alpha}}(\alpha P_m(t))^{\frac{1}{1-\alpha}} + \varepsilon\frac{\dot{Q}_m(t)}{Q_m(t)}, \quad (14)$$

where the rates of entry are symmetric across industries $I_m(t) = I_m(k_m(j_m,t))$. Expression (14) represents the vertical R&D arbitrage condition, from which the optimal vertical innovation rate can be derived.

After solving equation (11) for $R_{v,m}(j_m,t) = R_{v,m}(k_m)$ and aggregating across industries j_m, we determine optimal total resources devoted to vertical R&D, $R_{v,m}(t) = \int_0^{A_m(t)} R_{v,m}(j_m,t)\,dj_m$. As the innovation rate is industry independent, then:

$$R_{v,m}(t) = \zeta \cdot \lambda^{\frac{\gamma}{1-\gamma}} \cdot I_m(t) \cdot Q_m(t)^{(\epsilon+1)}. \quad (15)$$

2.4.2 Horizontal R&D free-entry and dynamic arbitrage conditions

We derive the horizontal R&D free-entry and dynamic arbitrage condition following the same reasoning as in the vertical R&D case. However, as we emphasized in the beginning of this section, contrarily to the vertical R&D case, where each new successful innovation increases the quality of an existent good by $\lambda^{\frac{\gamma}{1-\gamma}}$, a successful horizontal innovation, i.e. the newly created intermediate-good line, will possess the *average* quality level of already existing varieties in the m-complementary intermediate-goods sector. Therefore, before we proceed to the derivation of the horizontal R&D arbitrage condition, we need to define the *average industry quality index* and specify the variety expansion cost function.

Average quality index So far, in our model, we have been working with the m-complementary intermediate-goods sector aggregate quality index, $Q_m(t)$, defined by(6). However, recalling that we have $A_m(t)$ m-complementary intermediate-goods sector industries, we can alternatively consider an *average* industry quality index, which may be defined as follows:

$$\overline{z}(t) \equiv \int_0^{A_m(t)} \frac{\left(\lambda^{k_m(j_m,t)}\right)^{\frac{\gamma}{1-\gamma}}}{A_m(t)}dj_m \equiv \frac{Q_m(t)}{A_m(t)}$$

The average industry quality level, $\overline{z}(t)$, will be considered in the derivation of the free-entry (profits and expected value respectively) and arbitrage conditions for horizontal R&D.

Variety expansion production and cost functions The production function of new varieties of intermediate goods by firm f, exhibiting constant returns to scale at firm level and involving a horizontal innovation cost symmetric across all firms, is of the form:

$$\dot{A}_m^f(t) = \frac{1}{\eta_m(t)}R_{h,m}^f(t)$$

where $\dot{A}_m^f(t)$ denotes the contribution of an individual firm to the total of new intermediate-good varieties that are being created at time t; $R_{h,m}^f$ denotes the resources an intermediate-good firm devotes to creating new product lines, and $\eta_m(t)$ may be interpreted as the horizontal R&D entry cost (in units of final good). Then, aggregating across firms, total resources devoted to horizontal R&D are given by:

$$R_{h,m}(t) = \eta_m(t)\dot{A}_m(t) \tag{16}$$

Following Barro and Sala-i-Martin (2004, Ch.4), let the horizontal R&D entry cost, $\eta_m(t)$, be given by $\eta_m(t) = \beta_1 A_m(t)^{\beta_2}$, where $\beta_1 > 0$ and $\beta_2 > 0$ are the fixed-flow cost and elasticity (can be also interpreted as an entry barrier index) parameters respectively. Taking into account the average quality index defined above and following a similar line of reasoning as in the vertical R&D case, the expected present value from investing in variety expansion, $V_m(\bar{z})$, is given by:

$$V_m(\bar{z}) = \pi^0 M \cdot \bar{z}(t) \int_t^{\infty} P_m(s)^{\frac{1}{1-\alpha}} \cdot Q_m(s)^{\epsilon} \cdot e^{-\int_t^s [r(v) - I(\bar{z}(v))]dv} ds$$

Then, analogously to the vertical R&D case, the free-entry condition for horizontal R&D is $\dot{A}_m(t) \cdot V_m(\bar{z}) = \eta_m(t)\dot{A}_m(t)$, which by (16) implies $V_m(\bar{z}) = \eta_m(t)$. Time-differentiating the latter yields the horizontal R&D arbitrage condition:

$$r(t) + I(t) = \frac{\bar{\pi}_m(t)}{\eta_m(t)} \tag{17}$$

where $\bar{\pi}_m(t) = \pi_0 M P_m(t)^{\frac{1}{1-\alpha}} \bar{z}(t) Q_m(t)^{\varepsilon}$.

2.4.3 Consistency and inter-technology arbitrage conditions

Using the above derived vertical and horizontal R&D arbitrage conditions given by equations (14) and (17) respectively, we can now derive the *consistency* arbitrage condition, by equating the effective rate of return, $r(t)+I(t)$, for both R&D dimensions for $m = L$, H in every t. The consistency arbitrage condition reflects the idea that, in equilibrium, the competitive capital market is equally willing to finance R&D in either variety expansion or quality improvement. Thus, using equations (14) and (17) and recalling the above expressions for $\bar{\pi}_m(t)$, $\bar{z}(t) \equiv \frac{Q_m(t)}{A_m(t)}$, $\eta_m(t)$ and π_0, the consistency arbitrage condition is:

$$\frac{Q_m(t)^{(\varepsilon+1)}}{\eta_m(t) \cdot A_m(t)} = \frac{1}{\varsigma} + \varepsilon \frac{\dot{Q}_m(t)}{Q_m(t) \cdot \pi_0 M \cdot P_m(t)^{\frac{1}{1-\alpha}}} \tag{18}$$

For the case of vertical R&D we can also derive an *inter-technology* arbitrage condition, defining a situation of indifference between innovating in L- or H-technology intermediate-goods sector. We do that by simply equating the effective rate of return for each $m = L$, H in (14), which yields:

$$I_H(t) - I_L(t) = \frac{\pi_0}{\varsigma}\left(hHP_H(t)^{\frac{1}{1-\alpha}} - lLP_L(t)^{\frac{1}{1-\alpha}}\right) + \varepsilon\left(\frac{\dot{Q}_H(t)}{Q_H(t)} - \frac{\dot{Q}_L(t)}{Q_L(t)}\right) \tag{19}$$

3 General equilibrium

In this section we derive the general equilibrium and characterize the interior BGP. We specifically discuss the steady-state values for the economy's equilibrium growth rates, technological-knowledge bias, threshold final good, skill premium and final-good prices.

In a BGP equilibrium we must have that aggregate macroeconomic variables – output, consumption, intermediate goods and resources spent on R&D – all grow at a constant rate: $g_Y = g_C = g_X = g_R = g$. Similarly, in a BGP, aggregate quality indices for both m-type technologies grow at the same constant rate, i.e. $g_{Q_L} = g_{Q_H}$. Consequently, in equilibrium we have:

(i) a constant interest rate, which implies a constant equilibrium rate of successful vertical R&D for both m-type technologies;

(ii) a constant technological-knowledge bias, $\frac{Q_H}{Q_L}$, which on its turn implies constant levels for the threshold final good, $\overline{n}$, final-good prices, P_L and P_H, and skill premium, $W = \frac{W_H}{W_L}$;

(iii) a steady-state linear relationship between aggregate output and aggregate quality index growth rates, and between vertical and horizontal R&D growth rates.

Let us start by deriving the equilibrium values (for a given technological-knowledge bias) for the threshold final good, $\overline{n}$, final-good prices, P_H and P_L, and wages paid to skilled and unskilled labour, W_H and W_L.

Given that the existence of an endogenous threshold reflects the idea that the production of final goods $n \in [0, \overline{n}]$ is more efficient using L-technology, and of final goods $n \in [\overline{n}, 1]$ is more efficient using H-technology, we can rewrite expression (5) as:

$$Y(n,t) = \begin{cases} \left(\frac{\alpha P(n,t)}{p}\right)^{\frac{\alpha}{1-\alpha}} (1-n) l L(n) Q_L(t)^{(\varepsilon+1)}, \; for \; n \in [0, \overline{n}] \\ \left(\frac{\alpha P(n,t)}{p}\right)^{\frac{\alpha}{1-\alpha}} n h H(n) Q_H(t)^{(\varepsilon+1)}, \; for \; n \in [\overline{n}, 1] \end{cases} \tag{20}$$

By perfect competition assumption, it must be true that for $\overline{n}$ the L- and the H- technology firms must break even. Then, using expression (20), rewriting price normalization (9), and normalizing labour such that $L(n) = \frac{L}{\overline{n}}$, $H(n) = \frac{H}{1-\overline{n}}$, we can derive the endogenous final-good price ratio as a function of $\overline{n}$, and an endogenous relationship between final-good price ratio, $\frac{P_H}{P_L}$, technological-knowledge bias, $\frac{Q_H}{Q_L}$, and threshold final good, $\overline{n}$:

$$\left(\frac{P_H}{P_L}\right)^{\frac{1}{1-\alpha}} = \frac{\overline{n}}{1-\overline{n}} \tag{21}$$

$$\frac{1-\overline{n}}{\overline{n}} = \left(\frac{P_H}{P_L}\right)^{\frac{1}{1-\alpha}} \frac{hH}{lL} \left(\frac{Q_H(t)}{Q_L(t)}\right)^{(\varepsilon+1)} \tag{22}$$

Expression (21) reflects the idea that in the production of the final good $n = \overline{n}$, in each t both a firm that uses L-technology and a firm that uses H-technology should break even. Combining expressions (21) and (22), we can endogenously derive the threshold final good $\overline{n}$:

$$\overline{n} = \left[1 + \left(\frac{hH}{lL} \cdot \left(\frac{Q_H(t)}{Q_L(t)}\right)^{(\varepsilon+1)}\right)^{\frac{1}{2}}\right]^{-1} \tag{23}$$

In order to obtain the final-good prices, P_L and P_H, we use our expression (3) for normalized (following Acemoglu and Zilibotti, 2001) aggregate output, which, combined with price normalization (9) and equations (21) and (23), after some algebra yields:

$$\begin{aligned} P_H &= \exp(-(1-\alpha)) \cdot (1-\overline{n})^{-(1-\alpha)} \\ P_L &= \exp(-(1-\alpha)) \cdot \overline{n}^{-(1-\alpha)} \end{aligned} \tag{24}$$

Finally, assuming that the wage per unit of m-type labour equals its marginal product, we use the production function (20) aggregated across n (keeping in mind that $\int_0^{\overline{n}(t)} L(n)dn = L$ and $\int_{\overline{n}(t)}^1 H(n)dn = H$ hold at any moment t) to derive the skilled and unskilled labour wages:

$$Y(t) = Y_L(t) + Y_H(t), \text{ where } \begin{cases} Y_H(t) = \left(\frac{\alpha}{p}\right)^{\frac{\alpha}{1-\alpha}} hHP_H^{\frac{1}{1-\alpha}} Q_H(t)^{(\varepsilon+1)} \\ Y_L(t) = \left(\frac{\alpha}{p}\right)^{\frac{\alpha}{1-\alpha}} lLP_L^{\frac{1}{1-\alpha}} Q_L(t)^{(\varepsilon+1)} \end{cases} \tag{25}$$

$$\begin{aligned} W_H &= \frac{\partial Y_H}{\partial H} = \left(\frac{\alpha}{R}\right)^{\frac{\alpha}{1-\alpha}} hP_H^{\frac{1}{1-\alpha}} Q_H(t)^{(\varepsilon+1)} \\ W_L &= \frac{\partial Y_L}{\partial L} = \left(\frac{\alpha}{R}\right)^{\frac{\alpha}{1-\alpha}} lP_L^{\frac{1}{1-\alpha}} Q_L(t)^{(\varepsilon+1)} \end{aligned} \tag{26}$$

The skill premium, W, our wage inequality measure, is then given by the ratio of skilled to unskilled labour wages from (26):

$$W = \frac{W_H}{W_L} = \frac{h}{l} \left(\frac{P_H}{P_L}\right)^{\frac{1}{1-\alpha}} \left(\frac{Q_H(t)}{Q_L(t)}\right)^{(\varepsilon+1)} \tag{27}$$

where $\varepsilon + 1 = \frac{\phi-\alpha}{1-\alpha}$.

Also, using expressions (3), (20) and (23), we can define equilibrium aggregate output (as a function of aggregate technological-knowledge indexes) as:

$$Y(t) = \exp^{-1}\left(\frac{\alpha\gamma}{rq}\right)^{\frac{\alpha}{1-\alpha}} \left[\left(lLQ_L(t)^{(\varepsilon+1)}\right)^{\frac{1}{2}} + \left(hHQ_H(t)^{(\varepsilon+1)}\right)^{\frac{1}{2}}\right]^2 \tag{28}$$

We have derived the equilibrium values for final-good prices as functions of the threshold final good, which on its turn was derived as a function of the endogenous technological-knowledge bias, while equilibrium skill premium is a function of both price ratio and technological-knowledge bias.

To complete our derivation of the steady-state equilibrium, we now turn to deriving the steady-state technological-knowledge bias, $\left(\frac{Q_H}{Q_L}\right)^*$, which will allow us to fully specify the equilibrium threshold final good, $\overline{n}^*$, final-good prices, P_L^* and P_H^*, and skill premium, W^*.

3.1 Equilibrium technological-knowledge bias, $\left(\frac{Q_H}{Q_L}\right)^*$

Recalling our inter-technology arbitrage condition (19) and keeping in mind that in steady state we must have $g_{Q_L} = g_{Q_H}$ and $I_L = I_H$, we can derive the equilibrium price ratio as a function of labour endowments and absolute productivity parameters:

$$\left(\frac{P_H}{P_L}\right)^{\frac{1}{1-\alpha}} = \left(\frac{lL}{hH}\right) \tag{29}$$

Combining this with expressions (21) and (23), after some algebra we obtain:

$$\left(\frac{Q_H}{Q_L}\right)^* = \left(\frac{hH}{lL}\right)^{\frac{1-\alpha}{\phi-\alpha}} \tag{30}$$

Expression (30) defines the equilibrium technological-knowledge bias $\left(\frac{Q_H}{Q_L}\right)^*$ in particular as a function of the relative labour endowment, $\frac{H}{L}$. This is a common result in the *SBTC* literature, reflecting the endogeneity of the *skill*-bias of newly adopted technologies. In fact, recalling expression (14) we can see that labour endowments influence the direction of technological-knowledge development through two channels. On the one hand, the market-size channel, acting through $H(L)$, increases the probability of *H(L)*-technology R&D, since an increase in the labour supply broadens the market for the respective technology type. On the other hand, the price channel, acting through P_m, reduces the probability of *H(L)*-technology R&D, given that the structure of labour endowments favours developing technologies that save the relatively scarce type of labour, since the prices for the final goods that they produce will be higher. Expression (30) shows that the market-size effect dominates the price-channel effect, since technological-knowledge bias is increasing in the relative skilled-labour supply. Acemoglu (2009, Ch. 15) refers to this result as a ***weak equilibrium (relative) bias***, implying that an increase in the relative supply of a factor – in our case the relative supply of H – always induces technological-knowledge change that is biased in favour of this factor. Additionally, as expression (30) shows, in equilibrium $\left(\frac{Q_H}{Q_L}\right)^*$ is directly affected by the complementarity degree, ϕ. This result will be analyzed in more detail in our discussion on steady-state effects.

Using equation (30) we can now complete the derivation of equilibrium threshold final good, final-good prices and skill premium, previously calculated for a given technological-knowledge bias.

3.2 Steady-state equilibrium threshold final good, final-good prices, and skill premium

Threshold final good, $\overline{n}$. The steady-state equilibrium value for the threshold final good $\overline{n}$ is derived by substituting (30) in (23), which gives us:

$$\overline{n}^* = \left[1 + \left(\frac{hH}{lL}\right)\right]^{-1} \tag{31}$$

From expression (31), the equilibrium threshold final good, $\overline{n}$, is decreasing in the relative labour endowment, and consequently it is decreasing in the technological-knowledge bias. Once again, this result is consistent with the *SBTC* literature. Namely, when technological knowledge is skill-biased (there is a large relative supply of H), the fraction of industries using H-technology is large and so $\overline{n}$ is small.

Final-good prices, P_L and P_H. Next, we can complete the derivation of the equilibrium final-good prices, P_L and P_H, by substituting (31) in equations (24), which yields:

$$\begin{aligned} P_H^* &= \exp^{(\alpha-1)} \cdot \left[1 + \left(\tfrac{lL}{hH}\right)\right]^{-(\alpha-1)} \\ P_L^* &= \exp^{(\alpha-1)} \cdot \left[1 + \left(\tfrac{hH}{lL}\right)\right]^{-(\alpha-1)} \end{aligned} \tag{32}$$

Here again, the result we obtain regarding prices is consistent with the *SBTC* literature – when technological knowledge is skill-biased, $\overline{n}^*$ is small and equations (32) verify the relation $P_L^* > P_H^*$. As we have previously mentioned, this is what the price-channel effect supposes.

Skill premium, W. Using equation (29) in (27), our expression for skill premium becomes:

$$W = \frac{L}{H} \left(\frac{Q_H(t)}{Q_L(t)} \right)^{(\varepsilon+1)} \tag{33}$$

Expression (33) illustrates the two mechanisms through which changes in relative labour endowments influence the skill premium in equilibrium. Initially, an increase in the supply of the H factor reduces the skill premium, which is consistent with the basic producer theory, i.e. increasing supply reduces prices. This immediate effect is followed by a change in the opposite direction induced through the technological-knowledge bias (as mentioned above, technological-knowledge bias increases as a result of a higher supply of H, because the market-size effect dominates the price-channel effect). If the technological-knowledge bias effect on W outweighs the initial supply effect, then, as is the case in most *SBTC* models, a positive cumulative effect on W is verified. Acemoglu (2009, Ch.15) refers to this result as the *strong equilibrium (relative) bias*, implying that a greater relative supply of a factor causes sufficiently strong technological-knowledge change as to make the resulting relative price of this (more abundant) factor increase. In equilibrium however, this result does not verify in our model. In particular, substituting the above derived expressions (30) in (33) we get:

$$W^* = \frac{L}{H} \left(\frac{hH}{lL} \right) = \frac{h}{l} \tag{34}$$

Thus, our steady-state skill premium, W^*, does not depend on relative labour endowments, $\frac{H}{L}$, because the two above referred mechanisms exactly offset each other. Following Acemoglu (2009, Ch.15), we attribute this result to the effect of elasticity of substitution between technologies in the production function. More specifically, in the absence of state dependence, an elasticity of substitution higher than 2 is required to ensure an overall positive effect on W.[4] Recalling our expression (28), we can see that this condition is not verified in our case, since our aggregate output is characterized by constant elasticity of substitution between H- and L- technologies equal to 2. Although appearing in line with the existing empirical evidence (as referred by Acemoglu (2009, Ch.15), in the context of substitution between skilled and unskilled workers an elasticity of substitution close to 2 is most probable), this specific feature of our aggregate output function implies that the market-size effect can not create sufficiently powerful

[4] State dependence implies that, to achieve sustained growth, factors allocated to R&D need to become more and more productive over time, because of spillovers from past research (the path of past innovations affects the relative costs of different types of innovations). According to Acemoglu (2009, Ch.15), an elasticity of substitution greater than 2 is not always crucial for obtaining the strong equilibrium (relative) bias. Provided a directed technological change model with knowledge spillovers and state dependence, the result of upward-sloping relative demand curves requires that the elasticity of substitution be higher than a certain threshold (determined by the state dependence parameter).

technological-knowledge change as to induce an increase in the relative marginal product, and thus in the relative price, of the factor that has become more abundant. Consequently, the *strong equilibrium (relative) bias* is not verified in our case.

As expression (34) shows, in our case the steady-state skill premium is determined solely by the productivity advantage ratio of skilled to unskilled labour, $\frac{h}{l}$. Although the role of $\frac{h}{l}$ is fairly intuitive, since workers' productivity has a direct positive impact on wages and consequently on the skill premium, the result that it is the only determinant of equilibrium skill premium deserves a more thorough investigation.

3.3 Equilibrium growth rate and industrial structure

Here, we derive the BGP growth rates for aggregate output and vertical and horizontal R&D, and specify our measure of industrial structure.

Growth rate The equilibrium growth rate of aggregate output can be derived log-differentiating expression (25) for each m-technology, which gives us:

$$g = (1+\varepsilon) g_{Q_m} \tag{35}$$

Also, from our consistency arbitrage condition (18), a balanced-path relationship between vertical and horizontal R&D growth rates can be defined, keeping in mind that in a BGP the right-hand side of expression (18) is constant. Log-differentiating equation (18) yields: $g_{A_m} = \frac{(1+\varepsilon)}{(1+\beta_2)} g_{Q_m}$, implying that variety expansion is sustained by endogenous technological-knowledge accumulation. In particular, expected higher profits from increased intermediate-good quality generated by vertical R&D make it attractive for intermediate-good firms to invest in variety expansion, despite the negative spillovers in horizontal entry (entry cost increases with the number of newly created product lines). Thus, regarding our view that economic growth proceeds both along an intensive and an extensive margin, the derived balanced-path relationship between vertical and horizontal R&D growth rates together with equation (35), imply that vertical innovation drives economic growth, in the sense that it sustains both variety expansion and aggregate output growth.

Next, using expressions (6), assuming $Q_m(t)$ deterministic and recalling the above defined average industry quality level, we can also define the aggregate quality index growth rate as: $g_{Q_m} = I_m(\lambda^{\frac{\gamma}{1-\gamma}} - 1) + g_{A_m}$, which is positive if $I_m > 0$ (with I_m derived from (14)). Then, using the above defined relations we can now complete the derivation of the economy's equilibrium aggregate quality index, g_{Q_m}, and output growth rates, g. In particular, keeping in mind that both r and q are functions of g,[5] we obtain the following implicit expressions for the optimal growth rates g_{Q_m} and g:

$$g_{Q_m} = \psi \lambda^{\frac{\gamma}{1-\gamma}} \left(\lambda^{\frac{\gamma}{1-\gamma}} - 1\right) \left((1-\gamma) M \left(\frac{rq}{\gamma}\right)^{-\frac{\alpha}{1-\alpha}} (\alpha P_m)^{\frac{1}{1-\alpha}} \frac{1}{\varsigma} - r\right) \tag{36}$$

$$g = (1+\varepsilon)\, \psi \lambda^{\frac{\gamma}{1-\gamma}} \left(\lambda^{\frac{\gamma}{1-\gamma}} - 1\right) \left((1-\gamma) M \left(\frac{rq}{\gamma}\right)^{-\frac{\alpha}{1-\alpha}} (\alpha P_m)^{\frac{1}{1-\alpha}} \frac{1}{\varsigma} - r\right) \tag{37}$$

[5]Recalling our consumption and investment optimization problems for the equilibrium interest rate and market capital values as functions of equilibrium growth rate, given by (1) and (7) respectively.

where $\psi \equiv \frac{\beta_2+1}{(\beta_2+1)\varepsilon\lambda^{\frac{\gamma}{1-\gamma}}(1-\lambda^{\frac{\gamma}{1-\gamma}})-\beta_2+\varepsilon}$, and both r and q are functions of g: $r = g\sigma + \rho$ and $q = 1 + \theta g$.

Regarding equations (36) and (37), it is important to emphasize the following. Given that, apart from the production of intermediate goods further supplied to the final-good firms, intermediate-good producers also decide on the amount of R&D expenditures, if equation (36) verified $g_{Q_m} < 0$, intermediate-good firms would have no incentives to invest in vertical R&D. Consequently, there would be no quality improvements of intermediate goods over time, i.e. g_{Q_m} would be zero, which by equation (35) would lead to $g = 0$. Therefore, equations (36) and (37) are assumed to verify $g_{Q_m} \geq 0$ and (by consequence) $g \geq 0$ respectively.[6]

Following from the above discussion on the equilibrium growth rate and recalling equations (18), (15) and (16), we can see that in BGP Y_m, $R_{v,m}$ and $R_{h,m}$ all grow with $Q_m^{\epsilon+1}$. Then, from the aggregate resource constraint, $Y = C + \dot{K}_L + \dot{K}_H + R_{v,L} + R_{v,H} + R_{h,L} + R_{h,H}$, it can be shown that in BGP C and $\dot{K}_m$ also grow with $Q_m^{\epsilon+1}$. Thus, the physical and R&D capital stocks that result from $\dot{K}_m$, $R_{v,m}$ and $R_{h,m}$ must also grow with $Q_m^{\epsilon+1}$, and hence the total capital stock Z, which is a linear combination of physical and R&D capital. It follows from here that $g_Z = g_Y$, as considered in our derivations in Subsection 2.3 above.

Industrial structure Recalling that, in alternative to vertical R&D, intermediate-good firms can also invest in horizontal R&D, i.e. creation of new product lines in each m-type sector, we associate the latter to industrial structure and correspondingly define it by the aggregate variety indexes ratio, $\frac{A_H}{A_L}$.[7] This ratio can be endogenously derived from the consistency arbitrage condition (18) and accounting for our horizontal R&D cost function specification:

$$\left(\frac{A_H}{A_L}\right) = \left(\frac{Q_H}{Q_L}\right)^{\frac{\varepsilon+1}{(\beta_2+1)}} \left(\frac{\varsigma^{-1} + \varepsilon g_{Q_H} \left(\pi^0 hH\right)^{-1} P_H^{-\frac{1}{1-\alpha}}}{\varsigma^{-1} + \varepsilon g_{Q_L} \left(\pi^0 lL\right)^{-1} P_L^{-\frac{1}{1-\alpha}}}\right)^{-\frac{1}{(\beta_2+1)}} \tag{38}$$

Recalling the derived expression for steady-state final-good prices, and the fact that in steady state aggregate quality indexes for both m-type technologies grow at the same constant rate, the component with exponent $-\frac{1}{(\beta_2+1)}$ becomes equal to 1. Thus, industrial structure is determined by technological-knowledge bias, adjusted by the elasticity parameter β_2 (from horizontal R&D cost function). Then, taking into account equation (30), the above derived expression for the steady-state industrial structure becomes:

$$\left(\frac{A_H}{A_L}\right)^* = \left(\frac{hH}{lL}\right)^{\frac{1}{(\beta_2+1)}} \tag{39}$$

[6]As regards the vertical R&D optimal growth rate, it is easy to see that replacing in equation (36) M and P_m for each m-type technology (with P_m given by equation (32)), in steady state aggregate quality indexes for both m-type technologies grow at the same constant rate, i.e. $g_{Q_L} = g_{Q_H}$.

[7]Note that in the sense that the endogenous threshold final good, $\overline{n}$, determines when the switch from one technology to another becomes advantageous, it can be also regarded as an alternative measure of final goods production structure.

4 Steady-state effects: comparative analysis

In this section we analyze the steady-state effects on equilibrium aggregate output growth rate, g, technological-knowledge bias, $\left(\frac{Q_H}{Q_L}\right)^*$, skill premium, W^*, and industrial structure, $\left(\frac{A_H}{A_L}\right)^*$. In particular, we assess the impact of an increase in the complementarity degree, ϕ, internal investment costs, θ, and skilled-labour supply, H.

4.1 Steady-state effects on equilibrium growth rate, g

Effects of an increase in complementarity between intermediate goods, $\triangle\phi$. One of the basic assumptions of our model is the feature of complementarity between intermediate goods in production, reflecting the idea that having more of one type of good raises the marginal productivity of the others. Let us see what happens to the economy's equilibrium growth rate if the degree of complementarity intensifies.

In order to assess the effects of an increase in the complementarity degree parameter, ϕ, we use the Implicit Function Theorem to derive $\frac{\partial g}{\partial \phi} = -\frac{\frac{\partial F}{\partial \phi}}{\frac{\partial F}{\partial g}}$. Recalling that $\psi \equiv \frac{\beta_2+1}{(\beta_2+1)\varepsilon\lambda^{\frac{\gamma}{1-\gamma}}(1-\lambda^{\frac{\gamma}{1-\gamma}})-\beta_2+\varepsilon}$ and $\varepsilon \equiv \frac{\phi-1}{1-\alpha}$ and defining:

$$F(\cdot) \equiv g - \psi\left(1+\varepsilon\right)\lambda^{\frac{\gamma}{1-\gamma}}\left(\lambda^{\frac{\gamma}{1-\gamma}}-1\right)\left((1-\gamma)M\left(\frac{rq}{\gamma}\right)^{-\frac{\alpha}{1-\alpha}}\left(\alpha P_m\right)^{\frac{1}{1-\alpha}}\frac{1}{\varsigma}-r(t)\right)=0$$

then, taking the corresponding partial derivatives we have:

$$\begin{aligned}\frac{\partial F}{\partial g} = \quad & 1+\psi\left(1+\varepsilon\right)\lambda^{\frac{\gamma}{1-\gamma}}\left(\lambda^{\frac{\gamma}{1-\gamma}}-1\right)\left(\tfrac{\alpha}{1-\alpha}\right)\left(g\sigma+\rho\right)^{-\frac{1}{1-\alpha}}\cdot \\ & \cdot\left((1-\gamma)M\gamma^{\frac{\alpha}{1-\alpha}}\left(\alpha P_m\right)^{\frac{1}{1-\alpha}}\tfrac{1}{\varsigma}\left(1+\theta g\right)^{-\frac{\alpha}{1-\alpha}}\left[\left(g\sigma+\rho\right)\left(1+\theta g\right)\theta+\sigma\right]-\sigma\right)>0\end{aligned}$$

In $\frac{\partial F}{\partial \phi}$ the derivative of the relevant product, $\left[\psi\left(1+\varepsilon\right)\right]_{\phi}^{'}$, is positive and plugged in the initial expression yields:

$$\begin{aligned}\frac{\partial F}{\partial \phi} = \quad & -\left[\tfrac{1}{1-\alpha}\psi\left(1+(\phi-\alpha)\left(\tfrac{(\beta_2+1)\lambda^{\frac{\gamma}{1-\gamma}}(\lambda^{\frac{\gamma}{1-\gamma}}-1)-1}{(\beta_2+1)\lambda^{\frac{\gamma}{1-\gamma}}(1-\lambda^{\frac{\gamma}{1-\gamma}})\varepsilon-\beta_2+\varepsilon}\right)\right)\right]\cdot \\ & \cdot\left(\lambda^{\frac{\gamma}{1-\gamma}}-1\right)\left((1-\gamma)M\left(\tfrac{rq}{\gamma}\right)^{-\frac{\alpha}{1-\alpha}}\left(\alpha P_m\right)^{\frac{1}{1-\alpha}}\tfrac{1}{\varsigma}-r(t)\right)<0\end{aligned}$$

Thus, we have $\frac{\frac{\partial F}{\partial \phi}}{\frac{\partial F}{\partial g}} = \frac{-}{+} < 0$, and by $\frac{\partial g}{\partial \phi} = -\frac{\frac{\partial F}{\partial \phi}}{\frac{\partial F}{\partial g}}$ it follows that $\frac{\partial g}{\partial \phi} > 0$. We can therefore conclude that an increase in the degree of complementarity between intermediate goods in production positively affects the economy's equilibrium growth rate. The intuition behind this result is fairly simple. When intermediate goods used in production are complements, an increased quantity of some goods raises the marginal productivity of the others.[8] This implies that, at constant prices, the quantity demanded goes up, production increases and output grows faster than it would otherwise. Thus, as we have shown, an increase in the complementarity degree induces an increase in the economy's aggregate output growth

[8] In this sense, the effect of complementarities between intermediate goods in production is equivalent to increasing returns.

rate.

Effects of an increase in internal investment costs, $\triangle\theta$. Analogously to the previous case, we take the partial derivatives of $F(\cdot)$ to determine the sign of $\frac{\partial g}{\partial \theta} = -\frac{\frac{\partial F}{\partial \theta}}{\frac{\partial F}{\partial g}}$:

$$\frac{\partial F}{\partial \theta} = \psi\left(1+\varepsilon\right)\lambda^{\frac{\gamma}{1-\gamma}}\left(\lambda^{\frac{\gamma}{1-\gamma}}-1\right)\left(\frac{\alpha}{1-\alpha}\right)\left((1-\gamma)Mr^{-\frac{\alpha}{1-\alpha}}\gamma^{\frac{\alpha}{1-\alpha}}\left(\alpha P_m\right)^{\frac{1}{1-\alpha}}\frac{1}{\varsigma}\right)\left(1+\theta g\right)^{-\frac{1}{1-\alpha}}g > 0$$

Recalling that $\frac{\partial F}{\partial g} > 0$, we have $\frac{\frac{\partial F}{\partial \theta}}{\frac{\partial F}{\partial g}} = \frac{+}{+} > 0$, and by $\frac{\partial g}{\partial \theta} = -\frac{\frac{\partial F}{\partial \theta}}{\frac{\partial F}{\partial g}}$ it follows that $\frac{\partial g}{\partial \theta} < 0$. Thus, we can conclude that aggregate output growth is negatively affected by internal investment costs, an increase in which decreases the economy's equilibrium growth rate. The explanation for this result steams directly from the internal investment costs theory. In particular, accounting for capital installation costs in their optimization problem, firms control their rate of investment (not the capital stock) in each t. Consequently, when internal investment costs increase, i.e. installing new capital becomes more expensive, firms will tend to reduce investment. This will reduce production of intermediate goods, leading to a slowdown in the economy's aggregate output growth.

Effects of an increase in skilled-labour supply, $\triangle H$. Here again, we apply the Implicit Function Theorem to calculate $\frac{\partial g}{\partial H} = -\frac{\frac{\partial F}{\partial H}}{\frac{\partial F}{\partial g}}$:

$$\frac{\partial F}{\partial H} = -\psi\left(1+\varepsilon\right)\lambda^{\frac{\gamma}{1-\gamma}}\left(\lambda^{\frac{\gamma}{1-\gamma}}-1\right)(1-\gamma)\alpha^{\frac{1}{1-\alpha}}h\left(\frac{rq}{\gamma}\right)^{-\frac{\alpha}{1-\alpha}}\frac{1}{\varsigma}\exp^{-1} < 0$$

Again, recalling that $\frac{\partial F}{\partial g} > 0$, we have $\frac{\frac{\partial F}{\partial H}}{\frac{\partial F}{\partial g}} = \frac{-}{+} < 0$ and by $\frac{\partial g}{\partial H} = -\frac{\frac{\partial F}{\partial H}}{\frac{\partial F}{\partial g}}$ it follows that $\frac{\partial g}{\partial H} > 0$. Thus, an increase in the skilled-labour supply has a positive effect on output growth rate in steady state due to the market-size channel: an increase in H raises the steady-state vertical innovation probability, thus accelerating the rate of technological-knowledge progress and growth.

4.2 Steady-state effects on equilibrium technological-knowledge bias, $\left(\frac{Q_H}{Q_L}\right)^*$.

As regards technological-knowledge bias, we analyze the effects of complementarity and relative labour supply variations; the effects of costly investment can not be directly assessed.

Effects of an increase in complementarity between intermediate goods, $\triangle\phi$. Recalling expression (30), the effect of complementarity degree on equilibrium technological-knowledge bias can be directly assessed. In particular, taking the derivative of $\frac{Q_H}{Q_L}$ with respect to ϕ we have:

$$\left[\left(\frac{Q_H}{Q_L}\right)^*\right] = \left[\left(\frac{hH}{lL}\right)^{\frac{1-\alpha}{\phi-\alpha}}\right]_\phi^{'} = \left(\frac{hH}{lL}\right)^{\frac{1-\alpha}{\phi-\alpha}}\ln\left(\frac{hH}{lL}\right)\left(-\frac{1-\alpha}{\left(\phi-\alpha\right)^2}\right) < 0$$

Thus, an increase in the degree of complementarity between intermediate goods in production reduces technological-knowledge bias. We emphasize that this result is due to a specific economic mechanism that relates the threshold final good and the technological-knowledge bias through the price-channel effect. In particular, recalling our expression (23), which defines the threshold final good as a decreasing function of technological-knowledge bias, and also recalling that $\varepsilon \equiv \frac{\phi-1}{1-\alpha}$, we can see that an increase in the complementarity degree ϕ increases ε, which on its turn increases the negative effect of $\frac{Q_H}{Q_L}$ on the threhold final good $\overline{n}$. As previously noted, a lower threshold value implies that there are more final goods produced with H-technology. Once they become more abundant, their market price, P_H, decreases. This reduces final-good firms' economic motivation for their production, and results in a lower demand for the corresponding type of labour, i.e. skilled labour. Consequently, the dimensions of skill-biased technological change are reduces, and so is the technological-knowledge bias. In sum, variations in the complementarity degree influence $\overline{n}$, which in turn affects technological-knowledge bias via the price-channel effect.

Effects of an increase in internal investment costs, $\triangle\theta$. As it can be seen from expression (30), equilibrium technological-knowledge bias does not depend on the internal investment costs parameter, θ, for the same reason that we will refer for the equilibrium skill premium, in Section 4.3.

Effects of an increase in skilled-labour supply, $\triangle H$. As the *SBTC* literature predicts, an increase in the skilled-labour supply has a positive effect on technological-knowledge bias, also verified in our case:

$$\left[\left(\frac{Q_H}{Q_L}\right)^*\right] = \left[\left(\frac{hH}{lL}\right)^{\frac{1-\alpha}{\phi-\alpha}}\right]'_H = \left(\frac{1-\alpha}{\phi-\alpha}\right)\frac{h}{lL}\left(\frac{hH}{lL}\right)^{\frac{1-\phi}{\phi-\alpha}} > 0$$

That is, increasing the supply of skilled workers induces technological-knowledge change biased in favour of skilled labour, as it was shown in our discussion on general equilibrium.

4.3 Steady-state effects on equilibrium skill premium, W^*

Effects of an increase in complementarity degree, $\triangle\phi$, internal investment costs, $\triangle\theta$, and skilled-labour supply, $\triangle H$. Recalling our expression (34), we can see that neither complementarities nor investment costs have a direct effect on W^*. In particular, the costly investment parameter, θ, does not explicitly enter our equilibrium skill premium equation, which is due to its implicit equitable effects on both available technologies, and consequently on the derived skilled and unskilled labour prices, thus canceling out in the relative factor price calculation. As regards the effect of an increase in the skilled-labour supply, it has no impact on equilibrium skill premium because of the offsetting of the initial supply, price-channel effect and market-size effect, as already discussed in Section 3.2. For the same reason, the complementarity degree parameter, ϕ, does not affect the skill premium in equilibrium.

4.4 Steady-state effects on equilibrium industrial structure, $\left(\frac{A_H}{A_L}\right)^*$

Effects of an increase in complementarity degree, $\triangle\phi$, and in internal investment costs, $\triangle\theta$. As expression (39) illustrates, complementarities and costly investment do not affect the steady-state industrial structure $\left(\frac{A_H}{A_L}\right)^*$ for the same reasons as referred for the case of equilibrium skill premium. In particular, due to its implicit equitable effects on both available technologies, the effect of internal investment costs cancels out, as does the effect of complementarities induced through the aggregate quality index growth rates for each m-type technology, g_{Q_H} and g_{Q_L} (see equations (36), (38) and (39)).

Effects of an increase in skilled-labour supply, $\triangle H$. Taking the partial derivative of $\left(\frac{A_H}{A_L}\right)^*$ with respect to H we have:

$$\frac{\partial\left(\frac{A_H}{A_L}\right)^*}{\partial H} = \frac{1}{(\beta_2+1)}\left(\frac{hH}{lL}\right)^{-\frac{\beta_2}{(\beta_2+1)}}\frac{h}{lL} > 0$$

Thus, we can conclude that an increase in the skilled-labour supply biases industrial structure in favour of the H-technology sector.

5 Conclusions

In this work, we have developed an extended directed technological change model and studied the growth rate, technological-knowledge bias, industrial structure and skill-premium behaviour in a framework of R&D driven growth, complementarities between intermediate goods in production, and internal investment costs. We have found that equilibrium output growth rate is affected both by complementarities and internal investment costs. We have also found that the complementarity degree directly influences technological-knowledge bias in equilibrium, and defined the underlying mechanism; and we have determined that neither of the two elements have a direct impact on steady-state skill premium and industrial structure. We have also shown that while in the proposed framework the Acemoglu's (2009) *weak equilibrium (relative) bias* is verified, i.e. technological-knowledge is biased in favour of the more abundant skilled labour, the *strong equilibrium (relative) bias* is not verified because the resulting technological-knowledge change is not strong enough as to induce an increase in the skill premium. This is particularlydue to our model's aggregate production function with a constant elasticity of substitution equal to 2, implying that labour endowments do not influence the steady-state skill premium, which is determined solely by the absolute productivities of skilled and unskilled labour.

We have also performed an analysis of the steady-state effects of an increase in complementarity degree, internal investment costs and skilled-labour supply on our model's key variables. As regards the steady-state effects on growth rate, we have found that it is positively influenced by both complementarities and skilled-labour endowments, and negatively by internal investment costs. We have also found that while in line with the *SBTC* literature an increase in skilled-labour supply has a positive effect on equilibrium technological-knowledge bias, an increase in the complementarity degree reduces it via the price channel, and neither of these two parameters influence the skill premium, because the initial supply, price-channel and market-size effect exactly offset each other. On its turn, an increase in internal investment costs affects neither the technological-knowledge bias nor the skill premium, due to its implicit equitable effects on both available technologies.

Finally, we believe that there are several topics that would be interesting to consider for future research. Our model generates a number of results that call for empirical confirmation, namely, those referred in our steady-state analysis. Furthermore, the result that equilibrium skill premium depends solely on the productivity advantage of skilled over unskilled labour deserves a more thorough investigation. In particular, given that, having a direct impact on workers productivity, skills both increase the capacity to innovate and apply new ideas, and stimulate technological development, it is plausible that the absolute productivity advantages of skilled and unskilled workers are related to technological-knowledge bias and labour supply. Therefore, a challenging task would be to endogenize the productivity parameters in the framework of a $SBTC$ model.

References

Acemoglu, D., (2002). Directed technical change. Review of Economic Studies 69(4), 781-810.

Acemoglu, D., (2009). Introduction to modern economic growth. Princeton University Press.

Acemoglu, D. and Zilibotti, F., (2001). Productivity differences. Quarterly Journal of Economics 116(2), 563-606.

Afonso, O., (2006). Skill-biased technological knowledge without scale effects. Applied Economics 38(1), 13-21.

Aghion, P. and Howitt, P., (1992). A model of growth through creative destruction. Econometrica 60, 323-351.

Anagnostopoulou, S., (2008). R&D expenses and firm valuation: a literature review. International Journal of Accounting and Information Management 16(1), 5-24.

Barro, R., and Sala-i-Martin, X., (2004). Economic Growth. Cambridge, Massachusetts. MIT Press, second edition.

Benavie, A., Grinols, E. and Turnovsky, S., (1996). Adjustment costs and investment in a stochastic endogenous growth model. Journal of Monetary Economics 38, 77-100.

Evans, G., Honkapohja, S., and Romer, P., (1998). Growth cycles. American Economic Review 88(3), 495-515.

François, P., and Lloyd-Ellis, H., (2009). Schumpeterian business cycles with pro-cyclical R&D. Review of Economic Dynamics 12(4), 567-591.

Hayashi, F., (1982). Tobin's marginal q and average q: a neoclassical interpretation. Econometrica 50, 213-224.

Howitt, P., (1999). Steady endogenous growth with population and R&D inputs growing. Journal of Political Economy 107(4), 715-730.

Matsuyama, K., (1995). Complementarities and cumulative processes in models of monopolistic competition. Journal of Economic Literature 33, 701-729.

Peretto, P. and Connolly, M., (2007). The Manhattan metaphor. Journal of Economic Growth 12(4), 329-350.

Romer, D., (1996). Advanced macroeconomics. Mc-Graw Hill Companies, Inc.

Segerstrom, P., (2000). The long-run growth effects of R&D subsidies. Journal of Economic Growth 5(3), 277-305.

Thompson, M., (2008). Complementarities and costly investment in a growth model. Journal of Economics 94(3), 231-240.

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