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THE RELATIONSHIP BETWEEN TRIGGER PRICE AND PUNISHMENT PERIOD IN GREEN AND PORTER (1984) GAME MADE ENDOGENOUS*

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Abstract

Green and Porter (1984) made a huge contribution to Industrial Organization Theory where a trigger price is defined by firms and whenever the price falls below this trigger price, the firms cease to produce at the monopoly level and enter into a punishment period. Our goal with this paper is to define, endogenously in the model, relationships between the trigger price and the punishment period, which were set exogenously in the original paper.

Keywords: Green and Porter (1984); trigger price; punishment period.

JEL classification: L13, L20

1. INTRODUCTION

Green and Porter (1984) made a huge contribution to Industrial Organization Theory by considering a dynamic model in which the firms of an industry are confronted with the problems of detecting and deterring cheating in an agreement in a context of imperfect monitoring. Firms decide the quantity to produce and observe the market price to imperfectly infer the aggregate production. This imperfection results from the existence of uncertainty in the demand of the goods which may lead to lower prices even if the firms do not overproduce.

In the model presented in Green and Porter (1984), a trigger price is defined by firms and whenever the price falls below this trigger price, the firms cease to produce at the monopoly level (or collusion previously defined output) and enter into a punishment What Green and Porter (1984) suggest is that the existence of low prices and higher production for certain periods of time are consistent with the existence of a

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noncooperative collusion and may not be a result of price wars and/or a result of abortive attempts to form a cartel. It represented a new insight on collusive behavior. They defended that public intervention may be required in order to increase competition.

In Green and Porter (1984) model, both the trigger price and the punishment period T are exogenous. In our point of view, there would be gains by making endogenous these parameters since then we can see how they depend on the strategies of the firms. Then, this represents the objective of this paper, that is, to create combinations of trigger price and punishment period that are optimal in the point of view of firms. In addition, it is our objective to show that these different combinations may lead to lower or higher volatility in the markets in analysis.

A similar exercise to ours was made in Tirole (1988) where it is tried to find a optimal value for the punishment period. However, Tirole (1988) has different assumptions specially regarding competition which is made on prices instead of quantities like in Green and Porter (1984) and on the demand which has only two possible outcomes each period. In addition, Tirole (1988) follows a different approach. In the paper's model, it is determined the optimal punishment period with a per-period analysis instead of pre-game implicit contract as in our case (for a wider clarification see note 2). This yields different results as one can see point 3.

2. The Model

When we consider different combinations of T and $\bar{p}$, we are implicitly changing the relationship between the expected long run profits while colluding and the expected long run profits when producing the Cournot levels. For instance, if the time in reversionary phase is longer, that is if T is higher, the total long-run profits in Cournot increases, ceteris paribus. In a given period of time, the total profits in collusion will be the product of the collusion profits in each period times the number of periods it occurs[1]. The same occurs for the Cournot case but the expected number of periods we are in Cournot once we enter in the reversionary period is given and is T-1 in Green and Porter (1984).

[1] No discounting considered for simplicity.

From the definition of the trigger price and the punishment period results a proportion of long-run profits in collusion relatively to long-run profits in Cournot (given by K in the equation below). Naturally, firms would like to have this proportion as big as possible if they stay in collusion since they earn more profits. In order to maximize this proportion, given the profit levels, this would imply changes in the expected number of periods staying in Cournot and in Collusion. More specifically, this would imply a lower expected number of periods staying in Cournot and the opposite in the case of collusion. However, for a given time-horizon, increasing the number of periods in collusion relatively to Cournot would imply lower welfare and public discontentment. This means that firms have to face restrictions when maximizing this proportion of long run profits like the possibility of changes in habits of consumption. In addition, the public discontentment may lead to government and public institutions pressure and also the possibility of new entrants. Restrictions to K may be found in several papers like in Harrington (2004a, 2004b, 2005). In the first two, the author develops theoretical models in which pricing in cartels is analyzed when antitrust authorities exist. In Harrington (2005), firms face a profit maximization profit problem where they maximize their present value of income flow which depends on the profits while colluding, the penalties if caught cheating and the probability of being caught.

More formally:

$$\Pi_{Colluding} * E\left(t_{colluding}\right) = K * \Pi_{Cournot} * (T-1) \Leftrightarrow$$

$$\Leftrightarrow E\left(t_{colluding}\right) = \frac{K*\Pi_{Cournot}*(T-1)}{\Pi_{Colluding}} \quad (1)$$
[2]

Where $\Pi_{Colluding}$ is the profit while producing the amount of the collusive behavior. $E\left(t_{colluding}\right)$ represents the expected number of periods of collusion. $\Pi_{Cournot}$ corresponds to the Cournot Profits and T-1 is the length of the punishment. K represents the proportion of the profits. [3]

[2] In this equation it is not considered an Incentive Compatibility Constraint of keeping in collusion as is considered in Tirole (1988). One must consider this equality to be an implicit contract between all firms before entering the market. As a result, all firms have an incentive to accept this kind of contract since it yields higher profits in the long-run than staying in Cournot. This must not be confused with the decision taken by each firm in each period regarding deviation or not from the collusive behavior.

[3] Note, however, that the profits in collusion and in punishment periods are general, that is, they do not depend on the modelization they are built upon. For example, in the punishment period, instead of Cournot Profits one may use, for instance, perfect competition prices.

It is not direct from the formula above the relationship between T and $\bar{p}$. However, $E(t_{colluding})$ depends on $\bar{p}$ and by solving this connection we may find the relationship between T and $\bar{p}$.

We know from statistics that

$$E(t_{colluding}) = \frac{1}{Prob(P<\bar{P})} \quad (2)$$

Proof:

$$E(t_{colluding}) = 1 * Prob(P < \bar{P}) + 2 * Prob(P > \bar{P}) * Prob(P < \bar{P}) + 3 * Prob(P > \bar{P})^2 * Prob(P < \bar{P}) + \cdots + n * Prob(P > \bar{P})^{n-1} * Prob(P < \bar{P})$$

Assuming that the firms are colluding in the first period as is assumed in Green and Porter (1984) this expected value is built as follows. In the first term of the right-hand side, the price is below the trigger price so in the following period, firms enter into the punishment phase. In the second term, in the first period the price is above the trigger price while in the second one it falls below the trigger price implying that the collusion remains for two periods. Considering this for n periods yield the equality above. Continuing the resolution of the proof:

$$Prob(P < \bar{P}) * [1 + 2 * Prob(P > \bar{P}) + 3 * Prob(P > \bar{P})^2 + \cdots] =$$

$$= Prob(P < \bar{P}) * \left[\frac{1}{1 - Prob(P > \bar{P})} + \frac{Prob(P > \bar{P})}{1 - Prob(P > \bar{P})} + \cdots\right] =$$

Given that $Prob(P > \bar{P}) = 1 - Prob(P < \bar{P})$

$$= 1 + Prob(P > \bar{P}) + Prob(P > \bar{P})^2 + \cdots =$$

$$= \frac{1}{1 - Prob(P > \bar{P})} = \frac{1}{Prob(P < \bar{P})} = E(t_{colluding})$$

Like in the Green and Porter (1984), the Demand Function is of the type:

$$P_t = \theta_t P(\sum x_{it}) \quad (3)$$

We make here an additional assumption regarding θ_t distribution. Just for calculus simplification and easy of interpretation we assume that θ_t has an uniform distribution like

$$\theta \sim U[\underline{\theta}, \overline{\theta}] \quad (4)$$

When setting $\Pi_{Colluding}$ it is implicit that all firms follow the collusive behavior so each firm produces a constant x_i[4]. As a result, the total economy production, $\sum x_i$, is constant. Moreover the function *P(.)* does not change with time so we using equations (3) and (4) can state that:

$$\frac{P_t}{P(\sum x_{it})} \sim U[\underline{\theta}, \overline{\theta}] = U\left[\frac{P_{min}}{P(\sum x_i)}, \frac{P_{max}}{P(\sum x_i)}\right] \quad (5)$$

Additionally from (2):

$$\frac{1}{Prob(P<\overline{P})} = E(t_{colluding}) \Leftrightarrow \frac{1}{Prob\left(1<\overline{P}/_P\right)} = E(t_{colluding}) \quad (6)$$

Using the properties of the uniform distribution, equation (5) and given that *P*=$P(\sum x_i)$:

$$Prob\left(1 < \overline{P}/_P\right) = \frac{\frac{\overline{P}}{P(\sum x_{it})} - \frac{P_{min}}{P(\sum x_i)}}{\frac{P_{max}}{P(\sum x_i)} - \frac{P_{min}}{P(\sum x_i)}}$$

Using (6):

$$E(t_{colluding}) = \frac{\frac{P_{max}}{P(\sum x_i)} - \frac{P_{min}}{P(\sum x_i)}}{\frac{\overline{P}}{P(\sum x_{it})} - \frac{P_{min}}{P(\sum x_i)}} \Leftrightarrow$$

Using (1):

$$\frac{K * \Pi_{Cournot} * (T-1)}{\Pi_{Colluding}} = \frac{\frac{P_{max}}{P(\sum x_i)} - \frac{P_{min}}{P(\sum x_i)}}{\frac{\overline{P}}{P(\sum x_{it})} - \frac{P_{min}}{P(\sum x_i)}}$$

$$\Leftrightarrow \frac{\overline{P}}{P(\sum x_{it})} = \frac{\left(\frac{P_{max}}{P(\sum x_i)} - \frac{P_{min}}{P(\sum x_i)}\right) * \Pi_{Colluding}}{K * \Pi_{Cournot} * (T-1)} + \frac{P_{min}}{P(\sum x_i)}$$

[4] Note that the time subscript is not necessary since the level of production is pre-determined like in Green and Porter (1984) and all firms are assumed to abide by the contract requisites leading to unchangable production through time.

Following footnote 4, we are again not taking into account the idiosyncrasies of each period so the time subscript is not required. This together with $P(\sum x_{it}) = P(\sum x_i)$, leads to:

$$\overline{P} = \frac{(P_{max} - P_{min}) * \Pi_{Colluding}}{K * \Pi_{Cournot} * (T-1)} + P_{min} \quad (7)$$

3. CONCLUSION

In order to understand equation (7) is important to note first that there is an inverse relationship between $\overline{P}$and $E(t_{colluding})$. As one can conclude the higher is the proportion of profits (K), ceteris paribus, the lower is $\overline{P}$ in order to guarantee that the expected number of periods while colluding is greater. Given that all the remaining variables are constant, to increase the proportion of profits, each firm has to be in collusion for a longer period implying a lower trigger price.

Additionally, the lower is the ratio of profits $\Pi_{Colluding}/\Pi_{Cournot}$, ceteris paribus, the higher has to be $\overline{P}$ to increase the expected long run profits while colluding. For instance, if the profits in collusion increases, the number of periods in collusion must decrease implying a higher trigger price.

The most important relationship is that the higher is T, ceteris paribus, the lower has to be $\overline{P}$ so that our condition holds. The higher is the punishment period the higher has to be the expected number of periods of colluding so production would less likely change from one period to other, meaning that a lower trigger price is required. This implies that the volatility on the market regarding prices and quantities would depend on the definition of the combination of trigger price and T. In the particular case considered, with higher punishment period, the volatility would decrease ceteris paribus.

These results differ from Tirole (1988) where it is obtained an incentive incompatibility constraint. Thisis used as constraint to the maximization of the present discounted value of a firm's profits from period t on, leading to the conclusion that the punishment period should be as low as possible. This conclusion results from the fact that, the longer is the punishment period the lower are the expected profits. As one can see, this differs from our results. In our case, however, a low punishment period would lead to higher market volatility and no gains in profits.

References

- Green, Edward J and Porter, Robert H, 1984. Noncooperative Collusion under Imperfect Price Information,Econometrica, Econometric Society, vol. 52(1), pages 87-100, January.
- Harrington, J., 2005. Optimal Cartel Pricing in the Presence of an Antitrust Authority, International Economic Review, vol.46, pages 145-169.
- Harrington, J., 2004a. Cartel Pricing Dynamics in the Presence of an Antitrust Authority, RAND Journal of Economics, vol.35, pages 651-673.
- Harrington, J., 2004b. Post-Cartel Pricing during Litigation, The Journal of Industrial Economics, pages 517-533 LII.
- Tirole, J., 1988, The theory of industrial organization, The MIT Press: Cambridge, Massachussets, 1988. ISBN: 0-262-20071-6

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