Spatial Competition and Firms’ Location Decisions under Cost Uncertainty

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Abstract

This paper considers a two-staged Location-Price game à la Hotelling, where firms first choose location and then set prices. A lack of information arises, because before choosing location firms are not sure about the marginal costs. However, they know the two possible outcomes in advance. We conclude that contrary to the perfect information case, firms may agglomerate, given that the difference between their possible marginal costs is sufficiently high for either firm. Also, we find that in most cases both firms are better off with imperfect information, which explains firms’ incentives to develop cost reduction activities, such as R&D.

JEL Codes: L13, R12
1. INTRODUCTION

Quite often firms have to decide their exact location without having full knowledge about their costs, even considering that previously they carried out detailed market and technical studies to support their location decisions (as it is expected). Only after locating the plant and starting the production will firms understand their true costs. This is the situation that we study in this work. We analyze the behavior of two competitors in the framework of Hotelling (1929). We depart from the classical two-staged location-price game by introducing uncertainty regarding the firms’ costs. Firms are only fully aware of their production technology, and therefore about their costs, between the two stages of the game, that is, only after deciding where to locate in the linear city. After having this information firms compete in prices and maximize their profits.

Uncertainty is an important feature to consider. Firms are not aware of all the conditions they are going to face before effectively entering the market. Some conditions are crucial; the fixed costs needed to enter the market; the behavior of demand; the relationship with suppliers or with possible rival firms; the technological capacity that they own or the technological capacity of other firms; other variables affecting the market, such as interest rates, consumer confidence and many more.

In spite of having various determinants, as mentioned above, the marginal cost uncertainty will be referred to as being caused by uncertainties in the technology needed to produce the good. Before setting up their plant in a given location firms are expecting a certain productive behavior, given their (exogenous) investment in technology/capacity. However, only after locating the plant and starting the production of goods for sale will firms understand the true marginal costs of production that the good has.

To simplify the problem, we assume that firms have only two possible outcomes regarding their marginal cost: a high and a low marginal cost. The high marginal cost is assumed to be equal for both firms and it is a fixed constant, while the low marginal costs are allowed to be different between them. For instance, when firms are expecting the results from cost-reducing R&D activities, if these activities fail their objective, firms maintain high marginal costs. However, if they are successful a low marginal cost production technology is available.

We assume that with perfect information firms are certain to get the lowest marginal cost outcome, while with imperfect information firms are only aware of the probabilities of each of the four possible outcomes, depending on the success of the cost reducing activities
for each firm. These assumptions are justified by the fact that uncertainty usually means that there is a probability of a negative result for the firms. Note, however, that the assumption for the perfect information case does not change the location results, but has some implications when it comes to comparing the profits and pricing policies between both cases of perfect and imperfect information.

The main conclusion of the paper is that agglomeration between firms is now a possible optimal outcome when the low marginal cost outcome is sufficiently lower than the high marginal cost for both firms. This result is a clear contrast with the perfect information model of Ziss (1993), in which firms are either dispersed throughout the linear city or are located randomly, as no location equilibrium arises when the difference between the marginal costs of the firms is too high.

Also, we conclude that prices are higher in the imperfect information case when location equilibrium exists for both cases. However, this essentially derives from the nature of the assumption of imperfect information rather than deriving from the change in the location patterns of firms.

Concerning profits and when a comparison between both cases is possible, firms prefer to be in imperfect information setting if the difference between the high and low marginal costs is large enough. Under this setting, firms earn a higher expected profit than in the perfect information case. When the differences between the possible marginal costs are small, there is a small region where both firms would like to be in imperfect information, but most of the cases indicate that one of the firms would want to be in perfect information while the other firm would not.

The above conclusions highlight the importance of considering imperfect information about costs when studying firms’ location decisions. Agglomeration is a frequent phenomenon in some industries\(^1\), and our model presents an explanation for this observed behavior. We conclude that when firms are uncertain about the costs they will face before choosing where to locate, they may prefer to agglomerate. This is in clear contrast with the perfect information case, in which no agglomeration equilibrium can be found.

Section 2 fits the paper in the literature. Section 3 presents the model and both perfect and imperfect information cases. Section 4 makes a brief analysis of the results, while section 5 concludes.

\(^1\) For further developments on firms’ agglomeration see Ellison et al. (2010) and the references therein.
2. THEORETICAL BACKGROUND

The model of Hotelling (1929) is a well-known approach when it comes to justifying the location choices of two firms. The Hotelling model is based on the following assumptions: two firms are the players in a two-staged location-price game. In the first stage, firms must choose their location in a linear and bounded city, and in the second stage they compete in prices. The good sold by the firms is homogenous except for the location they have chosen in the first stage, and the firms have the same cost structure. Demand is perfectly inelastic, that is, consumers in that city must buy one unit of the good, while incurring in a linear transportation cost in order to buy the good. Hotelling concluded in his original model that firms would optimally agglomerate in the center of the city. This is the basis for the “Principle of Minimum Differentiation” called so by Boulding (1966).

However, the original model of Hotelling had some tractability problems, due to demand discontinuity for the firms when their chosen location was outside the extremes of the city. This discontinuity extended to the profit function, which implied that no price-equilibrium existed for all the possible locations of the firms in the city.

An important approach to the Hotelling model was made by d’Aspremont et al. (1979). By introducing quadratic transportation costs (instead of linear) with respect to distance, the result of central agglomeration disappeared, with firms optimally locating in each of the extremes of the city. This result became known as the “Principle of Maximum Differentiation”. That paper is also very important because the introduction of this type of transportation cost eliminates the tractability problems of the Hotelling model, allowing a price equilibrium for all possible locations of the firms.

The location behavior raised by Hotelling and d’Aspremont et al. attracted the interest of the academic world. Using the same model in a more regional and urban economics approach, some scientists attempted to explain the conditions in which firms agglomerate in cities, while other scientists were more concerned about the horizontal differentiation problem, approaching this model from the interests of Industrial Organization.

After the work by d’Aspremont et al., the field expanded significantly, and more important publications appeared about this issue. Most of these publications focused on
changing the assumptions of the original Hotelling model and then concluding about the new location decisions of the firms.²

The most direct reference for the present work is by Ziss (1993), who derives the results for the two-stage location-price game of Hotelling by allowing for different marginal costs between the two firms. Ziss concludes that when the difference between the marginal costs is small, firms prefer to locate in different extremes of the city. However, when the difference is higher than a given threshold, the low-cost firm prefers to locate as close as possible to the high-cost firm which, in a rival fashion, wishes to locate as far as possible from the other firm. This leads to the absence of location equilibrium. However, if the difference between both marginal costs is high enough, the low-cost firm drives the high-cost firm out of the market. It is shown throughout our paper how these results are extended by our analysis.

Matsumura and Matsushima (2009) later extended the Location-Price game of Ziss (1993), but allowing for mixed strategies in the location stage. They conclude that there is a mixed-strategy for all cost differentials in the game. For the cost differentials where no pure strategy equilibrium exists, the mixed equilibrium involves each firm choosing to locate in each extreme of the market 50% of the time.

Along the same line of allowing different marginal costs between firms, Boyer et al. (2003) studied the location-price decision when firms choose locations sequentially and then choose prices simultaneously. As with Boyer et al., our work departs from the complete information assumption. Boyer et al. assume that the first mover has perfect information, while the second mover does not know if the opponent firm has a low or high marginal cost. Although the nature of our imperfect information is slightly different, that is, it does not come from a firm’s secrecy with respect to their production process but rather from the uncertainty of the production process itself, the approach of Boyer et al. (2003) signals the importance of imperfect information in this framework, as results change significantly with the introduction of this feature.

3. THE MODEL

The basic assumptions of the model are as follows: Two firms compete in a two-staged Location-Price game. In the first stage, firms simultaneously choose their locations within the linear and bounded city of Hotelling, which is assumed to have a length equal to

² See Biscaia and Mota (2011) and the references therein for a detailed review.
one. These location variables are represented by $x_i$ (for i=1,2) for firm 1 and firm 2 respectively, where $x_1$ is expressed as the distance from the left extreme of the market and $x_2$ is the distance from the right extreme. Note that under these specifications of distance, when $x_1 + x_2 = 0$, firms are fully differentiated and when $x_1 + x_2 = 1$, firms are agglomerated in any point of the market. We will assume also, without loss of generality, that firm 1 is never located to the right of firm 2, which is equivalent to imposing that $x_1 \leq 1 - x_2$. In the second stage, firms simultaneously choose prices, which are represented by $p_i$ (for i=1,2). Firms are assumed to have no initial advantage over the other except for the parameter $c_i$ (for i=1,2), which is the exogenous marginal cost of production for firm $i$.

The remaining assumptions are equal to Hotelling (1929), with the proposed extension by d’Aspremont et al. (1979): The goods produced by the firms are homogenous to the eyes of the consumers except for the location that firms will choose. Consumers are uniformly distributed across the linear city and are obliged to buy one unit of good in order to survive. Therefore, the reservation price of the consumers is assumed to be high enough that the market is always fully covered by the existing firms. Consumers incur a transportation cost to buy the good, which is quadratic with respect to distance. Without loss of generality, we assume that the unit transportation cost is equal to one.

### 3.1 Perfect Information

In the perfect information case, firms participate in the two stage game without any uncertainty regarding the outcome of the marginal cost. This section is basically a review of the results of Ziss (1993), with the addition of some insights that will be useful when analyzing the imperfect information case.

The demand of each firm is derived directly from the location of the indifferent consumer. Typically in price competition in models a la Hotelling, each firm has its own market area, that is, each firm has an area in which all the consumers located there buy in their store. The boundary of that area is given by the point where the indifferent consumer is located. Firm 1 will have a demand equal to $x$, while firm 2 will have a demand equal to $1 - x$. The indifferent consumer is located at:

$$x = \frac{1 - 2x_2 - x_1^2 + x_2^2 - p_1 + p_2}{2 - 2x_1 - 2x_2}$$
However, when drawing the profit functions for each firm we must ensure that the value for the indifferent consumer’s location does not become lower than 0 or higher than 1. The profit function is represented by $\Pi_1$ and $\Pi_2$ respectively and is given by:

$$
\Pi_i(x_i, x_j, p_i, p_j) = \begin{cases} 
(p_i - c_i)(1 - 2x_j - x_i^2 + x_j^2 - p_i + p_j) & , \hat{c}_i < 0 \\
\frac{2 - 2x_i - 2x_j}{2 - 2x_i - 2x_j} & , 0 \leq \hat{c}_i \leq \bar{\theta} \\
0 & , \hat{c}_i > \bar{\theta}
\end{cases}
$$

Where $\hat{c}_i$ is the marginal cost difference between both firms, expressed by $c_i - c_j$, $\bar{\theta}$ is given by $-x_i^2 + 4x_i + x_j^2 + 2x_j - 3$ and $\bar{\theta}$ is given by $-x_i^2 - 2x_i + x_j^2 - 4x_j + 3$. Also, the indifferent consumer function and the profit function are only valid for the case of $x_1 < 1 - x_2$. When $x_1 = 1 - x_2$, which means that the firms are in the same location, the idea of the indifferent consumer stops making sense and the resulting profits are similar to the ones in Bertrand (1883).

3.1.1 Price Stage

When firms have all of the market their price behavior is obvious: firms want to set the highest price that allows them to keep the entire market. This “threshold” price increases with the distance from the store of the opponent, ceteris paribus. When a firm does not have any consumer, the price set is equal to the marginal cost.

The interesting case is when both firms have their own market area. After differentiating the profit function with respect to the price for each firm and after equalizing to zero, we obtain the best-response functions in terms of pricing. Combining the expressions for both firms we get, for each firm:

$$
p_i(x_i, x_j) = 1 - \frac{2}{3}x_i - \frac{4}{3}x_j - \frac{1}{3}x_i^2 + \frac{1}{3}x_j^2 + \frac{2}{3}c_i + \frac{1}{3}c_j
$$

Also, the second-order conditions for this maximization problem are satisfied. Replacing both prices in the profit function, we get:

$$
\Pi_i(x_i, x_j) = \begin{cases} 
-1 + 2x_i - x_i^2 + x_j^2 - \hat{c}_i & , \hat{c}_i < 0 \\
(-3 + 2x_i + 4x_j + x_i^2 - x_j^2 + \hat{c}_i)^2 & , 0 \leq \hat{c}_i \leq \bar{\theta} \\
0 & , \hat{c}_i > \bar{\theta}
\end{cases}
$$
3.1.2 Location Stage

To solve for the location stage, the usual methodology of deriving the profit function with respect to the decision variable and equalizing to zero is not the most adequate, mainly for two reasons: The resulting values and equations are too complicated to draw direct conclusions from and the optimal result does not take into consideration the natural boundaries for the locations of the firms, that is \( x_1 \leq 1 - x_2 \) and \( 0 \leq x_1, x_2 \leq 1 \). Some previous insights on the optimal result can be used in order to simplify procedures and retrieve the results for the model.

This framework is equivalent to that of d’Aspremont et al. (1979) if the marginal cost of the firms is equal. That means that firms will optimally locate in different extremes of the markets \((x_1 = x_2 = 0)\) for similar marginal costs. Moreover, we know from Tabuchi and Thisse (1995) that if firms are unrestricted in choosing a location inside the linear city they choose to be even more distant, optimally choosing \( x_1 = x_2 = -\frac{1}{4} \). The reason for this location outcome is known. Firms face two effects when choosing where to locate; a demand effect, which leads firms to locate closer to their rival in order to capture more demand for their good; and a price/competition effect, in which firms wish to locate with the farthest separation possible so that price competition is softened, allowing both firms to increase their profits. In d’Aspremont et al. and Tabuchi and Thisse’s setting, the price effect clearly dominates and leads the firms to maximum differentiation and even more differentiation, respectively.

It is expected that given a marginal cost difference between the firms, the low-cost firm may have the incentive to change its location, as the demand and price effects change in a different manner when this marginal cost difference appears).

Next we will prove that the low-cost firm is only interested in either the maximal differentiation spot, or in agglomerating with its rival. This means that the low-cost firm (given that the rival is located at one of the extremes) is never interested in locating in a middle spot of the city. We depart from d’Aspremont et al., in which the marginal cost difference is zero and firms optimally choose to locate in both extremes. Analyzing the derivative of the profit with respect to the location variable of one firm (say, firm \( i \)) and fixing the location of the other firm to the extreme of the city \((x_j = 0)\) we have:

\[
\frac{\partial \Pi_i}{\partial x_i}(x_j = 0) = \frac{\hat{c}_i^2 + (-2x_i^2 + 4x_i - 2)(\hat{c}_i) - 3x_i^4 - 4x_i^2 + 14x_i^2 - 4x_i - 3}{18(x_i - 1)^2}
\]
The fact that this derivative can be expressed in terms of the difference of the marginal costs justifies that the choice of an initial marginal cost is irrelevant, because it does not affect the location and investment results.

Figure 1 presents this derivative using different values for the difference in the marginal costs. Note, however, that this derivative is not valid for all the feasible values for the location of firm $i$. When the difference in the marginal cost becomes too big, the low-cost firm has all the demand, which implies that the expression for the profit of the firms changes for further increases in the differences between the marginal costs, namely, the profit function becomes $\Pi_i = -1 + 2x_i - x_i^2 + x_f^2 + c_j - c_i$. The first derivative in this case is simply given by $2 - 2x_i$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{First derivative of the profit function with respect to location for $\hat{c}_i = 0$, $\hat{c}_i = -0.5$, $\hat{c}_i = -1$, $\hat{c}_i = -1.5$, $\hat{c}_i = -2$ for the low-cost firm $i$}
\end{figure}

From Figure 1, there is an immediate conclusion that there can only be two optimal locations for a low-cost firm in this range of favorable cost differentials: to locate in one of the extremes, either fully agglomerating or fully dispersing relative to the other firm. Looking at the case of $\hat{c}_i = 0$ (dotted line), it is clear that the optimal solution for firm $i$ is to locate at the left extreme, since the derivative throughout all the relevant values of location is always negative, meaning that the firm can amplify its profits by “diminishing the value” of location. It is also clear that when $\hat{c}_i = -1.5$ and $\hat{c}_i = -2$ (the thicker lines), firm $i$ clearly wants to
agglomerate with the other firm, as the derivative is always positive through all the possible values of location.

So, the problem of optimal location is reduced to finding which one is more profitable for the low-cost firm between two possible locations. Therefore, by the straight comparison of both profits, we find that firm $i$ prefers to agglomerate when:

$$\Pi_i(x_i = 1, x_j = 0) > \Pi_i(x_i = 0, x_j = 0)$$

$$-\hat{c} > \frac{(-3 + \hat{c})^2}{18}$$

$$\hat{c} < -(6 - 3\sqrt{3})$$

The resulting value is the result of Ziss (1993), but considering fixed city length and transportation costs. Note that this result is only valid because of a lower difference in the marginal costs. The high-cost firm does not wish to leave its own extreme.

The high-cost firm does not wish to be agglomerated with the low-cost firm, since that would imply zero market and profits. The high-cost firm always wants to be far from the low-cost firm since it benefits in two ways: it increases its demand and increases the price of its good, due to the softening of price competition. Therefore, in this case there is no location equilibrium, as one of the firms prefers to be as close as possible to the opponent, while the other firm prefers to be as far as possible.

Ziss (1993) also considers a first stage, in which firms have to choose if they are going to enter the market or not. In this work we do not consider that option. The high-cost firm has to remain in the city even if it has no profits. The choice of keeping the high-cost firm in the market has also a useful purpose: to limit the maximum price that the monopolist can set for its good.

So, the optimal location pattern is given by:

\[
\begin{cases} 
0 & \text{for } |\hat{c}| < (6 - 3\sqrt{3}) \\
\text{No Location Equilibrium} & \text{for } |\hat{c}| > (6 - 3\sqrt{3})
\end{cases}
\]

Considering only the possible interval $c_i \in [0,10]$ for the marginal costs of the firms, Figure 2 represents the resulting location equilibrium of the game.
Therefore, the profits of the firms, after replacing for the optimal location decision, are given by:

\[ \Pi_i = -\hat{\epsilon}_i + 2x_i - x_i^2 + x_j^2 - 1 \quad \text{for } \hat{\epsilon}_i < -3 \]

\[ \Pi_i = \frac{\left( x_i^2 + 2x_i - x_i^2 + 4x_j^2 - 3 + \hat{\epsilon}_i \right)^2}{18(1-x_i-x_j)} \quad \text{for } -3 \leq \hat{\epsilon}_i \leq -(6 - 3\sqrt{3}) \]

\[ \Pi_i = \frac{(-3+\hat{\epsilon}_i)^2}{18} \quad \text{for } -(6 - 3\sqrt{3}) < \hat{\epsilon}_i < (6 - 3\sqrt{3}) \]

\[ \Pi_i = \frac{\left( x_i^2 + 2x_i - x_i^2 + 4x_j^2 - 3 + \hat{\epsilon}_i \right)^2}{18(1-x_i-x_j)} \quad \text{for } (6 - 3\sqrt{3}) \leq \hat{\epsilon}_i \leq 3 \]

\[ \Pi_i = 0 \quad \text{for } \hat{\epsilon}_i > 3 \]

The first and last branches of the function are relative to the case where only one of the firms sells the good to the entire market, independently of the final location outcome. That interval for the difference of marginal costs is also identified by Ziss (1993), and that is the case when the high-cost firm, even when fully differentiated relative to its rival, is not able to obtain any demand.

### 3.2 Imperfect Information

In the imperfect information case, we assume that firms participate in the same two-staged location-price game. The difference is that firms are not entirely sure what their marginal cost is when they choose where to locate in the linear city. Firm \( i \) has a chance,
given by \( v \), that its marginal cost will turn out to be low and equal to \( c_i \) and a \( 1-v \) chance to have a high marginal cost, given by \( C \). To simplify and without loss of generality (as the optimal location decision depends on the difference between marginal costs), we assume that the high-cost marginal cost \( C \) is the same for both firms and equal to 10. Also, firms are assumed to be risk-neutral. This assumption is represented in the fact that their expected utility is equal to their expected profit and, therefore, throughout the paper the term profit can be interpreted as utility for the firms.

So, the timing of the game with imperfect information is; before taking any decision, firms know the two possible outcomes for their own marginal cost and for the marginal cost of the competitor. The outcomes are either a high marginal cost, given by 10, or a low marginal cost, given by \( c_i \) for their own firm and \( c_j \) for the competitor. Firms also know the probability of occurrence of each outcome. Then, in the first stage firms have to choose their location within the linear city. After playing the first stage, firms become aware of the final outcome of the marginal costs for both firms. Finally, in the second stage, firms compete in prices. As in the previous subsection, the game is solved using backward induction.

### 3.2.1 Price Stage

The computations are basically similar to the perfect information case. The optimal price chosen is exactly the same, given by:

\[
p_i(x_i, x_j) = 1 - \frac{2}{3} x_i - \frac{4}{3} x_j - \frac{1}{3} x_i^2 + \frac{1}{3} x_j^2 + \frac{2}{3} c_i + \frac{1}{3} c_j
\]

This is the general expression for prices. Note that firms have perfect information when arriving at the beginning of this stage; therefore, firms choose a price that maximizes a profit function instead of an expected profit function. In order to find the optimal price policies we just need to correctly replace the marginal costs of the firms depending on the outcome revealed between both stages.

However, note that this previous expression is only valid when the indifferent consumer is located between 0 and 1. For each of the four outcomes, the threshold value for this optimal price decision varies, which complicates the calculations of the expected profit function. So, the general profit function expression that allows for the calculation of each of the possible outcomes is given by:
Therefore, for each marginal cost outcome, the profit function becomes different. The differences occur both in the profits and in the thresholds for its different branches. Nevertheless, since firms already have perfect information when setting the prices, the results for this stage become simpler after knowing the optimal location decision of the firms and their marginal cost outcome.

### 3.2.2. Location Stage

Only after their location decision do firms learn their own and their opponent’s final marginal cost outcomes. This means that when choosing their location, firms have to consider an expected profit function dependent on the four outcomes. These four outcomes are: Both firms manage to have low marginal cost, with probability $v^2$; firm $i$ has a low marginal cost while firm $j$ has a high marginal cost with probability $v(1 - v)$; firm $i$ has a high marginal cost while firm $j$ has a low marginal cost with probability $(1 - v)v$; finally both firms have a high marginal cost with probability $(1 - v)^2$. Note that when both firms have a high marginal cost, their marginal cost is equal and we are exactly in the case of d’Aspremont et al. (1979), with an equal marginal cost of 10. On the other hand, when both firms have a low-marginal cost, we have the case of Ziss (1993), and firms are allowed to have different exogenous marginal costs between them.

The expected profit function is therefore given by:

$$E(\Pi_i) = v^2(\Pi_i(c_i, c_j)) + v(1 - v)(\Pi_i(c_i, 10)) + (1 - v)^2(\Pi_i(10, 10))$$

As we previously mentioned in the case of perfect information, the usual method is to discover the first derivative of the profit function with respect to the location choice variable and equalize to zero in order to find the optimal values of location for each firm that maximizes their profit function. However, drawing on some intuition in the results proves to be a less complex way of solving the same problem, due to the reasons mentioned the previous section. In addition, in this case the profit function is not available, making the usual method unfeasible.

There is, however, a problem in setting this expected profit function, which was mentioned in the previous subsection; the branches of each profit function (for each of the outcomes) have threshold values that are dependent on each of the four possible outcomes.
Therefore, when combining these four outcomes into a single function all the possible combinations of these branches must be considered, which results in an expected profit function with a lot of different cases. Moreover, these threshold values are dependent on the location of both firms, which does not allow for a simple representation of this function.

This absence of an expected profit function hinders full analysis of the profit function derivatives, which would allow a better assessment of the location behavior of both firms, as in the perfect information case. We will, nevertheless, assume that the low-cost firm is still not interested in picking a middle spot in the city, preferring only to either agglomerate with or disperse from the high-cost firm. This assumption is based on the fact that, when analyzing each of the four outcomes separately (except for the high-cost outcome, where firms always prefer to locate in different extremes of the city), this property is verified. Moreover, when all the middle branches of each outcome are linearly combined into a branch of the expected profit function this property is also verified, that is, the low-cost firm either agglomerates or disperses relative to its rival.

Assuming that the low-cost firm only wishes to agglomerate or disperse similar to the perfect information case, we have to compare:

\[ \Pi_i(x_i = 1, x_j = 0) > \Pi_i(x_i = 0, x_j = 0) \]

The comparison for this case is now feasible, since for fixed locations it is possible to build the expected profit function combining for the thresholds of the four possible outcomes. In spite of its feasibility, the function still has a lot of cases, derived directly from the four possible outcomes.

The expected profit function for the agglomeration case is simpler, as it is simply the Bertrand case allowing for different marginal costs for the firms. The function is given by, for firm 1:

\[
E(\Pi_1(c_1, c_2)) = \begin{cases} 
\nu^2(-\hat{c}_1) + \nu(1 - \nu)(10 - c_1) + (1 - \nu)(\nu)(0) + (1 - \nu)^2(0), & \hat{c}_1 < 0 \\
\nu^2(0) + \nu(1 - \nu)(10 - c_1) + (1 - \nu)(\nu)(0) + (1 - \nu)^2(0), & \hat{c}_1 \geq 0
\end{cases}
\]

The dispersion case is a bit more complicated, as the function changes towards 8 different branches. Nevertheless, both profit functions (for agglomeration and dispersion, separately) are continuous throughout their entire domain. Both cases are presented in Tables 2 and 3 of Appendix A1.

Summing up, the joint profits of agglomeration and dispersion have 10 regions within the relevant values for the low marginal costs \( c_i \in [0,10] \). These regions are presented in
Figure 3. Notice that the setting of these regions (which are the branches of the expected profit function) is independent from the probability of firms having a low marginal cost $v$.

**Figure 3 – Regions of the Expected Profit Function**

For each of these regions, there is a different expression for the expected profit function, based on the different combinations of the branches of each profit function for each outcome. Our next step was to analyze the location results of both firms for each of the regions represented for a fixed value of $v = 0.5$ and to calculate the Nash Equilibrium results for the location stage.
Figure 4 – Location Equilibrium with imperfect information, for $v=0.5$

![Diagram showing location equilibrium with imperfect information for $v=0.5$.](image)

Figure 4 displays the results. We can see that while for some regions the resulting preferred location of the firms is unique, which is the case of regions 1, 3.1, 3.2 and 7 in which both firms prefer to be agglomerated, for other regions there is more than one possible outcome. In regions 6.1 and 6.2, firms can be either fully dispersed or unable to find location equilibrium. In regions 2, 4, 5 and 8, firms can be either fully agglomerated or unable to find location equilibrium. The results are fully shown in Appendix A.2. Bear in mind that just as in the case of Ziss (1993), the absence of location equilibrium occurs if one of the firms wants to be as close as possible to the opponent and the other wishes to be the farthest.

Some properties of these final location results are easily understandable in Figure 4. First, the results are symmetric with respect to the $c_i = c_j$ curve. This is an expected result, as firms have no other relative advantage in the model than the difference in their marginal costs of production. Secondly, the continuity of the profit function implies that the four regions in the Figure are closed sets. This property happens in spite of the borders of each set being located in different regions of the profit function. Thirdly, when the difference between the possible low and high marginal costs of both firms ($C - c_i$ and $C - c_j$) is equal to 3, the firms are indifferent between choosing to disperse or to agglomerate. This critical value was identified by Ziss (1993) and corresponds to the situation in which the low-cost firm drives the high-cost firm out of the market for each possible location chosen by both firms. That is, in this case when a firm gets the low marginal cost and the opponent gets the high one, the low marginal cost firm will always have the entire market, independently of the location of
both firms. Fourthly, the d’Aspremont et al. (1979) case is also in the figure – being the point (10,10) in which firms choose to fully disperse in the city.

The lines between regions mean that firms are indifferent between choosing either of the options that the line is separating. The obvious exception is line $c_i = c_j$ till point (7,7). In this case, firms are indifferent between agglomerating or dispersing, as it yields the same profit for them. As detailed in the following section, this occurs because the additional profits obtained in the case of a positive outcome when both firms are agglomerated is exactly compensated by the additional profits obtained in the case of an equal outcome when firms are dispersed in the city.

To conclude the location result, when the difference between the low and high marginal costs for both firms is small, firms will optimally choose to disperse. When the difference between the low and high marginal costs is too small for one firm and too big for the other, firms are unable to find a location equilibrium, as the firm with the lowest low marginal cost firm wishes to be located close to the rival and the firm with the higher low-marginal cost wants to be located as far away as possible. If both firms have a high difference between their low and high marginal costs (as long as firms have different low costs), firms will optimally choose to agglomerate at any point of the city.

4. MAIN RESULTS

4.1 Location

The differences between the perfect and imperfect information case are striking, and can be better assessed when comparing Figure 2 with Figure 4. The first difference is that in Figure 2 in most cases the location result is uncertain between agglomeration and dispersion while in Figure 4 firms agglomerate or disperse more often. The second difference is that the agglomeration result is now a possible profit-maximizing outcome for both firms, while in perfect information firms would only either disperse or locate randomly, in a no location equilibrium result. This is a very important result, as geographical agglomeration of firms is a more realistic occurrence than full dispersion.

So, the main question is what motivates firms to agglomerate when the variability of their marginal costs gets so high? Firms wish to agglomerate due to the possibility of having a strong monopoly, which will happen when firm $i$ achieves a significant cost reduction, and firm $j$ does not (this happens with probability $\nu(1 - \nu)$, since they will have a great difference between marginal costs in that outcome, which, when agglomerated, allows having high
profits. If firm $i$ believes that it will achieve a significant cost reduction while firm $j$ will not, then firm $i$ desires to agglomerate and capture all the demand. The introduction of this behavior for both firms in the model explains the existence of agglomeration equilibria. The question can be better answered with an example. The comparison of the profits of firms when $(c_i, c_j) = (8,8)$ and when $(c_i, c_j) = (6,6)$ is presented in Table 1. The case when $(c_i, c_j) = (8; 8,5)$ and $(c_i, c_j) = (6; 6,5)$ is presented in Table 2. In these tables, (L,L), (L,H),(H,L) and (H,H) represent the four possible combinations between the marginal cost outcome of the two firms. For instance, (L,H) refers to the outcome where firm 1 has a low marginal cost (8 in the example $(c_i, c_j) = (8,8)$) and firm 2 has a high marginal cost (10, by assumption). In both cases we are assuming $v = 0.5$, so the probability of occurrence of each outcome is 25%.

Table 1 – Profit of both firms by outcome for equal low marginal costs, with $v = 0.5$

<table>
<thead>
<tr>
<th>Outcome</th>
<th>(L,L)</th>
<th>(L,H)</th>
<th>(H,L)</th>
<th>(H,H)</th>
<th>Expected Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dispersion</td>
<td>1/2</td>
<td>3</td>
<td>0</td>
<td>1/2</td>
<td>1</td>
</tr>
<tr>
<td>Agglomeration</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Dispersion</td>
<td>1/2</td>
<td>25/18</td>
<td>1/18</td>
<td>1/2</td>
<td>11/18</td>
</tr>
<tr>
<td>Agglomeration</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
</tr>
</tbody>
</table>

When the low marginal costs of both firms are equal, we find that dispersion dominates the agglomeration outcome. For the case $(c_i = c_j < 7)$, when firms have a similar outcome (H,H) or (L,L), dispersion is more profitable than agglomeration since firms are able to soften price competition and share the market equally. However, this difference in profits in the dispersion case are exactly compensated by the higher profits obtained by the firms in the case of agglomeration, when the outcome is the most favorable for firm $i$ (see the first two lines of Table 1). Therefore, in this case, the expected profit of dispersion and agglomeration is equal. In the case of $(c_i = c_j > 7)$, when the outcomes are equal the explanation of the profits are similar to the previous case. However, when the outcomes are different due to the softening of price competition, prices are higher in the differentiation case, which allows both
firms to obtain a higher profit in spite of having less demand. Note also that the less fortunate firm (H,L) still earns profits when fully differentiated from its rival.

That the threshold value of this relationship between dispersion and agglomeration is $7$ has a reason: the threshold marginal cost difference above which the low-cost firm takes the high-cost firm out of the market when both firms are located in the extremes of the city is $3$. This value has been previously identified by Ziss (we assume a given city length and unit transportation costs). Therefore, when both firms have a low-marginal cost below $7$, in the case of firms having different outcomes, the low-cost firm captures the whole market. This implies that the derivative of the expected profit function with respect to the difference in the marginal costs is equal for both agglomeration and dispersion cases, for $(c_i = c_j < 7)$.

**Table 2 – Profit of both firms by outcome for different low marginal costs, with $\nu = 0.5$**

<table>
<thead>
<tr>
<th>$(c_i, c_j)$</th>
<th>Outcome</th>
<th>(L,L)</th>
<th>(L,H)</th>
<th>(H,L)</th>
<th>(H,H)</th>
<th>Expected Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(6;6,5)</strong></td>
<td>Dispersion Firm i</td>
<td>49/72</td>
<td>3</td>
<td>0</td>
<td>1/2</td>
<td>301/288</td>
</tr>
<tr>
<td></td>
<td>Agglomeration Firm i</td>
<td>1/2</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>9/8</td>
</tr>
<tr>
<td><strong>(8;8,5)</strong></td>
<td>Dispersion Firm i</td>
<td>49/72</td>
<td>25/18</td>
<td>1/18</td>
<td>1/2</td>
<td>21/32</td>
</tr>
<tr>
<td></td>
<td>Agglomeration Firm i</td>
<td>1/2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>5/8</td>
</tr>
<tr>
<td><strong>(6;6,5)</strong></td>
<td>Dispersion Firm j</td>
<td>25/72</td>
<td>5/2</td>
<td>0</td>
<td>1/2</td>
<td>241/288</td>
</tr>
<tr>
<td></td>
<td>Agglomeration Firm j</td>
<td>0</td>
<td>7/2</td>
<td>0</td>
<td>0</td>
<td>7/8</td>
</tr>
<tr>
<td><strong>(8;8,5)</strong></td>
<td>Dispersion Firm j</td>
<td>25/72</td>
<td>9/8</td>
<td>1/18</td>
<td>1/2</td>
<td>73/144</td>
</tr>
<tr>
<td></td>
<td>Agglomeration Firm j</td>
<td>0</td>
<td>3/2</td>
<td>0</td>
<td>0</td>
<td>3/8</td>
</tr>
</tbody>
</table>

For the case $(c_i, c_j) = (6; 6,5)$, both firms prefer to agglomerate. This happens because compared to the dispersion case, the gain in the profits when the outcome is favorable for firm $i$ or $j$ ((L,H) or (H,L), depending on the firm) is higher than the losses caused by all other outcomes, even for the firm that has higher low marginal cost. However, for the case $(c_i, c_j) = (8; 8,5)$, both firms prefer to disperse. Similar to the case above, the fact is that when dispersed and the difference between the low and high marginal costs is small, the
demand effect is still in operation, which makes up for the difference in the higher profits of agglomeration when firms have a favorable outcome.

4.2 Prices

The comparison between the price policies is only interesting for the case when there is location equilibrium for both cases of perfect and imperfect information. The differences in the price policies between both cases arise because of the probability of non-occurrence of low-marginal costs for both firms and because of the different location patterns chosen in the previous stage.

Regarding location patterns when in a monopoly, the difference between the prices is because the monopolist has to set a lower price when dispersed in order to cover the whole market compared to the agglomeration situation. That difference between the prices is constant and equal to 1. On the other hand, in a duopoly when firms are agglomerated, price is always equal to the higher marginal cost existent between the two firms. In the case of dispersion, the optimal pricing policy can be rewritten in terms of their own marginal cost and of the difference between the marginal costs of both firms, that is:

\[ p_t = c_t - \frac{1}{3} \langle \hat{c}_i \rangle + 1, \text{for } |\hat{c}_i| < 3 \]

For the low-cost firm, we find that the price set when firms are dispersed is higher than when firms are agglomerated if:

\[ c_t - \frac{1}{3} \langle \hat{c}_i \rangle + 1 > c_j \iff \hat{c}_i > -\frac{3}{2} \]

Since we are only comparing pure-strategy equilibrium results, that is, the cases when \(|\hat{c}_i| < (6 - 3\sqrt{3})\), this result means that that the prices when firms are dispersed are always higher than prices when firms are agglomerated.

However, the main determinant of the pricing policy is the probability of the outcomes. As stated before, perfect information is the particular case of imperfect information when the outcome is always low-marginal cost for both firms (\(v = 1\)). All other outcomes imply higher marginal costs for at least one of the firms, a fact that is expected to bring higher prices in the market. Table 3 displays the pricing policy for the same marginal costs set in Table 2 and for \(v = 0.5\).
Table 3 – Pricing policies of both firms by outcome for different low marginal costs

<table>
<thead>
<tr>
<th>((c_i, c_j))</th>
<th>Outcome</th>
<th>((L,L))</th>
<th>((L,H))</th>
<th>((H,L))</th>
<th>((H,H))</th>
<th>Average Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>((6;6,5))</td>
<td>Perfect Information Firm i</td>
<td>43/6</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>43/6</td>
</tr>
<tr>
<td></td>
<td>Imperfect Information Firm i</td>
<td>39/6</td>
<td>60/6</td>
<td>N/A</td>
<td>60/6</td>
<td>53/6</td>
</tr>
<tr>
<td>((8;8,5))</td>
<td>Perfect Information Firm i</td>
<td>55/6</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>55/6</td>
</tr>
<tr>
<td></td>
<td>Imperfect Information Firm i</td>
<td>55/6</td>
<td>58/6</td>
<td>63/6</td>
<td>66/6</td>
<td>60.5/6</td>
</tr>
<tr>
<td>((6;6,5))</td>
<td>Perfect Information Firm j</td>
<td>44/6</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>44/6</td>
</tr>
<tr>
<td></td>
<td>Imperfect Information Firm j</td>
<td>N/A</td>
<td>N/A</td>
<td>60/6</td>
<td>60/6</td>
<td>60/6</td>
</tr>
<tr>
<td>((8;8,5))</td>
<td>Perfect Information Firm j</td>
<td>54/6</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>54/6</td>
</tr>
<tr>
<td></td>
<td>Imperfect Information Firm j</td>
<td>54/6</td>
<td>62/6</td>
<td>60/6</td>
<td>66/6</td>
<td>60.5/6</td>
</tr>
</tbody>
</table>

We can see that the perfect information case results in average lower prices for the market. This happens mainly because of the nature of the perfect information assumed in this model, which determines that firms have lower marginal costs when in perfect information, allowing for the setting of lower prices. On the other hand, there is another effect changing the prices, which results from changes in the location patterns of the firms. Firms now agglomerate for lower values of low-marginal costs and agglomeration itself forces firms to practice lower prices in the market. Note that this effect only occurs when firms decide to agglomerate, that is, when \(c_i, c_j < 7\). However, the dominant effect proves to be the assumed nature of perfect information.

4.3 Profits

The comparison of profits can only be analyzed when there is location equilibrium in both cases, since the absence of that equilibrium implies multiple possibilities for the profits of the firms, depending on the locations they are randomly assigned in.

Location equilibrium in both cases only occurs in the shaded area shown in Figure 5. This area is clearly defined in Appendix A.3. In the upper region, both firms choose to disperse with perfect and imperfect information, while in the lower region, in perfect information firms disperse and in imperfect information firms agglomerate, as is seen in Figures 2 and 4.
Figure 5 – Set of parameter values when there is location equilibrium for both cases, and profit comparisons for firm $i$.

In the lower region, both firms always prefer to be in imperfect information, in spite of having a higher expected marginal cost compared to the perfect information case. This happens because in the lower region, there is always a 25% (when $\nu = 0.5$) possibility of both firms having a very profitable monopoly, since the difference between the marginal costs of both firms becomes very high in each firm’s most favorable outcome.

By comparing the profits between both cases we find that in the upper region, when in imperfect information firms have a higher probability of having higher profits. The line that separates both cases is given by:

$$\frac{1}{8} \left( \frac{1}{3} c_j - \frac{7}{3} \right)^2 + \frac{1}{8} \left( \frac{1}{3} c_j - \frac{1}{3} c_i + 1 \right)^2 + \frac{1}{8} \left( \frac{1}{3} c_i - \frac{13}{3} \right)^2 + \frac{1}{8} > \frac{1}{18} (c_i - c_j - 3)^2$$

$$c_i > \frac{3}{2} c_j - \frac{1}{2} \sqrt{5c_j^2 - 88c_j + 416 - 2}$$

Note, however, that firm j’s preferences are symmetric with respect to the line $c_i = c_j$.

This means that there is no situation where both firms would like to be in perfect information. The reason is rather obvious: when a firm has a low marginal cost disadvantage, being in imperfect information would allow the firm to have a cost advantage with some probability relative to its rival, which makes up for the possibility of negative outcomes where the cost disadvantage becomes higher than in perfect information. When the low marginal costs between firms are similar, both firms would prefer to be in imperfect information.
In the upper region, firms are dispersed, independently of being in perfect or imperfect information. The differences in the profits arise because when firm $i$ has a lower marginal cost, it prefers to be in perfect information where it is sure it will have a good profit. However, as its marginal cost becomes relatively higher, firm $i$ prefers imperfect information as the mix of the 4 outcomes becomes more profitable, as discussed above. Figure 6 presents a close-up of the upper shaded region of the final result for both firms, where it is better shown that the perfect information outcome is never a preferred outcome for both firms at the same time.

**Figure 6 – Profit comparison for both firms in the upper region $c_i, c_j \in [7, 10]$**

The result that firms have a higher profit when in imperfect information is a puzzling one. In this model, it occurs because the absence of knowledge of the firms relative to their own and their opponent’s marginal costs induces firms to risk more in terms of their location, agglomerating more often and leading them to a higher expected profit.

Firms have higher expected profit under imperfect information than under perfect information, because under the first hypothesis it is possible that one firm obtains a stronger cost reduction than the rival. Hence, the firm would have a cost advantage, which explains why the agglomeration decision might appear under imperfect information. This is a very interesting result as it contributes to explaining the willingness to invest in uncertain cost reducing activities (such as R&D). Firms invest, because they have the expectation of obtaining a high cost advantage, which is not possible under the perfect information scenario considered here.
4.4 Changes in the probability of the low cost outcomes

This section discusses what would happen for changes in the value of the probability of occurrence of low-marginal costs for both firms. This discussion is mainly focused on the location patterns of firms rather than on the pricing behavior and profit results.

Figure 4 and all the subsequent analysis is based on $v = 0.5$. Also, when $v = 1$, the model turns out to be the perfect information case (Figure 2), while when $v = 0$, we have the d’Aspremont et al. (1979) model (dispersion for all the values of $c_l$ and $c_H$, since these values are never taken into account).

Departing from $v = 0.5$ and from Figure 4, a small increase in this parameter increases the probability of both firms having a low marginal cost outcome at the expense of a decrease of the probability of both (L,H) and (H,L) outcomes and a larger decrease in the probability of (H,H) outcome. If the low-marginal costs are different between firms, one of the firms will more probably be at a disadvantage compared with the rival and will prefer to disperse more often, while the firm with an advantage will prefer to agglomerate more often. This will increase the possibilities of absence of location equilibrium. Concerning agglomeration vs. dispersion, firms will more often have similar outcomes between them ($(L,L) + (H,H) > (L,H) + (H,L)$), which makes firms disperse more often. Summing up, the areas of agglomeration are expected to decrease and the areas of dispersion and “no location equilibrium” are expected to increase with a small increase of $v$ when departing from $v = 0.5$. As expected, the results are close to Ziss’s (1993) conclusions.

Similarly, a small decrease in parameter $v$ raises the odds of firms being located in the (H,H) outcome. This result induces both firms to disperse more often, which has two consequences: the firms will be at the no location equilibrium situation more often, as the firm with the higher low-marginal cost will disperse more often in the cases when the advantaged firm still prefers to agglomerate; the second consequence is similar to the previous case, as firms are expected to have more similar outcomes than before, which pushes firms to disperse rather to agglomerate. Therefore, the resulting pattern is going to be very similar to the case of an increase in parameter $v$. The exception is that since dispersion occurs more often than agglomeration, the area of no location equilibrium diminishes at the expense of the area of dispersion. It seems that $v = 0.5$ maximizes the region of agglomeration. This happens because that value for the parameter maximizes the probability of both firms of having a low
marginal cost while the other firm has the high marginal cost, which is the outcome that gives the most profit for agglomerated firms.

As an example, the results for \( v=0.4 \) and \( v=0.75 \) are displayed in Figures 7 and 8. These specific values are chosen to provide a better display of the resulting conditions and not because of any sort of “parameter mining”.

**Figure 7 - Location Equilibrium with imperfect information, for \( v=0.4 \)**

![Figure 7](image)

**Figure 8 – Location Equilibrium with imperfect information, for \( v=0.75 \)**

![Figure 8](image)
When comparing case $v=0.75$ (Figure 8) with the Ziss model ($v=1$, Figure 2), we conclude that when $v$ diminishes departing from $v=1$, not only does the “No location Equilibrium” area shrink because of agglomeration but the number of cases where dispersion occurs are also much lower, due to the existence of incentives of agglomeration for one of the firms (which means no location equilibrium) in the central strip of the figure.

Moreover, Ziss’s equilibrium changes drastically in the case of a small decrease in $v$. This happens because in the set of parameters that we have called region 7 (see Figure 3) and when $v = 1$, the firm that has a higher low marginal cost is indifferent between agglomerating and dispersing, since it has no demand either way. However, as $v$ lowers marginally, the firm has a very small probability of having a positive demand, which occurs in the case of having the positive outcome ($\text{L,H}$ for firm $i$). In this case, the firm prefers to agglomerate to maximize that small possibility of having profits throughout all the set of parameters contained in region 7. Since the other firm (which is symmetrically located in region 1) prefers also to agglomerate when having such an advantage, the agglomeration result occurs in both regions 1 and 7.

Also, when departing from the lower values of $v$ the result of d’Aspremont et al. holds, that is, both firms wish to disperse independent of the value of their low marginal cost until $v$ reaches the value of $2 - \sqrt{3}$, which is approximately 0.26795. As $v$ rises, when the difference between the low marginal costs is sufficiently large (region 1 and 2 of Figure 3), the lower low marginal cost firm starts preferring to agglomerate, which leads to a “no location equilibrium” area in part of these regions (and also in regions 7 and 8, due to the agglomeration desires of the other firm). However, when $v$ reaches $1/3$, region 7 once again becomes crucial as it fully changes from dispersion to agglomeration: the higher low marginal cost firm wishes to agglomerate because of all the parameters expressed in region 7, due to the increasing possibility of having a monopoly of its own if the marginal cost outcome is favorable to it ($\text{L,H}$ for firm $i$).

These values ($v=1/3$ and $v=1$) are the only discontinuities in the location decision function, as the regions of agglomeration, dispersion and no location equilibrium react continuously to the remaining changes in the values of the parameter $v$.

So, agglomeration for both firms may occur in this setting when $v \in [1/3; 1]$, with the highest area of agglomeration occurring when $v = 0.5$. 
5. CONCLUSIONS

In this paper, we introduce imperfect information in the framework of Hotelling (1929), d’Aspremont et al. (1979) and Ziss (1993), in the sense that firms are unaware of which marginal cost they will have before choosing their location. We have opted for a simplifying case of imperfect information as firms have two possible outcomes for their marginal cost: a high marginal cost, equal for both firms, and a low marginal cost not necessarily equal for both firms.

The main conclusion drawn from this work is that agglomerative location equilibrium becomes possible when the low-marginal cost of both firms is sufficiently different from their high-marginal cost, and when the probability of both firms having a low marginal cost is between 1/3 and 1. This result is not possible in the perfect information models of d’Aspremont et al. (1979) and Ziss (1993). The agglomeration result happens mainly because firms are able to risk agglomerating and face a typical Bertrand (1883) competition setting in which they could possibly conquer all of the market rather than dispersing, where the possibility of low-cost firms conquering the whole market when they have a sufficiently high marginal cost advantage forces them to set a lower price, which naturally lowers profits.

Other conclusions, although dependent on the assumed nature of imperfect information, is that prices are on average lower on perfect information and profits are higher on imperfect information when firms choose to agglomerate in the location stage and in some cases of dispersion. Also, if firms had the opportunity to choose, both would never agree to be in a perfect information situation.

This paper proves that imperfect information matters to the analysis of the two-stage location equilibrium game a la Hotelling, a claim also made by Boyer et al. (2003). Moreover, the lack of information is an important issue nowadays, as uncertainty arises in some important processes that firms face in many different areas of their organization. For instance, estimation of demand; estimation of the impact of a certain marketing strategy; uncertainty about worker productivity; uncertainty about innovation outcomes; uncertainty about the conditions that other firms in the market face… and many more.

In order to better understand the results of this model, future research should focus on making endogenous the uncertainties of the model, for instance, considering an investment in marginal cost reduction, considering a suppliers’ market or considering different assumptions.
for the demand. This would allow for a better understanding of how different types of uncertainties would affect the market and firms’ decisions.

References


Appendix

A.1 Branches of the expected profit function

The expected profit function when firms are dispersed becomes complicated due to the profit branches for each of the four possible outcomes. Table 4 presents the behavior of the expected profit function for each of the branches. These branches, combined with the case of firms’ expected profit function in the case of agglomeration (Figure 3), allowed for the comparison of profits between agglomeration and dispersion, and subsequent construction of Figure 4.

Table 4 – Expected profit for each of the regions of the profit function for firm i, in the case of dispersion.

<table>
<thead>
<tr>
<th>Restrictions/Outcome</th>
<th>Expected profit (Firm i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Θi, εi ≤ j ≤ i; i ≠ j; c_i ≠ j;</td>
<td>(c_i, c_j) ≤ 10 ≤ i ≠ j</td>
</tr>
<tr>
<td>Θi, εi ≤ j ≤ i; i ≠ j; c_i ≤ j;</td>
<td>(c_i, c_j) ≤ 10 ≤ i ≠ j</td>
</tr>
<tr>
<td>Θi, εi ≤ j ≤ i; i ≠ j; c_i ≥ j;</td>
<td>(c_i, c_j) ≤ 10 ≤ i ≠ j</td>
</tr>
<tr>
<td>Θi, εi ≤ j ≤ i; i ≠ j; c_i ≥ j;</td>
<td>(c_i, c_j) ≤ 10 ≤ i ≠ j</td>
</tr>
</tbody>
</table>

Table 5 – Expected profit for each of the regions of the profit function for firm i, in the case of agglomeration.

<table>
<thead>
<tr>
<th>Restrictions/Outcome</th>
<th>Expected profit (Firm i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>c_i ≤ i; i ≠ j; 0 ≤ j;</td>
<td>(c_i, c_j) ≤ 10 ≤ i ≠ j</td>
</tr>
<tr>
<td>c_i ≤ i; i ≠ j; 0 ≤ j;</td>
<td>(c_i, c_j) ≤ 10 ≤ i ≠ j</td>
</tr>
</tbody>
</table>

A.2 Profit Comparison and results for the imperfect information case

Tables 4, 5 and 6 present the expected profits results for firm i given different values for v. The final location result is constructed after comparing the results for firm j (which are symmetric with respect to firm i) and checking whether both firms prefer to agglomerate (agglomeration equilibrium), to disperse (dispersion equilibrium) or if one prefers to disperse and the other to agglomerate (No location equilibrium).
Table 6 – Profit comparison between dispersion and agglomeration for firm $i$, for $v=0.5$

<table>
<thead>
<tr>
<th>Region</th>
<th>Expected Profit</th>
<th>Dispersion</th>
<th>Agglomeration</th>
<th>Solution ($v=0.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>($c_j - (6 - 3\sqrt{3}) &lt; c_i &lt; c_j$) &amp; $c_i &lt; \frac{c_j+\sqrt{(c_j+6)(c_j-7)+7}}{2}$ &amp; $c_i &lt; \frac{c_j+\sqrt{(c_j+10)(c_j-9)+15}}{2}$ &amp; $c_j &gt; \frac{c_i+\sqrt{(c_i+6c_j)(c_i-36)}}{2}$) and $c_i &gt; \frac{c_i+\sqrt{(c_i+6c_j)(c_i-36)}}{2}$</td>
<td>$\frac{1}{4}c_i \cdot \frac{1}{2}c_i \cdot \frac{1}{4}c_i$</td>
<td>$\frac{1}{4}c_i \cdot \frac{1}{2}c_i \cdot \frac{1}{4}c_i$</td>
<td>$\text{Disp} \leq \text{Agg}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{4}(c_i + \frac{1}{2}c_i + \frac{1}{4}c_i)^3 \cdot \frac{1}{2}c_i$</td>
<td>$\frac{1}{4}(c_j + \frac{1}{2}c_j + \frac{1}{4}c_j)^3 \cdot \frac{1}{2}c_j$</td>
<td>$\frac{1}{4}(c_j + \frac{1}{2}c_j + \frac{1}{4}c_j)^3 \cdot \frac{1}{2}c_j$</td>
<td>$\text{Disp} \leq \text{Agg}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{1}{4}(c_i + \frac{1}{2}c_i + \frac{1}{4}c_i)^3 \cdot \frac{1}{2}c_i$</td>
<td>$\frac{1}{4}(c_j + \frac{1}{2}c_j + \frac{1}{4}c_j)^3 \cdot \frac{1}{2}c_j$</td>
<td>$\frac{1}{4}(c_j + \frac{1}{2}c_j + \frac{1}{4}c_j)^3 \cdot \frac{1}{2}c_j$</td>
<td>$\text{Disp} \leq \text{Agg}$</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{1}{4}(c_i + \frac{1}{2}c_i + \frac{1}{4}c_i)^3 \cdot \frac{1}{2}c_i$</td>
<td>$\frac{1}{4}(c_j + \frac{1}{2}c_j + \frac{1}{4}c_j)^3 \cdot \frac{1}{2}c_j$</td>
<td>$\frac{1}{4}(c_j + \frac{1}{2}c_j + \frac{1}{4}c_j)^3 \cdot \frac{1}{2}c_j$</td>
<td>$\text{Disp} \leq \text{Agg}$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{1}{4}(c_i + \frac{1}{2}c_i + \frac{1}{4}c_i)^3 \cdot \frac{1}{2}c_i$</td>
<td>$\frac{1}{4}(c_j + \frac{1}{2}c_j + \frac{1}{4}c_j)^3 \cdot \frac{1}{2}c_j$</td>
<td>$\frac{1}{4}(c_j + \frac{1}{2}c_j + \frac{1}{4}c_j)^3 \cdot \frac{1}{2}c_j$</td>
<td>$\text{Disp} \leq \text{Agg}$</td>
</tr>
<tr>
<td>6</td>
<td>$\frac{1}{4}(c_i + \frac{1}{2}c_i + \frac{1}{4}c_i)^3 \cdot \frac{1}{2}c_i$</td>
<td>$\frac{1}{4}(c_j + \frac{1}{2}c_j + \frac{1}{4}c_j)^3 \cdot \frac{1}{2}c_j$</td>
<td>$\frac{1}{4}(c_j + \frac{1}{2}c_j + \frac{1}{4}c_j)^3 \cdot \frac{1}{2}c_j$</td>
<td>$\text{Disp} \leq \text{Agg}$</td>
</tr>
<tr>
<td>7</td>
<td>$\frac{1}{4}(c_i + \frac{1}{2}c_i + \frac{1}{4}c_i)^3 \cdot \frac{1}{2}c_i$</td>
<td>$\frac{1}{4}(c_j + \frac{1}{2}c_j + \frac{1}{4}c_j)^3 \cdot \frac{1}{2}c_j$</td>
<td>$\frac{1}{4}(c_j + \frac{1}{2}c_j + \frac{1}{4}c_j)^3 \cdot \frac{1}{2}c_j$</td>
<td>$\text{Disp} \leq \text{Agg}$</td>
</tr>
<tr>
<td>8</td>
<td>$\frac{1}{4}(c_i + \frac{1}{2}c_i + \frac{1}{4}c_i)^3 \cdot \frac{1}{2}c_i$</td>
<td>$\frac{1}{4}(c_j + \frac{1}{2}c_j + \frac{1}{4}c_j)^3 \cdot \frac{1}{2}c_j$</td>
<td>$\frac{1}{4}(c_j + \frac{1}{2}c_j + \frac{1}{4}c_j)^3 \cdot \frac{1}{2}c_j$</td>
<td>$\text{Disp} \leq \text{Agg}$</td>
</tr>
</tbody>
</table>

A.3 Profit Comparison between perfect and imperfect information

This expression defines the area when both firms have location equilibrium in both cases of imperfect and perfect information. It corresponds to the area represented in Figure 5.

$((c_j - (6 - 3\sqrt{3}) < c_i < c_j) \& c_i < \frac{c_j+\sqrt{(c_j+6)(c_j-7)+7}}{2} \& (c_j - (6 - 3\sqrt{3}) < c_i < c_j) \& c_i > \frac{c_j+\sqrt{(c_j+10)(c_j-9)+15}}{2})$ and $((c_j < c_i < c_j + (6 - 3\sqrt{3})) \& c_j < c_i + \sqrt{c_j^2 + 8c_j + 3} \& (c_i < c_i < c_j + (6 - 3\sqrt{3})) \& c_i > \frac{c_i+\sqrt{(c_i+6c_j)(c_i-36)}}{2})$
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