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Does Vertical Integration Promote
Downstream Incomplete Collusion?
An Evaluation of Static and Dynamic Stability

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Abstract

This paper analyzes the impact of vertical integration on the static and dynamic stability of downstream incomplete collusion. It is shown that a vertical merger between an upstream firm and a downstream cartel or fringe firm promotes downstream collusion, under certain conditions on the market size. However, for low market concentration, a vertical merger with a cartel firm hinders collusion. Moreover, a welfare analysis shows that consumer surplus increases with the vertical merger because the merger partially eliminates the double marginalization problem.

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1 Introduction

The theoretical and empirical literature on vertical integration has gained a great contribution from researchers on industrial economics over many years. It has been very important for economists to understand the main determinants of vertical integration, to identify the type of transactions that are mediated within firms through vertical integration or conducted through the market. In this paper we focus on the coordinated effects of vertical integration that have been ignored for several years. Chicago scholars emphasized the pro-competitive effects of vertical integration (Stigler, 1963, 1971). However, with the introduction of game-theory tools, post-Chicago scholars claimed that vertical integration could also have anti-competitive effects (Riordan, 2008; Riordan and Salop, 1995; Salop and Scheffman, 1987). Therefore, the analysis of the anticompetitive effects of vertical integration has been a debated topic of research in Industrial Organization. Sometimes, vertical integration is used as a strategy to eliminate the rivals from the market, allowing the achievement of greater efficiency and higher profits. Also, occasionally, firms adopt this type of organization in order to create barriers to entry, to increase market power and to facilitate collusion. Antitrust authorities have remained concerned that vertical integration might facilitate collusion at upstream and downstream levels (Riordan, 2008). Vertical integration might facilitate collusion by supporting the punishment and monitoring mechanisms and also by allowing agreement between those firms. Vertical integration facilitates collusion if, after the merger, upstream or downstream firms are able to coordinate in a more effectively way than if they were operating separately.

The main contribution of this paper is to highlight the importance of considering, simultaneously, two types of firm's strategies: vertical integration and collusion, in a context where some firms have no incentives to collude (incomplete collusion). This analysis is useful to antitrust authorities in order to more properly evaluate situations where vertical integration is used to enforce collusion, even when there are fringe firms involved and also to quantify its impacts on social welfare. When evaluating a vertical merger, usually, antitrust authorities focus on the coordinate effects, on the impacts on competition and on market foreclosure, which in turn, reduce social welfare. However, here we show that vertical merger and incomplete collusion can actually promote social welfare.

There are a few real examples of situations where this question is important. For example, in August 2011, the UK's Office of Fair Trading (OFT) issued an infringement decision against Arla, Asda, Dairy Crest, McLelland (prior to its acquisition by Groupe Lactalis), Safeway (prior

to its acquisition by Morrisons), Sainsbury's, Tesco, The Cheese Company and Wiseman. The OFT found that the parties had infringed the Competition Act 1998 by coordinating increases in the final prices of certain dairy products (cheese, milk, butter) in 2002 and/or 2003.¹ This is one important example of downstream incomplete collusion, in an industry characterized by significant vertical relations.

We could not find any academic literature that considered simultaneously both incomplete collusion and vertical integration. The related literature considers either collusion and vertical integration or only incomplete collusion, separately. Therefore, our contribution is twofold. Firstly, our paper adds to the literature that studies how vertical integration facilitates downstream and upstream collusion. Chen (2001), Nocke and White (2007, 2010) and Normann (2009) analyzed this question in relation to upstream collusion while, Chen and Riordan (2007) and Mendi et al. (2011) investigate the effects on downstream market collusion. However, the referred works assumed that all the firms in the industry accept the collusive agreement.

Secondly, our paper adds to the literature on the sustainability of incomplete collusion. Despite the limited literature, there are some studies of incomplete cartels available such as: Selten (1973), D'Aspremont et al. (1983), Donsimoni et al. (1986), Martin (1990), Shaffer (1995), ESCRIHUELA-VILLAR (2004, 2008). These papers differ not only on the type of framework (dynamic or static) but also on the type of competition (price or quantities) and the type of game (simultaneous or sequential). However, in these studies, it is not assumed that the industry has a vertically integrated structure.

In this paper we combine these two issues to study how vertical integration affects downstream collusion when there are fringe downstream firms. In the static framework our paper is closely related to D'Aspremont et al. (1983) that studies the necessary internal and external conditions for horizontal stability. More recently, ESCRIHUELA-VILLAR (2008) studied how the number of firms in the cartel affects the possibility that its members can sustain a collusive agreement. Following D'Aspremont et al. (1983), Martin (1990), ESCRIHUELA-VILLAR (2008) we also endogeneized the cartel formation by analyzing the number of firms that are willing to accept the collusive agreement. Also, our paper is close to Shaffer (1995)'s who analyzed the size and uniqueness of the stable cartel when the fringe is Cournot and the cartel behaves as a Stackelberg leader. Moreover, like ESCRIHUELA-VILLAR (2009) we assume that, when there is collusion, fringe firms choose the output that maximizes their profit, taking the output of cartel firms as given. In the dynamic framework, our paper is closely related to Martin (1990) that analyzed the conditions for dynamic cartel sta-

¹See OFT (2011) for more details.

bility when collusion is enforced with two alternative punishment strategies: a trigger strategy (Friedman, 1971) or a stick-and-carrot strategy (Abreu, 1986). Martin (1990) found that either a trigger strategy or a stick-and-carrot strategy sustain cartel joint profit maximization in the presence of a fringe. Furthermore, Escrihuela-Villar (2004) analyzed the effects of the cartel size on the sustainability of a collusive agreement, using trigger and stick-and-carrot strategies and concluded that, with both strategies, collusion is easier to sustain the larger the cartel is.

In order to analyze the effects of vertical integration on downstream incomplete collusion, we construct three models: a model without vertical integration, a model where there is vertical integration with a cartel downstream firm and, finally, a model where there is vertical integration with a fringe downstream firm. In particular, we study each model alone and then we compare the results between these models. Moreover, we provide a numerical example to illustrate the results of these three theoretical models.

We find that, in general, a vertical merger with a cartel or a fringe downstream firm always enforces collusion. The main reason which explains this result is that the vertical merger decreases not only the number of firms in the industry, but also the collusive firms' incentives to deviate from the agreement. However, we find the opposite outcome for low market concentration. Vertical merger with a cartel firm might hurt downstream collusion because cartel firms earn lower individual profits the higher the number of firms in the cartel is. Finally, a welfare analysis shows that social welfare can increase with vertical integration due to the partial elimination of double marginalization.

The remainder of the paper is organized as follows: Section 2 introduces the common assumptions of three models and analyzes the general static and dynamic stability conditions for the industry without vertical integration (Section 2.1), with vertical integration with a downstream cartel firm (Section 2.2.) and with vertical integration with a fringe downstream firm (Section 2.3). Section 3 discusses the results of the static and dynamic stability conditions obtained for the three models and Section 4 presents a welfare analysis. These two sections are also based on the results of the numerical simulation. Finally, Section 5 concludes.

2 The Model

We consider a framework where, in each period, two upstream firms ($N = 2$) produce a homogeneous intermediate good that is used by M downstream firms to produce the homogeneous final good. We assume that one unit of the input to produce one unit of the output (production function with fixed proportions) is necessary. Each upstream firm has a constant marginal cost c per unit. Each downstream firm has a constant marginal cost of w per unit, where w is the input price and, by assumption, we normalize all the other downstream costs to zero. Both upstream and downstream firms compete in quantities in their respective markets. The interaction between upstream and downstream firms is repeated forever and we assume that there is complete information. We analyze the stationary collusive equilibrium in which some downstream firms form an incomplete cartel. That is, there are F ($F < M$) firms that form the fringe and they have no incentives to collude and the remaining K ($K < M$) firms have incentives to collude. Also, like in the cartel and fringe literature (Shaffer, 1995; Donsimoni, 1985; Escriuella-Villar, 2009) we assume that the cartel behaves as a Stackelberg leader. Hence, fringe firms choose the output that maximizes their profit, taking the output of cartel firms as given.

The inverse demand for the final consumption good is given by the linear function $P(Q) = \max(0, a - bQ)$, where $Q = Q_K + Q_F$ is the output produced by cartel and fringe firms, P is the output price. We assume that $a > c$.

The timing of the game is the following:

Stage 1. The upstream firms decide simultaneously the wholesale quantities.

Stage 2. The downstream firms decide their quantities.

The game is solved by backward induction.

In order to assess if vertical integration promotes downstream collusion when there are fringe downstream firms, we compare the results obtained for three situations: without vertical integration (model 1), with vertical integration with a cartel downstream firm (model 2) and with vertical integration with a fringe downstream firm (model 3). For each model we analyze first the static and then the dynamic stability of the cartel. Following D'Aspremont et al. (1983) a cartel is horizontally stable² if i) a member of the cartel earns at least as great a profit as it would have earned if it joins the fringe (internal stability condition - ISC) and ii) a fringe firm earns at least as great a profit by staying in the fringe as it would if it joins the cartel (external

²We use the expressions static stability and horizontal stability indistinctly.

stability condition - ESC). Therefore, the cartel is stable if it is both internally and externally stable.

Moreover, we examine the dynamic stability of the cartel. We assume that downstream cartel firms compete repeatedly in an infinite number of periods (Time is discrete: $t = 1, 2, \dots$) and seek to maximize the present discounted value of its profits, using a discount factor $\delta \in [0, 1]$. Following Escrihuella-Villar (2009), a pure strategy for a downstream cartel member k_j is an infinite sequence of functions $\{S_{k_j}^t\}_{t=1}^\infty$, with $S_{k_j}^t : \Sigma^{t-1} \rightarrow \Omega$, where Σ^{t-1} is the set of all downstream cartel output choices until $t - 1$, with each element of Ω , σ_j^τ , $j = 1, \dots, K$, $\tau = 1, \dots, t - 1$, and Ω is the set of output choices available to each downstream cartel firm.

Following Friedman (1971), we assume that firms adopt grim trigger strategies. Therefore, under these strategies the cartel downstream firms collude in the first period, producing $q_{k_j}^{Collusion}$, and continue to do so in the following periods if all the firms accept the collusive agreement. If at least one downstream cartel firm deviates from the collusive agreement, then there is a reversion to the Nash equilibrium forever, and each cartel firm produces $q_{k_j}^{Cournot-Nash}$. Since we restrict attention to trigger strategies, $\{S_{k_j}^t\}_{t=1}^\infty$ can be specified as follows: At $t = 1$, $S_{k_j}^1 = q_{k_j}^{Collusion}$, at $t = 2, 3, \dots$:

$$S_{k_j}^t(\sigma_j^\tau) = \begin{cases} q_{k_j}^{Collusion}, & \text{if } \sigma_j^\tau = q_{k_j}^{Collusion} \text{ for all } j = 1, \dots, K \text{ and } \tau = 1, \dots, t - 1 \\ q_{k_j}^{Cournot-Nash}, & \text{otherwise} \end{cases}$$

Regarding fringe firms, the optimal response consists of maximizing their current period's payoff, acting as Cournot quantity-setters. A fringe firm produces q_{f_i} if all downstream cartel members accept the collusive agreement and $q_{f_i}^{Cournot}$ if cartel members have reverted to Cournot behavior. Under both cases, fringe downstream firms maximize their profits, given the outputs of other firms. However, when a downstream cartel firm deviates from the collusive agreement, fringe firms react to that deviation, maximizing their profits, taking into account the output of the deviating firm. We denote the profit function of a cartel downstream firm and of a fringe downstream firm, respectively, as π_k and π_f . As shown by Friedman (1971), cartel firms colluding in each period can be sustained as a Subgame Perfect Nash Equilibrium (SPNE) of the repeated game with the strategy profile defined above if and only if the Incentive Compatibility Constraint (ICC) is satisfied: $\frac{1}{1-\delta}\pi_k^C \geq \pi_k^d + \frac{\delta}{1-\delta}\pi_k^N$, where π^d , π^C and π^N are, respectively, the cartel downstream deviation, collusion and punishment (Cournot-Nash) profits.

For each model (without vertical integration, vertical integration with a cartel firm, vertical integration with a fringe firm) we find the optimal quantities and the corresponding profits in the

three cases a) collusion in the downstream market, obtaining π^C b) Cournot competition in the downstream market, obtaining π^N and c) deviation from the collusion by one firm, obtaining π^d . In sections 2.1, 2.2 and 2.3 we first present the three models in a general framework, that is, for K and F downstream firms. Then, for each model, we provide a numerical simulation in order to improve the analysis of the effects of vertical integration in the dynamic stability of collusion. Here we assume a limited number of cartel and fringe firms ($K \in [0, 12]$ and $F \in [0, 10]$) because these represent the most significant real world cases of industries where the firms collude and it is difficult to have real cases with more than 12 firms in the cartel. Furthermore, some cartel and fringe firms are removed from the analysis because they do not meet the stability conditions.

2.1 The Model without Vertical Integration

a) Collusive Results

The profit function of one representative downstream fringe (for instance, firm 1) is given by:

$$\pi_{f_1}^D = (a - b \sum_{l=1}^F q_{f_l} - b \sum_{j=1}^K q_{k_j} - w) q_{f_1}, \text{ with } l = 1, \dots, F \text{ and } j = 1, \dots, K$$

where q_{k_j} and q_{f_l} represent, respectively, the outputs produced by each cartel downstream firm and each fringe downstream firm. By symmetry, all the fringe firms choose the same output and, therefore, the best reply of each fringe firm is given by $q_f = \frac{a-w-bQ_K}{b(F+1)}$.

Acting as a Stackelberg leader against the downstream Cournot fringe firms, the cartel produces the monopoly output: $Q_K = \frac{a-w}{2b}$ and each member of the cartel produces $q_{k_j} = \frac{a-w}{2bK}$, with $j = 1, \dots, K$. Substituting Q_K into the best reply function of the fringe firms we obtain $q_{f_l} = \frac{a-w}{2b(F+1)}$, with $l = 1, \dots, F$. The total output in the downstream market and the price are given by $Q = \frac{(a-w)(2F+1)}{2b(F+1)}$ and $P = \frac{a+w(2F+1)}{2(F+1)}$, respectively. The total quantity of the downstream market is also the quantity bought from upstream firms. Then, the inverse demand function for the upstream firms is given by $w = \frac{a(2F+1)-2bQ(F+1)}{2F+1}$.

The individual profit function for an upstream firm is given by:

$$\pi_i^U = (w - c) q_i^U = \left(\frac{a(2F+1)-2bQ(F+1)}{2F+1} - c \right) q_i^U \text{ with } i = 1, 2 \text{ and } Q = q_1^U + q_2^U$$

By symmetry $q_1^U = q_2^U$, then the quantity that maximizes the individual profit of the upstream firm is: $q_i^U = \frac{A(2F+1)}{6(F+1)}$ and the total output is $Q = \frac{A(2F+1)}{3(F+1)}$, where $A = \frac{a-c}{b}$.

Considering the above results, the upstream and the downstream prices are $w = \frac{a+2c}{3}$ and $P = \frac{a(2+F)+c(2F+1)}{3(F+1)}$. The equilibrium downstream cartel and fringe quantities are $q_{k_j} = \frac{A}{3K}$ and $q_{f_l} = \frac{A}{3(F+1)}$, respectively.

The profits are the following:

- $\pi_i^U = \frac{A^2b(2F+1)}{18(F+1)}$, for upstream firms;
- $\pi_{k_j}^D(K, F) = \frac{A^2b}{9K(F+1)}$, for downstream cartel firms; and
- $\pi_{f_l}^D(K) = \frac{A^2b}{9(F+1)^2}$, for downstream fringe firms.

To prove that the cartel is horizontally stable we have to verify both internal and external stability conditions.

◦ **Internal Stability Condition:** a member of the cartel earns at least as great a profit as it would have earned if it joins the fringe: $\pi_k(K, F) \geq \pi_f(F+1)$. For this particular case, we get: $\frac{A^2b}{9K(F+1)} \geq \frac{A^2b}{9(F+2)^2} \Leftrightarrow K \leq \frac{(F+2)^2}{F+1}$.

◦ **External Stability Condition:** a fringe firm earns at least as great a profit by staying in the fringe as it would if it join the cartel: $\pi_k(K+1, F-1) \leq \pi_f(F)$. For this particular case, we get: $\frac{A^2b}{9F(K+1)} \leq \frac{A^2b}{9(F+1)^2} \Leftrightarrow K \geq \frac{(F+1)^2}{F} - 1$.

Proposition 1. Without vertical integration, downstream collusion is horizontally stable if and only if:³

$$\frac{(F+1)^2}{F} - 1 \leq K \leq \frac{(F+2)^2}{F+1}$$

As we can see, without vertical integration the profits are the same for both upstream firms. Also, comparing the cartel's firms profits with fringe's firms profits we conclude that the first is higher than the latter if and only if the number of cartel firms is lower than $F+1$. However, as we show above, this is incompatible with the static stability conditions. Therefore, although the difference is small, the individual profit of a cartel downstream firm is lower than the individual profit of a fringe firm. Moreover, we conclude that the cartel of K firms is stable if and only if there are $K-2$ or $K-3$ fringe firms.⁴ Then, the cartel should have, respectively, 50% – 75% or

³This result is the same as in Martin's (1990) paper.

⁴This conclusion is obtained from the simulation results with $K \in 3, \dots, 12$ and $F \in 1, \dots, 10$. We consider that this simulation represents the most significant cases of collusion, as it is difficult to have real cases with more than 12 firms in the cartel. For more details see Appendix A1.

50% – 80% firms in the industry.⁵

From the results we can ask: what is the incentive to constitute a cartel? Before answering this question we need to verify if there is dynamic stability. Therefore, we also need to obtain the Cournot and the deviation results.

b) Cournot Equilibrium

The punishment profit is the downstream profit obtained under the static Nash-Cournot Equilibrium. The profits for downstream firms are given by:⁶

$$\pi_{j=l}^D = \frac{4A^2b}{9(F+K+1)^2}, j = 1, \dots, K \text{ and } l = 1, \dots, F.$$

Comparing the Cournot profit with the collusive profit of a cartel firm, it is possible to see that the collusive profit is higher than the Cournot profit for $K > F+1$, which is compatible with the static stability conditions. Then, the cartel firms have always incentives to form a cartel. The fact that firms have an incentive to free-riding, could explain why, under collusion, cartel firms will, generally, have lower individual profits than fringe firms. Each firm will prefer that their rivals form the cartel in order to have higher profit.⁷

Moreover, when we move from Cournot to the collusive scenario, the profitability of the cartel under collusion depends on two opposite forces: on the one hand the cartel firms produce lower quantity and thereby, increase its profits. On the other hand, the fringe firms react by increasing their quantity which reduces the profitability of the cartel firms. In this case the second effect dominates the first, which could also explain why the profit of cartel firm, under collusion, is lower than the profit of a fringe firm.

c) Deviation Results

Consider that one firm from the cartel, firm D_{k_1} for example, deviates while the others downstream cartel firms maintain their quantities and the fringe firms react to the deviation. In this case the downstream profits are:⁸

$$\begin{aligned} & - \pi_{k_1}^{deviation} = \frac{A^2b(K+1)^2}{36K^2(F+1)}, \text{ for the deviation downstream cartel firm;} \\ & - \pi_{k_j}^D = \frac{A^2b(K+1)}{18K^2(F+1)}, \text{ for the } j = 2, \dots, K \text{ downstream cartel firms; and} \end{aligned}$$

⁵This result is similar to Salant et al. (1983) that indicate that mergers are only likely to occur when 80% of the firms in a market are included in the merger.

⁶For a detailed description of the results with Cournot competition in the downstream level see Appendix A1 and Appendix B2.

⁷This result is similar to Horizontal Mergers' already known in the literature.

⁸For a detailed description of the results with deviation from the cartel see Appendix A2.

$$- \pi_{f_l}^D = \frac{A^2 b (K+1)^2}{36 K^2 (F+1)^2}, \text{ for the } l = 1, \dots, F \text{ downstream fringe firms.}$$

By deviating from the collusive agreement, the cartel downstream firm gains higher profit than if it was in collusion, in part due to the high deviation quantity. The remaining cartel downstream firms continue to produce the same collusive quantity, however, fringe firms, by reacting to the deviation, reduce their quantity. Moreover, both cartel and fringe firms earn a lower profit, than if they were in collusion.

To study the dynamic stability of collusion we replace the deviation, collusion and punishment profits in the ICC and get the critical discount factor for collusion at downstream level with fringe downstream firms, when there is no vertical integration. The result is summarized in Proposition 2.

Proposition 2. Without vertical integration, downstream collusion is supported as a SPNE if and only if:

$$\delta \geq \frac{(K-1)^2 (F+K+1)^2}{(F+K+1)^2 (K+1)^2 - 16 K^2 (F+1)} \equiv \delta_{withoutVI}^*$$

Table 1 displays the values of K and F (with K and F lower than 12), obtained by numeric simulation, that satisfy both static and dynamic stability conditions.⁹

K	F	Static Stability	$\delta_{withoutVI}^*$	K	F	Static Stability	$\delta_{withoutVI}^*$
3	1	ESC & ISC	0.892857	8	6	ESC & ISC	0.997106
4	1	ESC & ISC	0.835052	9	6	ESC & ISC	0.991288
4	2	ESC & ISC	0.964989	9	7	ESC & ISC	0.998057
5	2	ESC & ISC	0.927536	10	7	ESC & ISC	0.993940
5	3	ESC & ISC	0.984802	10	8	ESC & ISC	0.998634
6	3	ESC & ISC	0.963020	11	8	ESC & ISC	0.995619
6	4	ESC & ISC	0.992129	11	9	ESC & ISC	0.999003
7	4	ESC & ISC	0.978852	12	9	ESC & ISC	0.996732
7	5	ESC & ISC	0.995419	12	10	ESC & ISC	0.999251
8	5	ESC & ISC	0.986848	...	...	...	...

Table 1: Static and Dynamic Stability Results - Model 1

⁹Note: ESC - External Stability Condition and ISC - Internal Stability Condition.

With a constant number of cartel firms, if F increases, then the region above which collusion is sustainable (δ) decreases which imply that $\delta_{without VI}^*$ is higher. Therefore, an increase in the number of fringe firms hinders collusion.

$$\frac{\partial \delta}{\partial F} = \frac{-16K^2(K-1)^2(F-K+1)(F+K+1)}{((F+K+1)^2(K+1)^2-16K^2(F+1))^2} < 0$$

Also, in a constant the number of fringe firms, if K increases, then the region above which collusion is sustainable (δ) increases which implies that $\delta_{without VI}^*$ is lower. Therefore, an increase in the number of cartel firms enforces collusion.

$$\frac{\partial \delta}{\partial K} = \frac{4(K-1)(F-K+1)(F+K+1)((2K^2+1)(2F+1)+K(F^2-K^2-3)+(F-K)^2)}{((F+K+1)^2(K+1)^2-16K^2(F+1))^2} > 0$$

2.2 The Model of Vertical Integration with a cartel member

Here we assume that there is vertical integration between one downstream cartel firm (D_{k_1} , for instance) and one upstream firm (U_1 , for instance). The total profit for the vertically integrated firm now depends on both upstream and downstream activities.

a) Collusive Results

We assume, as in the model without vertical integration, that the cartel firms behave as a Stackelberg leader and the fringe firms as followers. Additionally, we assume that although firm D_{k_1} is vertically integrated with firm U_1 , it has autonomy from the parent firm regarding the quantity decision. Therefore, firm D_{k_1} is allowed to collude with some rivals considering only the downstream profits. We are considering here a particular type of vertical integration where the upstream firm is the parent firm that maximizes the total profit while the downstream firm decides considering only the downstream profits.¹⁰

Under the above assumptions we obtain for the downstream market the same results as in the model without vertical integration. The difference from the previous model is due to the different decision process of the parent firm U_1 . At Stage 1, firm 1 maximizes the total profit that includes the profits from the upstream and downstream activities. Hence, the profit function of firm U_1 is:

$$\pi_1^{VIK} = (w - c)q_1^U + (a - bQ_K - bQ_F - w)q_{k_1}$$

¹⁰For instance, in UK telecommunications industry, the downstream firm Openreach is run and managed separately from the economic group British Telecom (Whalley and Curwen, 2008). Openreach is an example of functional separation.

The equilibrium upstream quantities are given by $q_1^U = \frac{A(2F+1)(K(2F+1)+1)}{2B(F+1)}$ and $q_2^U = \frac{A(2F+1)(K(2F+1)-1)}{2B(F+1)}$, where $A = \frac{a-c}{b}$ and $B = (3K + 6FK - 1)$. By substitution we obtain the equilibrium prices $w = \frac{a(K(2F+1)-1)+2Kc(2F+1)}{B}$ and $P = \frac{a((F+1)B-K(2F+1)^2)+K(2F+1)^2c}{B(F+1)}$, the equilibrium quantities for the cartel and fringe firms $q_{k_l} = \frac{A(2F+1)}{B}$ and $q_{f_l} = \frac{AK(2F+1)}{B(F+1)}$ and the profits:

- $\pi_1^{VJ^K} = \frac{A^2b(2F+1)[K(2F+1)(K+2FK+2)-1]}{2(F+1)B^2}$, for the integrated firm;
- $\pi_2^U = \frac{A^2b(2F+1)(K+2FK-1)^2}{2(F+1)B^2}$, for the independent upstream firm;
- $\pi_{k_j}^D = \frac{A^2bK(2F+1)^2}{(F+1)B^2}$, for $j = 2, \dots, K$ independent downstream cartel firms; and
- $\pi_{f_l}^D = \frac{A^2bK^2(2F+1)^2}{(F+1)^2B^2}$, for $l = 1, \dots, F$ downstream fringe firms.

From the numerical results¹¹ we verify that there is an incentive for vertical integration because the total profit of D_{k_1} and U_1 is higher than the independent upstream and than the sum of the profits of D_{k_1} and U_1 without vertical integration. Although the profit of the independent upstream firm is lower than in the case without vertical integration, the profits for the remaining downstream firms (cartel and fringe) are now higher. Once again, the profits of cartel firms are lower than the profits of the fringe firms, for the same reasons explained before.

To verify that the cartel is horizontally stable we analyze both internal and external stability conditions and the conclusions are summarized by Proposition 3.

Proposition 3. When U_1 and D_{k_1} are vertically integrated, downstream collusion is horizontally stable if and only if:

◦ **Internal Stability Condition:**

$$\frac{K(2F+1)^2}{(F+1)B^2} \geq \frac{(2F+3)^2(K-1)^2}{(F+2)^2(6F-9K-6FK+10)^2}$$

◦ **External Stability Condition:**

$$\frac{(K+1)(2F-1)^2}{F[3(K+1)(2F-1)-1]^2} \leq \frac{K^2(2F+1)^2}{(F+1)^2B^2}$$

Using the results from the numerical simulation¹² we conclude that for this model, collusion is sustainable if and only if there are $F+2$ or $F+3$ cartel firms and if $K > 3$. This is the same result as in the model without vertical integration, however, in this case, there is a lower bound for the number of cartel firms.

¹¹See Appendix B3.

¹²See Appendix B1.

b) Cournot equilibrium

When all downstream firms decide their quantities à la Cournot we obtain the following profits in equilibrium:¹³

$$\begin{aligned}
- \pi_1^{VI^K} &= \frac{A^2 b [(F+K)((F+K+1)^2 (F+K)^2 - 4) + 4(F+K)^2 (F+K+1)]}{(F+K+1)E^2}, \text{ for the integrated firm, where} \\
E &= 3(F+K+1)(F+K) - 2; \\
- \pi_2^U &= \frac{A^2 b (F+K)(F+K-1)^2 (F+K+2)^2}{E^2 (F+K+1)}, \text{ for the independent upstream firm; and} \\
- \pi_{f_l=k_j}^D &= \frac{4A^2 b (F+K)^2}{E^2}, \text{ for the independent } j = 2, \dots, K \text{ and } l = 1, \dots, F \text{ downstream firms.}
\end{aligned}$$

c) Deviation Results

Regarding the deviation from the agreement in the downstream market we analyze two different cases: 1) when the deviating firm is the firm that is vertically integrated; 2) when the deviating firm is one of the independent colluding firms, for instance firm D_{k_2} . As before we assume that the fringe firms react to the deviation.

For the first case the profits are the following:¹⁴

$$\begin{aligned}
- \pi_1^{VI^K} &= \frac{A^2 b H [(-4K + 8FK^2 + 5K^2 - 1)(8FK^2 + 7K^2 + 1) + 4KH(K+1)^2]}{4K(F+1)G^2}, \text{ for the integrated firm, where} \\
H &= 3K + 4FK - 1 \text{ and } G = -8K + 24FK^2 + 17K^2 - 1. \\
- \pi_2^U &= \frac{A^2 b H (-4K + 8FK^2 + 5K^2 - 1)^2}{4K(F+1)G^2}, \text{ for the independent upstream firm;} \\
- \pi_{k_1}^{deviation} &= \frac{A^2 b (K+1)^2 H^2}{(F+1)G^2}, \text{ for the deviation downstream cartel firm;} \\
- \pi_{k_j}^D &= \frac{2A^2 b (K+1) H^2}{(F+1)G^2}, \text{ for the } j = 2, \dots, K \text{ independent downstream cartel firms and;} \\
- \pi_{f_l}^D &= \frac{A^2 b (K+1)^2 H^2}{(F+1)^2 G^2}, \text{ for the } l = 1, \dots, F \text{ downstream fringe firms.}
\end{aligned}$$

By deviating from the collusive agreement, the cartel downstream firm, that is also vertically integrated with an upstream firm, earns higher profit than if it was in collusion, therefore it has an incentive to deviate. Also, the total profit of the vertical integrated firm and the profit of the independent upstream firm are higher than under collusion or without vertical integration. The profits of the independent cartel firms and of fringe firms are lower than if they were in collusion.

Thereafter we present the constraint on the discount factor that ensures the dynamic stability of collusion. When U_1 and D_{k_1} are vertically integrated, downstream collusion is dynamically stable if and only if:

$$\delta \geq \frac{E^2 [(K+1)^2 H^2 B^2 - K(2F+1)^2 G^2]}{B^2 [(K+1)^2 H^2 E^2 - 4(F+K)^2 (F+1) G^2]} \equiv \delta_{D_{k_1}}$$

¹³Appendix A3 presents the detailed description of the results with Cournot competition in the downstream market.

¹⁴Appendix A4 presents the detailed description of the results with deviation from the agreement for both cases.

Under the second case D_{k_2} deviates from the collusive agreement, assuming that D_{k_j} , with $j = 1, \dots, K \setminus \{2\}$ produce the collusive quantities. The profits are the following:

$$\begin{aligned}
& - \pi_1^{VI^F} = A^2 b \frac{2K(K+1)H^2 + H(-2K+4FK^2+3K^2-1)(4FK^2+3K^2+1)}{4K(F+1)I^2} \text{ for the integrated firm, where} \\
& I = -4K + 12FK^2 + 9K^2 - 1 ; \\
& - \pi_2^U = \frac{A^2 b H(-2K+4FK^2+3K^2-1)^2}{4K(F+1)I^2}, \text{ for the independent upstream firm;} \\
& - \pi_{k_2}^{deviation} = \frac{A^2 b H^2 (K+1)^2}{4(F+1)I^2}, \text{ for the deviation downstream cartel firm;} \\
& - \pi_{k_j}^D = \frac{A^2 b (K+1)H^2}{2(F+1)I^2}, \text{ for the independent } j = 1, \dots, K \setminus \{2\} \text{ downstream cartel firms and;} \\
& - \pi_{f_l}^D = \frac{Ab^2 H^2 (K+1)^2}{4(F+1)^2 I^2}, \text{ for } l = 1, \dots, F \text{ downstream fringe firms.}
\end{aligned}$$

By deviating from the collusive agreement, the cartel downstream firm obtains higher profit. However, also the vertical integrated firm's profit and the independent upstream firm's profits are higher. The remaining cartel firms' profits and the fringe firms' profits are lower.

Subsequently we present the constraint on the discount factor that ensures dynamic stability of collusion, in the second case. When U_1 and D_{k_2} are vertically integrated, downstream collusion is dynamically stable if and only if:

$$\delta \geq \frac{E^2[(K+1)^2 H^2 B^2 - 4K(2F+1)^2 I^2]}{B^2[(K+1)^2 H^2 E^2 - 16(F+K)^2 (F+1)I^2]} \equiv \delta_{D_{k_2}}$$

Proposition 4 summarizes the necessary conditions of dynamic stability of the cartel when there is vertical integration with a cartel firm.

Proposition 4. Under vertical integration with a cartel firm, collusion is dynamically stable if and only if:

$$\delta \geq \max(\delta_{D_{k_1}}, \delta_{D_{k_2}}) \equiv \delta_{withVI^K}^*$$

From numeric simulation, we verify that $\delta_{D_{k_1}}$ is always higher than $\delta_{D_{k_2}}$ and therefore $\delta_{withVI^K}^* = \delta_{D_{k_1}}$. The values of the critical discount factors above which collusion is sustainable and the results for the static stability conditions, for different numbers of F and K are shown on Table 2:

K	F	Static Stability	$\delta_{withVIK}^*$	K	F	Static Stability	$\delta_{withVIK}^*$
3	1	No ESC & ISC	0.882065	8	6	ESC & ISC	0.997111
4	1	ESC & ISC	0.831454	9	6	ESC & ISC	0.991340
4	2	ESC & ISC	0.963883	9	7	ESC & ISC	0.998063
5	2	ESC & ISC	0.927391	10	7	ESC & ISC	0.993976
5	3	ESC & ISC	0.984628	10	8	ESC & ISC	0.998638
6	3	ESC & ISC	0.963152	11	8	ESC & ISC	0.995644
6	4	ESC & ISC	0.992100	11	9	ESC & ISC	0.999007
7	4	ESC & ISC	0.978966	12	9	ESC & ISC	0.996750
7	5	ESC & ISC	0.995419	12	10	ESC & ISC	0.999253
8	5	ESC & ISC	0.986926	...	...	...	...

Table 2: Static and Dynamic Stability Results - Model 2

Analyzing Table 2 we conclude that, in order for collusion to be sustainable both statically and dynamically, we have to impose a lower bound for the number of cartel firms ($K > 3$). As we can see, for $K = 3$ and $F = 1$ the external stability condition (ESC) is not satisfied and therefore we do not have static stability. Once again, collusion is easier to sustain the higher the number of cartel firms is and the lower the number of fringe firms is.

2.3 The Model of Vertical Integration with a fringe firm

Here we assume that there is vertical integration between one downstream fringe firm (D_{f_1} , for instance) and one upstream firm (U_1 , for instance).

a) Collusive Results

Again, we assume that although firm D_{f_1} is vertically integrated with firm U_1 , it has autonomy from the parent firm regarding the quantity decision.

The profit function of firm U_1 is:

$$\pi_1^{VI^F} = (w - c)q_1^U + (a - bQ_K - bQ_F - w)q_{f_1}$$

The equilibrium upstream quantities are given by: $q_1^U = \frac{A(2F+1)(3F+2F^2+2)}{2(F+1)J}$, where $J = 9F + 6F^2 + 2$ and $q_2^U = \frac{AF(2F+3)(2F+1)}{2(F+1)J}$. By substitution we obtain the equilibrium prices

$w = \frac{2c(2F^2+3F+1)+a(2F^2+3F)}{J}$ and $P = \frac{a(2F^2+5F+1)+c(4F^2+4F+1)}{J}$, the equilibrium quantities for the cartel and fringe firms $q_{k_j} = \frac{A(F+1)(2F+1)}{KJ}$ and $q_{f_l} = \frac{A(2F+1)}{J}$ and the profits:

- $\pi_1^{VF} = A^2b \frac{2(F+1)(2F+1)^2+F(2F+1)(2F+3)(3F+2F^2+2)}{2(F+1)J^2}$, for the integrated firm;
- $\pi_2^U = \frac{a^2bF^2(2F+1)(2F+3)^2}{2(F+1)J^2}$, for the independent upstream firm;
- $\pi_{k_j}^D = \frac{A^2b(F+1)(2F+1)^2}{KJ^2}$, for $j = 1, \dots, K$ downstream cartel firms; and
- $\pi_{f_l}^D = \frac{A^2b(2F+1)^2}{J^2}$, for $l = 2, \dots, F$ independent downstream fringe firms.

Looking at the numerical results¹⁵ we conclude that the profit of the vertically integrated firm is always higher than the profit of the independent upstream firm U_2 . Once again, there is an incentive for firms to vertically integrate because the total profit of the vertically integrated firm is higher than the sum of the profits of D_{f_1} and U_1 without vertical integration. The profits of the fringe firms are, as in the previous models, higher than the profits of cartel firms and both are also higher than in the case without vertical integration. That is, when there is vertical integration with a fringe firm, cartel firms have higher incentives to collude and as we can see from the simulation, their profits are higher than the Cournot case or than the case where there is no vertical integration.

Once again to verify that the cartel is horizontally stable we have to ensure both internal and external stability conditions:

◦ **Internal Stability Condition:**

$$K \leq \frac{(2F+1)^2(F+1)(21F+6F^2+17)^2}{(2F+3)^2J^2}$$

◦ **External Stability Condition:**

$$K \geq \frac{F(2F-1)^2J^2}{(-3F+6F^2-1)^2(2F+1)^2} - 1$$

Using the results from the numerical simulation¹⁶ we conclude that collusion is sustainable if and only if there are $F + 2$ or $F + 3$ cartel firms and if $F > 1$. This is the same result as in the model without vertical integration, however here there is a lower bound for the number of fringe firms.

¹⁵See Appendix B4.

¹⁶See Appendix B1.

Proposition 5. When U_1 and D_{f_1} are vertically integrated, downstream collusion is horizontally stable if and only if:

$$\frac{F(2F-1)^2 J^2}{(-3F+6F^2-1)^2(2F+1)^2} - 1 \leq K \leq \frac{(2F+1)^2(F+1)(21F+6F^2+17)^2}{(2F+3)^2 J^2}$$

b) Cournot equilibrium

The Cournot results are the same as in the previous model because at the downstream market all firms have the same behavior.

c) Deviation Results

Considering that, for instance firm D_{k_1} deviates from the collusive agreement and, as before, the fringe firms react to the deviation, the profits are the following:¹⁷

$$\begin{aligned} - \pi_1^{VI^F} &= A^2 b \frac{4K(F+1)(K+1)^2 H^2 + (K+2FK-1)(5K+4FK+1)(14FK^2+8F^2K^2-2FK+7K^2+1)H}{4KL^2(F+1)}, \text{ for the} \\ &\text{integrated firm, with } L = -8K + 42FK^2 + 24F^2K^2 - 6FK + 17K^2 - 1; \\ - \pi_2^U &= A^2 b \frac{H(K+2FK-1)^2(5K+4FK+1)^2}{4KL^2(F+1)}, \text{ for the independent upstream firm;} \\ - \pi_{k_1}^{\text{deviation}} &= A^2 b \frac{(F+1)(K+1)^2 H^2}{L^2}, \text{ for the deviation downstream cartel firm;} \\ - \pi_{k_j}^D &= 2A^2 b \frac{(F+1)(K+1)H^2}{L^2}, \text{ for the } j = 2, \dots, K \text{ downstream cartel firms; and} \\ - \pi_{f_l}^D &= A^2 b \frac{H^2(K+1)^2}{L^2}, \text{ for the independent } l = 2, \dots, F \text{ downstream fringe firms.} \end{aligned}$$

By deviating from the collusive agreement, the cartel firm earns higher profit. However, if the firm does not deviate it will earn lower profit than if there were no vertical integration. Then the incentives to deviate from the collusive agreement are lower. The constraint described by Proposition 6 ensure that collusion is dynamically stable.

Proposition 6. When U_1 and D_{f_1} are vertically integrated, downstream collusion is dynamically stable if and only if:

$$\delta \geq \frac{E^2[(F+1)(K+1)^2 H^2 K J^2 - (F+1)(2F+1)^2 L^2]}{K J^2 [(F+1)(K+1)^2 H^2 E^2 - 4(F+K)^2 L^2]} \equiv \delta_{\text{with } VI^F}^*$$

¹⁷Appendix A5 presents the detailed description of the results with deviation.

Table 3 shows the values of the critical discount factors above which collusion is sustainable and the results of the static stability conditions, for different number of F and K firms:

K	F	Static Stability	$\delta_{withVIF}^*$	K	F	Static Stability	$\delta_{withVIF}^*$
3	1	No ESC & ISC	0.744721	8	6	ESC & ISC	0.996443
4	1	No ESC & ISC	0.724597	9	6	ESC & ISC	0.990245
4	2	ESC & ISC	0.940123	9	7	ESC & ISC	0.997678
5	2	ESC & ISC	0.901072	10	7	ESC & ISC	0.993323
5	3	ESC & ISC	0.977686	10	8	ESC & ISC	0.998401
6	3	ESC & ISC	0.953961	11	8	ESC & ISC	0.995231
6	4	ESC & ISC	0.989391	11	9	ESC & ISC	0.998853
7	4	ESC & ISC	0.974993	12	9	ESC & ISC	0.996476
7	5	ESC & ISC	0.994154	12	10	ESC & ISC	0.999149
8	5	ESC & ISC	0.984944	...	...	...	...

Table 3: Static and Dynamic Stability Results - Model 3

From the results in Table 3 we conclude that there is no collusion for $F \leq 1$ because the external stability condition is not satisfied. Then, only when there are $F + 2$ or $F + 3$ and when $F > 1$ collusion is stable. Again, collusion is more difficult to sustain the lower the number of cartel firms is and the higher the number of fringe firms is.

3 Discussion of Results

In order to verify if vertical integration promotes collusion, we compare the results of the dynamic and static stability conditions for the three models.¹⁸

From the numerical results we conclude that, for the first model, the participation conditions are verified if the number of cartel firms are $F + 2$ or $F + 3$, that is the cartel should have respectively, 50% – 75% or 50% – 80% firms in the industry. However, for models 2 and 3 we have to introduce an additional assumption, $F > 1$, otherwise, the external stability conditions (ESC) are not satisfied.

¹⁸See Appendix B1.

Figure 1 - Models' Critical Discount Values for $F=K-2$

Figure 2 - Models' Critical Discount Values for $F=K-3$

Moreover, after analyzing the critical discount values (Figures 1 and 2), we conclude that vertical integration always promotes collusion when the upstream firm merges with a downstream fringe firm. This is justified because vertical integration with a fringe firm reinforces the incentives for the cartel firms not to deviate from the collusive agreement. As the vertically integrated firm is now in a privileged position, cartel firms take this as a threat and therefore react defensively against this competitive firm. Furthermore, there is an incentive for firms to vertically integrate because the profit of the vertically integrated firm is higher than the sum of profits of a D_f and an U firms, without vertical integration.

However, vertical integration with a cartel firm could hinder collusion, for some specific number of firms in the downstream market. Although there is incentive to vertically integrate due to the possibility of getting higher profits, this could make collusion more difficult to sustain. For example, for $K = 4$ and $F = 2$, vertical integration either with a fringe or a cartel firm promotes collusion. Also, for this case, vertical integration with a fringe firm incentives more the collusion than vertical integration with a cartel firm. But for $K = 8$ and $F = 5$ vertical integration increases cartel sustainability only when it is with a fringe firm. One reason that could explain why collusion is difficult to sustain when there is vertical integration with a cartel firm, is the low level of market concentration. With a high number of firms in the market, the vertically integrated firm continues to have higher profit than its rivals, however, cartel individual profits will be lower because they have to share the cartel total profit with a high number of firms so they will earn more from deviating.

4 Welfare analysis

In this section we discuss the social welfare implications of the three models.

When there is vertical integration we can have two main effects. On one hand, we have the collusive effects, that is, vertical integration increases the incentives for downstream firms to collude, which have a negative effect on social welfare. On the other hand, vertical integration has pro-competitive effects: the total quantity is higher and the final price is lower for both models 2 and 3 when comparing with the model without vertical integration.¹⁹ This is due to the decrease of the retail prices (w). In both models 2 and 3 vertical integration reduces the problem of double marginalization, but it does not solve it because $w > c$. This reflects the fact that the quantity of the vertically integrated firm is always higher than the quantity of any independent firm. We conclude that the positive effects more than compensate the negative effects which explains why there is an improvement in welfare when we move from model 1 to models 2 and 3.

Table 4 displays the differences of social welfare (SW) between models 1 and 2 and between models 1 and 3, decomposing it into the variation of consumer surplus (CS) and producer surplus (PS).²⁰

K	F	Model 1 vs Model 2			Model 1 vs Model 3		
		Δ PS	Δ CS	Δ SW	Δ PS	Δ CS	Δ SW
3	1	-	-	-	-	-	-
4	1	-0.08%	5.80%	1.88%	-	-	-
4	2	-0.46%	3.42%	1.03%	-0.63%	4.60%	1.38%
5	2	-0.36%	2.72%	0.82%	-0.63%	4.60%	1.38%
5	3	-0.40%	1.93%	0.56%	-0.50%	2.42%	0.70%
6	3	-0.33%	1.61%	0.47%	-0.50%	2.42%	0.70%
6	4	-0.32%	1.25%	0.35%	-0.38%	1.50%	0.42%
7	4	-0.27%	1.07%	0.30%	-0.38%	1.50%	0.42%
7	5	-0.25%	0.87%	0.24%	-0.29%	1.02%	0.28%
8	5	-0.22%	0.76%	0.21%	-0.29%	1.02%	0.28%
8	6	-0.20%	0.64%	0.18%	-0.23%	0.74%	0.20%
9	6	-0.18%	0.57%	0.16%	-0.23%	0.74%	0.20%
9	7	-0.17%	0.50%	0.13%	-0.19%	0.56%	0.15%
10	7	-0.15%	0.45%	0.12%	-0.19%	0.56%	0.15%
10	8	-0.14%	0.39%	0.11%	-0.15%	0.44%	0.12%
11	8	-0.13%	0.36%	0.10%	-0.15%	0.44%	0.12%
11	9	-0.12%	0.32%	0.09%	-0.13%	0.35%	0.09%
12	9	-0.11%	0.29%	0.08%	-0.13%	0.35%	0.09%
12	10	-0.10%	0.27%	0.07%	-0.11%	0.29%	0.08%

Table 4: Welfare analysis

¹⁹In the comparison of prices and quantities for the three models, we assume that $a = 1$, $b = 1$ and $c = 0.5$.

See Appendix B5 for further information.

²⁰See Appendix B6.

As we can observe from Table 4, under collusion, social welfare increases when we move from an industry with no vertical integration to an industry where there is vertical integration either with a cartel or a fringe downstream firm. This is also true when we compare the results obtained under Cournot. With vertical integration, consumers see their welfare improving because of the increase of total quantity. Firms have lower profits due to the reduction of both upstream and downstream prices (w and P). The increase in the consumer surplus is higher than the decrease in the producer surplus, then the net effect is that vertical integration increases social welfare.

5 Concluding Remarks

This paper explores the effect of vertical integration (or vertical mergers) on the incentives to collude, when there is incomplete collusion in the downstream market. A central result of the paper is that in general by acquiring a downstream firm (a cartel or a fringe firm), the vertically integrated firm promotes collusion at downstream level. The vertical merger decreases not only the number of firms in the downstream market but also the cartel firm's deviation incentives from the collusive agreement.

However, for low market concentration, the opposite result is found. The larger the number of cartel firms is the lower the profit that each cartel firm earns and therefore the higher the incentive to disrupt the collusive agreement is. Hence, vertical integration with a cartel firm hinders downstream collusion for high number of cartel firms in downstream market.

In spite of collusion, we verify that vertical integration increases social welfare due to the partial elimination of double marginalization. These results could be important for antitrust authorities that are concerned with the effects of such mergers on downstream collusion.

The framework and the assumptions we have assumed are of a particular kind. Further research on the analysis of the relationship between vertical merger and incomplete collusion should consider what happens if there is vertical integration with more than one downstream firm. Also, it would be interesting to study the case where the downstream firm also takes into account the profits of the upstream firm, relaxing the assumption that it has an independent behavior from the upstream firm. We think that these are very important and useful subjects for further research.

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Appendix A. Equilibrium profits

Appendix A1. Cournot profits for non-integrated equilibrium - Model 1

Under this case all downstream firms decide individually their quantity. Knowing that the firms are symmetric, the quantity produced by each downstream firm is given by: $q_{k_j} = q_{f_l} = \frac{a-w}{b(F+K+1)}$, with $j = 1, \dots, K$ and $l = 1, \dots, F$. Therefore, the inverse demand function to upstream firms is $w = a - \frac{b(F+K+1)Q}{F+K}$. At stage 1, the upstream firms decide the quantity, maximizing their profits functions: $\pi_i^U = (w - c)q_i^U = (\frac{a(F+K) - b(F+K+1)(q_i^U + q_2^U)}{F+K} - c)q_i^U$, with $i = 1, 2$. The equilibrium results are:

$$\begin{aligned} q_i^U &= \frac{A(F+K)}{3(F+K+1)}, \text{ with } A = \frac{a-c}{b} \\ q_{k_j}^D &= q_{f_l}^D = \frac{2A}{3(F+K+1)} \\ P &= \frac{a(3+F+K)+2c(F+K)}{3(F+K+1)} & w &= \frac{a+2c}{3} \\ \pi_i^U &= \frac{A^2b(F+K)}{9(F+K+1)} & \pi_{k_j}^D &= \pi_{f_l}^D = \frac{4A^2b}{9(F+K+1)^2}, \end{aligned}$$

Appendix A2. Deviation profits for non-integrated firms - Model 1

Firm D_{k_1} deviates from the collusive agreement knowing that firms D_{k_j} , with $j = 2, \dots, K$, produce the collusive quantities and fringe firms D_{f_l} , with $l = 1, \dots, F$, react to the deviation.

Then residual demand for the deviating firm is given by: $P = a - bq_{k_1}^d - (K-1)\frac{a-w}{2K} - F\frac{a-w-bQ_K}{(F+1)}$ and the deviation quantity is $q_{k_1}^{deviation} = \frac{(a-w)(K+1)}{4bK}$.

At stage 1 the upstream firms decide the quantity, knowing that $w = a - \frac{4b(F+1)Q}{H}$, where $H = 3K + 4FK - 1$. Then the upstream firms will produce, in equilibrium, $q_i^U = \frac{AH}{12K(F+1)}$, with $i = 1, 2$. The equilibrium prices are given by $w = \frac{a+2c}{3}$ and $P = \frac{a(3K+2FK+1)+cH}{6K(F+1)}$ and the downstream quantities are $q_{k_1}^{deviation} = \frac{A(K+1)}{6K}$, $q_{k_j} = \frac{A}{3K}$, with $j = 2, \dots, K$ and $q_{f_l} = \frac{A(K+1)}{6K(F+1)}$, with $l = 1, \dots, F$. The equilibrium profits are:

$$\begin{aligned} \pi_i^U &= \frac{A^2bH}{36K(F+1)} & \pi_{k_1}^{deviation} &= \frac{A^2b(K+1)^2}{36K^2(F+1)} \\ \pi_{k_j}^D &= \frac{A^2b(K+1)}{18K^2(F+1)}, \text{ with } j = 2, \dots, K & \pi_{f_l}^D &= \frac{A^2b(K+1)^2}{36K^2(F+1)^2}, \text{ with } l = 1, \dots, F. \end{aligned}$$

Appendix A3. Cournot profits under Vertical Integration with a Cartel or Fringe downstream firm - Models 2 and 3

Assuming that the downstream integrated behaves as a subsidiary firm that decides its quantity in an independent way from the parent upstream firm, the first stage the results are the same as in the model without vertical integration. The profit functions for upstream firms are:

$$\begin{aligned}\pi_1^{VI^K} &= (w - c)q_1^U + (a - b(q_1^U + q_2^U) - w)q_{k_1} \\ \pi_2 &= (w - c)q_2^U\end{aligned}$$

which lead to the following results:

$$\begin{aligned}q_1^U &= \frac{A(F+K)[(F+K+1)(F+K)+2]}{(F+K+1)E}, \text{ where } E = 3(F+K+1)(F+K) - 2 \\ q_2^U &= \frac{A(F+K)(F+K-1)(F+K+2)}{(F+K+1)E} \\ w &= \frac{a[(F+K+1)(F+K)-2] + 2c(F+K)(F+K+1)}{E} \quad P = \frac{a[(F+K)^2 + 3(F+K) - 2] + 2c(F+K)^2}{E} \\ q_{k_j} &= q_{f_k} = \frac{2A(F+K)}{E}, \text{ with } j = 1, \dots, K \text{ and } l = 1, \dots, F. \\ \pi_2^U &= \frac{A^2b(F+K)(F+K-1)(F+K+2)[(F+K+1)(F+K)-2]}{E^2(F+K+1)} \\ \pi_{k_j}^D &= \pi_{f_l}^D = \frac{4A^2b(F+K)^2}{E^2}, \text{ with } j = 1, \dots, K \text{ and } l = 1, \dots, F. \\ \pi_1^{VI} &= \frac{A^2b(F+K)[(F+K+1)^2(F+K)^2 - 4]}{(F+K+1)E^2} + \frac{4A^2b(F+K)^2}{E^2}\end{aligned}$$

Appendix A4. Deviation profits under Vertical Integration with a Cartel downstream firm - Model 2

First Case: D_{k_1} , that is vertically integrated, deviates from the collusive agreement knowing that D_{k_j} , for $j = 2, \dots, K$ produce the collusive quantities and that fringe firms D_{f_l} react to the deviation.

The residual demand for firm D_{k_1} is: $P = a - bq_{k_1}^{deviation} - (K-1)b\frac{a-w}{2bK} - bF\frac{a-w-bQ_K}{b(F+1)}$ and the quantity is $q_{k_1}^{deviation} = \frac{(K+1)(a-w)}{4bK}$. The upstream firms maximize their profits, given by

$$\begin{aligned}\pi_1^{VI} &= (w - c)q_1^U + (a - b(q_1^U + q_2^U) - w)q_{k_1}^{deviation} \\ \pi_2^U &= (w - c)q_2^U\end{aligned}$$

knowing that $w = a - \frac{4bK(F+1)Q}{H}$.

Then the equilibrium quantities, prices and profits are given by:

$$\begin{aligned}q_1^U &= \frac{AH(8FK^2 + 7K^2 + 1)}{4K(F+1)G}, \text{ where } G = -8K + 24FK^2 + 17K^2 - 1 \\ q_2^U &= \frac{AH(-4K + 8FK^2 + 5K^2 - 1)}{4K(F+1)G} \\ w &= \frac{a(8FK^2 + 5K^2 - 4K - 1) + 4cKH}{G} \quad P = \frac{a(8F^2K^2 + 17FK^2 + 8K^2 - F - 2K - 2) + cH^2}{(F+1)G} \\ q_{k_1}^{deviation} &= \frac{AH(K+1)}{G} \quad q_{f_l} = \frac{AH(K+1)}{G(F+1)}, \text{ with } l = 1, \dots, F \quad q_{k_j} = \frac{2AH}{G}, \text{ with } j = 2, \dots, K\end{aligned}$$

$$\begin{aligned}
\pi_2^U &= \frac{A^2 b(-4K+8FK^2+5K^2-1)^2 H}{4KG^2(F+1)} & \pi_{k_1}^{deviation} &= \frac{A^2 b(K+1)^2 H^2}{(F+1)G^2} \\
\pi_{k_j}^D &= \frac{2A^2 b(K+1)H^2}{(F+1)G^2} & \pi_{f_l}^D &= \frac{A^2 b(K+1)^2 H^2}{(F+1)^2 G^2} \\
\pi_1^{VIK} &= \frac{A^2 b(-4K+8FK^2+5K^2-1)(8FK^2+7K^2+1)H}{4KG^2(F+1)} + \frac{A^2 b(K+1)^2 H^2}{(F+1)G^2}
\end{aligned}$$

Second Case: D_{k_2} deviates from the collusive agreement, knowing that D_{k_j} , with $j = 1, \dots, K \setminus \{2\}$ produce the collusive quantities and the fringe downstream firms react to that deviation.

The residual demand for firm D_{k_2} is given by $P = a - bq_{k_2}^{deviation} - (K-1) \frac{a-w}{2K} - bF \frac{a-w-bQ_K}{(F+1)b}$ and the deviation quantity is $q_{k_2}^{deviation} = \frac{(a-w)(K+1)}{4bK}$.

The upstream firms maximize their profits, knowing that $w = a - \frac{4K(F+1)Qb}{H}$. Then the equilibrium quantities, prices and profits are given by:

$$\begin{aligned}
q_1^U &= \frac{AH(4FK^2+3K^2+1)}{4K(F+1)I}, \text{ where } I = -4K+12FK^2+9K^2-1 & q_2^U &= \frac{A(-2K+4FK^2+3K^2-1)H}{4K(F+1)I} \\
w &= \frac{a(4FK^2+3K^2-2K-1)+2cKH}{I} & P &= \frac{a(8F^2K^2+18FK^2+9K^2-2F-2K-3)+cH^2}{2(F+1)I} \\
q_{k_2}^{deviation} &= \frac{AH(K+1)}{2I} & q_{f_l} &= \frac{AH(K+1)}{2I(F+1)} & q_{k_j} &= \frac{AH}{I} \\
\pi_2^U &= \frac{A^2 bH(-2K+4FK^2+3K^2-1)^2}{4KI^2(F+1)} \\
\pi_{k_2}^{deviation} &= \frac{A^2 b(K+1)^2 H^2}{4(F+1)I^2} & \pi_{k_j}^D &= \frac{A^2 b(K+1)H^2}{2(F+1)I^2}, \text{ with } j = 1, \dots, K \setminus \{2\} \\
\pi_{f_l}^D &= \frac{A^2 b(K+1)^2 H^2}{4(F+1)^2 I^2}, \text{ with } l = 1, \dots, F \\
\pi_1^{VIK} &= \frac{A^2 b(K+1)H^2}{2(F+1)I^2} + \frac{A^2(-2K+4FK^2+3K^2-1)(4FK^2+3K^2+1)H}{4KI^2(F+1)}
\end{aligned}$$

Appendix A5. Deviation profits under Vertical Integration with a Fringe downstream firm - Model 3

Firm D_{k_1} deviates from the collusive agreement knowing that D_{k_j} , for $j = 2, \dots, K$ produce the collusive quantities and that fringe firms D_{f_l} react to the deviation. The residual demand for firm D_{k_1} is given by $P = a - bq_{k_1}^{deviation} - (K-1)b\frac{a-w}{2bK} - bF\frac{a-w-bQ_K}{(F+1)b}$. Therefore, the quantity for this firm is $q_{k_1}^{deviation} = \frac{(a-w)(K+1)}{4bK}$ and the quantity for the fringe firms is $q_{f_l} = \frac{(a-w)(K+1)}{4b(F+1)K}$, with $l = 1, \dots, F$. The upstream firms maximize their profits:

$$\pi_1^{VI} = (w-c)q_1^U + (a-b(q_1^U + q_2^U) - w)q_{f_1} \text{ and } \pi_2^U = (w-c)q_2^U,$$

knowing that $w = a - \frac{4K(F+1)Qb}{H}$. Then the equilibrium quantities, prices and profits are given by:

$$\begin{aligned}
q_1^U &= \frac{AH(14FK^2+8F^2K^2-2FK+7K^2+1)}{4K(F+1)K}, \text{ where } L = -8K+42FK^2+24F^2K^2-6FK+17K^2-1 \\
q_2^U &= \frac{AH(5K+4FK+1)(K+2FK-1)}{4K(F+1)L}
\end{aligned}$$

$$\begin{aligned}
w &= \frac{a(K+2FK-1)(5K+4FK+1)+4cK(F+1)H}{L} & P &= \frac{2a(4F^2K^2+9FK^2+4K^2-K-1)+cH^2}{L} \\
q_{k_1}^{deviation} &= \frac{AH(F+1)(K+1)}{L} & q_{f_l} &= \frac{AH(K+1)}{L}, \text{ with } l = 1, \dots, F \\
q_{k_j} &= \frac{2AH(F+1)}{L}, \text{ with } j = 2, \dots, K \\
\pi_2^U &= \frac{A^2bH(K+2FK-1)^2(5K+4FK+1)^2}{4KL^2(F+1)} & \pi_{k_1}^{deviation} &= \frac{A^2bH^2(K+1)^2(F+1)}{L^2} \\
\pi_{k_j}^D &= \frac{2A^2bH^2(F+1)(K+1)}{L^2}, \text{ with } j = 2, \dots, K & \pi_f^D &= \frac{A^2bH^2(K+1)^2}{L^2}, \text{ with } l = 1, \dots, F \\
\pi_1^{VIF} &= \frac{A^2bH^2(K+1)^2}{L^2} + \frac{A^2bH(K+2FK-1)(5K+4FK+1)(14FK^2+8F^2K^2-2FK+7K^2+1)}{4KL^2(F+1)}
\end{aligned}$$

Appendix B. Numerical Simulation

Appendix B1. Results from Static and Dynamic Stability²¹

		Static Stability			Dynamic Stability			
		Model 1	Model 2	Model 3	Model 1	Model 2		Model 3
K	F				$\delta_{without VI}^*$	$\delta_{D_{k1}}^*$	$\delta_{D_{k2}}^*$	$\delta_{with VIF}^*$
3	1	ESC ISC	No ESC ISC	No ESC ISC	0.892857	0.882065	0.860994	0.744721
4	1	ESC ISC	ESC ISC	No ESC ISC	0.835052	0.831454	0.812044	0.724597
4	2	ESC ISC	ESC ISC	ESC ISC	0.964989	0.963883	0.960915	0.940123
5	2	ESC ISC	ESC ISC	ESC ISC	0.927536	0.927391	0.922718	0.901072
5	3	ESC ISC	ESC ISC	ESC ISC	0.984802	0.984628	0.983868	0.977686
6	3	ESC ISC	ESC ISC	ESC ISC	0.963020	0.963152	0.961592	0.953961
6	4	ESC ISC	ESC ISC	ESC ISC	0.992129	0.992100	0.991830	0.989391
7	4	ESC ISC	ESC ISC	ESC ISC	0.978852	0.978966	0.978320	0.974993
7	5	ESC ISC	ESC ISC	ESC ISC	0.9954188	0.9954187	0.995301	0.994154
8	5	ESC ISC	ESC ISC	ESC ISC	0.986848	0.986926	0.986615	0.984944
8	6	ESC ISC	ESC ISC	ESC ISC	0.997106	0.997111	0.997052	0.996443
9	6	ESC ISC	ESC ISC	ESC ISC	0.991288	0.991340	0.991174	0.990245
9	7	ESC ISC	ESC ISC	ESC ISC	0.998057	0.998063	0.998030	0.997678
10	7	ESC ISC	ESC ISC	ESC ISC	0.993940	0.993976	0.993879	0.993323
10	8	ESC ISC	ESC ISC	ESC ISC	0.998634	0.998638	0.998619	0.998401
11	8	ESC ISC	ESC ISC	ESC ISC	0.995619	0.995644	0.995584	0.995231
11	9	ESC ISC	ESC ISC	ESC ISC	0.999003	0.999007	0.998994	0.998853
12	9	ESC ISC	ESC ISC	ESC ISC	0.996732	0.996750	0.996711	0.996476
12	10	ESC ISC	ESC ISC	ESC ISC	0.999251	0.999253	0.999245	0.999149

²¹Note: ESC - External Stability Condition and ISC - Internal Stability Condition.

Appendix B2. Results for Model 1²²

a) Collusive Results							
K	F	U qt	K qt	F qt	U profit	K profit	F profit
3	1	0.25000	0.11111	0.16667	0.08333	0.01852	0.02778
4	1	0.25000	0.08333	0.16667	0.08333	0.01389	0.02778
4	2	0.27778	0.08333	0.11111	0.09259	0.00926	0.01235
5	2	0.27778	0.06667	0.11111	0.09259	0.00741	0.01235
5	3	0.29167	0.06667	0.08333	0.09722	0.00556	0.00694
6	3	0.29167	0.05556	0.08333	0.09722	0.00463	0.00694
6	4	0.30000	0.05556	0.06667	0.10000	0.00370	0.00444
7	4	0.30000	0.04762	0.06667	0.10000	0.00317	0.00444
7	5	0.30556	0.04762	0.05556	0.10185	0.00265	0.00309
8	5	0.30556	0.04167	0.05556	0.10185	0.00231	0.00309
8	6	0.30952	0.04167	0.04762	0.10317	0.00198	0.00227
9	6	0.30952	0.03704	0.04762	0.10317	0.00176	0.00227
9	7	0.31250	0.03704	0.04167	0.10417	0.00154	0.00174
10	7	0.31250	0.03333	0.04167	0.10417	0.00139	0.00174
10	8	0.31481	0.03333	0.03704	0.10494	0.00123	0.00137
11	8	0.31481	0.03030	0.03704	0.10494	0.00112	0.00137
11	9	0.31667	0.03030	0.03333	0.10556	0.00101	0.00111
12	9	0.31667	0.02778	0.03333	0.10556	0.00093	0.00111
12	10	0.31818	0.02778	0.03030	0.10606	0.00084	0.00092

b) Cournot Results					
K	F	U qt	K qt=F qt	U profit	K=F profit
3	1	0.26667	0.13333	0.08889	0.01778
4	1	0.27778	0.11111	0.09259	0.01235
4	2	0.28571	0.09524	0.09524	0.00907
5	2	0.29167	0.08333	0.09722	0.00694
5	3	0.29630	0.07407	0.09877	0.00549
6	3	0.30000	0.06667	0.10000	0.00444
6	4	0.30303	0.06061	0.10101	0.00367
7	4	0.30556	0.05556	0.10185	0.00309
7	5	0.30769	0.05128	0.10256	0.00263
8	5	0.30952	0.04762	0.10317	0.00227
8	6	0.31111	0.04444	0.10370	0.00198
9	6	0.31250	0.04167	0.10417	0.00174
9	7	0.31373	0.03922	0.10458	0.00154
10	7	0.31481	0.03704	0.10494	0.00137
10	8	0.31579	0.03509	0.10526	0.00123
11	8	0.31667	0.03333	0.10556	0.00111
11	9	0.31746	0.03175	0.10582	0.00101
12	9	0.31818	0.03030	0.10606	0.00092
12	10	0.31884	0.02899	0.10628	0.00084

²²Note: The quantities are multiplied by $A = \frac{a-c}{b}$ and the profits by A^2b .

c) Deviating Results									
K	F	U qt	K1 qt dev	K qt	F qt	U profit	K profit	K dev profit	F profit
3	1	0.27778	0.22222	0.11111	0.11111	0.09259	0.00617	0.02469	0.01235
4	1	0.28125	0.20833	0.08333	0.10417	0.09375	0.00347	0.02170	0.01085
4	2	0.29861	0.20833	0.08333	0.06944	0.09954	0.00347	0.01447	0.00482
5	2	0.30000	0.20000	0.06667	0.06667	0.10000	0.00222	0.01333	0.00444
5	3	0.30833	0.20000	0.06667	0.05000	0.10278	0.00222	0.01000	0.00250
6	3	0.30903	0.19444	0.05556	0.04861	0.10301	0.00154	0.00945	0.00236
6	4	0.31389	0.19444	0.05556	0.03889	0.10463	0.00154	0.00756	0.00151
7	4	0.31429	0.19048	0.04762	0.03810	0.10476	0.00113	0.00726	0.00145
7	5	0.31746	0.19048	0.04762	0.03175	0.10582	0.00113	0.00605	0.00101
8	5	0.31771	0.18750	0.04167	0.03125	0.10590	0.00087	0.00586	0.00098
8	6	0.31994	0.18750	0.04167	0.02679	0.10665	0.00087	0.00502	0.00072
9	6	0.32011	0.18519	0.03704	0.02646	0.10670	0.00069	0.00490	0.00070
9	7	0.32176	0.18519	0.03704	0.02315	0.10725	0.00069	0.00429	0.00054
10	7	0.32188	0.18333	0.03333	0.02292	0.10729	0.00056	0.00420	0.00053
10	8	0.32315	0.18333	0.03333	0.02037	0.10772	0.00056	0.00373	0.00041
11	8	0.32323	0.18182	0.03030	0.02020	0.10774	0.00046	0.00367	0.00041
11	9	0.32424	0.18182	0.03030	0.01818	0.10808	0.00046	0.00331	0.00033
12	9	0.32431	0.18056	0.02778	0.01806	0.10810	0.00039	0.00326	0.00033
12	10	0.32513	0.18056	0.02778	0.01641	0.10838	0.00039	0.00296	0.00027

Appendix B3. Results for Model 2²³

a) Collusive Results									
K	F	U qt	K1 qt dev	K qt	F qt	U profit	K profit	K dev profit	F profit
4	1	0.235714	0.2357143	0.085714	0.1714286	0.102245	0.074082	0.014694	0.029388
4	2	0.26836	0.26836	0.08475	0.11299	0.10509	0.08642	0.00958	0.01277
5	2	0.27027	0.27027	0.06757	0.11261	0.10257	0.08766	0.00761	0.01268
5	3	0.28606	0.28606	0.06731	0.08413	0.10468	0.09352	0.00566	0.00708
6	3	0.28700	0.28700	0.05600	0.08400	0.10343	0.09414	0.00470	0.00706
6	4	0.29627	0.29627	0.05590	0.06708	0.10496	0.09753	0.00375	0.00450
7	4	0.29681	0.29681	0.04787	0.06702	0.10425	0.09788	0.00321	0.00449
7	5	0.30290	0.30290	0.04783	0.05580	0.10539	0.10009	0.00267	0.00311
8	5	0.30323	0.30323	0.04183	0.05577	0.10495	0.10031	0.00233	0.00311
8	6	0.30753	0.30753	0.04180	0.04777	0.10583	0.10185	0.00200	0.00228
9	6	0.30776	0.30776	0.03714	0.04776	0.10553	0.10200	0.00177	0.00228
9	7	0.31095	0.31095	0.03713	0.04177	0.10623	0.10314	0.00155	0.00174
10	7	0.31111	0.31111	0.03341	0.04176	0.10602	0.10324	0.00140	0.00174
10	8	0.31358	0.31358	0.03340	0.03711	0.10659	0.10412	0.00124	0.00138
11	8	0.31369	0.31369	0.03036	0.03710	0.10644	0.10419	0.00113	0.00138
11	9	0.31565	0.31565	0.03035	0.03339	0.10690	0.10488	0.00101	0.00111
12	9	0.31574	0.31574	0.02782	0.03338	0.10679	0.10494	0.00093	0.00111
12	10	0.31734	0.31734	0.02781	0.03034	0.10718	0.10550	0.00084	0.00092

²³Note: The quantities are multiplied by $A = \frac{a-c}{b}$ and the profits by A^2b .

b) Cournot Results							
K	F	U1 -qt	U2 qt	K qt=F qt	VI - profit	U profit	K=F profit
4	1	0.30303	0.26515	0.11364	0.09965	0.08437	0.01291
4	2	0.30415	0.27650	0.09677	0.10045	0.08919	0.00937
5	2	0.30572	0.28464	0.08434	0.10123	0.09259	0.00711
5	3	0.30737	0.29076	0.07477	0.10194	0.09511	0.00559
6	3	0.30896	0.29552	0.06716	0.10258	0.09704	0.00451
6	4	0.31042	0.29933	0.06098	0.10314	0.09856	0.00372
7	4	0.31176	0.30245	0.05584	0.10364	0.09979	0.00312
7	5	0.31297	0.30505	0.05150	0.10409	0.10081	0.00265
8	5	0.31408	0.30725	0.04779	0.10449	0.10166	0.00228
8	6	0.31507	0.30913	0.04459	0.10485	0.10239	0.00199
9	6	0.31598	0.31076	0.04178	0.10518	0.10301	0.00175
9	7	0.31681	0.31218	0.03931	0.10547	0.10355	0.00155
10	7	0.31756	0.31344	0.03712	0.10574	0.10402	0.00138
10	8	0.31826	0.31456	0.03516	0.10598	0.10444	0.00124
11	8	0.31889	0.31555	0.03339	0.10620	0.10481	0.00112
11	9	0.31948	0.31645	0.03180	0.10641	0.10515	0.00101
12	9	0.32002	0.31726	0.03035	0.10660	0.10545	0.00092
12	10	0.32052	0.31800	0.02902	0.10677	0.10572	0.00084

c1) Deviating Results											
K	F	U1 qt	U2 qt	K1 qt deviation	K qt	F qt	VI - profit	U2 profit	K profit	K dev profit	F profit
4	1	0.326394	0.258678	0.04334	0.08668	0.10835	0.12354	0.07931	0.00939	0.02348	0.01174
4	2	0.328265	0.283784	0.04270	0.08540	0.07117	0.11918	0.08990	0.00608	0.01519	0.00506
5	2	0.327273	0.286364	0.03409	0.06818	0.06818	0.11808	0.09112	0.00465	0.01395	0.00465
5	3	0.328663	0.298168	0.03388	0.06777	0.05082	0.11628	0.09611	0.00344	0.01033	0.00258
6	3	0.328226	0.299429	0.02821	0.05642	0.04937	0.11576	0.09671	0.00279	0.00975	0.00244
6	4	0.329197	0.306235	0.02812	0.05623	0.03936	0.11480	0.09959	0.00221	0.00775	0.00155
7	4	0.328968	0.306945	0.02409	0.04818	0.03854	0.11452	0.09993	0.00186	0.00743	0.00149
7	5	0.32967	0.311355	0.02404	0.04808	0.03205	0.11394	0.10179	0.00154	0.00616	0.00103
8	5	0.329536	0.311794	0.02103	0.04205	0.03154	0.11377	0.10200	0.00133	0.00597	0.00099
8	6	0.330065	0.314878	0.02100	0.04200	0.02700	0.11338	0.10330	0.00113	0.00510	0.00073
9	6	0.32998	0.315169	0.01866	0.03732	0.02666	0.11327	0.10344	0.00099	0.00497	0.00071
9	7	0.33039	0.317444	0.01864	0.03729	0.02330	0.11300	0.10440	0.00087	0.00434	0.00054
10	7	0.330333	0.317646	0.01678	0.03355	0.02307	0.11292	0.10449	0.00077	0.00426	0.00053
10	8	0.330661	0.319392	0.01676	0.03353	0.02049	0.11272	0.10523	0.00069	0.00378	0.00042
11	8	0.33062	0.319538	0.01524	0.03048	0.02032	0.11266	0.10530	0.00062	0.00372	0.00041
11	9	0.330888	0.32092	0.01523	0.03046	0.01827	0.11251	0.10588	0.00056	0.00334	0.00033
12	9	0.330859	0.321029	0.01396	0.02792	0.01815	0.11247	0.10593	0.00051	0.00329	0.00033
12	10	0.331081	0.322149	0.013952	0.027905	0.016489	0.1123405	0.1064	0.00046	0.002991	0.000272

<i>c</i> ₂) Deviating Results											
K	F	U1 qt	U2 qt	K2 qt deviation	K qt	F qt	VI - profit	U2 profit	K profit	K dev profit	F profit
4	1	0.29888	0.27243	0.21160	0.08464	0.10580	0.10546	0.08796	0.00895	0.02239	0.01119
4	2	0.31030	0.29277	0.21037	0.08415	0.07012	0.10731	0.09568	0.00590	0.01475	0.00492
5	2	0.30896	0.29552	0.20149	0.06716	0.06716	0.10596	0.09704	0.00451	0.01353	0.00451
5	3	0.31504	0.30498	0.20109	0.06703	0.05027	0.10724	0.10056	0.00337	0.01011	0.00253
6	3	0.31445	0.30632	0.19530	0.05580	0.04882	0.10662	0.10121	0.00272	0.00954	0.00238
6	4	0.31822	0.31172	0.19512	0.05575	0.03902	0.10752	0.10319	0.00218	0.00761	0.00152
7	4	0.31792	0.31247	0.19103	0.04776	0.03821	0.10719	0.10355	0.00182	0.00730	0.00146
7	5	0.32049	0.31594	0.19093	0.04773	0.03182	0.10784	0.10481	0.00152	0.00608	0.00101
8	5	0.32032	0.31640	0.18789	0.04175	0.03131	0.10764	0.10503	0.00131	0.00588	0.00098
8	6	0.32218	0.31882	0.18783	0.04174	0.02683	0.10814	0.10590	0.00112	0.00504	0.00072
9	6	0.32207	0.31912	0.18547	0.03709	0.02650	0.10801	0.10605	0.00098	0.00491	0.00070
9	7	0.32348	0.32090	0.18543	0.03709	0.02318	0.10840	0.10668	0.00086	0.00430	0.00054
10	7	0.32340	0.32111	0.18355	0.03337	0.02294	0.10831	0.10678	0.00077	0.00421	0.00053
10	8	0.32451	0.32247	0.18353	0.03337	0.02039	0.10862	0.10726	0.00068	0.00374	0.00042
11	8	0.32446	0.32262	0.18199	0.03033	0.02022	0.10856	0.10734	0.00061	0.00368	0.00041
11	9	0.32535	0.32369	0.18197	0.03033	0.01820	0.10882	0.10771	0.00055	0.00331	0.00033
12	9	0.32531	0.32380	0.18070	0.02780	0.01807	0.10877	0.10777	0.00050	0.00327	0.00033
12	10	0.32604	0.32467	0.18068	0.02780	0.01643	0.10898	0.10807	0.00046	0.00297	0.00027

Appendix B4. Results for Model 3²⁴

a) Collusive Results										
K	F	U1 -qt	U2 qt	K qt	F qt	VI - profit	U2 profit	K profit	F profit	
4	2	0.30303	0.26515	0.08523	0.11364	0.10933	0.08437	0.00968	0.01291	
5	2	0.30303	0.27027	0.06818	0.11364	0.10933	0.08437	0.00775	0.01291	
5	3	0.30572	0.28606	0.06747	0.08434	0.10656	0.09259	0.00569	0.00711	
6	3	0.30572	0.28700	0.05622	0.08434	0.10656	0.09259	0.00474	0.00711	
6	4	0.30896	0.29627	0.05597	0.06716	0.10596	0.09704	0.00376	0.00451	
7	4	0.30896	0.29681	0.04797	0.06716	0.10596	0.09704	0.00322	0.00451	
7	5	0.31176	0.30290	0.04786	0.05584	0.10598	0.09979	0.00267	0.00312	
8	5	0.31176	0.30323	0.04188	0.05584	0.10598	0.09979	0.00234	0.00312	
8	6	0.31408	0.30753	0.04182	0.04779	0.10621	0.10166	0.00200	0.00228	
9	6	0.31408	0.30776	0.03717	0.04779	0.10621	0.10166	0.00178	0.00228	
9	7	0.31598	0.31095	0.03714	0.04178	0.10649	0.10301	0.00155	0.00175	
10	7	0.31598	0.31111	0.03343	0.04178	0.10649	0.10301	0.00140	0.00175	
10	8	0.31756	0.31358	0.03341	0.03712	0.10677	0.10402	0.00124	0.00138	
11	8	0.31756	0.31369	0.03037	0.03712	0.10677	0.10402	0.00113	0.00138	
11	9	0.31889	0.31565	0.03036	0.03339	0.10704	0.10481	0.00101	0.00112	
12	9	0.31889	0.31574	0.02783	0.03339	0.10704	0.10481	0.00093	0.00112	
12	10	0.32002	0.31734	0.02782	0.03035	0.10729	0.10545	0.00084	0.00092	

²⁴Note: The quantities are multiplied by $A = \frac{a-c}{b}$ and the profits by A^2b .

b) Cournot Results							
K	F	U1 -qt	U2 qt	K qt=F qt	VI - profit	U profit	K=F profit
4	2	0.30415	0.27650	0.09677	0.10045	0.08919	0.00937
5	2	0.30572	0.28464	0.08434	0.10123	0.09259	0.00711
5	3	0.30737	0.29076	0.07477	0.10194	0.09511	0.00559
6	3	0.30896	0.29552	0.06716	0.10258	0.09704	0.00451
6	4	0.31042	0.29933	0.06098	0.10314	0.09856	0.00372
7	4	0.31176	0.30245	0.05584	0.10364	0.09979	0.00312
7	5	0.31297	0.30505	0.05150	0.10409	0.10081	0.00265
8	5	0.31408	0.30725	0.04779	0.10449	0.10166	0.00228
8	6	0.31507	0.30913	0.04459	0.10485	0.10239	0.00199
9	6	0.31598	0.31076	0.04178	0.10518	0.10301	0.00175
9	7	0.31681	0.31218	0.03931	0.10547	0.10355	0.00155
10	7	0.31756	0.31344	0.03712	0.10574	0.10402	0.00138
10	8	0.31826	0.31456	0.03516	0.10598	0.10444	0.00124
11	8	0.31889	0.31555	0.03339	0.10620	0.10481	0.00112
11	9	0.31948	0.31645	0.03180	0.10641	0.10515	0.00101
12	9	0.32002	0.31726	0.03035	0.10660	0.10545	0.00092
12	10	0.32052	0.31800	0.02902	0.10677	0.10572	0.00084

c) Deviating Results											
K	F	U1 qt	U2 qt	K1 qt deviation	K qt	F qt	VI - profit	U2 profit	K profit	K dev profit	F profit
4	2	0.30833	0.29375	0.21003	0.08401	0.14002	0.10601	0.09632	0.00588	0.01470	0.00490
5	2	0.30896	0.29552	0.20149	0.06716	0.13433	0.10596	0.09704	0.00451	0.01353	0.00451
5	3	0.31335	0.30582	0.20081	0.06694	0.10041	0.10612	0.10111	0.00336	0.01008	0.00252
6	3	0.31377	0.30666	0.19519	0.05577	0.09760	0.10617	0.10143	0.00272	0.00952	0.00238
6	4	0.31692	0.31237	0.19491	0.05569	0.07797	0.10665	0.10362	0.00217	0.00760	0.00152
7	4	0.31719	0.31283	0.19092	0.04773	0.07637	0.10670	0.10379	0.00182	0.00729	0.00146
7	5	0.31948	0.31645	0.19078	0.04769	0.06359	0.10717	0.10515	0.00152	0.00607	0.00101
8	5	0.31966	0.31673	0.18779	0.04173	0.06260	0.10721	0.10525	0.00131	0.00588	0.00098
8	6	0.32138	0.31922	0.18771	0.04171	0.05363	0.10760	0.10617	0.00112	0.00503	0.00072
9	6	0.32151	0.31941	0.18539	0.03708	0.05297	0.10764	0.10624	0.00098	0.00491	0.00070
9	7	0.32283	0.32122	0.18534	0.03707	0.04633	0.10797	0.10690	0.00086	0.00429	0.00054
10	7	0.32293	0.32135	0.18348	0.03336	0.04587	0.10799	0.10694	0.00077	0.00421	0.00053
10	8	0.32398	0.32273	0.18345	0.03335	0.04077	0.10827	0.10744	0.00068	0.00374	0.00042
11	8	0.32405	0.32282	0.18193	0.03032	0.04043	0.10829	0.10747	0.00061	0.00368	0.00041
11	9	0.32490	0.32391	0.18191	0.03032	0.03638	0.10852	0.10786	0.00055	0.00331	0.00033
12	9	0.32496	0.32398	0.18065	0.02779	0.03613	0.10854	0.10788	0.00050	0.00326	0.00033
12	10	0.32567	0.32486	0.18063	0.02779	0.03284	0.10873	0.10820	0.00046	0.00297	0.00027

Appendix B5. Results for Prices and Quantities for Models 1, 2 and 3

		Price - P			Price - W			Q		
K	F	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
3	1	0.75000	0.74038	0.73529	0.66667	0.65385	0.64706	0.25000	0.25962	0.26471
4	1	0.75000	0.74286	0.73529	0.66667	0.65714	0.64706	0.25000	0.25714	0.26471
4	2	0.72222	0.71751	0.71591	0.66667	0.66102	0.65909	0.27778	0.28249	0.28409
5	2	0.72222	0.71847	0.71591	0.66667	0.66216	0.65909	0.27778	0.28153	0.28409
5	3	0.70833	0.70553	0.70482	0.66667	0.66346	0.66265	0.29167	0.29447	0.29518
6	3	0.70833	0.70600	0.70482	0.66667	0.66400	0.66265	0.29167	0.29400	0.29518
6	4	0.70000	0.69814	0.69776	0.66667	0.66460	0.66418	0.30000	0.30186	0.30224
7	4	0.70000	0.69840	0.69776	0.66667	0.66489	0.66418	0.30000	0.30160	0.30224
7	5	0.69444	0.69312	0.69289	0.66667	0.66522	0.66497	0.30556	0.30688	0.30711
8	5	0.69444	0.69328	0.69289	0.66667	0.66540	0.66497	0.30556	0.30672	0.30711
8	6	0.69048	0.68948	0.68934	0.66667	0.66559	0.66544	0.30952	0.31052	0.31066
9	6	0.69048	0.68959	0.68934	0.66667	0.66571	0.66544	0.30952	0.31041	0.31066
9	7	0.68750	0.68673	0.68663	0.66667	0.66584	0.66574	0.31250	0.31327	0.31337
10	7	0.68750	0.68680	0.68663	0.66667	0.66592	0.66574	0.31250	0.31320	0.31337
10	8	0.68519	0.68457	0.68450	0.66667	0.66601	0.66594	0.31481	0.31543	0.31550
11	8	0.68519	0.68462	0.68450	0.66667	0.66607	0.66594	0.31481	0.31538	0.31550
11	9	0.68333	0.68283	0.68278	0.66667	0.66613	0.66608	0.31667	0.31717	0.31722
12	9	0.68333	0.68287	0.68278	0.66667	0.66618	0.66608	0.31667	0.31713	0.31722
12	10	0.68182	0.68140	0.68136	0.66667	0.66623	0.66618	0.31818	0.31860	0.31864

Appendix B6. Results for Social Welfare

Model 1							
K	F	Total U Profits	Total K profits	Total F profits	Producer Surplus	Consumer Surplus	Social Welfare
3	1	0.04167	0.01389	0.00694	0.06250	0.03125	0.09375
4	1	0.04167	0.01389	0.00694	0.06250	0.03125	0.09375
4	2	0.04630	0.00926	0.00617	0.06173	0.03858	0.10031
5	2	0.04630	0.00926	0.00617	0.06173	0.03858	0.10031
5	3	0.04861	0.00694	0.00521	0.06076	0.04253	0.10330
6	3	0.04861	0.00694	0.00521	0.06076	0.04253	0.10330
6	4	0.05000	0.00556	0.00444	0.06000	0.04500	0.10500
7	4	0.05000	0.00556	0.00444	0.06000	0.04500	0.10500
7	5	0.05093	0.00463	0.00386	0.05941	0.04668	0.10610
8	5	0.05093	0.00463	0.00386	0.05941	0.04668	0.10610
8	6	0.05159	0.00397	0.00340	0.05896	0.04790	0.10686
9	6	0.05159	0.00397	0.00340	0.05896	0.04790	0.10686
9	7	0.05208	0.00347	0.00304	0.05859	0.04883	0.10742
10	7	0.05208	0.00347	0.00304	0.05859	0.04883	0.10742
10	8	0.05247	0.00309	0.00274	0.05830	0.04955	0.10785
11	8	0.05247	0.00309	0.00274	0.05830	0.04955	0.10785
11	9	0.05278	0.00278	0.00250	0.05806	0.05014	0.10819
12	9	0.05278	0.00278	0.00250	0.05806	0.05014	0.10819
12	10	0.05303	0.00253	0.00230	0.05785	0.05062	0.10847

Model 2								
K	F	U Profit	VI Profit	Total K-1 profits	Total F profits	Producer Surplus	Consumer Surplus	Social Welfare
4	1	0.01852	0.02556	0.01102	0.00735	0.06245	0.03306	0.09551
4	2	0.02161	0.02627	0.00718	0.00638	0.06144	0.03990	0.10134
5	2	0.02191	0.02564	0.00761	0.00634	0.06151	0.03963	0.10114
5	3	0.02338	0.02617	0.00566	0.00531	0.06052	0.04336	0.10388
6	3	0.02353	0.02586	0.00588	0.00529	0.06056	0.04322	0.10378
6	4	0.02438	0.02624	0.00469	0.00450	0.05981	0.04556	0.10537
7	4	0.02447	0.02606	0.00481	0.00449	0.05984	0.04548	0.10532
7	5	0.02502	0.02635	0.00400	0.00389	0.05926	0.04709	0.10635
8	5	0.02508	0.02624	0.00408	0.00389	0.05928	0.04704	0.10632
8	6	0.02546	0.02646	0.00349	0.00342	0.05884	0.04821	0.10705
9	6	0.02550	0.02638	0.00355	0.00342	0.05885	0.04818	0.10703
9	7	0.02578	0.02656	0.00310	0.00305	0.05850	0.04907	0.10757
10	7	0.02581	0.02651	0.00314	0.00305	0.05851	0.04905	0.10755
10	8	0.02603	0.02665	0.00279	0.00275	0.05822	0.04975	0.10797
11	8	0.02605	0.02661	0.00282	0.00275	0.05823	0.04973	0.10796
11	9	0.02622	0.02673	0.00253	0.00251	0.05799	0.05030	0.10829
12	9	0.02623	0.02670	0.00255	0.00251	0.05799	0.05029	0.10828
12	10	0.02637	0.02680	0.00232	0.00230	0.05779	0.05075	0.10855

Model 3								
K	F	U Profit	VI Profit	Total K profits	Total F-1 profits	Producer Surplus	Consumer Surplus	Social Welfare
4	2	0.02109	0.02733	0.00968	0.00323	0.06134	0.04035	0.10169
5	2	0.02109	0.02733	0.00968	0.00323	0.06134	0.04035	0.10169
5	3	0.02315	0.02664	0.00711	0.00356	0.06046	0.04357	0.10402
6	3	0.02315	0.02664	0.00711	0.00356	0.06046	0.04357	0.10402
6	4	0.02426	0.02649	0.00564	0.00338	0.05977	0.04567	0.10545
7	4	0.02426	0.02649	0.00564	0.00338	0.05977	0.04567	0.10545
7	5	0.02495	0.02650	0.00468	0.00312	0.05924	0.04716	0.10640
8	5	0.02495	0.02650	0.00468	0.00312	0.05924	0.04716	0.10640
8	6	0.02542	0.02655	0.00400	0.00286	0.05882	0.04826	0.10708
9	6	0.02542	0.02655	0.00400	0.00286	0.05882	0.04826	0.10708
9	7	0.02575	0.02662	0.00349	0.00262	0.05848	0.04910	0.10758
10	7	0.02575	0.02662	0.00349	0.00262	0.05848	0.04910	0.10758
10	8	0.02601	0.02669	0.00310	0.00241	0.05821	0.04977	0.10798
11	8	0.02601	0.02669	0.00310	0.00241	0.05821	0.04977	0.10798
11	9	0.02620	0.02676	0.00279	0.00223	0.05798	0.05032	0.10830
12	9	0.02620	0.02676	0.00279	0.00223	0.05798	0.05032	0.10830
12	10	0.02636	0.02682	0.00253	0.00207	0.05779	0.05077	0.10855

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