Technology Structure and Skill Structure: Costly Investment and Complementarity Effects Quantification

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Based on an extended model of endogenous directed technical change and on cross-country data, we identify and quantify the long-run link between: (i) the technology structure (high-versus low-tech sectors) and the skill structure (high-versus low-skilled workers), by considering an explicit role for the (potential) gross complementarity between technological goods; (ii) the Tobin-q and the technology characteristics of the firms through their impact on economic growth. Our estimation and calibration exercise suggests the existence of a moderate degree of gross complementarity and of an elastic relationship between the Tobin-q and key technology parameters.

Keywords: high-tech, low-tech, skills, complementarity, Tobin-q, technological-knowledge bias

JEL classification: O31, O41

1 Introduction

This paper aims to identify and quantify the long-run link between the technology structure and the skill structure, by considering an explicit role for the (potential) gross complementarity arising between technological goods. By considering internal investment costs, the paper also tackles the long-run relationship between the Tobin-q and the technology characteristics of the firms, such as the degree of complementarity between technological goods, through the impact of the latter on the long-run economic growth rate.

The importance of analysing the relationship between the technology structure and the skill structure under costly investment and complementarity effects is suggested by a number of empirical facts. From the extant literature, we retain that: (i) there is an impact of the technology structure, measured by either the number of firms or by production in high-vs-avvis low-tech manufacturing sectors, on industrial performance (Pilat et al., 2006) and on the skill premium (Cozzi and Impullitti, 2010); (ii) gross complementarities between technological goods are a relevant feature in explaining industrial growth (e.g., Matsuyama, 1995; Ciccone and Matsuyama, 1996); (iii) there is a positive relationship between the Tobin-q and the R&D

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and technology characteristics of the firms (e.g., Chan et al., 1990; Connolly and Hirschey, 2005). Moreover, by gathering data for a number of European countries between 1995 and 2007, we find that there is a positive relationship between the technology structure (regarding both the number of firms and production in high- vs. low-tech manufacturing sectors) and both the skill premium and the skill structure (the latter being measured as the ratio of high- to low-skilled manufacturing workers). In particular, we infer from this evidence that the skill structure featuring a higher proportion of high-skilled workers is associated with technological change directed towards the high-tech sectors, given the observed positive elasticity of the technology structure regarding the skill structure.

In the light of these facts, this paper develops an extended directed technological change model of endogenous growth that integrates a number of key ingredients. Firstly, the model incorporates endogenous directed technical change, such that final-goods production uses either low- or high-skilled labour with labour-specific intermediate goods, while R&D can be directed to either the low- or the high-skilled labour complementary technology; hence, the intermediate goods are the technological goods in the model, and “sector” denotes a group of firms producing the same type of labour-complementary intermediate goods. Since the data shows that the high-tech sectors are more intensive in high-skilled labour than the low-tech sectors, we consider the high- and low-skilled labour-complementary intermediate-good sectors in the model as the theoretical counterpart of the high- and low-tech sectors (e.g., Cozzi and Impullitti, 2010). This directed technical change setup allows us to have an endogenous high- versus low-tech technological bias.

We also consider simultaneously vertical and horizontal R&D. Under our R&D specification, the choice between vertical (increase of product quality) and horizontal innovation (creation of new products and industries) is related to the allocation of R&D expenditures, which are fully endogenous. Therefore, we endogenise the rate of growth along both the vertical and the horizontal direction, and thereby the number of industries/firms in each sector. Given the distinct nature of vertical and horizontal innovation (immaterial versus physical) and the consequent asymmetry in terms of R&D complexity costs, this framework then makes economic growth and firm dynamics closely related: vertical R&D is the ultimate growth engine, while horizontal R&D builds an explicit link between aggregate and technology-structure variables (the number of firms and production in high- and in low-tech sectors).

Finally, the model features two additional assumptions: (i) we allow for an interaction between the quantity of one intermediate good and the marginal productivity of the other intermediate goods, such that gross substitutability or gross complementarity may arise between the intermediate goods used in the production of final goods; (ii) we include internal investment costs à la Hayashi (1982) (e.g., Benavie et al., 1996; Thompson, 2008). Therefore, the model establishes a relationship between the technology structure and the skill structure regulated by the degree of gross substitutability/complementarity between technological goods (i.e., intermediate goods), together with a relationship between the Tobin-q and the technology characteristics of the firms.

By solving the model for the balanced-growth path (BGP), we uncover the analytical mechanism that explains the cross-country behaviour of the technological structure. Two types of effects exist: the market-size effect, through which a higher proportion of high-skilled workers...
induces technological change directed towards the high-tech sectors, and the skill-premium effect, reflecting the absolute productivity advantage of high-skilled workers. However, the interaction between the quantity of one technological good and the marginal productivity of the other technological goods plays a crucial role here. In particular, goods must be either gross substitutes or gross complements with the degree of complementarity below a certain threshold such that the elasticity of the technology structure with respect to the skill structure is positive, as observed in the cross-country data. That is, the increased incentives for allocating resources to R&D activities that come from gross complementarity must not be too large to offset the technological-knowledge bias channel. It is noteworthy that no upper limit seems to arise in related models featuring complementarities (e.g., Evans et al., 1998; Jones and Williams, 2000; Thompson, 2008).

Next, we take our model to the data and quantitatively associate the empirical facts on the technology structure and the skill structure to the degree of gross substitutability/complementarity between technological goods. In particular, we are interested in checking whether the case of gross complementarity, which features as a key assumption in a significant strand of the theoretical literature that studies poverty traps, growth and business cycles (e.g., Matsuyama, 1995; Ciccone and Matsuyama, 1996; Evans et al., 1998), has empirical support. Based on the analytical BGP relationships derived in the model and on the cross-country data for the technology structure and the skill structure, we compute indirect estimates of the value of the parameter that regulates the degree of gross substitutability/complementarity. Our results show that this parameter has a confidence interval between 1.08 and 1.78. Since the estimates are, in any case, larger than unity, our identification and estimation exercise suggests that there is gross complementarity between technological goods, thus backing up the complementarity assumption appearing in the referred to theoretical literature. However, our empirical results also suggest that the values that the literature tends to use to calibrate the complementarity parameter are apparently too high. Additionally, our estimations show that the elasticity regulating the horizontal-R&D complexity costs exceeds the one in the vertical-R&D case by between 22 and 130 percent, in line with what should be expected having in mind the distinct nature of the two types of R&D.

We are also interested in the long-run relationship between the Tobin-q and both the degree of complementarity between technology goods and the complexity effect pertaining to horizontal R&D, through the impact of the last two factors on the long-run economic growth rate. To obtain quantitative results, we calibrate the model bearing in mind the previous estimation exercise. Our results confirm the strong relationship between the Tobin-q and the R&D and technology characteristics of the firms that has been frequently reported by the empirical literature (e.g., Chan et al., 1990; Hall, 1999; Connolly and Hirschey, 2005). Nevertheless, our estimation and calibration exercise also underscores the importance of distinguishing between the technological determinants of the long-run market value of capital and explicitly considering their interaction, in as much as it may significantly dampen or enhance the impact on the Tobin-q.

The remainder of this work is organised as follows. Section 2 presents a dynamic general-equilibrium model of directed technological change with vertical and horizontal R&D, internal costly investment and gross substitutability/complementarity between the intermediate goods. In Sections 3 and 4, we derive the general equilibrium and analyse the BGP properties. In Section 5, we take the model to the data and quantitatively associate the empirical facts on the technology structure and the skill structure to the degree of gross substitutability/complementarity, and also quantify the long-run relationship between the Tobin-q and the technology characteristics of the firms. Concluding remarks are presented in Section 6.

2 Model specifications

Biased technical change is introduced in a closed-economy dynamic general-equilibrium setup. The economy consists of a competitive sector that produces a final good that can be used in consumption, production of intermediate goods and R&D. There are also two intermediate-good sectors, each having a large number of firms which operate in a monopolistic competitive framework. There are both vertical and horizontal R&D activities. If an innovation is successful, an incumbent is replaced by a new entrant in a given existing intermediate-good industry, or a monopolist emerges in a new intermediate-good industry, within a particular sector. Then successful R&D introduces, through creative destruction and variety expansion, both internal and external industry-wide limits to market power and generates endogenous economic growth. Thus, the model is an extension of Acemoglu and Zilibotti (2001), augmented with vertical R&D, as introduced in Afonso (2006). Moreover, with the purpose of introducing internal costly investment, we assume that final-good producers incur a Hayashi’s (1982) investment cost when accumulating total capital, which is composed by physical capital and technological-knowledge capital (i.e., capital accumulated through R&D). We also allow for an interaction between the quantity of one intermediate good and the marginal productivity of the other intermediate goods, such that gross complementarities may arise between the intermediate goods used in the production of final goods, as in, e.g., Evans et al. (1998) and Jones and Williams (2000). We will refer to intermediate goods also as technological goods, in the sense that they are the ones that embody successful R&D results.

The economy is populated by a fixed number of infinitely-lived households who maximise lifetime utility, which depends on consumption, bearing in mind the income generated by the supply one of two types of labour to final-good firms – low-skilled, \( L \), and high-skilled labour, \( H - (wages) \) and by the ownership of firms (interest).

2.1 Consumption side

The economy consists of a fixed number of identical and infinitely-lived households and has a zero population growth. Indexed with \( a \in [0,1] \) depending on their ability level, households consume final goods, own firms (equity) and inelastically supply low-skilled, \( L_a \) (if \( a \leq \bar{a} \)), or high-skilled labour, \( H_a \) (if \( a > \bar{a} \)) to final-good firms. Thus, total labour supply, \( L + H \), is exogenous and constant, with \( L = \int_0^\pi L_a da \) and \( H = \int_\bar{a}^1 H_a da \). All households have identical preferences and perfect foresight concerning the technological change over time, and they choose the time path of final-good consumption to maximise discounted lifetime utility:

\[
U(t) = \int_0^\infty e^{-\rho t} \cdot \frac{C_a(t)(1-\sigma)}{1-\sigma} \cdot \frac{1}{dt},
\]

subject to the flow budget constraint:

\[
\dot{B}_a(t) = r(t)B_a(t) + W_m(t) \cdot m_a(t) - C_a(t) \tag{1}
\]

(with \( m = L \) if \( a \leq \bar{a} \) and \( m = H \) if \( a > \bar{a} \)), where \( \sigma > 0 \) is the relative risk aversion coefficient, \( 0 < \rho < 1 \) is the subjective discount rate, \( r(t) \) is the market real interest rate, \( W_m(t) \) is the real wage, \( C_a(t) \) is the consumption of the representative household \( a \) and \( B_a \) denotes the household’s real equity holdings. The initial level of wealth, \( B_a(0) \), is given and the non-Ponzi games condition \( \lim_{t \to \infty} e^{-\int_0^t r(s) ds} B_a(t) \geq 0 \) is imposed.

The optimal consumption path is given by the standard Euler equation:

\[
\frac{C_a'(t)}{C_a(t)} = \frac{C(t)}{C_a(t)} = \frac{r(t) - \rho}{\sigma}, \tag{2}
\]
which is independent from $a$, and the transversality condition is $\lim_{t \to \infty} B_a(t) e^{-\rho t} = 0$. Along the balanced-growth path (BGP), with $\dot{C}/C$ constant, $r$ will also be constant.

2.2 Production side

**Final-good firms:** production The composite final-good, $Y$, is produced by competitive final-good firms continuously indexed by $n \in [0,1]$, such that $Y(t) = \int_0^1 P(n,t)Y(n,t)dn$, where $P(n,t)$ and $Y(n,t)$ are the relative price and the quantity of the final good produced by firm $n$. Firms produce employing a combination of labour and intermediate goods, using one of the two substitute production technologies available. In particular, firms can either use low-skilled labour, $L$, and a continuum of $L$-specific intermediate goods indexed by $j_L \in [0, A_L(t)]$, or high-skilled labour, $H$, and a continuum of $H$-specific intermediate goods indexed by $j_H \in [0, A_H(t)]$. Thus, each set of intermediate goods only combines with its corresponding type of labour, and each firm produces with one production technology exclusively, which we denote by $L$- and $H$-technology respectively. The production function for a representative firm $n$ is given by:

$$Y(n,t) = ((1 - n) \cdot L \cdot L(n))^{(1 - \alpha)} \cdot \left( \int_0^{A_L(t)} \lambda^k x_L(n, j_L, t) \gamma L \cdot dj_L \right)^{\phi} + (n \cdot h \cdot H(n))^{(1 - \alpha)} \cdot \left( \int_0^{A_H(t)} \lambda^k x_H(n, j_H, t) \gamma H \cdot dj_H \right)^{\phi},$$

where: $x_m(n, j_m, t)$ represents the quantity of the intermediate good $j_m$, $m = L, H$, and variables $H(n)$ and $L(n)$ denote the amount of high- and low-skilled labour used to produce the final good $n$ at time $t$; variables $A_H(t)$ and $A_L(t)$ represent the measure of variety of $H$- and $L$-specific intermediate goods available at time $t$; $\lambda > 1$ reflects the improvement in the quality of an intermediate good brought in by vertical innovation, and $k$ indicates both the number of quality improvements and the top quality rung at time $t$; $h > l \geq 1$ reflects the absolute productivity advantage of high- over low-skilled labour, while terms $n$ and $(1 - n)$ reflect the relative productivity advantage of each labour type, implying that high-skilled labour is relatively more productive for manufacturing final goods indexed by larger $n$s and vice versa; terms $(1 - \alpha)$ and $\alpha$, with $0 < \alpha < 1$, denote the share of labour and intermediate goods in production; and $\phi$ regulates the relationship between the quantity of one intermediate good and the marginal productivity of the other intermediate goods in the production function. We impose $\gamma = \alpha / \phi$ to ensure constant returns to scale, whereas $\phi > \alpha$ is required to guarantee that the model generates positive endogenous growth, as we will show below. If $\phi > 1$, then there is gross complementarity between intermediate goods used in production, in the sense that an increase in the quantity of one intermediate good increases the marginal productivity of the other intermediate goods. The terms $n$ and $(1 - n)$ imply that at each time $t$ there exists an endogenous threshold $\bar{n}$, at which switching from one production technology to another becomes advantageous, and consequently, each final good $n$ is produced with one technology $= H$ or $L$ – exclusively.

Operating in a perfect competition environment, each final-good firm $n$ seeks to maximise its profits at time $t$, taking as given $P(n,t)$, the prices of the intermediate goods $j_m, p_m(j_m, t)$, and the wages of high- or low-skilled labour employed in production, $W_m(t)$, $m = L, H$,

$$\max_{x_m(n,j_m,t)} \quad P(n,t) \cdot Y(n,t) - W_L(t) \cdot L(n) - W_H(t) \cdot H(n) - \int_0^{A_L(t)} p_L(j_L, t) \cdot x_L(n, j_L, t) dj_L - \int_0^{A_H(t)} p_H(j_H, t) \cdot x_H(n, j_H, t) dj_H.$$ 

Note that, since each final-good firm produces exclusively with one type of technology, profits at each $t$ are maximized with respect to $x_m(n,j_m,t)$. Then, from the first-order conditions of
each final-producer type’s maximization problem, we can derive the inverse demand functions for \(L\)- and \(H\)-specific intermediate goods used in final goods production:

\[
p_L(j_L, t) = P(n, t) \cdot \left(1 - n \right) \cdot l \cdot L(n) \cdot \alpha \cdot \lambda^{k_L(j_L, t)} \cdot x_L(n, j_L, t)^{\gamma - 1} \cdot \left(\int_0^{A_L(t)} \lambda^{k_L(j_L, t)} \cdot x_L(n, j_L, t)^{\gamma - 1} \cdot dj_L\right)^{\phi - 1}
\]

\[
p_H(j_H, t) = P(n, t) \cdot \left(1 - n \right) \cdot h \cdot H(n) \cdot \alpha \cdot \lambda^{k_H(j_H, t)} \cdot x_H(n, j_H, t)^{\gamma - 1} \cdot \left(\int_0^{A_H(t)} \lambda^{k_H(j_H, t)} \cdot x_H(n, j_H, t)^{\gamma - 1} \cdot dj_H\right)^{\phi - 1}
\]

Solving (4) for \(\int_0^{A_L(t)} \lambda^{k_L(j_L, t)} \cdot x_L(n, j_L, t)^{\gamma} dj_L\) and \(\int_0^{A_H(t)} \lambda^{k_H(j_H, t)} \cdot x_H(n, j_H, t)^{\gamma} dj_H\) respectively, and substituting in equation (3), final-good output for firm \(n\) can be re-written as:

\[
Y(n, t) = \left(\frac{\alpha \cdot P(n, t)}{p_m(j_m, t)}\right) \cdot \left(1 - n \right) \cdot l \cdot L(n) \cdot Q_L(t)^{\varepsilon + 1} + n \cdot h \cdot H(n) \cdot Q_H(t)^{\varepsilon + 1}
\]

where, as we will further show, \(p_L(j_L, t)\) and \(p_H(j_H, t)\) are equal, thus allowing us to place it in front of the brackets, and \(Q_H(t)\) and \(Q_L(t)\) are aggregate quality indices denoting the technological-knowledge stock at \(t\) for the \(H\)- and \(L\)-technology group respectively, being defined by:

\[
Q_m(t) = \int_0^{A_m(t)} \left(\frac{\lambda^{k_m(j_m, t)}}{\lambda^{k_m(j_m, t)}}\right)^{\frac{\gamma - 1}{\gamma}} dj_m, \quad m = L, H.
\]

The term \(\varepsilon = (\phi - 1) / (1 - \alpha)\) in (5) is a constant reflecting the interaction between the quantity of one intermediate good and the marginal productivity of the other intermediate goods in the final-good production function.

**Final-good firms: investment** Final-good producers are end-users of the stock of technological and of physical capital, as much as these stocks are embodied in the intermediate goods. Indeed, in line with Romer (1990) and others, the production of one unit of each intermediate good \(j_m\) takes one unit of physical capital, i.e., \(X_m(j_m, t) = K_m(j_m, t), m = L, H\), where \(X_L(j_L, t) = \int_0^{A_L(t)} x_L(n, j_L, t) dj_L\) and \(X_H(j_H, t) = \int_0^{A_H(t)} x_H(n, j_H, t) dj_H\), with \(x_m(n, j_m, t)\) computed from (4). Thus, the aggregate physical capital stock is the total quantity of the various types of intermediate goods, \(K(t) = K_L(t) + K_H(t)\), with \(K_L(t) = \int_0^{A_L(t)} X_L(j_L, t) dj_L\) and \(K_H(t) = \int_0^{A_H(t)} X_H(j_H, t) dj_H\). On the other hand, the production of intermediate goods also requires the use of designs (blueprints), where the source of new designs (new varieties or higher quality goods) are R&D activities, as detailed below. The technological-knowledge stock, (6), summarises the cumulative effect of R&D activities over time.

Following Thompson (2008), we assume that final-good producers incur a Hayashi’s (1982) investment cost when accumulating total capital, which is composed by physical capital and technological knowledge (i.e., capital accumulated through R&D). Total investment, \(\dot{Z}(t)\), is then given by the sum of investment in physical capital, \(\dot{K}(t)\), and investment in vertical, \(R_{c,m}(t)\), and horizontal, \(R_{h,m}(t)\), R&D:

\[
\dot{Z}(t) = \dot{K}(t) + R_{h,L}(t) + R_{h,H}(t) + R_{c,L}(t) + R_{c,H}(t)
\]

Assuming zero capital depreciation, installing \(\dot{I}(t) = \dot{Z}(t)\) new units of total capital requires spending an amount \(J(t) = \dot{I}(t) + \frac{1}{2} \theta Z(t)^2 / Z(t)\), where \(Z(t)\) is the total capital stock at \(t\), \(\frac{1}{2} \theta Z(t)^2 / Z(t)\)
represents the Hayashi’s internal installation cost, and \( \theta \) denotes the adjustment-cost parameter. In every period \( t \), final-good firms choose their investment rate so as to maximize the present discounted value of cash-flows. The current-value Hamiltonian for the optimal control problem is:

\[
H(t) = Y(t) - \mathcal{I}(t) - \frac{1}{2} \cdot \theta \cdot \frac{\mathcal{I}(t)^2}{\mathcal{Z}(t)} + q(t) \cdot \left( \mathcal{I}(t) - \mathcal{Z}(t) \right)
\]

where \( q(t) \) is the market value of total capital. The corresponding transversality condition is

\[
\lim_{t \to \infty} e^{-\int_0^t r(s) ds} \cdot q(t) \cdot \mathcal{Z}(t) = 0,
\]

where the market interest rate, \( r(s) \), equals the cost of capital. At this point, we anticipate the result, to be shown later on, that the economic growth rate, \( g \equiv g_Y \), is equal to the investment rate along the BGP, i.e., \( g = g_Z = \mathcal{I}(t)/\mathcal{Z}(t) \).

Then, from the first-order conditions, we have the long-run market value of capital:

\[
q = 1 + \theta g
\]

This ratio is known as Tobin’s (1969) marginal \( q \). With Hayashi’s investment cost function, marginal \( q \) equals Tobin’s average \( q \), the ratio of the market value of the firm to the replacement cost of its total capital stock. This equivalence is important because although it is marginal \( q \) that is relevant to investment, only average \( q \) is observable.

**Intermediate-good firms** Intermediate-good \( m \)-technology sector consists of a continuum \( A_m(t) \) of industries. There is monopolistic competition if we consider the whole sector: the monopolist in industry \( j_m \in [0, A_m(t)] \) fixes the price \( p_m(j_m, t) \) but faces an isoelessic demand curve, (4). Intermediate good \( j \) is produced with a cost function \( r(t)K(j, t) = r(t)X(j, t) \). Along the BGP, the cost of capital will also be constant, i.e., \( r(t) = r \). Profit in \( j_m \) is thus

\[
\pi_m(j_m, t) = (p_m(j_m, t) - rq) \cdot X_m(j_m, t),
\]

where \( rq \) denotes the production cost of one unit of the intermediate good \( j \). Depending on whether drastic or non-drastic innovations (e.g., Li, 2001; Barro and Sala-i-Martin, 2004) are the case, the profit maximising price is a markup over marginal cost, such that:

\[
p_L(j_L, t) = p_H(j_H, t) \equiv p = \frac{rq}{\chi}, \quad \text{where } \chi = \begin{cases} \gamma & \text{for } \lambda \geq \frac{1}{\gamma} \text{ (drastic innovation)} \\ \frac{1}{\lambda} & \text{for } \lambda < \frac{1}{\gamma} \text{ (non-drastic innovation)} \end{cases}
\]

Equation (9) implies that the price charged by intermediate-good producers for their differentiated goods is equal for all \( j \) and \( H(L) \)-technology sectors, i.e., it is *sector-invariant*. Moreover, on the BGP, where \( r \) and \( q \) will be constant, the markup is both *sector- and time-invariant*.

Next, using expressions (4) and (9), recalling that in any moment in time we have \( L = \int_0^1 L(n)dn \) and \( H = \int_1^\infty H(n)dn \), and normalising prices such that:

\[
P_L(t)^{\frac{1}{\gamma}} \equiv P(n, t)^{\frac{1}{\gamma}} \cdot (1 - n), \quad P_H(t)^{\frac{1}{\gamma}} \equiv P(n, t)^{\frac{1}{\gamma}} \cdot n,
\]

Firms producing different quality goods in industry \( j_m \) are engaged in Bertrand price competition. This assumption makes sure that only varieties of the lowest quality-adjusted price are kept in the market. Then, if \( p_m(k_m)/\lambda < rq \), where \( p_m(k_m) \) is the price of the top quality and \( p_m(k_m - 1) = rq \) due to Bertrand competition, lower grades are unable to provide any effective competition, and the top-quality producer can charge the unconstrained monopoly price. In this case, since the demand function (4) has a price elasticity of \(-1/(1 - \gamma)\), a top-quality firm sets \( p_m(k_m) = rq/\gamma \), which implies that \( \lambda > \frac{1}{\gamma} \), i.e., innovations are said to be “drastic”. In contrast, if \( \frac{1}{\gamma} > \lambda \), innovations are “non-drastic” and the producer of the leading-edge good has to engage in a limit-pricing strategy in order to drive his competitors out of the market, i.e., he/she charges \( p_m(k_m) = \lambda rq \). When final-good firms are indifferent between the top and second-highest goods on the quality ladder (i.e., if \( \frac{1}{\gamma} = \lambda \)), they are assumed to purchase the top quality.
we derive the total quantity that each \( H(L) \)-technology intermediate-good firm produces and sells, accounting for the threshold final good \( \pi \), and resulting profits:

\[
X_m(j_m, t) = M \cdot (\alpha \cdot P_m(t))^{\frac{1}{1-\alpha}} \cdot \left( \frac{rq}{\chi} \right)^{\frac{1}{1-\alpha}} \cdot (\chi h_m(j_{m,t}))^{\frac{1}{1-\alpha}} \cdot Q_m(t) \epsilon, \ m = L, H
\]

(11)

\[
\pi_m(t) = \pi_0 \cdot M \cdot P_m(t)^{\frac{1}{1-\alpha}} \cdot (\chi h_m(j_{m,t}))^{\frac{1}{1-\alpha}} \cdot Q_m(t) \epsilon, \ m = L, H
\]

(12)

with \( M = lL, hH \), and \( \pi_0 \equiv (1-\gamma)(rq/\chi)^{-\frac{\alpha}{1-\alpha}} \).

**Equilibrium aggregate output** Given that the existence of an endogenous threshold reflects the idea that the production of final goods \( n \in [\pi, 1] \) is more efficient using \( L \)-technology, and of final goods \( n \in [\pi, 1] \) is more efficient using \( H \)-technology, we can rewrite equation (5) as:

\[
Y(n, t) = \begin{cases} 
\left( \frac{\alpha}{r q} \cdot P(n, t) \right)^{\frac{1}{1-\alpha}} \cdot (1-n) \cdot L(n) \cdot Q_L(t)(\epsilon+1), & \text{for } n \in [0, \pi] \\
\left( \frac{\alpha}{r q} \cdot P(n, t) \right)^{\frac{1}{1-\alpha}} \cdot n \cdot h \cdot H(n) \cdot Q_H(t)(\epsilon+1), & \text{for } n \in [\pi, 1]
\end{cases}
\]

(13)

With competitive final-good producers, \( L \)- and the \( H \)-technology firms must break even at \( \bar{\pi} \). Then, using (13), rewriting price normalisation (10), and normalising labour such that \( L(n) = \bar{L}, H(n) = \frac{H}{\bar{L}} \), it can be shown that the endogenous final-good price ratio as a function of \( \bar{\pi} \) is (see Acemoglu and Zilibotti, 2001):

\[
\left( \frac{P_H}{P_L} \right)^{\frac{1}{1-\alpha}} = \frac{\bar{\pi}}{1-\bar{\pi}},
\]

(14)

where

\[
\bar{\pi} = \left[ 1 + \left( \frac{h}{L} \cdot \left( \frac{Q_H(t)}{Q_L(t)} \right)(\epsilon+1) \right)^{\frac{1}{2}} \right]^{-1}
\]

(15)

and

\[
P_H = P(n, t) \cdot n^{1-\alpha} = e^{-(1-\alpha)} \cdot (1 - \bar{\pi})^{-(1-\alpha)}
\]

\[
P_L = P(n, t) \cdot (1-n)^{1-\alpha} = e^{-(1-\alpha)} \cdot \bar{\pi}^{-(1-\alpha)}
\]

(16)

Assuming that the wage per unit of \( m \)-type labour equals its marginal product, we use the production function (13) aggregated across \( n \) (bearing in mind that \( \int_0^{\pi(t)} L(n)dn = L \) and \( \int_{\pi(t)}^1 H(n)dn = H \) hold at every \( t \)) to derive the high- and low-skilled labour wages:

\[
Y(t) = Y_L(t) + Y_H(t), \text{ where } \begin{cases} 
Y_H(t) = \left( \frac{\alpha}{r q} \cdot h \cdot H \cdot P_H^{\frac{1}{1-\alpha}} \cdot Q_H(t)(\epsilon+1) \right) \\
Y_L(t) = \left( \frac{\alpha}{r q} \cdot l \cdot L \cdot P_L^{\frac{1}{1-\alpha}} \cdot Q_L(t)(\epsilon+1) \right)
\end{cases}
\]

(17)

\[
W_H = \frac{\partial Y_H}{\partial H} = \left( \frac{\alpha}{r q} \cdot h \cdot P_H^{\frac{1}{1-\alpha}} \cdot Q_H(t)(\epsilon+1) \right)
\]

\[
W_L = \frac{\partial Y_L}{\partial L} = \left( \frac{\alpha}{r q} \cdot l \cdot P_L^{\frac{1}{1-\alpha}} \cdot Q_L(t)(\epsilon+1) \right)
\]

(18)
The skill premium, $W_H/W_L$, is then given by the ratio of high- and low-skilled labour wages from (18):

$$\frac{W_H}{W_L} = \frac{h}{l} \left( \frac{P_H}{P_L} \right)^{\frac{1}{\alpha}} \left( \frac{Q_H(t)}{Q_L(t)} \right)^{(\varepsilon+1)}$$

(19)

Finally, using (13) and (15), we can define equilibrium aggregate output as:

$$Y(t) = \frac{1}{\varepsilon} \left( \frac{\alpha \chi}{rq} \right)^{\frac{\alpha}{\varepsilon}} \cdot \left[ \left( \varepsilon \cdot l \cdot Q_L(t)^{(\varepsilon+1)} \right)^{\frac{1}{\varepsilon}} + \left( h \cdot Q_H(t)^{(\varepsilon+1)} \right)^{\frac{1}{\varepsilon}} \right]^2$$

(20)

From (20), it is clear that the dynamics of technological knowledge will drive economic growth, provided that $\varepsilon + 1 = (\phi - \alpha)/(1 - \alpha) > 0$; this is true, iff the condition $\phi > \alpha$ is satisfied. Moreover, if there is gross complementarity between intermediate goods, i.e., $\phi > 1$, then $\varepsilon + 1 > 1$, which implies that the effect of technological knowledge growth on economic growth is more than proportional.

2.3 R&D

We assume that R&D firms can choose between either vertical or horizontal innovation. By devoting their resources to vertical R&D, firms target qualitative improvements of already existing intermediate-good varieties, while in the case of horizontal R&D they aim at variety expansion, i.e. at creating a new variety. Each new design (a new variety or a higher quality good) is granted a patent and thus a successful innovator retains exclusive rights over the use of his/her good. We also assume, to simplify the analysis, that both vertical and horizontal R&D are performed by (potential) entrants, and that successful R&D leads to the set-up of a new firm in either an existing or in a new industry (as in, e.g., Howitt, 1999; Strulik, 2007; Gil et al., 2012). There is perfect competition among entrants and free entry in the R&D businesses.

**Vertical R&D**  
Vertical R&D is a standard creative destruction process, with the innovation arrival rate following a Poisson process. By improving on the current top quality level $k_{m}(j_{m}, t)$, a successful vertical R&D firm earns monopoly profits from selling the leading-edge input of $k_{m}(j_{m}, t) + 1$ quality to final-good firms. However, temporary exclusive rights over the top-quality intermediate good confer the successful innovator higher profits only for the duration of the patent, which is determined by the instantaneous vertical-innovation arrival rate, $I_{m}(j_{m}, t)$ – vertical innovation rate at time $t$, by potential entrant $i$ in industry $j_{m}$. The rate $I_{m}(j_{m}, t)$ is independently distributed across firms, across industries and over time, and depends on the flow of resources $R_{v,m}(j_{m}, t)$ committed by entrants at time $t$. As in, e.g., Barro and Sala-i-Martin (2004, ch. 7), $I_{m}(j_{m}, t)$ features constant returns in R&D expenditures, $I_{m}(j_{m}, t) = R_{v,m}(j_{m}, t)/\Phi_{m}(j_{m}, t)$. The cost $\Phi_{m}(j_{m}, t)$ is assumed to be symmetric within sector $m$, such that

$$\Phi_{m}(j_{m}, t) = \zeta \cdot \left( \lambda^{[k_{m}(j_{m}, t)+1]} \right)^{\frac{1}{\gamma}} \cdot Q_{m}(t)^{\varepsilon}, \quad m \in \{L, H\},$$

(21)

where $\zeta$ is a constant fixed vertical-R&D cost, assumed equal for both sectors $m$; and the term $\left( \lambda^{[k_{m}(j_{m}, t)+1]} \right)^{\frac{1}{\gamma}} \cdot Q_{m}(t)^{\varepsilon}$ accounts for the adverse effect resulting from the increasing complexity of quality improvements, implying that research becomes progressively more difficult with each new vertical innovation.\(^6\) Aggregated across firms $i$ in $j_{m}$, the instantaneous arrival rate of a new quality improvement is given by:

\(^6\)The assumption of complementarity between intermediate goods further adds to the progressive complexity of new quality improvements, captured by the term $Q_{m}^{\varepsilon}$. Note that our complementarity specification implies that incentives for discovering new goods would grow rapidly over time (more complements raise
\[ I_m(j_m, t) = R_{v,m}(j_m, t) \cdot \frac{1}{\zeta} \cdot \left( \lambda^{-[k_m(j_m,t)+1]} \right)^{\frac{1}{1-\gamma}} \cdot Q_m(t)^{-\varepsilon}, \quad m = L, H, \quad (22) \]

As the terminal date of each monopoly arrives as a Poisson process with frequency \( I_m(j_m, t) \) per (infinitesimal) increment of time, the present value of a monopolist’s profits is a random variable. Let \( V_m(j_m, t) \) and \( V_m^+(j_m, t) \) denote the expected value of an incumbent with current quality level \( k_m(j_m, t) \) and \( k_m(j_m, t) + 1 \), respectively.\(^7\) By focusing on the BGP, we take into account that, as will be shown later, \( P_m, r \) and \( I \) are constant over time. We then have \( V_m(j_m, t) = \pi_0 \cdot M \cdot (\lambda^{k_m(j_m,t)})^{\frac{1}{1-\gamma}} \cdot Q_m(t)^{-\varepsilon} \), given by (12), are constant in-between innovations; hence, we can further write

\[ V_m(j_m, t) = \frac{\pi_m(j_m, t)}{r + I_m(j_m) - \varepsilon \cdot \frac{Q_m}{Q_m}}, \quad m = L, H, \quad (23) \]

where \( \dot{Q}_m/Q_m \) is also constant on BGP. On the other hand, free-entry prevails in vertical R&D such that the condition \( I_m(j_m, t) \cdot V_m^+(j_m, t) = R_{v,m}(j_m, t) \) holds, which implies that

\[ V_m^+(j_m, t) = \Phi_m(j_m, t) = \zeta \cdot \left( \lambda^{k_m(j_m,t)+1} \right)^{\frac{1}{1-\gamma}} \cdot Q_m(t)^{-\varepsilon}, \quad m = L, H. \quad (24) \]

By substituting (23) into (24) and using (12) to simplify, we get the no-arbitrage condition facing a vertical innovator

\[ r + I_m = (1 - \gamma) \cdot \frac{M}{\zeta} \cdot \left( \frac{rq}{\chi} \right)^{\frac{1}{1-\alpha}} \cdot (\alpha P_m)^{\frac{1}{1-\alpha}} + \frac{\dot{Q}_m}{Q_m}, \quad m = L, H, \quad (25) \]

where the rates of entry are symmetric across industries \( I_m = I_m(j_m) \).

After solving equation (22) for \( R_{v,m}(j_m, t) \) and aggregating across industries \( j_m \), we determine optimal total resources devoted to vertical R&D, \( R_{v,m}(t) = \int_0^{A_m(t)} R_{v,m}(j_m, t) dj_m \). As the innovation rate is industry independent, then:

\[ R_{v,m}(t) = \zeta \cdot \lambda^{\frac{1}{1-\gamma}} \cdot I_m \cdot Q_m(t)^{(\varepsilon+1)}, \quad m = L, H. \quad (26) \]

**Horizontal R&D** Variety expansion arises from (horizontal) R&D aimed at creating a new intermediate good. We derive the horizontal R&D arbitrage condition following the same reasoning as in the vertical R&D case. However, contrarily to the vertical R&D case, where each new successful innovation increases the quality of an existing good by \( \lambda^{1/\gamma} \), each horizontal innovation results in a new intermediate good whose quality level is drawn randomly from the distribution of existing varieties (e.g., Howitt, 1999). Thus, the expected quality level of the horizontal innovator is

\[ \bar{z}_m(t) = \int_0^{A_m(t)} \frac{\left( \lambda^{k_m(j_m,t)} \right)^{\frac{1}{1-\gamma}}}{A_m(t)} dj_m = \frac{Q_m(t)}{A_m(t)}, \quad m = L, H. \quad (27) \]

\(^7\)Entrants behave as risk neutral agents, and, thus, only care about the expected value of the firm, since \( V_m(j_m, t) \) is the perfectly diversified stock-market valuation of the current incumbent in industry \( j \).
The production function of new varieties of intermediate goods, exhibiting constant returns to scale at firm level and involving a horizontal innovation cost symmetric across firms, is 
\[ \hat{A}_m(t) = \frac{R_{h,m}(t)}{\eta_m(t)} \]
where \( \hat{A}_m \) denotes the contribution of R&D firm \( f \) to the total of new \( m \)-complementary intermediate-good varieties that are being created at time \( t \) at a cost of \( \eta_m \) units of the final good, and \( R_{h,m} \) denotes the resources a firm devotes to creating new product lines. Then, aggregating across firms \( f \), total resources devoted to horizontal R&D are given by:

\[ R_{h,m}(t) = \eta_m(t) \cdot \frac{\hat{A}_m(t)}{A_m(t)}, \quad m = L, H. \]  

(28)

We assume that the cost of setting up a new variety is:

\[ \eta_m(t) = \beta_1 \cdot A_m(t)^{\beta_2+1}, \]

(29)

where \( \beta_1 > 0 \) is a constant fixed (flow) cost and \( \beta_2 > \epsilon \). The latter condition jointly with \( \phi > \alpha \), which guarantees that \( \epsilon > -1 \), implies that \( \beta_2 + 1 > 0 \). Thus, similarly to vertical R&D, horizontal entry cost function \( \eta_m \) incorporates an R&D complexity effect that arises through the dependence of \( \eta_m \) on \( A_m \) (e.g., Evans et al., 1998; and Barro and Sala-i-Martin, 2004, ch. 6), implying that the larger the number of existing varieties, the costlier it is to introduce new varieties.

Taking into account the expected quality level (27) and following a similar line of reasoning as in the vertical R&D case, the expected value of a monopolist is

\[ V_m(t) = \pi_0 \cdot M \cdot \bar{T}_m(t) \cdot \int_t^\infty Q_m(s)^\epsilon \cdot e^{-[r+\bar{I}_m(s)]s} ds, \]

where \( \pi_0 \cdot M \cdot \bar{T}_m(t) \cdot \int_t^\infty Q_m(s)^\epsilon \cdot e^{-[r+\bar{I}_m(s)]s} ds = \pi_0(t) \cdot Q_m(t)^{-\epsilon} \). The free-entry condition, \( A_m(t) \cdot V_m(t) = R_{h,m}(t) \), by (28) simplifies to:

\[ V_m(t) = \frac{\eta_m(t)}{A_m(t)}, \quad m = L, H. \]

(30)

Time-differentiating (30) yields the horizontal R&D no-arbitrage condition:

\[ r + I_m = \frac{\dot{\pi}_m(t)}{\eta_m(t)/A_m(t)} + \epsilon \frac{\dot{Q}_m}{Q_m}, \quad m = L, H. \]

(31)

**Intra and inter-sector no-arbitrage conditions** Using the above derived vertical and horizontal R&D arbitrage conditions given by equations (25) and (31) respectively, we can now derive the intra-sector arbitrage condition, by equating the effective rate of return, \( r + I_m \), \( m = L, H \), for both R&D. This no-arbitrage condition reflects the idea that, in equilibrium, the competitive capital market is equally willing to finance vertical and horizontal R&D in each sector \( m \). Thus, using equations (25) and (31), the intra-sector no-arbitrage conditions are:

\[ \zeta \cdot Q_m(t)^\epsilon \cdot \frac{Q_m(t)}{A_m(t)} = \frac{\eta_m(t)}{A_m(t)} = \beta_1 A_m^{\beta_2}(t), \quad m = L, H. \]

(32)

No-arbitrage conditions, within the \( H \)- and \( L \)-technology R&D sectors, equate the average cost of horizontal R&D, \( \eta_m/A_m \), to the average cost of vertical R&D, \( \zeta Q_m^{\pi_m} \).

We must also derive an inter-sector no-arbitrage condition, defining a situation of indifference between, e.g., vertical innovating in \( L \)- or \( H \)-technology R&D sector. Thus, by equating the effective rate of return for each \( m = L, H \) in (25), we get:

\[ I_H - I_L = \frac{\pi_0}{\zeta} \left( h \cdot H \cdot P_H^{\frac{1}{\epsilon_1}} - l \cdot L \cdot P_L^{\frac{1}{\epsilon_1}} \right) \]

(33)

\( ^8 \)It will be shown later that the condition \( \beta_2 > \epsilon \) is required for positive economic growth to exist with simultaneous vertical and horizontal R&D along the BGP.
3 The general-equilibrium BGP

In this section, we derive and characterise the general-equilibrium interior BGP.

The aggregate financial wealth held by households is composed by equity of intermediate good producers $B(t) = B_L(t) + B_H(t)$, where $B_m(t) = K_m(t) + \int_0^t Y_m(j_m, t)dj_m$, $m = L, H$. From the no-arbitrage condition between vertical and horizontal entry, we have equivalently $B(t) = K_L(t) + K_H(t) + \eta_L(t) \cdot A_L(t) + \eta_H(t) \cdot A_H(t)$. Taking time derivatives and comparing with (1), the aggregate flow budget constraint is equivalent to the final-good market equilibrium condition

$$Y(t) = C(t) + \dot{Z}(t)$$

(34)

The dynamic general equilibrium is defined by the paths of allocations and price distributions: $(\{X_m(j_m, t), p_m(j_m, t)\}, j_m \in [0, A_m(t)])_{t \geq 0}$ and of the number of firms, quality indices and vertical-innovation rates $(\{ A_m(t), Q_m(t), \bar{I}_m(t) \})_{t \geq 0}$ for sectors $m = L, H$, and by the aggregate paths $(\bar{I}(t), C(t), q(t), r(t))_{t \geq 0}$ such that: (i) final-good firms, intermediate-good firms and consumers solve their problems; (ii) free-entry and no-arbitrage conditions are met; and (iii) markets clear. Total supplies of high- and low-skilled-labour are exogenous.

Along the general-equilibrium BGP, the aggregate resource constraint (34) is satisfied with $Y$, $C$ and $\dot{Z}$ growing at the same constant rate, $g_Y = g_C = g_Z$. Since a constant $g_Z$ implies $g_Z = g_Z$, then $g_Y = g_Z$, as considered in our derivations in Subsection 2.2 above. By considering (20) and by time-differentiating (32), we show that a general-equilibrium BGP exists only if the following conditions hold among the asymptotic growth rates, which are all constant:

(i) the growth rates of the quality indices are equal, $g_{Q_L} = g_{Q_H}$, and are monotonously related to the endogenous growth rate of the economy, $g_Y = g = (1 + \varepsilon)g_{Q_m}$, $m = L, H$;

(ii) the growth rates of the number of varieties are equal, $g_{A_L} = g_{A_H}$;

(iii) the vertical-innovation rates and the final-good price indices are trendless, $g_{I_L} = g_{I_H} = g_{P_L} = g_{P_H} = 0$;

(iv) the growth rates of the quality indices and of the number of varieties are monotonously related as $g_{Q_L} / g_{A_L} = g_{Q_H} / g_{A_H} = (1 + \beta_2) / (1 + \varepsilon)$. Then $g_{A_H} = g_{A_L} = g / (1 + \beta_2)$;

(v) the market interest rate and the market value of total capital are trendless, $g_r = g_q = 0$.

The transversality conditions hold if $g > 0$. Thus, it is clear from (i) and (iv) that variety expansion is sustained by endogenous technological-knowledge accumulation. In particular, expected higher profits from increased intermediate-good quality generated by vertical R&D make it attractive for intermediate-good firms to invest in variety expansion, despite the negative spillovers in horizontal entry (entry cost increases with the number of newly created varieties). Thus, vertical innovation drives economic growth, in the sense that it sustains both variety expansion and aggregate output growth.

The technological-knowledge bias Necessary condition (i) implies that the trendless levels for the vertical-innovation rates verify $I_L = I_H = I$, along the BGP. Introducing this in equation (33) we derive (where $*$ denotes BGP values):

$$\left( \frac{P_H}{P_L} \right)^* = \left( \frac{l}{h} \frac{L}{H} \right)^{1-\alpha}$$

(35)

Combining this with expressions (14) and (15), after some algebra we obtain:
Equation (36) defines the long-run technological-knowledge bias \((Q_H/Q_L)^*\) as a function of the relative supply of skills, \(H/L\). This is a common result in the Skill Biased Technological Change (SBTC) literature, reflecting the endogeneity of the skill-bias of newly adopted technologies. In fact, recalling expression (25) we can see that the skill structure influences the direction of technological-knowledge development through two channels. On the one hand, the market-size channel, acting through \(H(L)\), increases the innovation rate of \(H(L)\)-technology R&D, since an increase in the labour supply broadens the market for the respective technology type. On the other hand, the price channel, acting through \(P_m\), reduces the innovation rate of \(H(L)\)-technology R&D, i.e., this channel favours developing technologies that complement the relatively scarce type of labour, since the prices for the final goods that they produce will be higher. Expression (36) shows that the market-size effect dominates the price-channel effect, since the technological-knowledge bias is increasing in the relative supply of skills. Additionally, as (36) shows, \((Q_H/Q_L)^*\) is directly affected by the degree of gross substitutability/complementarity, \(\phi\). This result will be analysed in more detail below.

**The skill premium** Using equation (35) in (19), the expression for the skill premium becomes:

\[
\frac{W_H}{W_L} = \frac{L}{H} \left( \frac{Q_H(t)}{Q_L(t)} \right)^{(\varepsilon + 1)}
\]  
(37)

Equation (37) illustrates the two mechanisms through which changes in the relative supply of skills influence the skill premium in equilibrium. Initially, an increase in the supply of the \(H\) factor reduces the skill premium, which is consistent with the basic producer theory, i.e. increasing supply reduces prices. This immediate effect is then followed by a change in the opposite direction induced through the technological-knowledge bias. By substituting (36) in (37), we get:

\[
\left( \frac{W_H}{W_L} \right)^* = \frac{L}{H} \left( \frac{h H}{l L} \right) = \frac{h}{l}
\]  
(38)

Thus, the long-run skill premium does not depend on the relative supply of skills, \(H/L\), because the two above referred to mechanisms exactly offset each other. As noted by Acemoglu and Zilibotti (2001), this results from the fact that the aggregate production function is characterized by a constant elasticity of substitution between \(H\)- and \(L\)-technologies equal to 2 (see (20)). As emphasised by Acemoglu (2009, Ch.15), in the context of substitution between high- and low-skilled workers an elasticity of substitution close to 2 seems to be the most probable according to the empirical evidence. As (38) shows, the long-run skill premium is determined solely by the productivity advantage ratio of high- to low-skilled labour, \(h/l\) (e.g., Acemoglu and Zilibotti, 2001).

**The economic growth rate** If we assume that the number of industries, \(A_m\), is large enough to treat \(Q_m\) as time-differentiable and non-stochastic, we can time-differentiate (6) to get \(\dot{Q}_m\). Then, if \(\beta_2 > \varepsilon\), we can derive another relationship between the growth rate of the quality indices and of the number of varieties,

\[
g_{Q_m}(t) = I_m(t) \cdot (\lambda^{m\varepsilon} - 1) + g_{A_m}(t), m = L, H.
\]  
(39)

By applying (39) to the BGP, and by combining it with the BGP condition \(g_{A_m}^* = (1 + \varepsilon)/(1 + \beta_2)g_{Q_m}^*\), we find that the constraint \(\beta_2 > \varepsilon\) is required for (39) to be well defined,
i.e., for positive BGP economic growth to exist with both positive \( g^*_m \) and \( I^*_m \). Under this condition, the economic growth rate, \( g^* = (1 + \varepsilon)g^*_m \), is proportional to the growth rate of intermediate-good quality plus the growth rate of the number of varieties, in line with the well-known view that industrial growth proceeds both along an intensive and an extensive margin. Note also that, since \( \varepsilon \equiv (\phi - 1) / (1 - \alpha) \), that condition is equivalent to \( \phi < \beta_2 \cdot (1 - \alpha) + 1 \). This upper limit on the parameter that regulates gross substitutability/complementarity, \( \phi \), is necessary because of the roundabout effect linking horizontal and vertical R&D: given the distinct nature of vertical and horizontal innovation (immaterial versus physical) and the consequent asymmetry in terms of R&D complexity costs (see (21) and (29)), vertical innovation sustains variety expansion, which, in turn, expands the technological-knowledge stock, thereby increasing the amount of resources available for both types of R&D. Such a constraint on \( \phi \) does not arise in, e.g., Evans et al. (1998), Thompson (2008), where only horizontal R&D exists, whereas in Jones and Williams (2000) vertical innovation is just a spillover of horizontal R&D activities.

Using the relations presented above, we complete the derivation of the growth rate of the aggregate quality indices, \( g^*_m \), and the economic growth rate, \( g \), for the BGP. We obtain the following implicit equations for \( g^*_m \) and \( g \):

\[
g^*_m = \psi (\Omega - r^*) , \tag{40}
\]

\[
g^* = (1 + \varepsilon) \psi (\Omega - r^*) , \tag{41}
\]

where

\[
\Omega \equiv \frac{1 - \gamma}{\zeta} M \left( \frac{r^* q^*}{\chi} \right)^{-\frac{\alpha}{1 - \alpha}} (\alpha P^*_m)^{\frac{1}{1 - \alpha}} , \tag{42}
\]

\[
\psi \equiv \frac{(1 + \beta_2) \left( \lambda \frac{\gamma}{\beta} - 1 \right)}{\beta_2 - \varepsilon - \varepsilon (\beta_2 + 1) (\lambda \frac{\gamma}{\beta} - 1)} , \tag{43}
\]

and both \( r^* \) and \( q^* \) are functions of \( g^* \), by (2) and (8), respectively.\(^9\)

Next, we analyse the BGP effects on the long-run economic growth rate of shifts in the technological parameters \( \psi \), \( \theta \) and \( \beta_2 \).

**Growth effects of shifts in the technological parameters** In order to assess the effects of an increase in the degree of gross substitutability/complementarity, \( \phi \), we use the Implicit Function Theorem to derive \( \frac{\partial F}{\partial \phi} = -\frac{\partial F}{\partial g} / \frac{\partial g}{\partial \phi} \). Recalling (43), (42) and \( \varepsilon \equiv (\phi - 1) / (1 - \alpha) \), and defining \( F(\cdot) \equiv g - \psi (1 + \varepsilon) (\Omega - r) = 0 \), we first compute \( \frac{\partial F}{\partial g} \), evaluated at the BGP:

\[
\frac{\partial F}{\partial g} = 1 + \psi (1 + \varepsilon) \left( \frac{\alpha}{1 - \alpha} \right) (g\sigma + \rho)^{\frac{1}{1 - \alpha}} \cdot (1 - \gamma) \left[ \frac{\chi^{\frac{\alpha}{1 - \alpha}} (\alpha P^*_m)^{\frac{1}{1 - \alpha}}}{\xi^2} (\sigma q + \theta r) + \sigma (\frac{1 - \alpha}{\alpha}) \right] > 0
\]

Then we find that \( \frac{\partial F}{\partial \phi} \), also evaluated at the BGP, is:

\[
\frac{\partial F}{\partial \phi} = -\frac{\psi^2}{\frac{\alpha}{1 - \alpha}} \left( \frac{1 + (\beta_2 + 1) (\lambda \frac{\gamma}{\beta} - 1)}{\beta_2 + 1} \right) \cdot (\Omega - r) < 0 \tag{45}
\]

\(^9\)Despite the nonlinearity of (41), there are no multiple general-equilibria interior BGP. To see this, observe that (41) is negatively sloped in the first quadrant of the space \((g, r, q)\), while (2) and (8) are always non-negatively sloped.
Consequently, we have \( \frac{\partial F}{\partial \phi} \frac{\partial \phi}{\partial g} < 0 \), and thus \( \frac{\partial g}{\partial \phi} > 0 \), i.e., an increase in \( \phi \) – a reduction in the degree of gross substitutability or an increase in the degree of gross complementarity between technological goods – positively affects the long-run economic growth rate. The intuition behind this result is fairly simple. When technological goods used in production are gross complements, an increased quantity of some goods raises the marginal productivity of the others.\(^{10}\) This implies that, at constant prices, the quantity demanded goes up, production increases and output grows faster than it would otherwise.

We now assess the effects of an increase in the horizontal R&D complexity-cost parameter, \( \beta_2 \), by deriving \( \frac{\partial g}{\partial \beta_2} = -\frac{\partial F}{\partial \beta_2} / \frac{\partial F}{\partial g} \). Since \( \frac{\partial F}{\partial \beta_2} \), evaluated at the BGP, is:

\[
\frac{\partial F}{\partial \beta_2} = - \left( \frac{1+\varepsilon}{[\beta_2 - \varepsilon(\beta_2+1)(\lambda^{1/\gamma} - 1)]} \right) \cdot (1+\varepsilon) \cdot \left( \lambda^{\frac{1}{1-\gamma}} - 1 \right) \cdot (\Omega - r) > 0 \tag{46}
\]

it follows that \( \frac{\partial F}{\partial \beta_2} / \frac{\partial F}{\partial g} > 0 \), and thus \( \frac{\partial g}{\partial \beta_2} < 0 \), i.e., an increase in the horizontal R&D complexity-cost parameter, \( \beta_2 \), negatively affects the long-run economic growth rate. The intuition behind this result is the following. Recalling our horizontal R&D entry cost function, \( \eta_a(t) \), we can see that entry costs are higher as a result of an increase in \( \beta_2 \) (see Section 2.3), which makes investments in the creation of new product lines less attractive for the intermediate-good firms. And given that the economic growth rate increases with the aggregate quality index growth rate, which on its turn is positively influenced by the variety expansion growth rate, the negative effect on \( g \) is verified.

Finally, in order to analyse the impact of changes in the adjustment-cost parameter, \( \theta \), we determine the sign of \( \frac{\partial g}{\partial \theta} = -\frac{\partial F}{\partial \theta} / \frac{\partial F}{\partial g} \). Since \( \frac{\partial F}{\partial \theta} \), evaluated at the BGP, is:

\[
\frac{\partial F}{\partial \theta} = \psi(1+\varepsilon) \left( \frac{\alpha}{1-\alpha} \right) \left( \frac{1-\gamma}{\zeta} \right) M r^{-\frac{\alpha}{1-\alpha}} \lambda^{\frac{\alpha}{1-\alpha}} (\alpha P_m)^{\frac{1}{1-\alpha}} \cdot (1+\theta g)^{-\frac{1}{1-\alpha}} g > 0 \tag{47}
\]

then \( \frac{\partial F}{\partial \theta} / \frac{\partial F}{\partial g} > 0 \) and thus \( \frac{\partial g}{\partial \theta} < 0 \), i.e., the long-run economic growth rate is negatively affected by the internal investment costs. The explanation for this result stems directly from the internal investment costs theory. In particular, accounting for capital installation costs in their optimisation problem, firms control their rate of investment (not the capital stock) at each \( t \). Consequently, when internal investment costs increase, i.e. installing new capital becomes more expensive, firms will tend to reduce investment. This will slowdown the accumulation of technological knowledge, leading to a reduction in the economic growth rate.

\section*{4 Long-run technology structure}

In order to take our model to the data and, in particular, to quantitatively associate the empirical facts on the technology structure and the skill structure to the complexity costs and the complementarity between technological goods (i.e., the intermediate goods), a task to carry out in Section 5, we need to consider convenient measures of the technology-structure variables.

The long-run technology structure of our model is characterised by the technological-knowledge bias, \( (Q_H/Q_L)^* \), the relative intermediate-good production \( (X_H/X_L)^* \), and the relative number of firms \( (A_H/A_L)^* \) (i.e., \( H \)-vis-à-vis \( L \)-technology sector). In equation (36), we show that the technological-knowledge bias is a function of the skill structure, which is characterised by the relative supply of skills, \( H/L \). The same can be proved as regards relative

\(^{10}\)In this sense, the effect of complementarities between technological goods in production is equivalent to increasing returns.
production and the relative number of firms. From the no-arbitrage condition (32) and taking into account equation (36), we derive the relative number of firms:

\[
\left( \frac{A_H}{A_L} \right)^* = \left( \frac{h}{l} \frac{H}{L} \right)^{D_0} = \left( \frac{W_H}{W_L} \right)^{D_0} \cdot \left( \frac{h}{l} \frac{H}{L} \right)^{D_0},
\]

where:

\[
D_0 \equiv \frac{1}{\beta_2 + 1}.
\]

From the expressions for \(X_L\) and \(X_H\) (obtained by aggregating (11)) and (36), we get relative production as:

\[
\left( \frac{X_H}{X_L} \right)^* = \left( \frac{h}{l} \frac{H}{L} \right)^{D_0} = \left( \frac{W_H}{W_L} \right)^{D_0} \cdot \left( \frac{h}{l} \frac{H}{L} \right)^{D_0}.\]

(50)

However, since we wish to confront our theoretical results with the data on production for a number of countries and the data is presented in a quality-adjusted base by the national statistics offices (see, e.g., Eurostat, 2001), we find it convenient to compute production also in quality-adjusted terms. Reiterating the steps as in Section 2.2, we find total intermediate-good quality-adjusted production to be:

\[
X_m = \int_0^A_m \int_0^H \lambda \chi_{m,jm} \cdot m njm = M \cdot \left( \frac{1}{\alpha} \cdot r \cdot q/\chi \right)^{\frac{1}{\alpha - 1}} Q_m \cdot \frac{Q_m}{Q_m}, m = L, H,
\]

where \(Q_m = \int_0^A_m \chi_{m,jm} \frac{1}{\gamma} djm\). We cannot find an explicit algebraic expression for the BGP value of \(Q_m\). Nevertheless, as shown in Gil et al. (2012b), we can build an adequate proxy for \(Q_m\), \(\hat{Q}_m = \frac{1}{\alpha} (\bar{Q}_m \cdot A_m)^{\frac{1}{\alpha - 1}}\). Accordingly, we define the proxy \(\hat{X}_m = X_m \cdot (Q_m/A_m)^{\frac{1}{\alpha - 1}}\) for \(X_m\). Thus, bearing in mind (36), (48) and (50), we consider the following quality-adjusted measure of relative production:

\[
\left( \frac{\hat{X}_H}{\hat{X}_L} \right)^* = \left( \frac{X_H}{X_L} \right)^* \cdot \left[ \left( \frac{Q_m}{Q_m^*} \right)^* \right] = \left( h \frac{H}{L} \right)^{D_1} = \left( \frac{W_H}{W_L} \right)^{D_1} \cdot \left( \frac{h}{l} \frac{H}{L} \right)^{D_1},
\]

where:

\[
D_1 \equiv 1 + \left( \frac{1 - \alpha}{\alpha} - \frac{\phi - \alpha}{\alpha (\beta_2 + 1)} \right).
\]

In the above equations, we used the fact that, on the BGP, the skill premium relates one-to-one to the absolute productivity advantage of high-over low-skilled labour \(h/l\) (see (38)), in order to replace the latter with \(W_H/W_L\). Thus, we see that, according to the model, two types of effects determine the cross-country behaviour of the technology structure depicted by Figure 2 in the Appendix: the market-size effect, through which a higher proportion of high-skilled workers is associated with technological change directed towards the high-tech sectors and the skill-premium effect, reflecting the absolute productivity advantage of high-skilled workers.

Anticipating the empirical results detailed in Section 5, we focus on the conditions under which: (i) the elasticities \(D_0\) and \(D_1\) are positive and (ii) \(D_0 < D_1\). As already shown in Section 2.3, given the constraints \(\phi > \alpha\) and \(\beta_2 > \varepsilon\), \(\varepsilon \equiv (\phi - 1)/(1 - \alpha)\), we have \(\beta_2 > -1\) and thus \(D_0 > 0\). On the other hand, the constraint \(\beta_2 > \varepsilon \iff \phi < \beta_2(1 - \alpha) + 1\) is sufficient for both \(D_1 > 0\) and \(D_0 < D_1\) to be satisfied (the former inequality is equivalent to \(\phi < \beta_2 + 1 + \alpha\) and the latter to \(\phi < \beta_2 + 1\)). Thus, when the degree of gross substitutability/complementarity, \(\phi\), is compatible with BGP economic growth both along an intensive and an extensive margin (see Section 3), it also implies a positive elasticity of both technology-structure variables – the relative number of firms and relative production – with respect to the skill structure and the skill premium, as observed in the data. Moreover, an elasticity of relative production
larger than that of the relative number of firms reflects the fact that the vertical-innovation mechanism ultimately commands the horizontal entry dynamics, meaning that a BGP with increasingly costly horizontal R&D occurs only because entrants expect the incumbency value to grow propelled by quality-enhancing R&D. This mechanism generates a roundabout effect that amplifies the impact of the market-size channel on relative (quality-adjusted) production, when compared to the impact on the relative number of firms.

5 Estimation and calibration

The degree of complementarity, horizontal R&D complexity cost, and the markup. We now turn to a quantitative assessment of the model and investigate how important our analytical mechanism may be in accounting for the cross-country pattern in the distribution of firms and production between high- and low-tech sectors. Although it should be clear that our mechanism does not account for all the variation in the technology structure across countries, we abstract from all other potential sources of variation. Thus, this exercise provides an upper bound on how much of the referred to cross-country differences can be explained by the variation in the skill structure. Then, given the estimated effect of the skill structure on the technology structure, we establish the implicit estimate of the horizontal R&D complexity-cost parameter, $\beta_2$, and thereby of the degree of complementarity between technological goods, $\phi$.

In particular, we are interested in checking whether the case of $\phi > 1$, which features as a key assumption in the theoretical literature that studies poverty traps, growth and business cycles based on the mechanism of gross complementarity in technological goods (e.g., Matsuyama, 1995; Ciccone and Matsuyama, 1996; Evans et al., 1998), has empirical support, by using recent cross-country data on the technology structure (i.e., relative production and the relative number of firms) and the skill structure (i.e., the relative supply of skills). Recalling, from Section 2, that the only assumption on $\phi$ imposed by our theoretical model is $\phi > \alpha$, with $0 < \alpha < 1$, such that either gross substitutability ($\phi < 1$) or gross complementarity ($\phi > 1$) may arise a priori.

We adopt the following strategy. Firstly, we consider equations (48) and (51) to get:

$$
\ln A_H/A_L = D_0 \ln (H/L) + D_0 \ln (W_H/W_L) \quad (53)
$$
$$
\ln \hat{X}_H/\hat{X}_L = D_1 \ln (H/L) + D_1 \ln (W_H/W_L) \quad (54)
$$

Then, based on (53) and (54), we run the regressions:

$$
\ln A_H/A_L = S_0 + D_{01} \ln (H/L) + D_{02} \ln (W_H/W_L) + e_0 \quad (55)
$$
$$
\ln \hat{X}_H/\hat{X}_L = S_1 + D_{11} \ln (H/L) + D_{12} \ln (W_H/W_L) + e_1 \quad (56)
$$

where $e_i, i = 0, 1$, are the error terms, to get the estimates $\hat{D}_{01}, \hat{D}_{02}, \hat{D}_{11}$, and $\hat{D}_{12}$. Table 1 reports the OLS estimates of the coefficients in regressions (55) and (56), which are run on the cross-section data in the Appendix. Bearing in mind that, in light of the theoretical model, $D_{01} = D_{02} = D_0$ and $D_{11} = D_{12} = D_1$, we overlap the confidence intervals for $D_{11}$ and $D_{12}, i = 0, 1$, in order to get a single confidence interval for the elasticities $D_t$. This is depicted by Table 2.

---

11 See the Appendix for more details on the data.
12 The log-log relationships arising from our BGP results for the relative number of firms and for relative production (equations (53) and (54)) have no intercept, in contrast to the regression equations (55) and (56). We can easily modify our model in order to have a non-zero intercept in (53) and (54), by considering heterogeneous flow fixed costs of R&D across L- and H-technology sectors in (22) and (28), such that $\zeta_H \neq \zeta_L$ and $\beta_{1H} \neq \beta_{1L}$.
As $D_0$ is a function of $\beta_2$ and $D_1$ is a function of $(\alpha, \phi, \beta_2)$, we let $\alpha = 0.4$, as usual in the literature, and then use the confidence intervals for $D_0$ and $D_1$ to get the implicit confidence intervals for $(\phi, \beta_2)$. Figure 1 juxtaposes the confidence intervals for $\phi$ and $\beta_2$, together with the subset of values in space $(\phi, \beta_2)$ that results from the constraint $\beta_2 > \varepsilon$, $\varepsilon \equiv (\phi - 1)/(1 - \alpha)$. Our calculations show values for $\beta_2$ between 0.22 and 1.30 and for $\phi$ between 1.08 and 1.78.\textsuperscript{13} The fact that we get a confidence interval with positive values for $\beta_2$ indicates that the elasticity regulating the horizontal-R&D complexity costs is larger than the one in the vertical-R&D case (i.e., $\beta_2 + 1 > 1$), in line with what should be expected bearing in mind the distinct nature of the two types of R&D. On the other hand, since our estimates of $\phi$ are larger than unity, our identification and estimation exercise suggests that there is gross complementarity between technological goods in production, thus supporting the key complementarity assumption appearing in the theoretical literature mentioned earlier. However, it is also noteworthy that, according to our exercise, calibrated values of $\phi$ as high as 4, as found in the literature (e.g., Evans et al., 1998; Thompson, 2008), seem to lack empirical support – Figure 1 makes clear that this observation is true irrespective of the constraint $\beta_2 > \varepsilon \iff \phi < \beta_2 \cdot (1 - \alpha) + 1$.

\textsuperscript{13}Notice that, even though the goodness of fit of the regressions in Table 1 might most likely increase if we added explanatory variables, the approach followed herein reflects the fact that the log-log linear relationships between the technology-structure variables, the skill premium and the relative supply of skills have an exact analytical counterpart in terms of the BGP equilibrium of the model, as already shown. We take advantage of this fact to pursue an identification and estimation strategy for the key technology parameters using the empirical results on the technology-structure variables presented in Table 1.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>ln Relative number of firms</th>
<th>ln Relative production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$-1.890$</td>
<td>$-0.908$</td>
</tr>
<tr>
<td>(s.e.): (p-value)</td>
<td>(0.489); (0.001)</td>
<td>(0.442); (0.061)</td>
</tr>
<tr>
<td>ln Relative supply of skills</td>
<td>$0.506$</td>
<td>$0.832$</td>
</tr>
<tr>
<td>(s.e.): (p-value)</td>
<td>(0.312); (0.120)</td>
<td>(0.280); (0.011)</td>
</tr>
<tr>
<td>ln Skill premium</td>
<td>$0.766$</td>
<td>$1.717$</td>
</tr>
<tr>
<td>(s.e.): (p-value)</td>
<td>(0.332); (0.032)</td>
<td>(0.842); (0.062)</td>
</tr>
<tr>
<td>Observations</td>
<td>23</td>
<td>16</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.144</td>
<td>0.291</td>
</tr>
</tbody>
</table>

Table 1: OLS regressions of the technology-structure variables (relative production and the relative number of firms) on the relative supply of skills and the skill premium, in logs, run on cross-country data. Standard errors (s.e.) are heteroskedasticity consistent.

<table>
<thead>
<tr>
<th></th>
<th>$i = 0$</th>
<th>$i = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence interval for $D_{i1}$</td>
<td>$0.506 \pm 0.312$</td>
<td>$0.832 \pm 0.280$</td>
</tr>
<tr>
<td>Confidence interval for $D_{i2}$</td>
<td>$0.766 \pm 0.332$</td>
<td>$1.717 \pm 0.842$</td>
</tr>
<tr>
<td>Confidence interval (juxtaposed) for $D_i$</td>
<td>$[0.434; 0.818]$</td>
<td>$[0.875; 1.112]$</td>
</tr>
</tbody>
</table>

Table 2: Two-standard-error confidence intervals for the slope coefficients in the regression equations (55) and (56), based on the results in Table 1.
Figure 1: Confidence intervals for $\phi$ and $\beta_2$ implicit in the confidence intervals for the elasticities $D_i$, $i = 0, 1$ in Table 2 (dashed lines). The bold line depicts the boundary in space $(\phi, \beta_2)$ imposed by the constraint $\beta_2 > \varepsilon$, $\varepsilon \equiv (\phi - 1) / (1 - \alpha)$. $\alpha = 0.4$.

Finally, something may be said about our estimates of the markup, since, recalling (9) and that $\gamma = \alpha / \phi$ is imposed to ensure constant returns to scale, we are able to compute $\gamma$ at the expense of $\phi$. Thus, if we consider the drastic-innovation case in our model, such that the markup is $1 / \chi = 1 / \gamma$, the implicit confidence interval for the markup is [2.70; 4.45]. Confronting with the cross-section studies that are known to have robust direct estimates for the markup, we find that our (indirect) estimates are too high (e.g., Nurrbin, 1993, and Basu and Fernald, 2002, indicate values within the interval [1.016; 1.837]). However, since those studies do not distinguish between drastic and non-drastic innovations, one may conjecture that the gap between the interval we computed, [2.70; 4.45], and the one found by the empirical literature, [1.016; 1.837], is a consequence of the quite small share of drastic innovations in the total number of innovations.\footnote{If we take the mid-point of those two intervals and also consider that the average markup for non-drastic innovations varies between say 1 and 1.3, then the share of drastic innovations will vary between 4.82 and 15.53 percent.}

Alternatively, if we consider the case of non-drastic innovations in our model, then the markup is given by $1 / \chi = \lambda < 1 / \gamma$, where now $1 / \gamma \in [2.70; 4.45]$ defines the interval of critical values for the size of the quality jump, $\lambda$, that is implicit in our cross-country data.

**Sensitivity of Tobin-q to technology parameters** We are also interested in the long-run relationship between the Tobin-q and both the degree of complementarity between technology goods, $\phi$, and the complexity effect pertaining to horizontal R&D, $\beta_2$, through the impact of the last two factors on the long-run economic growth rate. We focus on the extreme values of the confidence intervals for $\phi$ and $\beta_2$ depicted by Figure 1, such that four scenarios are analysed corresponding to four different pairs $(\phi, \beta_2)$. Table 3 summarises the results.

To obtain quantitative results, we calibrate the model with the following values for the remaining parameters and exogenous values: $\rho = 0.02$, $\sigma = 1.5$, $\alpha = 0.4$, $\theta = 1$, $\beta_1 = 1$, $\beta_3 = 0$.1
Table 3: Calibration results. $\rho = 0.02, \sigma = 1.5, \alpha = 0.4, \theta = 1, \beta_1 = 1, \zeta = 2.8, \lambda = 1.4, h/l = 1.7$ and $H/L = 0.18$.

$\zeta = 2.8, \lambda = 1.4, h/l = 1.7$ and $H/L = 0.18$. The values for $\rho$, $\sigma$ and $\alpha$ are set in line with the standard literature (see, e.g., Barro and Sala-i-Martin, 2004), while the values for $H/L$ and $h/l$ correspond to the average value of, respectively, the relative supply of skills and the skill premium in our cross-country data (see the Appendix). To reconcile the empirical literature on the markup with our estimates of the degree of complementarity, we consider the case of non-drastic innovations, the one that seems to be more common in practice, such that the markup is given by $1/\chi = \lambda < 1/\gamma$; then, bearing in mind the average estimate of the markup in Nurrbin (1993) and Basu and Fernald (2002), we let $\lambda = 1.4$. Finally, the values of the parameters $\theta$, $\beta_1$ and $\zeta$ are chosen in order to calibrate the BGP aggregate growth rate, $g^*$, around 3.4 percent/year on average across scenarios (see Table 3), which corresponds to the average growth rate of the cross-country sample in the Appendix. Then, the implied average value for the real interest rate, $r^*$, is 7.2 percent, in line with the empirical value for the long-run average real return on the stock market, and which should be taken as the equilibrium rate of return to R&D, as argued by Jones and Williams (2000).

By using equations (44)-(46) and (8), our calibration results show that the elasticity of the Tobin-q with respect to $q^*$, $E_{q^*}$, takes values between 1.64 and 3.34, and with respect to $\lambda$, $E_{\lambda}$, takes values between $-0.28$ and $-1.72$, as depicted by Table 3. It is noteworthy that $E_{q^*}$ is always greater than unity, meaning that a shift in the degree of gross complementarity between technological goods, $\phi$, has a more than proportional effect on the long-run market value of capital (i.e., there is an elastic relationship). In turn, a change in the complexity effect pertaining to horizontal R&D, $\beta_2$, which may also be interpreted as a measure of the barriers to entry through R&D activities, also induces a more than proportional (negative) effect on the long-run market value of capital, but only if the complementarity degree is large enough.

A strong positive relationship between the Tobin-q and the R&D and technology characteristics of the firms has been frequently reported by the empirical literature (e.g., Chan et al., 1990; Hall, 1999; Connolly and Hirschey, 2005). This result is apparently supported by our estimation and calibration exercise, which, however, also suggests that it is important to distinguish between the technological determinants of the long-run market value of capital and explicitly analyse their interaction.

6 Concluding remarks

This paper builds an endogenous growth model of directed technical change with simultaneous vertical and horizontal R&D, internal costly investment and gross substitutability/complementarity between technological goods, with a view to identify and quantify the
long-run link between: (i) the technology structure and the skill structure, by considering an explicit role for the degree of complementarity between technological goods; (ii) the Tobin-q and the technology characteristics of the firms through the impact of the latter on the long-run economic growth rate.

By solving the model for the BGP equilibrium, we show that it entails an analytical mechanism that is consistent with the observed cross-country pattern in the technology structure. Then, based on the analytical BGP relationships derived in the model and on the cross-country data for the technology structure and the skill structure, we compute indirect estimates of the value of the parameters that regulate the degree of gross substitutability/complementarity and the complexity costs pertaining to horizontal R&D. In particular, our identification and estimation exercise suggests that there is gross complementarity between technological goods, thus supporting the key complementarity assumption appearing in the theoretical literature. Furthermore, a strong relationship between the Tobin-q and the R&D and technology characteristics of the firms is obtained, in line with the related empirical literature.

The study of the transitional dynamics should be pursued in future work. The qualitative characterisation of the local dynamics properties might allow one to find to what extent variations in a country’s initial conditions lead to non-monotonic time paths of the technology structure and the economic growth rate towards the BGP in face of a shock in the skill structure, as observed in the 80s and 90s in a number of developed countries. Moreover, this will allow us to accommodate the fact that the time-span of the available data is relatively short and hence may entail transitional dynamics effects.

References


Appendix

In this appendix, we present the cross-country data with respect to the technology structure, measured by the number of firms and by production in high- vs. low-tech manufacturing sectors, by considering the OECD high-tech low-tech classification (see Hatzichronoglou, 1997). We will call these ratios the relative number of firms and relative production, respectively. We also collected data on the skill structure, i.e., the ratio of high- to low-skilled workers or the relative supply of skills, measured as the ratio of college to non-college graduates among persons employed in manufacturing, and the skill premium, measured as the mean annual earnings of the college graduates employed in manufacturing vs. the mean annual earnings of the non-college graduates. “College graduates” refers to those who have completed tertiary education (corresponding to the International Standard Classification of Education [ISCED] levels 5 and 6), while “non-college graduates” refers to those who have completed higher-secondary education or less (ISCED levels from 0 to 4).

The data concerns the 1995-2007 average and covers 25, 16 and 29 European countries regarding, respectively, the number of firms, production, and the supply of skills (educational attainment), whereas the data for the mean annual earnings respects to 2002 and covers 28 European countries. The source is the Eurostat on-line database on Science, Technology and Innovation – tables “Economic statistics on high-tech industries and knowledge-intensive services at the national level” and “Annual data on employment in technology and knowledge-intensive sectors at the national level, by level of education” (available at http://epp.eurostat.ec.europa.eu). At the aggregate level, we gathered data on the per capita real GDP growth rates for the same period, also from the same Eurostat on-line database.

Figure 2 depicts the data by country. We notice that, in all countries in the sample: (i) the relative number of firms and relative production are below unity; (ii) the relative supply of skills is below unity; and (iii) the skill premium is above unity.

[Figure Appendix goes about here]

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15 High-tech industries are, e.g., aerospace, computers and office machinery, electronics and communications, and pharmaceuticals, while the low-tech sector comprises, e.g., petroleum refining, ferrous metals, paper and printing, textiles and clothing, wood and furniture, and food and beverages.

16 According to our theoretical model, we should restrict our analysis to the production of intermediate and capital goods. However, we were not able to find data according to the OECD classification of high- and low-tech sectors detailed by type of good and thus focused on total production in each sector.

17 Data with respect to output (Gross Value Added) is also available from the on-line OECD STAN Indicators Database (link at http://stats.oecd.org). According to the latter, the patterns depicted by Figure 2 are also verified for the non-European countries belonging to the OECD, namely the US, Canada, Mexico, Australia, Korea and Japan.
Figure 2: The technology-structure variables (the relative number of firms and relative production), the relative supply of skills and the skill premium, in a sample of European countries, 1995-2007 average (except for the skill premium, whose data is from 2002).
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