Theory of Semi-Collusion in the Labor Market

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Abstract

We study the effects of cooperative wage setting in industries that use two different types of labor. In particular, we consider a two-stage game where firms hire non-specialized workers in a perfectly competitive labor market and specialized workers that are more productive and expensive, but whose wages can be cooperatively determined by firms. It is shown that semi-collusion leads to lower wages and employment of specialized labor, lower production levels and higher prices, due to the elimination of the business stealing effect, labor force stealing effect and as a result of a dynamic effect that is specific to semi-collusive games.

Key Words: Semi-collusion, labor market, oligopsony, business stealing effect, labor force stealing effect, price war effect, shooting the moon strategy.

JEL Classification: L11, L13, L41, L44, J01, J08

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1. Introduction

There is nowadays little disagreement about the role of free competition in the efficiency of the markets and social welfare. In the current free market system, a high degree of competition not only guarantees the supply of a great number and variety of goods at low prices, but also the dispersion of economic power among all individuals of society. Indeed, in very competitive markets each individual is able to earn a salary according to his skills and productivity, as well as a “fair” rent for the capital he was able to save along his life. Unfortunately, the ideal concept of perfect competition so often discussed in economic theory is not always present in real industries. Sometimes, when countries are not sufficiently opened to international trade and transport costs are high, markets are simply not large enough to promote competition. Other times, due to large economies of scale or network effects some goods can only be efficiently produced by one firm. That is the case of the natural monopoly. But perhaps most often, the free market system is jeopardized by the cooperative action of some individuals who conspire against society to reduce the level of competition and to get a monopoly rent. This type of behavior is usually referred to as collusion or cartel.

Since the creation of a Theory of Games by von Neumann (1943) and the development of important contributions to collusion theory by Reinhard Selten (1973), James Friedman (1971) and Dilip Abreu (1984), economists have dedicated many research work to the extensive analysis of collusion in the final good market, usually in the form of cooperative price setting, centralized determination of quotas or division of markets. In the meantime, after the publication of the Sherman Act in USA and similar anti-trust laws in other countries, competition authorities have used considerable resources to bring down such type of cartels.

Notwithstanding, economic researchers have rarely focused until now on alternative forms of collusion, as cooperative actions of firms to reduce wages or agreements not to steal employees from each other, which we define as collusion in the labor market. The few works published on the subject include a paper by Mukherjee, Selvaggi & Vasconcelos (2012) about the impact of exclusive contracts on the incentive of firms to cooperatively fix wages of highly productive workers and another paper by Shelkova

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In this work we only analyze collusion in the demand side of the labor market. We do not address collusion in the supply side in the form of labor unions, since this is a topic already much discussed in the literature.
Collusion in the labor market does not seem to be a major concern of competition authorities either. Almost all cartels uncovered so far have been charged of fixing prices or undertaking any other form of collusive deals in the final good market, while cartels who fix wages are hardly ever investigated, probably due to the enormous concern of competition authorities with the welfare of the consumer relatively to the welfare of the worker. We believe, however, that competition authorities should also be responsible for the prosecution of cartels in labor markets, for at least two important reasons. Firstly, in the absence of any other regulatory authority in charge of preserving competition between employers, the protection of the welfare of the worker is an important mean to achieve justice and efficiency, particularly in modern economies where the large part of the population works for somebody else. Secondly, as we will show later, collusion in the labor market leads to collusion in the final good market and so it has severe impacts on consumer’s welfare as well.

The impact of collusion in the labor market on workers and consumers’ welfare was already studied in a companion paper, *Theory of Collusion in the Labor Market* (Gonzaga, Brandão e Vasconcelos 2013), where it was established that cooperative agreements between firms cause wages, employment levels and quantities transacted to fall and prices of the final good to rise. Yet those results were obtained under the assumption that firms are able to cooperatively determine the price of all inputs. In opposition, in this paper we consider a semi-collusive model where firms are only able to undertake cooperative agreements about the price of some inputs used in their production functions.

The economic concept of “semi-collusion” is not new in economic literature and appears to have been used for the first time by Fershtman and Muller (1986), who

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5 Indeed, in United States nine out of ten working individuals are paid employees, whereas only one is self employed (see Hipple, 2010).
defined semi-collusive markets as those “where rivals compete in one variable (or set of variables) and collude in another”. Nonetheless there are some earlier contributions which, despite not using explicitly the term “semi-collusion”, end up discussing the same subject.

According to Steen and Sørgard (2009), typical models of semi-collusion are composed of a two-stage game, where in the first stage firms set the non-price variables (which are usually more rigid) and in the second stage they choose prices. Then there are two groups of models, those where firms collude on prices and compete on non-price variables and those where the opposite occurs.

In the first group of models, collusion on prices typically leads to tougher competition in the non-price variables and injures both consumers and firms. Some of the earlier examples include Bloch (1932) and Lorange (1973), who respectively describe a German coal industry cartel in the 20s and a cartel of Norwegian cement producers in the 60s, whose members cooperatively fixed prices and total production levels, but competed on productive capacity. Because the market share of every producer was defined as a function of their productive capacity, collusion led to an inefficient over-investment by all firms. The members of price cartels may also compete in other variables, as advertising (Eckard, 1991) and research and development (Brod and Shivakumar, 1999).

In the second group of models, where firms compete on prices and collude on other variables, semi-collusion always improves profits, but may benefit or hurt consumers depending on the particular characteristics of the game. Some examples include semi-collusion on non-price variables as quality (Deltas and Serfes, 2002), advertising (Simbanegavi, 2009) and research and development (d’Aspremont and Jacquemin, 1988). However, in the words of Steen and Sørgard (2009):

“(...) except for collusion on R&D there are few examples in the literature on collusion on non-price variables. As far as we know, there are only a few studies of collusion on advertising and a study of investment in infrastructure in telecom.”

Our work belongs to this second group of models and studies collusion on wages (which can also be seen as collusion on the productive capacity), contributing to cover what, to our knowledge, constitutes a gap in the extant literature.
To understand the empirical relevance of the semi-collusion hypothesis, we briefly address in what follows two examples of cooperative agreements in the labor market that we discussed in more detail in our previous paper. In 1997 fifteen oil companies were sued for exchanging detailed salary information and discussing the budgets for wages paid to managerial, professional and technical employees. And in 2011 several high technology companies like Google, Apple, Adobe, Intel, Intuit and Pixar were accused of undertaking no-solicitation agreements (also known as no-poaching agreements) against their technical engineers. Although in both cases firms succeeded in suppressing the wages paid to highly productive workers with specific technical skills, they were not able to affect the wages of the many non-specialized workers they employ to perform routine activities and less demanding tasks, who are usually hired in larger and more competitive labor markets. Once the two types of workers have some degree of substitutability and can be used in different combinations to achieve the same final output, this clearly suggests that, in many cases, firms are only able to collude about the price of some inputs.

Interestingly, we find in this paper that the effects of semi-collusion in the labor market do not differ too much from those obtained when firms fix the price of all inputs. Indeed, we show not only that semi-collusion causes the wages and employment of specialized workers to fall, but it also indirectly leads to collusion in the final good market, by creating an incentive for firms to reduce production and to increase prices. The different levels of the economic variables under semi-collusion and competition are the result of the elimination of the business stealing effect and labor force stealing effect, as well as a dynamic effect that is specific to semi-collusive games.

The remainder of the paper is organized as follows. In section 2 we present the general formal model consisting in a two-stage game with price and wage competition, which will be used to analyze and compare the non-cooperative equilibrium with the collusive equilibrium. In the first stage firms either decide cooperatively or individually the wages paid to specialized workers and in the second stage firms set simultaneously (without neither cooperate nor communicate) the prices of the final good and the number of non-specialized employees hired. Because the model is solved by backward induction, we begin by solving the second stage in section 3, next we provide the competitive solution for the first stage in section 4 and we describe the collusive solution for the first stage in section 5. In section 6 we compare the results obtained in
the two previous sections to identify the impact of semi-collusion on wages, employment, prices and quantities transacted. In section 7 we discuss how results would change if, in the competitive scenario, the control variables were all set simultaneously. Finally, section 8 concludes.

2. The model

Consider an industry composed by \( n \) firms producing close substitute goods with technologies that combine two types of labor: highly qualified workers \( (L_{1i}) \), paid at the wage rate \( W_i \), and non-specialized workers \( (L_{2i}) \), hired in a perfectly competitive labor market at the wage exogenously fixed at \( \bar{W} \). The production function \( h_i(L_{1i}, L_{2i}) \) is concave and increases at decreasing rates with respect to any input. However any increase in one of the inputs raises the marginal productivity of the other. In mathematical notation:

\[
\frac{\partial h_i(L_{1i}, L_{2i})}{\partial L_{1i}} > 0, \quad \frac{\partial h_i(L_{1i}, L_{2i})}{\partial L_{2i}} > 0, \\
\frac{\partial^2 h_i(L_{1i}, L_{2i})}{\partial L_{1i}^2} \leq 0, \quad \frac{\partial^2 h_i(L_{1i}, L_{2i})}{\partial L_{2i}^2} \leq 0, \quad \frac{\partial^2 h_i(L_{1i}, L_{2i})}{\partial L_{1i} \partial L_{2i}} \geq 0.
\]

Because the final goods are not perfect substitutes, any firm \( i \) of the industry faces a continuous demand function \( Q_i = f_i(P_1, ..., P_n) \) decreasing with respect to its own price and increasing with respect to the prices of the other firms. Similarly, we consider the job posts to be differentiated and hence each firm also faces a continuous specialized labor supply function \( L_{1i} = g_i(W_1, ..., W_n) \) that rises with \( W_i \) and falls with \( W_j \):

\[
\frac{\partial f_i(P_1, ..., P_n)}{\partial P_i} < 0, \quad \frac{\partial g_i(W_1, ..., W_n)}{\partial W_i} > 0, \quad \forall \ i = 1, ..., n \\
\frac{\partial f_i(P_1, ..., P_n)}{\partial P_j} > 0, \quad \frac{\partial g_i(W_1, ..., W_n)}{\partial W_j} < 0, \quad \forall \ j \neq i.
\]

The market demand of the final good, \( Q = F(P_1, ..., P_n) \), corresponds to the sum of firms’ individual demands and it is negatively correlated with any \( P_i \). In turn, the market supply of specialized labor, \( L_1 = G(W_1, ..., W_n) \), is the sum of firms’ labor supplies and is positively correlated with any \( W_i \).
Given their production technologies and the information available about market demand and labor supply, the firms of the industry repeatedly play a two-stage game in an infinite time horizon, which we will now describe.

In the first stage of the game, all firms set the wage of specialized workers and hire the specialized labor force under two possible equilibrium behaviors. At the non-cooperative equilibrium firms compete with each other and set the wage that optimizes their individual profits, taking the decisions of the other players as given. If, on the other hand, they are able to coordinate their strategies (labor market collusive equilibrium), firms cooperatively set the wages of specialized workers that maximize joint profits. In the latter case, as long as every firm earns a share of the collusion gains and the discount rate is sufficiently close to one, the collusive equilibrium can be sustained with trigger strategies (Friedman 1971) or any similar strategies.

In the second stage, firms hire any amount of non-specialized labor they wish at the wage exogenously fixed, set the price of the good produced with the two types of labor and sell it to the final consumer. Naturally they are always subject to the constraint that total sales cannot overcome the total production level. At this stage firms are not able to collude in any dimension.6

It is important to briefly discuss why was the model set up in this particular sequence of events, that is, why have we assumed that the wage of specialized workers is determined before the remaining control variables. As it is commonly considered in the economic literature, wages are a relatively rigid variable that cannot be changed very often, especially in the case of high-skilled workers who are the hardest to attract and contract, while prices and employment of non-skilled labor can be more easily adapted to the short run. And so it seems reasonable to consider that once the wages are determined in the industry they cannot be changed again until the next period, while firms can still freely modify prices and employment of non-skilled workers. But if we think about the specific case of the collusive equilibrium, it is easy to understand that this particular

\[
\frac{\partial F(P_1, ..., P_n)}{\partial P_i} < 0, \quad \frac{\partial G(W_1, ..., W_n)}{\partial W_i} > 0. 
\]

6 The inability of firms to collude in the second stage may result from the absence of market power in the non-specialized labor market and from the actions of competition authority to prevent cooperative price-fixing. Alternatively we can assume that firms are not able to coordinate all decision variables due to asymmetry of information or that they simply prefer to compete in some dimensions, so that they can easily react to exogenous idiosyncratic shocks.
sequence of events is, in fact, the only possible solution. Indeed, once the wages are centrally determined by the cartel, they cannot be changed anymore, but firms are still able to take advantage of any unilateral profitable deviations by changing their prices and amounts of non-specialized labor to improve their individual profits. Nevertheless, we cannot absolutely reject the hypothesis that all the decision variables can be set simultaneously in the non-cooperative equilibrium. In section 7 we discuss how the results would be affected in such case.

Next we solve the standard two-stage model following the usual backward induction procedure and thereby we start by determining the equilibrium at the second stage.

3. Second stage

At the second stage of the game the rational firm chooses the price of the final good and the amount of non specialized labor that optimize its profits, holding the decisions of the other firms fixed. The firm is also constrained by its production technology, as it must acquire the necessary inputs to produce any output sold. At this point the wages of specialized workers can be treated as mere parameters of the model, because they have already been determined in the previous stage and cannot be changed anymore. Then the optimizing problem can be formally expressed as:

\[ \text{Max}_{P_i, L_{2i}} \text{Profit}_i = P_i f_i(P_1, \ldots, P_n) - W_i g_i(W_1, \ldots, W_n) - \bar{W} L_{2i} \]

\[ \text{s.t.} \quad f_i(P_1, \ldots, P_n) = h_i(g_i(W_1, \ldots, W_n), L_{2i}) \]

To solve this problem we set up the Lagrangian function and introduce the Lagrange multiplier \( \lambda_i \), which can be interpreted as the shadow price of the final good.

\[ L_{\text{Firm}} = P_i f_i(P_1, \ldots, P_n) - W_i g_i(W_1, \ldots, W_n) - \bar{W} L_{2i} + \lambda_i [h_i(g_i(W_1, W_{-i}), L_{2i}) - f_i(P_i, P_{-i})] \]

As long as the sufficient conditions hold \( \left( \frac{\partial^2 f_i(.)}{\partial P_i^2} \leq 0 \text{ and } \frac{\partial h_i(.)}{\partial L_{2i}^2} \leq 0 \right) \), the optimization problem is well defined and has an interior solution, which can be found using the following first order conditions:

First order condition with respect to \( P_i \):

\[ \frac{\partial L_{\text{Firm}}}{\partial P_i} = 0 \iff f_i(.) + P_i \frac{\partial f_i(.)}{\partial P_i} - \lambda_i \frac{\partial f_i(.)}{\partial P_i} = 0 \iff f_i(.) + (P_i - \lambda_i) \frac{\partial f_i(.)}{\partial P_i} = 0. \quad (1) \]
First order conditions with respect to $L_{2i}$:
\[
\frac{\partial L_{Firm}}{\partial L_{2i}} = 0 \iff -\bar{W} + \lambda_i \frac{\partial h_i(.)}{\partial L_{2i}} = 0 \iff \lambda_i = \frac{\bar{W}}{\partial h_i}
\] (2)

First order conditions with respect to $\lambda_i$:
\[
\frac{\partial L_{Firm}}{\partial \lambda_i} = 0 \iff f_i(.) = h_i(g_i(.), L_{2i})
\] (3)

Because our variables of interest are $L_{2i}$ and $P_i$, equation (2) can be used to eliminate the shadow price from equation (1), so that first order conditions are rewritten as:
\[
f_i(P_1, ..., P_n) = h_i(g_i(W_1, ..., W_n), L_{2i})
\] (4)
\[
f_i(P_1, ..., P_n) + \left(P_i - \bar{W} \frac{\partial L_{2i}}{\partial h_i}\right) \frac{\partial f_i(P_1, ..., P_n)}{\partial P_i} = 0
\] (5)

Equation (4) is simply the production technology constraint. Equation (5) states that under optimal behavior the marginal impact on profits of a slight price change must be null. In other words, when an optimizing firm increases the price in one unit, the extra unit of money it earns for each unit of product sold must exactly offset the loss of sales resulting from the fall in demand.

Applying equations (4) and (5) to every firm gives the equilibrium prices and employment levels of non-specialized labor as a function of the wages of specialized workers, $P_i(W_1, ..., W_n)$ and $L_{2i}(W_1, ..., W_n)$.

4. Non-cooperative outcome in the first stage of the game

Now that we have predicted how the firms of the industry will behave in the second stage of the game for given wage levels, we have the necessary information to determine the solution in the first stage, which will crucially depend on whether the firms act non-cooperatively or successfully coordinate strategies. We begin by studying the non-cooperative solution.

As usual, in the non-cooperative equilibrium any firm of the industry chooses the wage level that optimizes its profits holding fixed the decisions of the other players. But since a rational firm is able to predict the behavior of the industry in the second stage, it will not only measure the direct impact of the wage on profits through total costs and...
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production capacity, but it will also account for the repercussions of the wage on future decisions about prices and employment levels of non-specialized workers, which affect profits as well. The maximizing problem of the firm can be therefore formally represented as:

\[
\max_{W_i} \text{Profit}_i = P_i(W_1, ..., W_n)f_i(P_i(W_1, ..., W_n), ..., P_n(W_1, ..., W_n)) - W_i g_i(W_1, ..., W_n) - \bar{W} L_{2i}(W_1, ..., W_n)
\]

s.t. \(f_i(P_i(W_1, ..., W_n), ..., P_n(W_1, ..., W_n)) = h_i(g_i(W_1, ..., W_n), L_{2i}(W_1, ..., W_n))\).

As we can see, the prices and employment levels of non-specialized workers appear in the expression as a function of wages. To solve this problem we set up again the Lagrange function, whose Lagrange multiplier is also a function of wages:

\[
L_i = P_i(W_1, ..., W_n)f_i(P_i(W_1, ..., W_n), ..., P_n(W_1, ..., W_n)) - W_i g_i(W_1, ..., W_n) - \bar{W} L_{2i}(W_1, ..., W_n) +
\]

\[+ \lambda_i(W_1, ..., W_n) [h_i (g_i(W_1, ..., W_n), L_{2i}(W_1, ..., W_n)) - f_i(P_i(W_1, ..., W_n), ..., P_n(W_1, ..., W_n))].\]

Given the assumptions we have made about the product demand, labor supply and production functions, the Lagrangian is concave and has an interior optimal solution at the point where its derivative with respect to the wage is equal to zero:

\[
\frac{\partial L_i}{\partial W_i} = \frac{\partial P_i(\cdot)}{\partial W_i} f_i(\cdot) + P_i(\cdot) \sum_{k=1}^{n} \left\{ \frac{\partial f_i(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} \right\} -
\]

\[-g_i(\cdot) - W_i \frac{\partial g_i(\cdot)}{\partial W_i} - \bar{W} \frac{\partial L_{2i}(\cdot)}{\partial W_i} + \frac{\partial \lambda_i(\cdot)}{\partial W_i} [h_i(\cdot) - f_i(\cdot)] +
\]

\[+ \lambda_i(\cdot) \left[ \frac{\partial h_i(\cdot)}{\partial g_i} \frac{\partial g_i(\cdot)}{\partial W_i} + \frac{\partial h_i(\cdot)}{\partial L_{2i}} \frac{\partial L_{2i}(\cdot)}{\partial W_i} - \sum_{k=1}^{n} \left\{ \frac{\partial f_i(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} \right\} \right] = 0.
\]

Splitting and rearranging the components of the last term allows us to rewrite the first order condition as:

\[
\frac{\partial P_i(\cdot)}{\partial W_i} f_i(\cdot) + (P_i(\cdot) - \lambda_i(\cdot)) \sum_{k=1}^{n} \left\{ \frac{\partial f_i(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} \right\} -
\]
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\[-g_i(.) + \left( \lambda_i(.) \frac{\partial h_i(.)}{\partial g_i} - W_i \right) \frac{\partial g_i(.)}{\partial W_i} + \left( \lambda_i(.) \frac{\partial h_i(.)}{\partial L_{2i}} - \bar{W} \right) \frac{\partial L_{2i}(.)}{\partial W_i} + \]
\[+ \frac{\partial \lambda_i(.)}{\partial W_i} [h_i(.) - f_i(.)] = 0.\]

The last expression can be further simplified using the first order conditions previously obtained for \(P_i, L_{2i}\), and \(\lambda_i(.)\). From equation (4) we know that the production level must meet the quantity demanded for the final good, that is, \(h_i(.) = f_i(.)\). Equation (2) can be used to express the shadow price as the cost of producing one unit of the final good using non-specialized labor, \(\lambda_i(.) = \bar{W} \frac{\partial L_{2i}(.)}{\partial h_i(.)}\). Replacing these two results in the previous equation:

\[
\frac{\partial P_i(.)}{\partial W_i} f_i(.) + \left( P_i(.) - \bar{W} \frac{\partial L_{2i}(.)}{\partial h_i} \right) \sum_{k=1}^{n} \left\{ \frac{\partial f_i(.)}{\partial P_k} \frac{\partial P_k(.)}{\partial W_i} \right\} - \]
\[\quad -g_i(.) + \left( \bar{W} \frac{\partial L_{2i}(.)}{\partial h_i} \frac{\partial h_i(.)}{\partial g_i} - W_i \right) \frac{\partial g_i(.)}{\partial W_i} = 0.\]

Breaking the terms inside the sum gives:

\[
\frac{\partial P_i(.)}{\partial W_i} \left[ f_i(.) + \left( P_i(.) - \bar{W} \frac{\partial L_{2i}(.)}{\partial h_i} \right) \frac{\partial f_i(.)}{\partial P_i} \right] + \]
\[+ \left( P_i(.) - \bar{W} \frac{\partial L_{2i}(.)}{\partial h_i} \right) \sum_{k \neq i}^{n-1} \left\{ \frac{\partial f_i(.)}{\partial P_k} \frac{\partial P_k(.)}{\partial W_i} \right\} - \]
\[\quad -g_i(.) + \left( \bar{W} \frac{\partial L_{2i}(.)}{\partial h_i} \frac{\partial h_i(.)}{\partial g_i} - W_i \right) \frac{\partial g_i(.)}{\partial W_i} = 0.\]

Finally, equation (5) can be used to replace \(f_i(.) + \left( P_i(.) - \bar{W} \frac{\partial L_{2i}(.)}{\partial h_i} \right) \frac{\partial f_i(.)}{\partial P_i}\) by zero, allowing us to rewrite the first order condition with respect to the specialized labor wage as:

\[
-g_i(.) + \left( \bar{W} \frac{\partial L_{2i}(.)}{\partial h_i} \frac{\partial h_i(.)}{\partial g_i} - W_i \right) \frac{\partial g_i(.)}{\partial W_i} + \]
\[+ \left( P_i(.) - \bar{W} \frac{\partial L_{2i}(.)}{\partial h_i} \right) \sum_{k \neq i}^{n-1} \left\{ \frac{\partial f_i(.)}{\partial P_k} \frac{\partial P_k(.)}{\partial W_i} \right\} = 0.\]
As one would expect, equation (10) states that at the non-cooperative equilibrium no profitable deviations exist, that is, a small variation in the wage does not affect the profits of the firm. In the left-handed side of equation 10 we observe the three different effects of raising the wage of specialized workers in one unit: firstly, the firm loses one unit of money for each worker employed (first term of equation 10); secondly, the firm is able to substitute some of the non-specialized labor force by specialized labor, saving thus in the wages paid to low-skilled employees (second term of equation 10); and thirdly the remaining firms of the industry will react to the new wage with different price levels, which in turn will affect the total amount of the final good that firm $i$ is able to sell (third term of equation 10).

The last effect of augmenting the wage is probably the most interesting and unexpected, since it results from the specific dynamics of this game. Its signal depends on how the equilibrium prices in the second stage react to a change in the wage level, $\partial P_k(.)/\partial W_i$, $k \neq i$.

In Appendix 1 we perform a comparative-static analysis to prove that the marginal effect of the wage of firm $i$ on its own price is negative, $\partial P_i(.)/\partial W_i < 0$. Intuitively a firm that has already committed to a higher wage and employed a greater specialized labor force in the first stage has an incentive to increase the production level in the second stage, which can only be sold at a lower price. Then we show in Appendix 1 that the marginal effect of the wage of firm $i$ on the prices of the other firms, $\partial P_k(.)/\partial W_i$, can be either positive or negative, depending on the specific characteristics of the industry. In fact, the signal of $\partial P_k(.)/\partial W_i$ relies on the dimension of two distinct forces operating in different directions: on the one hand, when firm $i$ increases the wage, it gives a clear signal that it will reduce prices in the second stage and so the rest of the industry is likely to respond with lower prices as well; on the other hand, all other firms are now able to hire less specialized workers for the same wage levels and face a greater marginal cost of production, which induces them to raise prices.

We classify industries in two different types according to the signal of $\partial P_k(.)/\partial W_i$. In type $A$ industries, job posts are well differentiated and final goods are very close substitutes (and so demand is very reactive to the prices of the alternative goods). As a result, the first force dominates the second and $\partial P_k(.)/\partial W_i$ is negative. In this case firms are usually more reluctant to increase wages as they do not want to trigger a price war – price war effect.
In *type B* industries, because final goods are sufficiently different at the eyes of consumers and post jobs are close substitutes, the second force dominates the first and $\partial P_k(.)/\partial W_i$ is positive. When this occurs, firms have an extra temptation to raise the wage as a strategic move to undermine the production capacity of the remaining firms, forcing them to produce less and to charge higher prices. We call this move a *shooting the moon strategy*, since it is extremely sophisticated and risky to increase the wage not to attract additional workers, but to steal business from the rival companies of the industry without having to decrease the price. Such strategy can easily backfire when the whole industry attempts to *shoot the moon*, case in which every firm ends paying a greater salary without being able to steal any sales from other companies.

As we will see later, the signal of $\partial P_k(.)/\partial W_i$ has an important role in the results of the paper.

### 5. Collusive outcome in the first stage of the game

To find the collusive equilibrium in the first stage of the game, we use similar procedures to those of the last section, except that the wage of specialized workers paid by a particular firm is now centrally determined to maximize the joint profits of the cartel. Here is the new optimization problem:

$$\begin{align*}
\text{Max}_{W_i} \quad \text{Total Profits} & = \sum_{j=1}^{n} P_j (W_1, \ldots, W_n)f_j \left( P_1 (W_1, \ldots, W_n), \ldots, P_n (W_1, \ldots, W_n) \right) - \\
& - \sum_{j=1}^{n} W_j g_j (W_1, \ldots, W_n) - \sum_{j=1}^{n} \bar{W}L_{2j} (W_1, \ldots, W_n) \\
\text{s.t.} f_j (P_1 (W_1, \ldots, W_n), \ldots, P_n (W_1, \ldots, W_n)) & = h_j \left( g_j (W_1, \ldots, W_n), L_{2j} (W_1, \ldots, W_n) \right), \forall j = 1, \ldots, n
\end{align*}$$

Note that, just as before, the wage is determined taking into account the impact it has on the future decisions of firms on prices and non-specialized labor hired. To solve the maximization problem we set the Lagrangian of the cartel.
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\[ L = \sum_{j=1}^{n} P_j(W_1, ..., W_n)f_j(P_1(W_1, ..., W_n), ..., P_n(W_1, ..., W_n)) - \]

\[ - \sum_{j=1}^{n} W_j g_j(W_1, ..., W_n) - \sum_{j=1}^{n} \bar{W} L_{2j}(W_1, ..., W_n) + \]

\[ + \sum_{j=1}^{n} \lambda_j (W_1, ..., W_n) \left[ h_j \left( g_j(W_1, ..., W_n), L_{2j}(W_1, ..., W_n) \right) - f_j(P_1(W_1, ..., W_n), ..., P_n(W_1, ..., W_n)) \right] \]

First order conditions require that at the interior optimal solution a small variation of the wage neither raises nor reduces the joint profits of the industry:

\[ \frac{\partial L}{\partial W_i} = \sum_{j=1}^{n} \left( \frac{\partial P_j(\cdot)}{\partial W_i} f_j(\cdot) \right) + \sum_{j=1}^{n} \left( P_j(\cdot) - \lambda_j(\cdot) \sum_{k=1}^{n} \frac{\partial f_j(\cdot)}{\partial P_k(\cdot)} \frac{\partial P_k(\cdot)}{\partial W_i} \right) - \]

\[ - g_i(\cdot) + \sum_{j=1}^{n} \left( \lambda_j(\cdot) \frac{\partial h_j(\cdot)}{\partial g_j} - W_i \right) \frac{\partial g_j(\cdot)}{\partial W_i} \]

\[ + \sum_{j=1}^{n} \left( \lambda_j(\cdot) \frac{\partial h_j(\cdot)}{\partial L_{2j}} - \bar{W} \right) \frac{\partial L_{2j}(\cdot)}{\partial W_i} + \sum_{j=1}^{n} \left( \frac{\partial \lambda_j(\cdot)}{\partial W_i} \left[ h_j(\cdot) - f_j(\cdot) \right] \right) = 0 \]

Once again, the expression above can be simplified with the first order conditions obtained in the second stage of the game. Using equation (2) to get rid of the Lagrangian multiplier and equation (4) to eliminate the last term:

\[ \sum_{j=1}^{n} \left( \frac{\partial P_j(\cdot)}{\partial W_i} f_j(\cdot) \right) + \sum_{j=1}^{n} \left( P_j(\cdot) - \bar{W} \right) \frac{\partial L_{2j}(\cdot)}{\partial h_j} \frac{\partial h_j(\cdot)}{\partial g_j} - W_i \frac{\partial g_j(\cdot)}{\partial W_i} \]

\[ - g_i(\cdot) + \sum_{j=1}^{n} \left( \frac{\partial L_{2j}(\cdot)}{\partial h_j} \frac{\partial h_j(\cdot)}{\partial g_j} - W_i \right) \frac{\partial g_j(\cdot)}{\partial W_i} \}

\[ = 0 \]

Splitting the term \[ \sum_{k=1}^{n} \frac{\partial f_j(\cdot)}{\partial P_k(\cdot)} \frac{\partial P_k(\cdot)}{\partial W_i} \] gives:

\[ \sum_{j=1}^{n} \left( \frac{\partial P_j(\cdot)}{\partial W_i} \left[ f_j(\cdot) + \left( P_j(\cdot) - \bar{W} \right) \frac{\partial L_{2j}(\cdot)}{\partial h_j} \frac{\partial f_j(\cdot)}{\partial P_j} \right] \right) + \]

\[ + \sum_{j=1}^{n} \left( \lambda_j(\cdot) \frac{\partial h_j(\cdot)}{\partial g_j} - W_i \right) \frac{\partial g_j(\cdot)}{\partial W_i} \]

\[ + \sum_{j=1}^{n} \left( \lambda_j(\cdot) \frac{\partial h_j(\cdot)}{\partial L_{2j}} - \bar{W} \right) \frac{\partial L_{2j}(\cdot)}{\partial W_i} \sum_{j=1}^{n} \left( \frac{\partial \lambda_j(\cdot)}{\partial W_i} \left[ h_j(\cdot) - f_j(\cdot) \right] \right) = 0 \]
At last, from equation (5) we know that $f_j(.) + \left( P_j(.) - W \frac{\partial L_{2j}(.)}{\partial h_j} \right) \sum_{k \neq j} \frac{\partial f_k(.) \partial P_k(.)}{\partial W_i} = 0$ and so the final expression for the first order condition with respect to the wage of highly skilled workers is:

$$-g_i(.) + \sum_{j=1}^{n} \left\{ \left( W \frac{\partial L_{2j}(.)}{\partial h_j} \frac{\partial h_j(.)}{\partial g_j} - W_j \right) \frac{\partial g_j(.)}{\partial W_i} \right\} +$$

$$+ \sum_{j=1}^{n} \left\{ \left( P_j(.) - W \frac{\partial L_{2j}(.)}{\partial h_j} \right) \sum_{k \neq j} \frac{\partial f_k(.) \partial P_k(.)}{\partial W_i} \right\} = 0 \quad (11)$$

In conclusion, when the firms of the industry form a cartel in the labor market, the wage paid by a particular firm is established to optimize the joint profits of the whole industry, which are affected through three different channels. Firstly, when the wage of firm $i$ increases in one unit, the cartel loses one unit of money for each worker employed at that firm. Secondly, an increase in the wage of firm $i$ has a positive effect on the specialized labor force of that firm, but a negative effect on the other companies, who are compelled to hire more unskilled workers. And finally, a change in the wage is replied with changes in prices and quantities, affecting the sales revenues of every firm.

6. Non cooperative equilibrium vs collusive equilibrium

Once it has been determined how the firms in the industry strategically interact under competitive and collusive settings, we can perform a comparative static analysis between the two equilibria to predict the effects of collusion in the labor market on the main economic variables. Starting by the impact of collusion on the wage of specialized workers, we use the first order conditions in equations (10) and (11) to compute the difference between the marginal effect of the wage on the joint profits of the industry and on the profit of the firm:
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\[
\frac{\partial L}{\partial W_i} - \frac{\partial L_i}{\partial W_i} = \sum_{j \neq i}^{n-1}\left\{ \left( P_j(\cdot) - \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \right) \frac{\partial f_j(\cdot)}{\partial P_i} \frac{\partial P_i(\cdot)}{\partial W_i} \right\} \\
+ \sum_{j \neq i}^{n-1}\left\{ \left( \frac{\partial L_{2j}(\cdot)}{\partial h_j} \frac{\partial h_j}{\partial g_j} - W_j \right) \frac{\partial g_j(\cdot)}{\partial W_i} \right\} \\
+ \sum_{j \neq i}^{n-1}\left\{ \left( P_j(\cdot) - \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \right) \sum_{k \neq j}^{n-2} \frac{\partial f_j(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} \right\}
\]

(12)

As long as that difference is negative, a positive variation in the wage paid by firm \(i\) to specialized workers has a greater impact on its own individual profits than on the profits of the whole cartel \(\frac{\partial L_i}{\partial W_i} > \frac{\partial L_i}{\partial W_i}\), meaning that under competition any firm is willing to set a higher wage and to employ more specialized workers. To prove that the difference between the two marginal effects is, indeed, negative, we must analyze the meaning and signal of each of the three components in the right-handed side of equation (12). For now we consider only type A industries where \(\frac{\partial P_k(\cdot)}{\partial W_i}\) is negative.

The first component of equation (12), \(\sum_{j \neq i}^{n-1}\left\{ \left( P_j(\cdot) - \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \right) \frac{\partial f_j(\cdot)}{\partial P_i} \frac{\partial P_i(\cdot)}{\partial W_i} \right\}\), can be interpreted as the business stealing effect: when a firm in the industry increases the wage paid to specialized workers and expands its productive capacity, it is forced to reduce the price of the final good to meet the new production levels. And, as a result, the firm ends up stealing indirectly some sales from the rival companies. At the collusive equilibrium firms refrain from hurting each other and eliminate the business stealing effect.

As regards to the signal of this component, the term \(\frac{\partial f_j(\cdot)}{\partial P_i}\) is proved to be negative in appendix 1, \(\frac{\partial f_j(\cdot)}{\partial h_j}\) is positive by definition and \(P_j(\cdot) - \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j}\) corresponds to the mark-up of the final good (price minus the marginal cost expressed in terms of non-specialized labor), which is clearly positive for any \(j\) or firms would be producing unprofitable units of product. Indeed, from equation (5):

\[
f_j(\cdot) + \left( P_j - \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \right) \frac{\partial f_j(\cdot)}{\partial P_j} = 0 \iff P_j - \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} = - \frac{f_j(\cdot)}{\partial f_j(\cdot)/\partial P_j}
\]
Because the product of the three terms in the first component of equation (12) is negative, the elimination of the *business stealing effect* leads to lower wages under collusion.

The second component of equation (12), $\sum_{j \neq i}^{n-1} \left\{ \left( \tilde{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \frac{\partial h_j(\cdot)}{\partial g_j} - W_j \right) \frac{\partial g_j(\cdot)}{\partial W_i} \right\}$, corresponds to the *(specialized) labor force stealing effect*: when the same firm of the industry raises the wage to its individually optimal level, it steals specialized workers from the other firms, who incur in extra costs to use non skilled labor instead. Under collusion the labor force stealing effect is eliminated, since firms account for the loss imposed on the rest of the industry when they attempt to steal workers from each other.

While the signal of $\partial g_j(\cdot)/\partial W_i$ is negative by definition, the term $\tilde{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \frac{\partial h_j(\cdot)}{\partial g_j} - W_j$ (which can be interpreted as the additional cost of using non specialized labor to make the job of one highly skilled worker) is always positive in type A industries, or firms would prefer to use only non specialized workers to produce the whole output. In fact, rearranging equation (10) gives:

$$\left( \tilde{W} \frac{\partial L_{2i}(\cdot)}{\partial h_i} \frac{\partial h_i(\cdot)}{\partial g_i} - W_i \right) \frac{\partial g_i(\cdot)}{\partial W_i} = g_i(\cdot) - \left( P_i(\cdot) - \tilde{W} \frac{\partial L_{2i}(\cdot)}{\partial h_i} \right) \sum_{k \neq i}^{n-1} \left\{ \frac{\partial f_i(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} \right\}$$

As we can see above, for $\partial P_k(\cdot)/\partial W_i < 0$ the right-handed side of the last equation is positive and so the term $\tilde{W} \frac{\partial L_{2i}(\cdot)}{\partial h_i} \frac{\partial h_i(\cdot)}{\partial g_i} - W_i$ in the left-handed side must be positive as well. Because the product of the two terms of the labor force stealing effect is negative, its elimination also leads to lower wages under collusion.

The final component of equation (12), $\sum_{j \neq i}^{n-1} \left\{ \left( P_j(\cdot) - \tilde{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \right) \sum_{k \neq j}^{n-2} \frac{\partial f_j(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} \right\}$, corresponds to an intensification of the *price war effect*. As we have already seen, at the non-cooperative equilibrium firms in type A industries avoid raising wages too much, as they do not want to trigger a price war in the second stage. But under collusion, an optimizing firm does not only internalize the costs of a price war on its own profits, but on the profitability of the whole industry. As a result, at the cooperative equilibrium
firms decrease wages paid to specialized workers even more due to an amplification of the price war effect.

It is easy to show that the signal of this component is also negative, since the mark-up 
\[ (P_j(\cdot) - W \frac{\partial L_{2j}(\cdot)}{\partial h_j}) \] 
is positive and the term \( \sum_{k \neq j}^{n-1} \frac{\partial f_j(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} \) is negative in type A industries.

Given the negative signals of the three different components in the right-handed side of equation (12) we verify that \( \frac{\partial L}{\partial W_i} - \frac{\partial L_i}{\partial W_i} < 0 \) and hence the wages paid to non specialized workers are lower under collusion. The difference between the wage levels in the two distinct scenarios is the result of the elimination of the business stealing effect, the labor force stealing effect and the amplification of the price war effect.

Interestingly things are slightly different as far as type B industries are concerned, where \( \partial P_k(\cdot)/\partial W_i \) is positive. For those firms, **shooting the moon strategies** are so appealing that at the non-cooperative equilibrium firms commit to very high wages and overly employ specialized workers in order to enforce the other companies to practice high prices, even though it would be cheaper to produce the same output level with more intensive combinations of non skilled labor. As we prove below, for \( \partial P_k(\cdot)/\partial W_i \) large enough, the term \( W \frac{\partial L_{2i}(\cdot)}{\partial h_i} \frac{\partial h_i(\cdot)}{\partial g_i} - W_i \) is negative and the second component in equation (12), which represents the elimination of the labor force stealing effect, becomes positive. This means that under collusion firms get an incentive to augment wages in order to relief the other firms from the excessive specialized labor force they have.

From equation (10):

\[
\left( W \frac{\partial L_{2i}(\cdot)}{\partial h_i} \frac{\partial h_i(\cdot)}{\partial g_i} - W_i \right) \frac{\partial g_i(\cdot)}{\partial W_i} = g_i(\cdot) - \left( P_i(\cdot) - W \frac{\partial L_{2i}(\cdot)}{\partial h_i} \right) \sum_{k \neq i}^{n-1} \left\{ \frac{\partial f_i(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} \right\}
\]

\[
\Rightarrow \left( W \frac{\partial L_{2i}(\cdot)}{\partial h_i} \frac{\partial h_i(\cdot)}{\partial g_i} - W_i \right) < 0, \quad \text{for } \partial P_k(\cdot)/\partial W_i \text{ large enough.}
\]

In addition, in type B industries where \( \partial P_k(\cdot)/\partial W_i \) is positive, the third component of equation (12) also becomes positive and can be now interpreted as an intensification of the **shooting the moon strategies**. In this type of industries, an optimizing firm operating in a cartel has an extra incentive to keep high wages, as it accounts for a positive side effect of the shooting the moon strategy on the profits of the other firms,
who are able to increase their sales due to the higher prices practiced in the rest of the industry.

Despite the positive signs of the labor force stealing effect and shooting the moon strategies in type B industries, we show in Appendix 4 that the business stealing effect component remains negative and offsets the other two components. As a result the wages paid to specialized workers prevail lower under collusion.

With regard to the impact of collusion on the other economic variables, we compute their average behavior using symmetry.

Given that the firms of a cartel pay on average lower wages, the effect of collusion on the employment of specialized workers can be directly inferred from the labor supply functions. Under symmetry,

\[ L_{1i} = g_i(W_1, \ldots, W_n) = \frac{G(W_1, \ldots, W_n)}{n} \]

As the total specialized labor supply \( G(W_1, \ldots, W_n) \) is positively correlated with every \( W \), when firms undertake cooperative agreements to reduce wages they are able to hire fewer specialized workers.

Concerning the prices of the final good, it has already been discussed how the firms of the industry react when one particular firm decreases the wage. Now we are interested in predicting the reaction of the industry when all firms decrease the wage or, similarly, when they all hire less specialized workers. In Appendix 5 we show that as a result of their smaller productive capacity their best reply is to charge a higher price. And given the market demand functions and the symmetry assumption, this means that firms are able to sell less:

\[ Q_i = f_i(P_1, \ldots, P_n) = \frac{F(P_1, \ldots, P_n)}{n} \]

Ultimately, the impact of collusion in the employment level of non specialized labor is undetermined. The ambiguity of the signal results from two opposing forces. When the industry forms a cartel and sets lower wages for specialized workers, firms are able to attract a smaller amount of highly skilled labor and, for the same production level, they need more non-specialized workers than before (substitution effect). On the other hand when firms set lower wages they have an incentive to produce less in the second stage.
of the game and thus to hire fewer non-specialized workers as well, whose marginal productivity is now smaller (scale effect).

7. Simultaneous competitive game

We have asserted that the sequential order of decisions in the two stage game previously described is the only adequate way to model the semi-collusive interaction between firms who cooperatively fix the wages of specialized workers. After all, if the prices and employment levels of non-specialized labor were non-cooperatively defined firstly, once the wages were centrally determined firms would choose new prices and employment levels to optimize their individual profits. The sequentiality of the two stage model also seems acceptable in the free competition scenario, because wages of specialized labor are often a variable more rigid than prices and low-skilled workers hired. Notwithstanding we cannot discard the hypothesis that under free competition all variables are set simultaneously. In that case, despite the first order conditions given in equations (4) and (5) remaining the same, the wage of specialized workers is now set to optimize individual profits without considering any future impact on prices and non-skilled labor. That is, wages are set to solve the following optimization problem:

$$\max_{W_i, P_i, L_2} Profit_i = P_i f_i(P_1, ..., P_n) - W_i g_i(W_1, ..., W_n) - \bar{W} L_{2_i}$$

s. t.  $f_i(P_1, ..., P_n) = h_i(g_i(W_1, ..., W_n), L_{2_i})$.

The Lagrangian function of the firm becomes

$$L_{Firm} = P_i f_i(P_1, ..., P_n) - W_i g_i(W_1, ..., W_n) - \bar{W} L_{2_i} + \lambda_i [h_i(g_i(W_1, W_{-i}), L_{2_i}) - f_i(P_1, P_{-i})]$$

and the first order condition is:

$$\frac{\partial L_{Firm}}{\partial W_i} = 0 \iff -g_i(.) - W_i \frac{\partial g_i(.)}{\partial W_i} + \lambda_i \frac{\partial h_i(.)}{\partial g_i} \frac{\partial g_i(.)}{\partial W_i} = 0 \iff$$

$$\iff -g_i(.) + \left(\lambda_i \frac{\partial h_i(.)}{\partial g_i} - W_i\right) \frac{\partial g_i(.)}{\partial W_i} = 0.$$ 

Substituting $\lambda_i$ by equation 2:

$$-g_i(.) + \left(\bar{W} \frac{\partial L_{2_i}(.)}{\partial h_i} \frac{\partial h_i(.)}{\partial g_i} - W_i\right) \frac{\partial g_i(.)}{\partial W_i} = 0. \quad (13)$$
The only difference between equations (10) and (13) is the absence in the last of the dynamic effect of wages on future decisions about prices. In fact, because all variables are now set simultaneously, the price war or shooting the moon effect disappears. Howsoever, when the cartel is formed, the artificial rigidity imposed on wages reintroduces the dynamic effect and firms become capable of affecting prices and employment levels of non-specialized workers through changes in wages of specialized workers. For that reason, the difference between the marginal effect of the wage on the profits of the industry and on the profit of the firm has now a slightly different expression:

\[
\frac{\partial L}{\partial W_i} - \frac{\partial L_i}{\partial W_i} = \sum_{j \neq i}^{n-1} \left\{ \left( P_j (.) - \bar{W} \frac{\partial L_{2j} (.)}{\partial h_j} \right) \frac{\partial f_j (.)}{\partial P_i} \frac{\partial P_i (.)}{\partial W_i} \right\} \\
+ \sum_{j \neq i}^{n-1} \left\{ \left( \bar{W} \frac{\partial L_{2j} (.)}{\partial h_j} \frac{\partial h_j (.)}{\partial g_j} - W_j \right) \frac{\partial g_j (.)}{\partial W_i} \right\} \\
+ \sum_{j=1}^{n} \left\{ P_j (.) - \bar{W} \frac{\partial L_{2j} (.)}{\partial h_j} \sum_{k \neq j}^{n-2} \frac{\partial f_j (.)}{\partial P_k} \frac{\partial P_k (.)}{\partial W_i} \right\}
\]

We observe in equation (14) that the formation of a cartel still eliminates the business stealing effect and the (specialized) labor stealing effect, but now it also creates the dynamic mechanism through which firms can use wages to influence the prices of the industry and total sales. Thus the third component of the right-handed side of equation (14) represents not a mere intensification of effects, but the creation of the price war effect or shooting the moon effect, which did not exist at the simultaneous non-cooperative equilibrium.

While it is easy to see that the three components of equation (14) are negative in type A industries, in Appendix 6 we prove that \( \frac{\partial L}{\partial W_i} - \frac{\partial L_i}{\partial W_i} \) remains negative in type B industries, so that wages are, as always, lower under collusion.
8. Conclusions

This paper investigates the relation between collusion in the labor market and collusion in the final good market, giving rise to unexpected and surprising results. When we came up with the idea of addressing this subject for the first time, we were immediately suggested by conventional wisdom that firms who cooperatively fix lower wages have access to cheaper cost technologies and are able, thereby, to produce more at lower prices. As we have already shown in a companion paper (Gonzaga, Brandão e Vasconcelos 2013), this is of course not the case under one input production functions, where there is a perfect and direct relation between the two types of collusion: any central decisions to decrease wages reduces the number of workers that firms are able to hire and their productive capacity, forcing them to raise prices. In this paper we attempted to break the direct relation between collusion in the labor market and collusion in the final good market by allowing firms to use a second input of production, non-specialized workers, over whom they have absolutely no power to affect wages. Nevertheless, the qualitative results remained surprisingly similar to the previous ones, although they were now obtained under a much more general and realistic setting where the interaction between players became substantially complex.

Indeed we have shown in a two-stage game model with two-input production functions that when firms interact cooperatively in the labor market, their only mechanism available to reduce wages is to accept lower levels of employment of specialized workers, which as before constrains their productive capacity. And even though firms can replace some of the specialized labor by unskilled workers, it is still in their own interest to produce less and to charge higher prices. And so collusion in the labor market leads, as always, to collusion in the final good market.

In the absence of more specific assumptions about demand, supply and production functions, it is not possible to determine in what direction collusion affects the employment of non-specialized labor. One the one hand, at the collusive equilibrium companies hire less specialized workers and thus they must use additional low-skilled employees as an alternative input to obtain the same output level (substitution effect). On the other hand, because at the collusive equilibrium production levels are smaller, firms employ less of every input, including non-specialized labor (scale effect).
The economic motivation for the cartel to coordinate lower wages and higher prices depends on the particular characteristics of the industry. We thus consider two types of industries which we discuss in turn.

In type A industries, where job posts are very differentiated and final goods are close substitutes, the low wage levels fixed under collusion result from the ability of the cartel to eliminate the business stealing effect and the labor force stealing effect and to amplify the price war effect. In other words, the cartel eliminates the individual temptation of firms to steal market quotas and specialized workers from each other and prevents them from triggering price wars.

In type B industries, where jobs posts are very close substitutes and final goods are sufficiently differentiated, wages are still lower under collusion due to the elimination of the business stealing effect, even though the cartel rationally prevents wages from falling too much, as it accounts for the positive side effects of the labor force stealing effect and shooting the moon strategies.
9. Appendix

9.1. Appendix 1

In this Appendix we do comparative statics to prove that a positive variation in the wage that firm $i$ offers to specialized workers has, in average, a negative impact on the price of the same firm and a positive or negative impact on the prices of the other firms, depending on the characteristics of the industry. In mathematical notation:

\[
\frac{\partial P_i(.)}{\partial W_i} < 0 \quad \text{and} \quad \begin{cases} 
\frac{\partial P_k(.)}{\partial W_i} < 0, & \text{if industry = type } A \ , \\
\frac{\partial P_k(.)}{\partial W_i} > 0, & \text{if industry = type } B 
\end{cases}
\]

Along our proof we assume symmetry, because this result may not hold for every firm when companies are too different. To understand why we must use comparative statics to compute the two derivatives, remark that when firm $i$ changes the wage paid to specialized workers in the first stage, this completely alters the equilibrium played in the second stage, in which firms choose different prices and levels of non specialized labor. Nevertheless, regardless of the wage fixed, firm $i$ optimizes profits in the second stage and so the first order conditions given by equations 4 and 5 are always verified. In other words, the two following functions $A_i(P_1, ..., P_n, L_{2i}, W_i)$ and $B_i(P_1, ..., P_n, L_{2i}, W_i)$ remain constant and equal to zero, because any change in $W_i$ is offset with changes in prices and $L_{2i}$:

\[
A_i(P_1, ..., P_n, L_{2i}, W_i) = f_i(P_1, ..., P_n) + \left( P_i - \bar{W} \frac{\partial L_{2i}}{\partial h_i} \right) \frac{\partial f_i(P_1, ..., P_n)}{\partial P_i} = 0.
\]

\[
B_i(P_1, ..., P_n, L_{2i}, W_i) = f_i(P_1, ..., P_n) - h_i(g_i(W_1, ..., W_n), L_{2i}) = 0.
\]

Using a generalization of the implicit function theorem for three variables, we can prove that:

\[
\begin{align*}
\frac{\partial A_i}{\partial P_1} dP_1 + \cdots + \frac{\partial A_i}{\partial P_n} dP_n + \frac{\partial A_i}{\partial L_{2i}} dL_{2i} + \frac{\partial A_i}{\partial W_i} dW_i &= 0 \\
\frac{\partial B_i}{\partial P_1} dP_1 + \cdots + \frac{\partial B_i}{\partial P_n} dP_n + \frac{\partial B_i}{\partial L_{2i}} dL_{2i} + \frac{\partial B_i}{\partial W_i} dW_i &= 0.
\end{align*}
\]
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Under symmetry:

\[
\begin{align*}
\frac{\partial A_i}{\partial P_i} dP_i + (n-1) \frac{\partial A_i}{\partial P_k} dP_k + \frac{\partial A_i}{\partial L_{2i}} dL_{2i} + \frac{\partial A_i}{\partial W_i} dW_i &= 0 \\
\frac{\partial B_i}{\partial P_i} dP_i + (n-1) \frac{\partial B_i}{\partial P_k} dP_k + \frac{\partial B_i}{\partial L_{2i}} dL_{2i} + \frac{\partial B_i}{\partial W_i} dW_i &= 0.
\end{align*}
\]

Dividing every term by \(dW_i\) gives

\[
\begin{align*}
\frac{\partial A_i}{\partial P_i} \frac{dP_i}{dW_i} + (n-1) \frac{\partial A_i}{\partial P_k} \frac{dP_k}{dW_i} + \frac{\partial A_i}{\partial L_{2i}} \frac{dL_{2i}}{dW_i} + \frac{\partial A_i}{\partial W_i} &= 0 \\
\frac{\partial B_i}{\partial P_i} \frac{dP_i}{dW_i} + (n-1) \frac{\partial B_i}{\partial P_k} \frac{dP_k}{dW_i} + \frac{\partial B_i}{\partial L_{2i}} \frac{dL_{2i}}{dW_i} + \frac{\partial B_i}{\partial W_i} &= 0.
\end{align*}
\]

Solving for \(\frac{dL_{2i}}{dW_i}\): 

\[
\begin{align*}
\frac{dL_{2i}}{dW_i} &= -\left( \frac{\partial A_i}{\partial P_i} \frac{dP_i}{dW_i} + (n-1) \frac{\partial A_i}{\partial P_k} \frac{dP_k}{dW_i} + \frac{\partial A_i}{\partial L_{2i}} \frac{dL_{2i}}{dW_i} + \frac{\partial A_i}{\partial W_i} \right) + \frac{\partial A_i}{\partial L_{2i}} \frac{dL_{2i}}{dW_i} \\
\frac{dL_{2i}}{dW_i} &= -\left( \frac{\partial B_i}{\partial P_i} \frac{dP_i}{dW_i} + (n-1) \frac{\partial B_i}{\partial P_k} \frac{dP_k}{dW_i} + \frac{\partial B_i}{\partial L_{2i}} \frac{dL_{2i}}{dW_i} + \frac{\partial B_i}{\partial W_i} \right) + \frac{\partial B_i}{\partial L_{2i}} \frac{dL_{2i}}{dW_i}.
\end{align*}
\]

Equating the right-handed side of the two previous equations to eliminate \(\frac{dL_{2i}}{dW_i}\) gives:

\[
(n-1) \left( \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k} \right) \frac{dP_k}{dW_i} + \left( \frac{\partial A_i}{\partial P_i} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_i} \right) \frac{dP_i}{dW_i} = 0.
\]

In the same way that firm \(i\) always optimizes individual profits in the second stage of the game for any \(W_i\), the other firms of the industry do the same. Then equations \(A_k(P_1, ..., P_n, L_{2k}, W_i)\) and \(B_k(P_1, ..., P_n, L_{2k}, W_i)\) also remain constant and equal to zero when the wage paid by firm \(i\) is changed.

\[
A_k(P_1, ..., P_n, L_{2k}, W_i) = f_k(P_1, ..., P_n) + \left( P_k - \bar{W} \frac{\partial L_{2k}}{\partial h_k} \right) \frac{\partial f_k(P_1, ..., P_n)}{\partial P_k} = 0.
\]

\[
B_k(P_1, ..., P_n, L_{2k}, W_i) = f_k(P_1, ..., P_n) - h_k(g_k(W_1, ..., W_n), L_{2k}) = 0.
\]
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Using the same reasoning as before:

\[
\begin{align*}
\frac{\partial A_k}{\partial p_1} dP_1 + \cdots + \frac{\partial A_k}{\partial p_n} dP_n + \frac{\partial A_k}{\partial L_{2k}} dL_{2k} + \frac{\partial A_k}{\partial W_i} dW_i &= 0 \\
\frac{\partial B_k}{\partial p_1} dP_1 + \cdots + \frac{\partial B_k}{\partial p_n} dP_n + \frac{\partial B_k}{\partial L_{2k}} dL_{2k} + \frac{\partial B_k}{\partial W_i} dW_i &= 0
\end{align*}
\]

Under symmetry,

\[
\begin{align*}
(n-2) \left( \frac{\partial A_k}{\partial p_1} dP_1 + \cdots + \frac{\partial A_k}{\partial p_n} dP_n + \frac{\partial A_k}{\partial L_{2k}} dL_{2k} + \frac{\partial A_k}{\partial W_i} dW_i = 0 \right) &= \implies \\
(n-2) \left( \frac{\partial B_k}{\partial p_1} dP_1 + \cdots + \frac{\partial B_k}{\partial p_n} dP_n + \frac{\partial B_k}{\partial L_{2k}} dL_{2k} + \frac{\partial B_k}{\partial W_i} dW_i = 0 \right)
\end{align*}
\]

Dividing every term by \(dW_i\) and solving for \(\frac{dL_{2k}}{dW_i}\) gives:

\[
\begin{align*}
\left( n-2 \right) \left( \frac{\partial A_k}{\partial p_1} \frac{dP_k}{dW_i} + \cdots + \frac{\partial A_k}{\partial p_n} \frac{dP_k}{dW_i} + \frac{\partial A_k}{\partial L_{2k}} \frac{dL_{2k}}{dW_i} + \frac{\partial A_k}{\partial W_i} = 0 \right) &= \implies \\
\left( n-2 \right) \left( \frac{\partial B_k}{\partial p_1} \frac{dP_k}{dW_i} + \cdots + \frac{\partial B_k}{\partial p_n} \frac{dP_k}{dW_i} + \frac{\partial B_k}{\partial L_{2k}} \frac{dL_{2k}}{dW_i} + \frac{\partial B_k}{\partial W_i} = 0 \right)
\end{align*}
\]

\[
\begin{align*}
\frac{dL_{2k}}{dW_i} &= \frac{-\frac{\partial A_k}{\partial p_1} + \left( n-2 \right) \frac{\partial A_k}{\partial p_n} + \frac{\partial A_k}{\partial L_{2k}} \frac{dL_{2k}}{dW_i} + \frac{\partial A_k}{\partial W_i} \frac{dP_k}{dW_i}}{\frac{\partial A_k}{\partial p_1} + \left( n-2 \right) \frac{\partial A_k}{\partial p_n} + \frac{\partial A_k}{\partial L_{2k}} \frac{dL_{2k}}{dW_i} + \frac{\partial A_k}{\partial W_i} \frac{dP_k}{dW_i}} \\
\frac{dL_{2k}}{dW_i} &= \frac{-\frac{\partial B_k}{\partial p_1} + \left( n-2 \right) \frac{\partial B_k}{\partial p_n} + \frac{\partial B_k}{\partial L_{2k}} \frac{dL_{2k}}{dW_i} + \frac{\partial B_k}{\partial W_i} \frac{dP_k}{dW_i}}{\frac{\partial B_k}{\partial p_1} + \left( n-2 \right) \frac{\partial B_k}{\partial p_n} + \frac{\partial B_k}{\partial L_{2k}} \frac{dL_{2k}}{dW_i} + \frac{\partial B_k}{\partial W_i} \frac{dP_k}{dW_i}}
\end{align*}
\]

Once again we eliminate \(\frac{dL_{2k}}{dW_i}\) and get another equation relating \(\frac{dP_k}{dW_i}\) and \(\frac{dP_k}{dW_i}\):

\[
\begin{align*}
\left( n-2 \right) \frac{\partial A_k}{\partial p_1} \frac{\partial B_k}{\partial L_{2k}} + \frac{\partial A_k}{\partial p_n} \frac{\partial B_k}{\partial L_{2k}} - \left( n-2 \right) \frac{\partial A_k}{\partial L_{2k}} \frac{\partial B_k}{\partial p_1} - \frac{\partial A_k}{\partial L_{2k}} \frac{\partial B_k}{\partial p_n} \frac{dP_k}{dW_i} + \\
+ \left( \frac{\partial A_k}{\partial p_1} \frac{\partial B_k}{\partial L_{2k}} - \frac{\partial A_k}{\partial p_n} \frac{\partial B_k}{\partial L_{2k}} \right) \frac{dP_k}{dW_i} = \frac{\partial A_k}{\partial L_{2k}} \frac{\partial B_k}{\partial W_i} - \frac{\partial A_k}{\partial W_i} \frac{\partial B_k}{\partial L_{2k}}.
\end{align*}
\]
The system of equations A and B can now be used to find \( \frac{dP_i}{dW_i} \) and \( \frac{dP_k}{dW_i} \). Solving equation A for \( \frac{dP_k}{dW_i} \):

\[
\frac{dP_k}{dW_i} = \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} + \left( \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_i} - \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} \right) \frac{dP_i}{dW_i} \]

Solving equation B for \( \frac{dP_k}{dW_i} \) and applying symmetry:

\[
\frac{dP_k}{dW_i} = \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} + \left( \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_i} - \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} \right) \frac{dP_i}{dW_i} - \left( n - 2 \right) \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_i} - \left( n - 2 \right) \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_i} \frac{dP_i}{dW_i} \]

Equating the two previous equations:

\[
\left( \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} - \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_i} \right) \left( n - 2 \right) \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_i} - \left( n - 2 \right) \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_i} \frac{dP_i}{dW_i} \]

\[
= \left( \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} - \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_i} \right) \left( n - 2 \right) \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_i} - \left( n - 2 \right) \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_i} \frac{dP_i}{dW_i} \]

\[
\frac{dP_k}{dW_i} = \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} + \left( \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_i} - \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial P_k} \right) \frac{dP_i}{dW_i} \]
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Solving for \( \frac{dP_i}{dW_i} \):

\[
\frac{dP_i}{dW_i} = -\left( \frac{\partial A_i}{\partial L_{2i}^k} \frac{\partial B_i}{\partial W_k} - \frac{\partial A_i}{\partial L_{2i}^k} \frac{\partial B_i}{\partial W_k} \right) \times (n-1) \left( \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} \right)
\]

It is possible to simplify this expression by factoring out the common factor:

\[
\frac{dP_i}{dW_i} = \frac{1}{(n-1)} \left( \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} \right) + \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}}
\]

Dividing out now the factor \( \frac{\partial A_i}{\partial L_{2i}^k} \frac{\partial B_i}{\partial W_k} - \frac{\partial A_i}{\partial L_{2i}^k} \frac{\partial B_i}{\partial W_k} \) from the terms inside the straight parentheses:

\[
\Leftrightarrow \frac{dP_i}{dW_i} = \frac{1}{(n-1)} \left( \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} \right) \times \frac{1}{\left( \frac{\partial A_i}{\partial L_{2i}^k} \frac{\partial B_i}{\partial W_k} - \frac{\partial A_i}{\partial L_{2i}^k} \frac{\partial B_i}{\partial W_k} \right)} \times \left( n-1 \right) \left( \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} \right)
\]
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\[
\frac{dP_i}{dW_i} = \frac{1}{(n-1)}\left( \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} + \frac{\partial B_i}{\partial P_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} \right) \times \\
(-n-1) \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial W_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} + (n-1) \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial W_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} + \\
\frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} - (n-1) \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} = \\
\frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} + \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} + \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k}.
\]

Rearranging the terms in brackets:

\[
\frac{dP_i}{dW_i} = \frac{1}{(n-1)}\left( \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial W_i} + (n-1) \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial W_i} + (n-1) \frac{\partial B_i}{\partial W_i} \right) \times \\
\frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} + \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} + \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} = \\
\frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} + \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} + \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k}.
\]
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From the signals of the partial derivatives of functions \(A_i\) and \(B_i\) given in Table 1 of Appendix 2, it follows that the right handed side of equation C is negative and so we conclude that:

\[
\frac{dP_k}{dW_i} < 0.
\]

To get a simplified expression for \(\frac{dP_k}{dW_i}\) we start by subtracting the terms in equation A to the terms in equation B:

\[
\left( (n - 2) \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} + \frac{\partial A_i}{\partial P_i} \frac{\partial B_i}{\partial L_{2i}} - (n - 2) \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_i} \right) \frac{dP_k}{dW_i} - (n - 1) \left( \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k} \right) \frac{dP_k}{dW_i} + \\
+ \left( \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k} \right) \frac{dP_i}{dW_i} - \left( \frac{\partial A_i}{\partial P_i} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_i} \right) \frac{dP_i}{dW_i} = \\
= \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial W_k} - \frac{\partial A_i}{\partial W_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial W_i} + \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_{2i}} \iff \\

\iff \left( \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k} - \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial P_i} \frac{\partial B_i}{\partial L_{2i}} + \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_i} \right) \frac{dP_k}{dW_i} - (n - 1) \left( \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k} \right) \frac{dP_k}{dW_i} + \\
+ \left( \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k} \right) \frac{dP_i}{dW_i} - \left( \frac{\partial A_i}{\partial P_i} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_i} \right) \frac{dP_i}{dW_i} = \\
= \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial W_k} - \frac{\partial A_i}{\partial W_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial W_i} + \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_{2i}}.
\]

Solving for \(\frac{dP_k}{dW_i}\):

\[
\frac{dP_k}{dW_i} = \frac{dP_i}{dW_i} + \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial W_k} - \frac{\partial A_i}{\partial W_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial W_i} + \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_{2i}}.
\]
Replacing $\frac{dP_i}{dW_i}$ by equation C and factoring out the two coefficients gives:

$$
\frac{dP_k}{dW_i} = \frac{1}{\frac{\partial B_i}{\partial L_2i} - \frac{\partial A_i}{\partial P_i} + (n-1) \frac{\partial B_i}{\partial P_k} + (n-1) \frac{\partial B_i}{\partial P_k}} \times 
$$

$$
\times \left[ - \frac{\partial A_i}{\partial L_2i} \frac{\partial B_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} + (n-1) \frac{\partial B_i}{\partial W_k} + \frac{\partial A_i}{\partial L_2i} \frac{\partial A_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} + (n-1) \frac{\partial B_i}{\partial W_k} \right] + 
$$

$$
+ \frac{\partial B_i}{\partial L_2i} \frac{\partial A_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} (\frac{\partial B_i}{\partial P_i} + (n-1) \frac{\partial B_i}{\partial P_k}) + \frac{\partial A_i}{\partial L_2i} \frac{\partial A_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} (\frac{\partial B_i}{\partial P_i} + (n-1) \frac{\partial B_i}{\partial P_k}) + 
$$

$$
- \frac{\partial A_i}{\partial L_2i} \frac{\partial B_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} \left[ \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_2i} + \frac{\partial B_i}{\partial W_i} \frac{\partial A_i}{\partial L_2i} \right]. 
$$

Rearranging the two last terms:

$$
\frac{dP_k}{dW_i} = \frac{1}{\frac{\partial B_i}{\partial L_2i} - \frac{\partial A_i}{\partial P_i} + (n-1) \frac{\partial B_i}{\partial P_k} + (n-1) \frac{\partial B_i}{\partial P_k}} \times 
$$

$$
\times \left[ - \frac{\partial A_i}{\partial L_2i} \frac{\partial B_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} + (n-1) \frac{\partial B_i}{\partial W_k} + \frac{\partial A_i}{\partial L_2i} \frac{\partial A_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} + (n-1) \frac{\partial B_i}{\partial W_k} \right] + 
$$

$$
+ \frac{\partial B_i}{\partial L_2i} \frac{\partial A_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} (\frac{\partial B_i}{\partial P_i} + (n-1) \frac{\partial B_i}{\partial P_k}) + \frac{\partial A_i}{\partial L_2i} \frac{\partial A_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} (\frac{\partial B_i}{\partial P_i} + (n-1) \frac{\partial B_i}{\partial P_k}) + 
$$

$$
- \frac{\partial A_i}{\partial L_2i} \frac{\partial B_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} \left[ \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_2i} + \frac{\partial B_i}{\partial W_i} \frac{\partial A_i}{\partial L_2i} \right]. 
$$
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\[ \frac{dA_i}{dW_i} \frac{dB_i}{dP_k} \left( \frac{\partial A_i}{\partial P_i} + (n-1) \frac{\partial A_i}{\partial P_k} \right) - \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial L_2} \frac{\partial A_i}{\partial P_i} (n-1) \frac{\partial B_i}{\partial P_k} - \]

\[ - \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_2} \frac{\partial A_i}{\partial P_i} + (n-1) \frac{\partial A_i}{\partial P_k} + \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_2} \frac{\partial A_i}{\partial P_i} (n-1) \frac{\partial B_i}{\partial P_k} + \]

\[ + \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_2} \frac{\partial A_i}{\partial P_i} (n-1) \frac{\partial A_i}{\partial P_k} - \frac{\partial A_i}{\partial B_i} \frac{\partial B_i}{\partial L_2} \frac{\partial A_i}{\partial P_i} + (n-1) \frac{\partial B_i}{\partial P_k} \].

Eliminating the repeated terms in brackets:

\[ \frac{dP_k}{dW_i} = \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial P_i} + (n-1) \frac{\partial A_i}{\partial P_k} - \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial L_2} \frac{\partial A_i}{\partial P_i} + (n-1) \frac{\partial B_i}{\partial P_k} + \]

\[ \frac{\partial B_i}{\partial L_2} \frac{\partial A_i}{\partial P_i} + (n-1) \frac{\partial A_i}{\partial P_k} - \frac{\partial A_i}{\partial B_i} \frac{\partial B_i}{\partial L_2} \frac{\partial A_i}{\partial P_i} + (n-1) \frac{\partial B_i}{\partial P_k} + \]

\[ - \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_2} \frac{\partial A_i}{\partial P_i} + (n-1) \frac{\partial A_i}{\partial P_k} + \frac{\partial A_i}{\partial B_i} \frac{\partial B_i}{\partial L_2} \frac{\partial A_i}{\partial P_i} + (n-1) \frac{\partial B_i}{\partial P_k} \]
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\[
+ \frac{\partial A_i}{\partial L_2} \left( \frac{\partial A_i}{\partial P_k} + (n-1) \frac{\partial A_i}{\partial W_k} \right) - \frac{\partial A_i}{\partial L_2} \frac{\partial A_i}{\partial L_2} \left( \frac{\partial B_i}{\partial P_k} + (n-1) \frac{\partial B_i}{\partial W_k} \right) - \\
- \frac{\partial B_i}{\partial L_2} \frac{\partial B_i}{\partial L_2} \left( \frac{\partial A_i}{\partial P_k} + (n-1) \frac{\partial A_i}{\partial W_k} \right) + \frac{\partial B_i}{\partial L_2} \frac{\partial B_i}{\partial L_2} \left( \frac{\partial B_i}{\partial P_k} + (n-1) \frac{\partial B_i}{\partial W_k} \right).
\]

Eliminating again the repeated terms:

\[
\frac{dP_k}{dW_i} = \frac{1}{\frac{\partial B_i}{\partial L_2} \frac{\partial A_i}{\partial L_2} \left( \frac{\partial A_i}{\partial P_k} + (n-1) \frac{\partial A_i}{\partial W_k} \right) - \frac{\partial A_i}{\partial L_2} \frac{\partial A_i}{\partial L_2} \left( \frac{\partial B_i}{\partial P_k} + (n-1) \frac{\partial B_i}{\partial W_k} \right) - \\
- \frac{\partial B_i}{\partial L_2} \frac{\partial B_i}{\partial L_2} \left( \frac{\partial A_i}{\partial P_k} + (n-1) \frac{\partial A_i}{\partial W_k} \right) + \frac{\partial B_i}{\partial L_2} \frac{\partial B_i}{\partial L_2} \left( \frac{\partial B_i}{\partial P_k} + (n-1) \frac{\partial B_i}{\partial W_k} \right) \times \\
\times \left[ \frac{\partial A_i}{\partial L_2} \frac{\partial B_i}{\partial L_2} \left( \frac{\partial A_i}{\partial P_k} + \frac{\partial A_i}{\partial W_k} \right) - \frac{\partial A_i}{\partial L_2} \frac{\partial A_i}{\partial L_2} \left( \frac{\partial B_i}{\partial P_k} + \frac{\partial B_i}{\partial W_k} \right) - \\
- \frac{\partial B_i}{\partial L_2} \frac{\partial B_i}{\partial L_2} \left( \frac{\partial A_i}{\partial P_k} + \frac{\partial A_i}{\partial W_k} \right) + \frac{\partial B_i}{\partial L_2} \frac{\partial B_i}{\partial L_2} \left( \frac{\partial B_i}{\partial P_k} + \frac{\partial B_i}{\partial W_k} \right) \right].
\]

(D)

If once again we use the information provided in Table 1 (Appendix 2) we verify that the signal of \(dP_k/dW_i\) is undetermined, as it depends on the relative dimension of the partial derivatives of functions \(A_i\) and \(B_i\) with respect to prices and wages. Nevertheless we can still analyze the expressions of the derivatives in Table 1 to identify the characteristics of the industry that are more likely to affect the signal of \(dP_k/dW_i\).

We conclude that for any \(k \neq i\) we have:

\[
\begin{cases}
\frac{\partial P_k}{\partial W_i} < 0, & \text{if industry = type A} \\
\frac{\partial P_k}{\partial W_i} > 0, & \text{if industry = type B}
\end{cases}
\]

where in type A industries \(\frac{\partial g_i}{\partial W_i}\) and \(\frac{\partial f_i}{\partial P_k}\) are sufficiently large relatively to \(\frac{\partial g_i}{\partial W_k}\) and \(\frac{\partial f_i}{\partial P_i}\), while in type B industries the opposite occurs.
## 9.2. Appendix 2 – Table 1

<table>
<thead>
<tr>
<th>Derivative</th>
<th>Value</th>
<th>Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{\partial A_i}{\partial P_i})</td>
<td>(2 \frac{\partial f_i(.)}{\partial P_i} + \left( P_i - W \frac{\partial L_{2i}}{\partial h_i} \right) \frac{\partial^2 f_i(.)}{\partial P_i^2})</td>
<td>(\leq 0)</td>
</tr>
<tr>
<td>(\frac{\partial A_i}{\partial P_k})</td>
<td>(\frac{\partial f_i(.)}{\partial P_k} + \left( P_i - W \frac{\partial L_{2i}}{\partial h_i} \right) \frac{\partial^2 f_i(.)}{\partial P_k^2})</td>
<td>(\geq 0)</td>
</tr>
<tr>
<td>(\frac{\partial A_i}{\partial P_i} + (n - 1) \frac{\partial A_i}{\partial P_k})</td>
<td>(\frac{\partial f_i(.)}{\partial P_i} + \frac{\partial F(.)}{\partial P_i} + \left( P_i - W \frac{\partial L_{2i}}{\partial h_i} \right) \frac{\partial^2 f_i(.)}{\partial P_i^2} + (n - 1) \frac{\partial^2 f_i(.)}{\partial P_i \partial P_k})</td>
<td>(\leq 0)</td>
</tr>
<tr>
<td>(\frac{\partial A_i}{\partial L_{2i}})</td>
<td>(\bar{W} \frac{\partial f_i(.)}{\partial P_i} \frac{\partial^2 h_i(.)}{\partial (L_{2i})^2} \left( \frac{\partial L_{2i}}{\partial h_i} \right)^2)</td>
<td>(\geq 0)</td>
</tr>
<tr>
<td>(\frac{\partial A_i}{\partial W_i})</td>
<td>(\bar{W} \frac{\partial f_i(.)}{\partial P_i} \frac{\partial^2 h_i(.)}{\partial L_{2i} \partial g_i} \frac{\partial g_i(.)}{\partial W_i} \left( \frac{\partial L_{2i}}{\partial h_i} \right)^2)</td>
<td>(\leq 0)</td>
</tr>
<tr>
<td>(\frac{\partial A_i}{\partial W_k})</td>
<td>(\bar{W} \frac{\partial f_i(.)}{\partial P_i} \frac{\partial^2 h_i(.)}{\partial L_{2i} \partial g_i} \frac{\partial g_i(.)}{\partial W_k} \left( \frac{\partial L_{2i}}{\partial h_i} \right)^2)</td>
<td>(\geq 0)</td>
</tr>
<tr>
<td>(\frac{\partial A_i}{\partial W_i} + (n - 1) \frac{\partial A_i}{\partial W_k})</td>
<td>(\bar{W} \frac{\partial f_i(.)}{\partial P_i} \frac{\partial^2 h_i(.)}{\partial L_{2i} \partial g_i} \frac{\partial G(.)}{\partial W_i} \left( \frac{\partial L_{2i}}{\partial h_i} \right)^2)</td>
<td>(\leq 0)</td>
</tr>
<tr>
<td>(\frac{\partial B_i}{\partial P_i})</td>
<td>(\frac{\partial f_i(.)}{\partial P_i})</td>
<td>(\leq 0)</td>
</tr>
<tr>
<td>(\frac{\partial B_i}{\partial P_k})</td>
<td>(\frac{\partial f_i(.)}{\partial P_k})</td>
<td>(\geq 0)</td>
</tr>
<tr>
<td>(\frac{\partial B_i}{\partial P_i} + (n - 1) \frac{\partial B_i}{\partial P_k})</td>
<td>(\frac{\partial F(.)}{\partial P_i})</td>
<td>(\leq 0)</td>
</tr>
<tr>
<td>(\frac{\partial B_i}{\partial L_{2i}})</td>
<td>(- \frac{\partial h_i(.)}{\partial L_{2i}})</td>
<td>(\leq 0)</td>
</tr>
<tr>
<td>(\frac{\partial B_i}{\partial W_i})</td>
<td>(- \frac{\partial h_i(.)}{\partial g_i} \frac{\partial g_i(.)}{\partial W_i})</td>
<td>(\leq 0)</td>
</tr>
<tr>
<td>(\frac{\partial B_i}{\partial W_k})</td>
<td>(- \frac{\partial h_i(.)}{\partial g_i} \frac{\partial g_i(.)}{\partial W_k})</td>
<td>(\geq 0)</td>
</tr>
<tr>
<td>(\frac{\partial B_i}{\partial W_i} + (n - 1) \frac{\partial B_i}{\partial W_k})</td>
<td>(- \frac{\partial h_i(.)}{\partial g_i} \frac{\partial G(.)}{\partial W_i})</td>
<td>(\leq 0)</td>
</tr>
</tbody>
</table>
9.3. Appendix 3

Here we prove that when firms are symmetric the term \( \sum_{k \neq j}^{n-1} \frac{\partial f_j(.)}{\partial P_k} \frac{\partial P_k(.)}{\partial W_i} \) is always negative:

\[
\sum_{k \neq j}^{n-1} \frac{\partial f_j(.)}{\partial P_k} \frac{\partial P_k(.)}{\partial W_i} = \frac{\partial f_j(.)}{\partial P_i} \left( \frac{\partial P_i(.)}{\partial W_i} + (n-2) \frac{\partial P_i(.)}{\partial W_k} \right) =
\]

\[
= \frac{\partial f_j(.)}{\partial P_i} \left( \frac{\partial P_i(.)}{\partial W_i} + (n-2) \frac{\partial P_i(.)}{\partial W_k} \right)
\]

Replacing \( \frac{\partial P_i(.)}{\partial W_i} \) and \( \frac{\partial P_k(.)}{\partial W_i} \) respectively by equations C and D in Appendix 1:

\[
\frac{\partial f_j(.)}{\partial P_i} \times \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} \times \frac{1}{1-n-2} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial W_k} \times \frac{1}{1-n-2} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial W_k} \times \frac{1}{1-n-2} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial W_k} \times \frac{1}{1-n-2} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial W_k} + \]

\[
- \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial W_k} - \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial W_k} - \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial W_k} + \]

\[
+ (n-2) \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial W_k} - (n-2) \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial W_k} - \]

\[
- (n-2) \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial W_k} + (n-2) \frac{\partial B_i}{\partial L_2_i} \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial W_k} - \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial W_k} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial W_k} - \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial W_k} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial L_2_i} \frac{\partial B_i}{\partial W_k} =
\]
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\[ \frac{\partial f_j(\cdot)}{\partial P_i} \times \frac{1}{\partial L_{2i}(\frac{\partial A_i}{\partial P_i} + (n-1)\frac{\partial A_i}{\partial P_k}) - \frac{\partial A_i}{\partial P_{L2i}}(\frac{\partial B_i}{\partial P_i} + (n-1)\frac{\partial B_i}{\partial P_k})} \]

\[ \times \frac{1}{\frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial P_{L2i}}} \frac{\partial A_i}{\partial P_{L2i}} \frac{\partial B_i}{\partial P_{L2i}} \frac{\partial A_i}{\partial P_{L2i}} \frac{\partial B_i}{\partial P_{L2i}} \]

\[ \times \left[ -\frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial L_{2i}} \frac{\partial A_i}{\partial W_i} \left( \frac{\partial A_i}{\partial P_i} + (n-1)\frac{\partial A_i}{\partial P_k} \right) + \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial L_{2i}} \frac{\partial A_i}{\partial W_i} \left( \frac{\partial B_i}{\partial P_i} + (n-1)\frac{\partial B_i}{\partial P_k} \right) + \frac{\partial B_i}{\partial L_{2i}} \frac{\partial B_i}{\partial L_{2i}} \frac{\partial A_i}{\partial W_i} \left( \frac{\partial B_i}{\partial P_i} + (n-1)\frac{\partial B_i}{\partial P_k} \right) + \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial L_{2i}} \frac{\partial A_i}{\partial W_i} \left( \frac{\partial B_i}{\partial P_i} + (n-1)\frac{\partial B_i}{\partial P_k} \right) - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial L_{2i}} \frac{\partial A_i}{\partial W_i} \left( \frac{\partial B_i}{\partial P_i} + (n-1)\frac{\partial B_i}{\partial P_k} \right) - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial L_{2i}} \frac{\partial A_i}{\partial W_i} \left( \frac{\partial B_i}{\partial P_i} + (n-1)\frac{\partial B_i}{\partial P_k} \right) - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial L_{2i}} \frac{\partial A_i}{\partial W_i} \left( \frac{\partial B_i}{\partial P_i} + (n-1)\frac{\partial B_i}{\partial P_k} \right) \right]. \]

Given the signals of the partial derivatives of functions A and B provided in the Table in Appendix 2, we conclude that:

\[ \sum_{k \neq j}^{n-1} \frac{\partial f_j(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} < 0 \]
9.4. Appendix 4

In this Appendix we show that \( \frac{\partial L}{\partial W_i} - \frac{\partial L}{\partial W_i} \) is negative in type B industries (industries where \( \frac{\partial P_k(.)}{\partial W_i} > 0 \)).

\[
\frac{\partial L}{\partial W_i} - \frac{\partial L}{\partial W_i} = \sum_{j \neq i}^{n-1} \left\{ \left( P_j(.) - \bar{W} \frac{\partial L_{2j}(.)}{\partial h_j} \right) \sum_{k \neq j}^{n-1} \frac{\partial f_j(.) \partial P_k(.)}{\partial W_i} \right\} + \\
+ \sum_{j \neq i}^{n-1} \left\{ \left( \bar{W} \frac{\partial L_{2j}(.) \partial h_j(.)}{\partial g_j} - W_j \right) \frac{\partial g_j(.)}{\partial W_i} \right\}.
\]

Under symmetry:

\[
\frac{\partial L}{\partial W_i} - \frac{\partial L}{\partial W_i} = (n - 1) \left( P_i(.) - \bar{W} \frac{\partial L_{2i}(.)}{\partial h_i} \right) \sum_{k \neq i}^{n-1} \frac{\partial f_i(.) \partial P_k(.)}{\partial W_i} + \\
+ \left( \bar{W} \frac{\partial L_{2i}(.) \partial h_i(.)}{\partial g_i} - W_i \right) (n - 1) \frac{\partial g_i(.)}{\partial W_i}.
\]

Using the fact that under symmetry \( \frac{\partial G(.)}{\partial W_i} = \frac{\partial g_i(.)}{\partial W_i} + (n - 1) \frac{\partial g_i(.)}{\partial W_i} \):

\[
\frac{\partial L}{\partial W_i} - \frac{\partial L}{\partial W_i} = (n - 1) \left( P_i(.) - \bar{W} \frac{\partial L_{2i}(.)}{\partial h_i} \right) \sum_{k \neq i}^{n-1} \frac{\partial f_i(.) \partial P_k(.)}{\partial W_i} + \\
+ \left( \bar{W} \frac{\partial L_{2i}(.) \partial h_i(.)}{\partial g_i} - W_i \right) \left( \frac{\partial G(.)}{\partial W_i} - \frac{\partial g_i(.)}{\partial W_i} \right).
\]

Equation 10 can now be used to replace the term \( \left( \bar{W} \frac{\partial L_{2i}(.) \partial h_i(.)}{\partial g_i} - W_i \right) \frac{\partial g_i(.)}{\partial W_i} \):

\[
\frac{\partial L}{\partial W_i} - \frac{\partial L}{\partial W_i} = (n - 1) \left( P_i(.) - \bar{W} \frac{\partial L_{2i}(.)}{\partial h_i} \right) \sum_{k \neq i}^{n-1} \frac{\partial f_i(.) \partial P_k(.)}{\partial W_i} + \\
+ \left( \bar{W} \frac{\partial L_{2i}(.) \partial h_i(.)}{\partial g_i} - W_i \right) \frac{\partial G(.)}{\partial W_i} + \left( P_i(.) - \bar{W} \frac{\partial L_{2i}(.)}{\partial h_i} \right) \sum_{k \neq i}^{n-1} \left\{ \frac{\partial f_i(.) \partial P_k(.)}{\partial W_i} - g_i(.) \right\}.
\]
Dividing out the common factor from the first and third terms:

\[
\frac{\partial L}{\partial W_i} - \frac{\partial L_i}{\partial W_i} = \left( p_j(.) - W \frac{\partial L_{ij}(.)}{\partial h_j} \right) \frac{\partial f_j(.)}{\partial p_k} \left[ (n - 1) \frac{\partial P_k(.)}{\partial W_i} + (n - 1) \left( \frac{\partial P_i(.)}{\partial W_i} + (n - 2) \frac{\partial P_k(.)}{\partial W_i} \right) \right] + \\
+ \left( W \frac{\partial L_{ij}(.)}{\partial h_j} \frac{\partial h_j(.)}{\partial g_j} - W_j \right) \frac{\partial G(.)}{\partial W_i} - g_j(.)
\]

Applying symmetry and factoring out the common factor from the terms in brackets:

\[
\frac{\partial L}{\partial W_i} - \frac{\partial L_i}{\partial W_i} = \\
= \left( p_j(.) - W \frac{\partial L_{ij}(.)}{\partial h_j} \right) \frac{\partial f_j(.)}{\partial p_k} (n - 1) \left( \frac{\partial P_i(.)}{\partial W_i} + (n - 1) \frac{\partial P_k(.)}{\partial W_i} \right) + \\
+ \left( W \frac{\partial L_{ij}(.)}{\partial h_j} \frac{\partial h_j(.)}{\partial g_j} - W_j \right) \frac{\partial G(.)}{\partial W_i} - g_j(.)
\]

The last expression is negative as long as \( \frac{\partial P_i(.)}{\partial W_i} + (n - 1) \frac{\partial P_k(.)}{\partial W_i} < 0 \), which we also prove here using equations (C) and (D) from Appendix 1:

\[
\frac{\partial P_i(.)}{\partial W_i} + (n - 1) \frac{\partial P_k(.)}{\partial W_i} = \\
= \frac{1}{\frac{\partial A_i}{\partial L_{2i}} \frac{\partial A_i}{\partial P_i} + (n - 1) \frac{\partial A_i}{\partial P_k}} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_i} + (n - 1) \frac{\partial A_i}{\partial P_k} \times \\
\times \frac{\frac{\partial A_i}{\partial L_{2i}} \frac{\partial A_i}{\partial P_k}}{\frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k} \frac{\partial B_i}{\partial L_{2i}} + \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k}} + (n - 1) \frac{\partial A_i}{\partial P_k} \times \\
\times \left[ \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial W_i} + (n - 1) \frac{\partial B_i}{\partial W_k} + \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial W_k} \frac{\partial B_i}{\partial L_{2i}} + (n - 1) \frac{\partial B_i}{\partial W_k} \right] + \\
\]

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\[ + \frac{\partial B_i \partial A_i \partial B_i}{\partial L_{2i} \partial P_k \partial L_{2i}} \left( \frac{\partial A_i}{\partial W_i} + (n-1) \frac{\partial A_i}{\partial W_k} \right) - \frac{\partial B_i \partial A_i \partial B_i}{\partial L_{2i} \partial L_{2i} \partial P_k} \left( \frac{\partial A_i}{\partial W_i} + (n-1) \frac{\partial A_i}{\partial W_k} \right) + \]

\[ + \frac{\partial A_i \partial B_i \partial B_i}{\partial L_{2i} \partial W_i \partial L_{2i}} \left( \frac{\partial A_i}{\partial P_i} + (n-1) \frac{\partial A_i}{\partial P_k} \right) - \frac{\partial A_i \partial B_i \partial A_i}{\partial L_{2i} \partial P_i \partial L_{2i}} \left( \frac{\partial A_i}{\partial W_i} + (n-1) \frac{\partial A_i}{\partial W_k} \right) - \frac{\partial A_i \partial B_i \partial A_i}{\partial L_{2i} \partial L_{2i} \partial P_k} \left( \frac{\partial A_i}{\partial W_i} + (n-1) \frac{\partial A_i}{\partial W_k} \right) + \]

\[ - \frac{\partial A_i \partial B_i \partial A_i}{\partial W_i \partial L_{2i} \partial L_{2i}} \left( \frac{\partial A_i}{\partial P_i} + (n-1) \frac{\partial A_i}{\partial P_k} \right) + \frac{\partial A_i \partial B_i \partial A_i}{\partial W_i \partial L_{2i} \partial P_k} \left( \frac{\partial A_i}{\partial W_i} + (n-1) \frac{\partial A_i}{\partial W_k} \right) + \]

\[ + (n-1) \frac{\partial A_i \partial B_i}{\partial L_{2i} \partial L_{2i}} \left( \frac{\partial A_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} - \frac{\partial A_i}{\partial P_k} \frac{\partial B_i}{\partial W_i} \right) - (n-1) \frac{\partial A_i \partial A_i}{\partial L_{2i} \partial L_{2i}} \left( \frac{\partial B_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} - \frac{\partial B_i}{\partial P_k} \frac{\partial B_i}{\partial W_i} \right) - \]

\[ -(n-1) \frac{\partial B_i \partial B_i}{\partial L_{2i} \partial L_{2i}} \left( \frac{\partial A_i}{\partial P_i} \frac{\partial A_i}{\partial W_i} - \frac{\partial A_i}{\partial P_k} \frac{\partial A_i}{\partial W_i} + (n-1) \frac{\partial B_i \partial A_i}{\partial L_{2i} \partial L_{2i}} \left( \frac{\partial B_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} - \frac{\partial B_i}{\partial P_k} \frac{\partial B_i}{\partial W_i} \right) \right) \]

Eliminating the repeated terms:

\[ \frac{\partial P_i(.)}{\partial W_i} + (n-1) \frac{\partial P_i(.)}{\partial W_k} = \]

\[ = \frac{1}{\frac{\partial A_i}{\partial L_{2i}} + (n-1) \frac{\partial A_i}{\partial P_k}} \times \frac{1}{\frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k} + \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_i} + \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial L_{2i}}} \times \]

\[ - \frac{\partial A_i \partial A_i \partial B_i}{\partial L_{2i} \partial P_k \partial L_{2i}} \left( \frac{\partial B_i}{\partial W_i} + (n-1) \frac{\partial B_i}{\partial W_k} \right) + \frac{\partial A_i \partial A_i \partial B_i}{\partial L_{2i} \partial L_{2i} \partial P_k} \left( \frac{\partial B_i}{\partial W_i} + (n-1) \frac{\partial B_i}{\partial W_k} \right) + \]

\[ + \frac{\partial B_i \partial A_i \partial B_i}{\partial L_{2i} \partial P_k \partial L_{2i}} \left( \frac{\partial A_i}{\partial W_i} + (n-1) \frac{\partial A_i}{\partial W_k} \right) - \frac{\partial B_i \partial A_i \partial B_i}{\partial L_{2i} \partial L_{2i} \partial P_k} \left( \frac{\partial A_i}{\partial W_i} + (n-1) \frac{\partial A_i}{\partial W_k} \right) + \]

\[ + \frac{\partial A_i \partial B_i \partial A_i}{\partial L_{2i} \partial L_{2i} \partial P_k} \left( \frac{\partial B_i}{\partial W_i} + (n-1) \frac{\partial B_i}{\partial W_k} \right) - \frac{\partial A_i \partial B_i \partial A_i}{\partial L_{2i} \partial L_{2i} \partial P_k} \left( \frac{\partial A_i}{\partial W_i} + (n-1) \frac{\partial A_i}{\partial W_k} \right) - \]

\[ - \frac{\partial B_i \partial B_i \partial A_i}{\partial L_{2i} \partial L_{2i} \partial P_k} \left( \frac{\partial A_i}{\partial W_i} + (n-1) \frac{\partial A_i}{\partial W_k} \right) + \frac{\partial B_i \partial B_i \partial A_i}{\partial L_{2i} \partial L_{2i} \partial P_k} \left( \frac{\partial A_i}{\partial W_i} + (n-1) \frac{\partial A_i}{\partial W_k} \right) \]

Given the information in Table 1 in Appendix 2, the last expression for \( \frac{\partial P_i(.)}{\partial W_i} + (n-1) \frac{\partial P_i(.)}{\partial W_k} \) is negative and hence \( \frac{\partial L_i}{\partial W_i} - \frac{\partial L_i}{\partial W_k} \) is negative in type B industries.
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9.5. Appendix 5

We prove now that when all the firms of the industry increase the wage in the same amount at the first stage of the game, the best reply for any firm is to decrease the price at the second stage. Under symmetry,

$$\sum_{k=1}^{n} \frac{\partial P_i(. \, .)}{\partial W_k} = \frac{\partial P_i(.)}{\partial W_i} + (n - 1) \frac{\partial P_i(.)}{\partial W_k} =$$

$$= \frac{\partial P_i(.)}{\partial W_i} + (n - 1) \frac{\partial P_i(.)}{\partial W_k}.$$

Replacing $\frac{\partial P_i(.)}{\partial W_k}$ and $\frac{\partial P_i(.)}{\partial W_i}$ by equations (C) and (D) respectively:

$$\sum_{k=1}^{n} \frac{\partial P_i(.)}{\partial W_k} = \frac{1}{\partial L_{2i} \, \partial W_k} \left( \frac{\partial A_i}{\partial L_{2i}} (\frac{\partial A_i}{\partial P_i} + (n - 1) \frac{\partial A_i}{\partial P_k}) - \frac{\partial A_i}{\partial L_{2i}} (\frac{\partial B_i}{\partial P_i} + (n - 1) \frac{\partial B_i}{\partial P_k}) \right) \times$$

$$\times \left[ -\frac{\partial A_i}{\partial L_{2i}} \frac{\partial A_i}{\partial P_k} (\frac{\partial B_i}{\partial W_i} + (n - 1) \frac{\partial B_i}{\partial W_k}) + \frac{\partial A_i}{\partial L_{2i}} \frac{\partial A_i}{\partial P_k} \left( \frac{\partial A_i}{\partial L_{2i}} + (n - 1) \frac{\partial B_i}{\partial W_k} \right) \right]$$

$$+ \frac{\partial B_i}{\partial L_{2i}} \frac{\partial A_i}{\partial P_k} \left( \frac{\partial A_i}{\partial L_{2i}} + (n - 1) \frac{\partial A_i}{\partial W_k} \right) - \frac{\partial B_i}{\partial L_{2i}} \frac{\partial A_i}{\partial P_k} \left( \frac{\partial B_i}{\partial L_{2i}} + (n - 1) \frac{\partial B_i}{\partial W_k} \right)$$

$$- \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_{2i}} \left( \frac{\partial A_i}{\partial W_i} + (n - 1) \frac{\partial A_i}{\partial P_k} \right) + \frac{\partial A_i}{\partial W_i} \frac{\partial B_i}{\partial L_{2i}} \left( \frac{\partial B_i}{\partial W_i} + (n - 1) \frac{\partial B_i}{\partial P_k} \right)$$

$$+ (n - 1) \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial L_{2i}} \left( \frac{\partial A_i}{\partial L_{2i}} - \frac{\partial B_i}{\partial P_k} \frac{\partial B_i}{\partial W_i} \right) - (n - 1) \frac{\partial A_i}{\partial L_{2i}} \frac{\partial A_i}{\partial L_{2i}} \left( \frac{\partial B_i}{\partial P_k} \frac{\partial B_i}{\partial W_i} \right) - \frac{\partial B_i}{\partial P_k} \frac{\partial B_i}{\partial W_i} \right].$$

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The last equation can be simplified to:

\[
\sum_{k=1}^{n} \frac{\partial P_i}{\partial W_k} = \frac{1}{\frac{\partial B_i}{\partial L_{2i}} + (n-1) \frac{\partial A_i}{\partial P_i} + \frac{\partial A_i}{\partial P_k} - \frac{\partial B_i}{\partial P_i} + (n-1) \frac{\partial B_i}{\partial P_k}} \times \\
\times \left[ -\frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k} \frac{\partial B_i}{\partial W_i} + (n-1) \frac{\partial B_i}{\partial W_k} + \frac{\partial A_i}{\partial L_{2i}} \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k} \frac{\partial B_i}{\partial W_i} + (n-1) \frac{\partial A_i}{\partial W_k} \right] \\
+ \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial L_{2i}} \frac{\partial A_i}{\partial P_i} \frac{\partial B_i}{\partial W_i} + (n-1) \frac{\partial B_i}{\partial W_k} - \frac{\partial A_i}{\partial L_{2i}} \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k} \frac{\partial B_i}{\partial W_k} + (n-1) \frac{\partial B_i}{\partial W_k} \\
- \frac{\partial B_i}{\partial L_{2i}} \frac{\partial B_i}{\partial L_{2i}} \frac{\partial A_i}{\partial P_k} \frac{\partial A_i}{\partial W_i} + (n-1) \frac{\partial A_i}{\partial W_k} + \frac{\partial B_i}{\partial L_{2i}} \frac{\partial A_i}{\partial L_{2i}} \frac{\partial B_i}{\partial P_k} \frac{\partial A_i}{\partial W_i} + (n-1) \frac{\partial A_i}{\partial W_k} \right].
\]

Given the signal of the derivatives in Table 1 in Appendix 2, the last expression is negative and the assertion is complete.
9.6. Appendix 6

We provide in this appendix a formal demonstration that in type B industries the wages centrally determined under collusion are lower than the wages fixed at the simultaneous non-cooperative equilibrium.

\[
\frac{\partial L}{\partial W_i} - \frac{\partial L_i}{\partial W_i} = \sum_{j \neq i} \left\{ \left( P_j(\cdot) - \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \right) \sum_{k \neq j} \frac{\partial f_k(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} \right\} + \\
+ \left( P_i(\cdot) - \bar{W} \frac{\partial L_{2i}(\cdot)}{\partial h_i} \right) \sum_{k \neq i} \frac{\partial f_k(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} + \sum_{j \neq i} \left\{ \left( \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \frac{\partial h_j(\cdot)}{\partial W_i} - W_j \right) \frac{\partial g_j(\cdot)}{\partial W_i} \right\}.
\]

Under symmetry,

\[
\frac{\partial L}{\partial W_i} - \frac{\partial L_i}{\partial W_i} = (n - 1) \left( P_j(\cdot) - \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \right) \sum_{k \neq j} \frac{\partial f_k(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} + \\
+ \left( P_i(\cdot) - \bar{W} \frac{\partial L_{2i}(\cdot)}{\partial h_i} \right) \sum_{k \neq i} \frac{\partial f_k(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} + \left( \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \frac{\partial h_j(\cdot)}{\partial W_i} - W_j \right) (n - 1) \frac{\partial g_j(\cdot)}{\partial W_i}.
\]

Factoring out the common factor from the first and second terms and using the fact that

\[
\frac{\partial G(\cdot)}{\partial W_i} = \frac{\partial g_i(\cdot)}{\partial W_i} + (n - 1) \frac{\partial g_j(\cdot)}{\partial W_i}:
\]

\[
\frac{\partial L}{\partial W_i} - \frac{\partial L_i}{\partial W_i} = \left( P_j(\cdot) - \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \right) \left[ \sum_{k \neq i} \frac{\partial f_k(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} + (n - 1) \sum_{k \neq j} \frac{\partial f_k(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} \right] + \\
+ \left( \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \frac{\partial h_j(\cdot)}{\partial W_i} - W_j \right) \frac{\partial G(\cdot)}{\partial W_i} - \frac{\partial g_i(\cdot)}{\partial W_i}.
\]

Substituting equation (13) in the last expression:

\[
\frac{\partial L}{\partial W_i} - \frac{\partial L_i}{\partial W_i} = \left( P_j(\cdot) - \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \right) \left[ \sum_{k \neq i} \frac{\partial f_k(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} + (n - 1) \sum_{k \neq j} \frac{\partial f_k(\cdot)}{\partial P_k} \frac{\partial P_k(\cdot)}{\partial W_i} \right] + \\
+ \left( \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \frac{\partial h_j(\cdot)}{\partial g_j} - W_j \right) \frac{\partial G(\cdot)}{\partial W_i} - g_j(\cdot).
\]
Applying symmetry to the terms in brackets:

\[
\frac{\partial L}{\partial W_i} - \frac{\partial L_i}{\partial W_i} = \left( P_i(\cdot) - \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \right) \frac{\partial f_j(\cdot)}{\partial P_k} \left[ (n-1) \frac{\partial P_k(\cdot)}{\partial W_i} + (n-1) \left( \frac{\partial P_i(\cdot)}{\partial W_i} + (n-2) \frac{\partial P_i(\cdot)}{\partial W_i} \right) \right] \\
+ \left( \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \frac{\partial h_j(\cdot)}{\partial g_j} - W_j \right) \frac{\partial G(\cdot)}{\partial W_i} - g_j(\cdot) \right) \Rightarrow
\]

\[
\Rightarrow \frac{\partial L}{\partial W_i} - \frac{\partial L_i}{\partial W_i} = \left( P_i(\cdot) - \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \right) \frac{\partial f_j(\cdot)}{\partial P_i} (n-1) \left[ \frac{\partial P_i(\cdot)}{\partial W_i} + (n-1) \frac{\partial P_i(\cdot)}{\partial W_i} \right] + \\
+ \left( \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \frac{\partial h_j(\cdot)}{\partial g_j} - W_j \right) \frac{\partial G(\cdot)}{\partial W_i} - g_j(\cdot).
\]

Because \( \frac{\partial P_i(\cdot)}{\partial W_i} + (n-1) \frac{\partial P_i(\cdot)}{\partial W_i} \) is always negative (see appendix 4) and \( \bar{W} \frac{\partial L_{2j}(\cdot)}{\partial h_j} \frac{\partial h_j(\cdot)}{\partial g_j} - W_j \) is negative in type B industries, the differential \( \frac{\partial L}{\partial W_i} - \frac{\partial L_i}{\partial W_i} \) is also negative. It is hence proved that the wages fixed at the collusive equilibrium are lower than the ones determined at the simultaneous non-cooperative equilibrium.
References


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