Strategic Delegation in Two-sided Markets

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Abstract. In a two-sided market duopoly, we investigate the effects of delegating long run restrictive and unrestrictive decisions to managers by the platforms’ owners, the effects of the platforms’ ownership establishing long run decisions without managers and the impacts of asymmetric regimes between platforms in terms of profitability, consumer surplus and total welfare. The fact that our analysis is focused on platforms introduces inter-group externalities. We find that for sufficiently low intensity of the inter-group externality, the owners of symmetric platforms should take the long run decisions by themselves. However, for an intermediate level of the inter-group externality, the owners of symmetric platforms should delegate the long run decision to their managers. Finally, for sufficiently high level of the inter-group externality, only tipping equilibria occur. Under an asymmetric environment, that is, one platform owner establishes long run decisions and the other owner delegate’s long run decisions to their manager the long run decisions should be taken by the platform’s owners.

Keywords: Two-sided markets, tipping, spatial competition, strategic delegation, managerial incentives.

JEL Classification Numbers: D43, L11, L13, R12, R32, R52.

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1 Introduction

A two-sided market is a market where intermediaries play a determinant role by allowing the interaction of two distinct sides that are mutually attracted. Most of the literature on two-sided markets focus on the waterbed effects, either under a monopoly or a duopoly model (e.g., Rochet and Tirole [18], Armstrong [2], Armstrong and Wright [3]). In such markets, tipping equilibria (or corner solutions) may arise when the intensity of the inter-group externality is sufficiently strong (e.g., Ambrus and Argenziano [1], among others). Our paper combines two strands of literature: the horizontal spatial differentiation topic and two-sided markets (in the spirit of Armstrong [2]) but assuming endogenous locations of platforms as in Zacarias and Serfes [19] to study the issue of delegation in two-sided markets.

Platforms (and firms) usually have to make two kinds of decisions: short-run and long-run decisions. Traditional economic theories assume that the single objective of the companies is profit maximization. Until now, two-sided markets researchers follow this idea. However, Baumol [5] shows that managers may be guided by objectives rather than pure profit-maximization and establishes a sales-maximization model as an alternative to traditional theories. Thus, to the best of our knowledge, no manuscript still devoted time to understand the role of strategic delegation in a two-sided market.

Strategic delegation implies that a platform has an owner and a manager. The literature on this topic establishes that owners have incentives to delegate short-run decisions (prices, marketing strategies or quantities) to their managers (e.g., Vickers [22], Fershtman and Judd [9], Sklivas [20]) by examining a two stage game where, in the first stage, the owner makes the manager’s contract and announces it publicly and in the second stage, after observing the contract, the manager maximizes their payoff given the reward contract\(^2\).

The discussion we promote here is two understand two questions, already covered for the case of traditional firms (e.g., Bárcena-Ruiz and Cazado-Izaga [4], Matsumura and

\(^2\)A realistic assumption in these manuscripts is that the managers’ rewards are proportional to a linear combination of profits and volume sales.
Matsushima [16, 17]) and now, solving them, on a two-sided market basis.

The first question we point out is the overall impact of the restrictions that managers face over long run decisions. The second relevant question is, given those long run restrictions, whether owners should delegate them to managers or to keep them under their scrutiny. In study such topics, we consider the location of the platform as the long run decision variable.

As the literature on spatial competition points out, the location of the firm can also be interpreted as product variety (e.g., Bárcena-Ruiz and Cazado-Izaga [4]). The restriction that managers face over long run decisions is defined by the Hotelling line length, that is, if the distance between the locations of the two platforms is within the unit line, it means that managers have to decide restricted long run decisions (in the spirit of d’Aspremont et al., [7]). However, if we let the locations of the platforms to be outside the unit line, the long run decision turns to be unrestrictive (in the spirit of Lambertini [13], Tabuchi and Thísse [21], among others).

Since we introduce this setting into a two-sided markets model, ultimately, we intend to understand the overall impact of the inter-group externalities on managers’ incentives and long run decisions. Thus, ignoring informational asymmetries, we focus on the owners’ incentives to delegate to managers long-run decisions (locations) in addition to short-run decisions (prices), with the presence of network effects, a key characteristic of two-sided markets. Each owner can delegate either location and price decisions, or only the price decisions.

Our manuscript finds that for sufficiently low intensity of the inter-group externality, the owners of symmetric platforms should take the long run decisions by themselves. However, and contrary to what traditional literature (that does not consider the presence of inter-group externalities) argues (e.g., Bárcena-Ruiz and Cazado-Izaga [4]), for an intermediate level of the inter-group externality, the owners of symmetric platforms should delegate the long run decision to their managers. Finally, for sufficiently high level of the inter-group externality, only tipping equilibria occur. In line with markets without network effects, under an asymmetric environment, that is, when one platform owner es-
tablishes long run decisions and the other delegate’s long run decisions to their manager. The long run decisions should be taken by the platform’s owners.

The remainder of the paper is organized as follows. Sections 2-8 present the distinct subgames and respective results. Finally, conclusions are drawn in section 9. Appendix for details is attached in section 10.

2 Model

Consider a two-sided market operating with two platforms, \( A \) and \( B \). We assume that each consumer buys exactly one unit of the good, which can be produced by either platform \( A \) or \( B \), with zero marginal costs\(^3\).

There is a unit mass of pure singlehoming consumers on both sides uniformly distributed across the linear city, \( x \in [0, 1] \). The two sides are symmetric in all respects: the strength of the intergroup externality, \( \alpha \geq 0 \) and the transportation cost parameter, \( t \) (Zacarias and Serfes [19]).

We follow Armstrong [2], with quadratic transportation costs à la d’Aspremont et al. [7]. Thus, the utilities of an agent of side \( j \) (\( j = 1, 2 \)) attending to platform \( i \), \( (i = A, B) \) are given by:

\[
\begin{align*}
u^A_1(x) &= v + \alpha D^A_2 - p^A_1 - t(x_1 - a)^2 \\
u^B_1(x) &= v + \alpha D^B_2 - p^B_1 - t(b - x_1)^2 \\
u^A_2(x) &= v + \alpha D^A_1 - p^A_2 - t(x_2 - a)^2 \\
u^B_2(x) &= v + \alpha D^B_1 - p^B_2 - t(b - x_2)^2,
\end{align*}
\]

where \( v \) is the stand-alone reservation value\(^4\), \( u^i_j \) is the utility level in platform \( i \in \{ A, B \} \) of an agent of group \( j \in \{ 1, 2 \} \) that is located at \( x \in [0, 1] \), \( p^i_j \) is the price that platform \( i \) charges to group \( j \) and \( D^i_j \) is the number of agents of group \( j \) that choose platform \( i \).

---

\(^3\)Notice that the results would be qualitatively equivalent if we consider a positive (and no discriminatory) marginal cost on both platforms.

\(^4\)The stand-alone is assumed to be sufficiently high such that the whole market is considered to be full covered.
The indifferent consumer of each side of the market is given by:

\[
\begin{align*}
\bar{x}_1 &= \frac{1}{2t(b-a)} \{ \alpha(D^2_2 - D^2_1) + p^B_1 - p^A_1 + t [b^2 - a^2] \} \\
\bar{x}_2 &= \frac{1}{2t(b-a)} \{ \alpha(D^1_2 - D^1_1) + p^B_2 - p^A_2 + t [b^2 - a^2] \}.
\end{align*}
\]

Thus, the resulting interior market shares of the platforms in each side of the market are given by:

\[
\begin{align*}
D^A_1 &= \frac{1}{2t(b-a)} \{ \alpha(D^2_2 - D^2_1) + p^B_1 - p^A_1 + t [b^2 - a^2] \} \\
D^A_2 &= 1 - D^A_1 \\
D^B_1 &= \frac{1}{2t(b-a)} \{ \alpha(D^1_2 - D^1_1) + p^B_2 - p^A_2 + t [b^2 - a^2] \} \\
D^B_2 &= 1 - D^A_2,
\end{align*}
\]

Thus, \(D^i_j\) is not only the quantity supplied by platform \(i\) on side \(j\) but also the market share of platform \(i\) on side \(j\) because the total inelastic demand is normalized to 1. Then, the profit of platform \(A\) is given by \(\pi^A = p^A_1 D^A_1 + p^A_2 D^A_2\) and the profit of platform \(B\) is given by \(\pi^B = p^B_1 D^B_1 + p^B_2 D^B_2\).

We follow Fershtman and Judd [9] by assuming that the owners offer a “take it or leave it” linear publicly observable incentive contract to their managers\(^5\). In this contract, the manager receives a fixed salary and an incentive related to the platform’s profits and market share. In particular, if profits are positive, the manager of platform \(i\) receive an incentive that is proportional to the linear combination \(U_i = \pi^i + \lambda^i \sum_{j=1}^{2} D^j_i\), where \(i = A, B\) and the weight \(\lambda^i\) is a real number (incentive parameter) chosen by owner \(i\) in order to maximize his profits.

Then, the consumer surplus is given by:

\[
CS = \int_0^{\bar{x}_1} \left( v + \alpha D^A_2 - p^A_1 - t(x_1 - a)^2 \right) dx + \int_{\bar{x}_1}^{1} \left( v + \alpha D^B_2 - p^B_1 - t(b - x_1)^2 \right) dx + \\
\int_0^{\bar{x}_2} \left( v + \alpha D^A_1 - p^A_2 - t(x_2 - a)^2 \right) dx + \int_{\bar{x}_2}^{1} \left( v + \alpha D^B_1 - p^B_2 - t(b - x_2)^2 \right) dx.
\]

\(^5\)The assumption that incentive contracts become common knowledge when the contract is signed is crucial to the results. If this assumption is not considered, the contracts cannot act as commitment devices (see Katz, 1991). Fershtman and Judd [9] argue that incentive contracts are costlier variables to change than prices, thus, remain unaltered for a substantial amount of time and are likely to be observed by rivals. Thus, we can say that the model setting in this paper follows that in Jansen et al. [12], but in a two-sided market setting.
and total social welfare is given by:

\[
TSW = \int_0^{\bar{x}_1} (v + \alpha D^A_2 - t(x_1 - a)^2) \, dx + \int_{\bar{x}_1}^1 (v + \alpha D^B_2 - t(b - x_1)^2) \, dx + \\
\int_0^{\bar{x}_2} (v + \alpha D^A_1 - t(x_2 - a)^2) \, dx + \int_{\bar{x}_2}^1 (v + \alpha D^B_1 - t(b - x_2)^2) \, dx.
\]

Rearranging, allow us to obtain:

\[
CS = 2 \left( v + \alpha \bar{x} (D^A_1 - D^B_2) + \alpha D^B_2 - \bar{x} (p^A - p^B) - p^B \right) - \frac{1}{3} + \bar{x}^2(a - b) + tb - \bar{x}(a - b)(a + b) - tb^2).
\]

and

\[
TSW = 2 \left( v + \alpha \bar{x} (D^A_1 - D^B_2) + \alpha D^B_2 - \frac{1}{3} + \bar{x}^2(a - b) + tb - \bar{x}(a - b)(a + b) - tb^2 \right).
\]

We assume that the owners of both the platforms may hire a manager to delegate the price and the location decisions (depending on the perfect subgame). We intend to solve six distinct perfect subgames: (i) the owners establish a restrictive policy on the long run decision that is not delegated to a manager, corresponding to the case where the owner has no manager or, alternatively, the owner and the manager correspond to the same person (one man show business model); (ii) the owners establish a restrictive policy on the long run decision that is delegated to a manager; (iii) the owners establish an unrestrictive policy on the long run decision that is delegated to a manager; (iv) the owners establish an unrestrictive policy on the long run decision that is not delegated to a manager (both platforms have a manager that only decide the short run decision); (v) the owners establish an unrestrictive policy on the long run decision when no manager is hired and, finally, (vi) we cover the case of asymmetric regimes between platforms, that is, one of the platforms delegates the long run decision to the manager and the other does not delegate the long run decision to the manager.

The timing of the game is similar in all the perfect subgames. As in Bárcena-Ruiz and Cazado-Izaga [4] and Matsumura and Matsushima [17], the timing considers that location decisions are chosen simultaneously and that prices are also chosen simultaneously. Only,
The incentive parameters are the variables which can be chosen sequentially in the case in which one owner delegate's location decisions but the other owner does not delegate.

Nonetheless, we shall first analyze the last stage of the game, the stage in which firms compete in prices, that is, when managers take the short run decisions. We also dispose in all the perfect subgames the (i) profit concavity condition and the (ii) interior equilibrium condition.

3 Restrictive policy

3.1 No delegation

This case corresponds to a "one man show business model" with a restrictive policy on the long run decision. Thus, the platforms' owners do not hire any manager, or, alternatively, the owner and the manager correspond to the same individual.

The timing of the game comes as follows. In stage one, the owners simultaneously choose their locations (long run decision) assuming that the platforms are not permitted to locate outside the unit Hotelling line (restrictive policy) and in stage two, the owners choose the prices (short run decision). Since no independent manager exists in the subgame, the incentive parameter corresponds to $\lambda^v = 0$.

We start by establishing the profit concavity condition that guarantees profit maximization.

Assumption 1 $\alpha < t$.

However, this condition does not guarantee that a platform will not tip the market. Thus, we establish the following Proposition.

Proposition 1 If $\alpha < \frac{1}{3} t$, maximum differentiation, i.e., $(a, b) = (0, 1)$ is a location equilibrium.
The equilibrium outcomes are obtained from Armstrong [2] considering the same level of the inter-group externality ($\alpha$) and degree of product differentiation ($t$) or, directly from Zacarias and Serfes [19].

$$\begin{align*}
    a^* &= 0 & b^* &= 1 \\
    p_{AR-ND^*}^* &= t - \alpha & p_{BR-ND^*}^* &= t - \alpha \\
    D_{AR-ND^*}^* &= \frac{1}{2} & D_{BR-ND^*}^* &= \frac{1}{2} \\
    \pi_{AR-ND^*}^* &= t - \alpha & \pi_{BR-ND^*}^* &= t - \alpha \\
    U_{AR-ND^*}^* &= 0 & U_{BR-ND^*}^* &= 0 \\
    CS_{R-ND^*}^* &= 2v + 3\alpha - \frac{13}{6}t & TSW_{R-ND^*}^* &= 2v + \alpha - \frac{t}{6} \\
    \bar{x}_{j}^{R-ND^*} &= \frac{1}{2}
\end{align*}$$

where the superscript "R - ND" denotes restricted policy on the long run decision under no delegation.

### 3.2 Delegation

This case corresponds to with a restrictive policy on the long run decision that is delegated by the owners to their correspondent managers.

The timing of the game is the following. In the first stage, each owner seeks to maximize their profits by choosing the incentive parameter in managers’ contract. In the second stage, manager $i$ chooses the location of its platform (long run decision) assuming the platforms are not permitted to locate outside the unit Hotelling line (restrictive policy) and in the third stage, manager $i$ chooses the short run decision (prices). The game is solved by backward induction.

We start by establishing the profit concavity condition.

**Assumption 2** $\alpha < t$.

However, the unique interior equilibrium holds under a more restrictive condition.
Proposition 2 If $\alpha < \frac{1}{2}t$, maximum differentiation, i.e., $(a, b) = (0, 1)$ is a location equilibrium.

Proof. see Appendix 10.3.

3.2.1 Stage three - price competition

Given the locations of the platforms, $a$ and $b$, the managers' face Bertrand competition in the last stage. The equilibrium prices are given by:

$$
\begin{align*}
p_A^A &= p_A^B = p_A^{A*}(\lambda^i, a, b, \alpha) = \frac{t(b-a)(2+b-a)-\lambda^A-2\lambda^A-3\alpha}{6(t(b-a)-\alpha)} \\
p_B^A &= p_B^B = p_B^{B*}(\lambda^i, a, b, \alpha) = \frac{t(b-a)(4-b-a)-\lambda^A-2\lambda^B-3\alpha}{6(t(b-a)-\alpha)}.
\end{align*}
$$

(1)

The equilibrium market shares are given by:

$$
\begin{align*}
D_A^A &= D_A^B = D_A^{A*}(\lambda^i, a, b, \alpha) = \frac{t(b-a)(2+b-a)+\lambda^A-\lambda^B-3\alpha}{6(t(b-a)-\alpha)} \\
D_B^A &= D_B^B = D_B^{B*}(\lambda^i, a, b, \alpha) = \frac{t(b-a)(4-b-a)+\lambda^A+\lambda^B-3\alpha}{6(t(b-a)-\alpha)}.
\end{align*}
$$

(2)

Therefore, the equilibrium profits are:

$$
\begin{align*}
\pi_A^{A*}(\lambda^i, a, b, \alpha) &= 2p_A^{A*}(\lambda^i, a, b, \alpha)D_A^{A*}(\lambda^i, a, b, \alpha) \\
\pi_A^{A*}(\lambda^i, a, b, \alpha) &= 2p_B^{B*}(\lambda^i, a, b, \alpha)D_B^{B*}(\lambda^i, a, b, \alpha)
\end{align*}
$$

(3)

The managers' rewards are given by:

$$
\begin{align*}
U_A^{A*}(\lambda^i, a, b, \alpha) &= \left[\frac{t(b-a)(2+b-a)+\lambda^A-\lambda^B-3\alpha}{9(t(b-a)-\alpha)}\right]^2 \\
U_B^{B*}(\lambda^i, a, b, \alpha) &= \left[\frac{t(b-a)(4-b-a)+\lambda^A+\lambda^B-3\alpha}{9(t(b-a)-\alpha)}\right]^2.
\end{align*}
$$

(4)

3.2.2 Stage two and stage one - location and managerial incentives

As Matsumura and Matsushima [15] point out, "when the difference in incentives is not so large, the horizontal maximum differentiation appears in equilibrium" and, thus, $a^* =$
$1-b^* = 0$. Then, the platforms’ prices (1), market shares (2), profits (3) and the managers’ rewards (4) at stage two are given by:

\[
\begin{align*}
    p^A(\lambda^i, \alpha) &= \frac{3(t-\alpha)-\lambda^B-2\lambda^A}{3} \\
    D^A(\lambda^i, \alpha) &= \frac{3(t-\alpha)-\lambda^B+\lambda^A}{6(t-\alpha)} \\
    \pi^A(\lambda^i, \alpha) &= \frac{(3(t-\alpha)-\lambda^B-2\lambda^A)(3(t-\alpha)-\lambda^B+\lambda^A)}{9(t-\alpha)} \\
    U^A(\lambda^i, \alpha) &= \frac{(3(t-\alpha)-\lambda^B+\lambda^A)^2}{6(t-\alpha)} \\
    \bar{x}_j(\lambda^i, \alpha) &= \frac{1}{2} + \frac{\lambda^A-\lambda^B}{6(t-\alpha)} \\
\end{align*}
\]

(5)

In the first stage, each owner $i$ independently chooses $\lambda^i$ to maximize its profit. The first-order conditions are given by:

\[
\begin{align*}
    \frac{\partial \pi^A}{\partial \lambda^A} &= 0 \Leftrightarrow \lambda^A(\lambda^B) = \frac{\lambda^B-3(t-\alpha)}{2} \\
    \frac{\partial \pi^B}{\partial \lambda^B} &= 0 \Leftrightarrow \lambda^B(\lambda^A) = \frac{\lambda^A-3(t-\alpha)}{2}.
\end{align*}
\]

Solving both equalities simultaneously, we obtain that in equilibrium:

\[
\lambda^A = \lambda^B = \lambda^* = \alpha - t.
\]

(6)

Substituting expression (6) in expression (5) yields the resulting equilibrium outcomes:

\[
\begin{align*}
    a^* &= 0 & b^* &= 1 \\
    p^{A^{R-D^*}} &= 2(t-\alpha) & p^{B^{R-D^*}} &= 2(t-\alpha) \\
    D^{A^{R-D^*}} &= \frac{1}{2} & D^{B^{R-D^*}} &= \frac{1}{2} \\
    \pi^{A^{R-D^*}} &= 2(t-\alpha) & \pi^{B^{R-D^*}} &= 2(t-\alpha) \\
    U^{A^{R-D^*}} &= t-\alpha & U^{B^{R-D^*}} &= t-\alpha \\
    C^{S^{R-D^*}} &= 2v + 5\alpha - \frac{25}{6}t & TSW^{R-D^*} &= 2v + \alpha - \frac{t}{6} \\
    \bar{x}_j^{R-D^*} &= \frac{1}{2} &
\end{align*}
\]

where the superscript "$R - D^n$" denotes restricted policy on the long run decision under delegation.

### 3.3 Comparison of the regimes

We perform a comparative-statics analysis of the delegation and the no delegation strategies under a restrictive policy.
According to Proposition 1 and Proposition 2, both strategies are comparable for \( \alpha < \frac{1}{3} t \).

**Proposition 3 (Restrictive policy: delegate or not long run decisions?)** Let \( \alpha \in [0, \frac{1}{3} t) \).

(i) The equilibrium prices charged by the platforms with a delegated decision are larger than those with a no-delegated decision. (ii) The equilibrium market shares remain unchanged. (iii) The consumer surplus in the delegation case is smaller than that in the non-delegation case. (iv) The equilibrium profits of the platforms in the delegation case are larger than those in the non-delegation case. (v) Total social welfare in the delegation case and in the non-delegation case is the same.

**Proof.** see Appendix 10.3. ■

Proposition 3 shows that, from the platform’s point of view, delegation strictly dominates no delegation under a restrictive policy towards the long run decision.

The platforms maximize the degree of product differentiation within the Hotelling line and the incentive contract parameter only affects the equilibrium prices (a higher value of \( \lambda^i \) leads to a lower price \( p^i \) because the manager \( i \) tends to put more stress on their volume of sales for profit maximization purposes).

The intuition of the result comes from the fact that, because of the strategic complementarities of prices, the competition between the two platforms is intensified. Anticipating this, each owner sets a lower \( \lambda^i \) to "mitigate" the subsequent price competition and, thus, the owners delegate the long run decision. As a result of this action, for a sufficiently low level of the inter-group externality, the strategy delegation strictly dominates no delegation.

For \( \alpha \in [\frac{1}{3} t, \frac{1}{2} t) \), tipping strictly dominates the no delegation strategy (the competitive environment turns into a monopoly environment). The owners try to conquering the whole market not by adopting a delegation strategy but instead with changes on its location position on the restrictive Hotelling line to force the rival to leave the market. Under delegation, the competitive equilibrium stands because of the higher managers’ aggressively that allows the two platforms to sustain a higher equilibrium profit in comparison
with tipping profitability.

However, for \( \alpha \geq \frac{1}{2} t \), tipping strictly dominates all the other strategies (delegation and no delegation). Thus, with a higher level of the inter-group externality, the competition is so severe that the managers’ aggressively on the volume of sales is not sufficient to guarantee the platforms survival.

**Lemma 4** Let \( \alpha \in \left[ \frac{1}{3} t, \frac{1}{2} t \right) \). Delegation of the long run decision is the optimal strategy to platforms under a restrictive policy on the long run decision. Let \( \alpha \geq \frac{1}{2} t \). Tipping is the optimal strategy under a restrictive policy on the long run decision.

4 **Unrestrictive policy**

4.1 **Delegation**

Consider the case in which the platforms’ long run decisions are unrestricted, that is, the restriction \( \{a, b\} \in [0, 1] \) is not binding for both platforms. This corresponds to with an unrestrictive policy on the long run decision that is delegated to the manager on both platforms.

The timing of the game is the following. In the first stage, each owner seeks to maximize their profits by choosing the incentive parameter in managers’ contract. In the second stage, manager \( i \) chooses the location of its platform (long run decision) assuming the platforms are permitted to locate outside the unit Hotelling line (unrestrictive policy) and in the third stage, manager \( i \) chooses the short run decision (prices). The game is solved by backward induction.

We start by establishing the profit concavity condition.

**Assumption 3** \( \alpha < \frac{3}{4} t \).

The unique interior equilibrium holds under this condition.
Proposition 5  If \( \alpha < \frac{3}{2}t \), maximum differentiation, i.e., \( (a, b) = \left( -\frac{1}{4}, \frac{5}{4} \right) \) is a location equilibrium.

Proof. see Appendix 10.4. ■

The third stage (price competition) is unchanged. Thus, the first-order conditions of stage two lead to:

\[
\begin{align*}
\frac{\partial U_A}{\partial a} = 0 & \iff a(\lambda^i, \alpha) = \frac{4(\lambda^A - \lambda^B + \alpha) - 3t}{12t - 16\alpha} \\
\frac{\partial U_B}{\partial b} = 0 & \iff b(\lambda^i, \alpha) = \frac{15t - 20\alpha - 4(\lambda^B - \lambda^A)}{12t - 16\alpha}.
\end{align*}
\]

The resulting equilibrium prices, market shares and profits of the platforms and the managers' rewards are given by:

\[
\begin{align*}
p^A(\lambda^i, \alpha) &= \frac{(3t - 4\alpha)[9t - 2(\lambda^B + 3\alpha)] + 6(\lambda^A - \lambda^B)}{6(3t - 4\alpha)} \\
p^B(\lambda^i, \alpha) &= \frac{(3t - 4\alpha)[9t - 2(\lambda^A + 3\alpha)] + 6(\lambda^B - \lambda^A)}{6(3t - 4\alpha)} \\
D^A(\lambda^i, \alpha) &= \frac{(3t - 4\alpha)[9t + 2(\lambda^A - \lambda^B - 3\alpha)] + 6(\lambda^A - \lambda^B)}{2(3t - 4\alpha)(9t - 6\alpha)} \\
D^B(\lambda^i, \alpha) &= \frac{(3t - 4\alpha)[9t + 2(\lambda^B - \lambda^A - 3\alpha)] + 6(\lambda^B - \lambda^A)}{2(3t - 4\alpha)(9t - 6\alpha)} \\
\pi^A(\lambda^i, \alpha) &= \frac{(3t - 4\alpha)[9t - 2(\lambda^B + 3\alpha)] + 6(\lambda^A - \lambda^B)}{6(3t - 4\alpha)(9t - 6\alpha)} \\
\pi^B(\lambda^i, \alpha) &= \frac{(3t - 4\alpha)[9t - 2(\lambda^A + 3\alpha)] + 6(\lambda^B - \lambda^A)}{6(3t - 4\alpha)(9t - 6\alpha)}.
\end{align*}
\]

In the first stage, each owner \( i \) independently chooses \( \lambda^i \) to maximize its profit. By symmetry, the first-order conditions are given by:

\[
\frac{\partial \pi^i}{\partial \lambda^i} = 0 \iff \frac{1}{6(3t - 4\alpha)^2(9t - 6\alpha)^2} \left[ 3t - 4\alpha \right] \left[ (3t - 4\alpha)(9t - 2(\lambda^i - \lambda^A + 3\alpha)) + 6t(\lambda^i - \lambda^i) \right] + \left[ 2(3t - 4\alpha) + 6t \right] \left[ (3t - 4\alpha)(9t - 2(\lambda^i - \lambda^A + 3\alpha)) + 6t(\lambda^i - \lambda^i) \right] = 0 \IFF
\]

which implies in equilibrium:

\[
\lambda^A = \lambda^B = \lambda^* = \frac{3}{4} (t - 4\alpha).
\]

[13]
The first stage equilibrium outcomes are the following:

\[
\begin{align*}
a^* &= -\frac{1}{4} & b^* &= \frac{5}{4} \\
p^{A_{U-D}} &= \frac{3}{4} t + 2\alpha & p^{B_{U-D}} &= \frac{3}{4} t + 2\alpha \\
D^{A_{U-D}} &= \frac{1}{2} & D^{B_{U-D}} &= \frac{1}{2} \\
\pi^{A_{U-D}} &= \frac{3}{4} t + 2\alpha & \pi^{B_{U-D}} &= \frac{3}{4} t + 2\alpha \\
CS^{U-D} &= 2v - 3\alpha - \frac{49}{24} t & TSW^{U-D} &= 2v + \alpha - \frac{13}{24} t \\
\bar{x}_{j}^{U-D} &= \frac{1}{2}
\end{align*}
\]

where the superscript “U - D” denotes unrestricted policy on the long run decision under delegation.

4.2 No delegation

This case corresponds to with an unrestrictive policy on the long run decision that is delegated by the owners to their correspondent managers.

The timing of the game is the following. In the first stage, each owner seeks to maximize their profits by choosing the incentive parameter in managers’ contract. In the second stage, manager \(i\) chooses the location of its platform (long run decision) assuming the platforms are permitted to locate outside the unit Hotelling line (unrestrictive policy) and in the third stage, manager \(i\) chooses the short run decision (prices). The game is solved by backward induction.

We start by establishing the profit concavity condition.

**Assumption 4** \(\alpha < \frac{3}{4} t\).

The unique interior equilibrium holds under this condition.

**Proposition 6** If \(\alpha < \frac{1}{2} t\), maximum differentiation, i.e., \((a, b) = (-\frac{1}{4}, \frac{5}{4})\) is a location equilibrium.
Proof. see Appendix 10.4. ■

From proposition 1, the no delegation strategy corresponds to the situation where \( \lambda^i = 0 \). Thus, the equilibrium outcomes come as follows:

\[
\begin{align*}
    a^* &= -\frac{1}{4} \\
    p_{A^{NU-ND^*}} &= \frac{3}{2} t - \alpha \\
    D_{A^{NU-ND^*}} &= \frac{1}{2} \\
    \pi_{A^{NU-ND^*}} &= \frac{3}{2} t - \alpha \\
    C_{S^{U-ND^*}} &= 2v + 5\alpha - \frac{85}{24}t \\
    x_{j}^{U-ND^*} &= 2v + \alpha - \frac{13}{24}t
\end{align*}
\]

where the superscript “U – ND” denotes unrestricted policy on the long run decision under no delegation.

4.3 Comparison of the regimes

We perform a comparative-statics analysis of the delegation and the no delegation strategies under a restrictive policy.

Accordingly to Proposition 5 and Proposition 6, both strategies are comparable for \( \alpha < \frac{1}{2} t \).

Proposition 7 (No restrictive decisions: delegate or not?) \( \alpha \in \left[ 0, \frac{1}{2} t \right) \) and suppose that the platforms are allowed to locate outside the linear city. (i) The equilibrium prices and profits in the delegation case are smaller than those in the non-delegation case for a low level of the intergroup externality. (ii) The consumer surplus in the delegation case is lower than that in the non-delegation case for sufficiently high inter-group externalities. (iii) Total welfare is the same in both regimes.

Proof. see Appendix 10.4. ■

Proposition 7 is distinct from Proposition 3. When the managers have no restrictions on the long run decision to be taken, price competition in the delegation case is tougher.
than that in the non-delegation case. Thus, as Matsumura and Matsushima [15] point out, "the incentive contract parameter $\lambda^i$ affects not only the equilibrium prices but also the location strategies" such that the rival platform moves far away from the center to escape the more severe competition that results from the higher value of $\lambda^i$ because of the strategic complementarities on prices. Anticipating this, each owner induces a higher $\lambda^i$ which accelerates competition.

The critical value for the equilibrium prices and profits is $\alpha^* = \frac{4}{5}$ and for the equilibrium consumer surplus is $\alpha^* = \frac{4}{5}$. Thus, from the platform’s point of view under an unrestricted policy, not delegating the long run decision is an optimal strategy for $\alpha \in \left[0, \frac{4}{5}\right)$ and delegation is an optimal strategy for $\alpha \in \left[\frac{4}{5}, \frac{1}{2}\right)$. This is due to the fact that the managers’ aggressively is enlarged with the presence of higher inter-group externalities, which amplifies the impacts of competition.

Indeed, for $\alpha \in \left[\frac{1}{2}, \frac{3}{2}t\right)$, tipping strictly dominates the no delegation strategy (the competitive environment turns into a monopoly environment). The owners try to conquering the whole market not by adopting a delegation strategy but instead with changes on its location position on the restrictive Hotelling line to force the rival to leave the market. Under delegation, the competitive equilibrium stands because of the higher managers’ aggressively that allows the two platforms to sustain a higher equilibrium profit in comparison with tipping profitability.

However, for $\alpha \geq \frac{3}{2}t$, tipping strictly dominates all the other strategies (delegation and no delegation). Thus, with a higher level of the inter-group externality, the competition is so severe that the managers’ aggressively on the volume of sales is not sufficient to guarantee the platforms survival.

**Lemma 8** Let $\alpha \in \left[\frac{1}{2}, \frac{3}{2}t\right)$. Delegation of the long run decision is the optimal strategy to platforms under an unrestricted policy on the long run decision. Let $\alpha \geq \frac{3}{2}t$. Tipping is the optimal strategy under an unrestricted policy on the long run decision.
5 Delegation vs no delegation: what is better for the owners?

This section investigates when delegation and no delegation are optimal strategies, whatever the policy on the long run decision. There is no agreement on the economic literature concerning this topic. Traditional literature points out that the long run decision taken by managers (delegation) generates higher profit if no restrictions emerge over the decision (e.g., Tabuchi and Thisse [21]). Some recent manuscripts establish the opposite, arguing that the existence of a restriction over the long run decision is profit enhancing under the managers’ supervision (e.g., Matsumura and Matsushima [15]).

For $\alpha \in \left[0, \frac{1}{3}t\right)$, from Proposition 3 we find that with a restrictive policy, the strategy delegation strictly dominates the strategy no delegation and from Proposition 7 we find that with an unrestrictive policy, the strategy delegation strictly dominates the strategy no delegation only for $\alpha \in \left[\frac{1}{4}t, \frac{1}{2}t\right)$.

For $\alpha \in \left[\frac{1}{4}t, \frac{1}{2}t\right)$, from Lemma 4 we find that with a restrictive policy, tipping strictly dominates the strategy delegation and no delegation. However, from Lemma 8 with an unrestrictive policy, the strategy delegation strictly dominates the strategy no delegation and both strategies strictly dominate tipping.

Our main finding stands for $\alpha \in \left[\frac{1}{3}t, \frac{1}{2}t\right)$. With a restrictive policy, tipping strictly dominates the strategy delegation and no delegation. However, with an unrestrictive policy, although tipping strictly dominates the strategy no delegation, the strategy delegation still strictly dominates the strategy tipping.

Just when the level of the inter-group externality is sufficiently high ($\alpha \geq \frac{1}{3}t$), the intensification of price competition leads tipping equilibria, that strictly dominates all other possible strategies (delegation and no delegation), whatever the policy adopted by the platforms owner’s on the long run decision (restrictive or unrestrictive).

**Lemma 9** For sufficiently low inter-group externality levels, $\alpha \in \left[0, \frac{1}{3}t\right)$: (i) under a restrictive policy, delegation is the only optimal strategy; (ii) under an unrestrictive policy,
both delegation and no delegation may be optimal strategies depending on the level of the inter-group externality. For an intermediate inter-group externality level, \( \alpha \in \left[ \frac{1}{3}t, \frac{2}{3}t \right] \):

(i) under a restrictive policy, tipping is the optimal strategy; (ii) under an unrestricted policy, delegation is an optimal strategy. For sufficiently high inter-group externality levels, \( \alpha \geq \frac{3}{2}t \), tipping is the optimal strategy. Globally, tipping is more likely to appear under a restrictive policy and delegation is more likely to appear under an unrestricted policy.

Bárcena-Ruiz and Cazado-Izagá [4] argue that long run decisions should always be taken by the owners of two symmetric firms. With the incorporation of the inter-group externalities, our result shows that if the owners adopt an unrestricted policy, then delegation is an optimal strategy for an intermediate level of the inter-group externality. The intuition (as explained previously) is that managers’ aggressively towards price competition is profit enhancing for both rivals.

6 Restrictive vs unrestricted policy: what is better for the managers?

We perform a comparative-static analysis to understand whether under delegation and under no delegation strategies, which of the policies are profit enhancing for both platforms.

Proposition 10 (i) **No delegation: restrict or not decisions?** Let \( \alpha \in \left[ 0, \frac{1}{3}t \right] \) and suppose that the owner never delegates long run decisions. The consumer surplus and the total welfare with a restrictive policy are always higher than with an unrestricted policy. However, the equilibrium profits with an unrestricted policy are always higher than with a restrictive policy. (ii) **Delegation: restrict or not decisions?** Let \( \alpha \in \left[ 0, \frac{1}{3}t \right] \) and suppose that the owner always delegates long run decisions. The consumer surplus with an unrestricted policy is higher than with restrictive decisions for sufficiently low inter-group externalities \( (\alpha \leq \frac{11}{102}t) \). The equilibrium profits with an unrestricted policy is higher than
with restrictive decisions for sufficiently high inter-group externalities \((\alpha \geq \frac{5}{16} t)\). Total welfare is always higher with a restrictive policy.

**Proof.** see Appendix 10.5. ■

Proposition 10 establishes that when no delegation occurs and from the platforms point of view, an unrestricted policy strictly dominates a restrictive policy, whatever the level of the inter-group externality. However, under delegation we observe that the domination of one policy relatively to the other depends on the level of the inter-group externality. In particular, from the platforms point of view, a restrictive policy strictly dominates an unrestricted policy for sufficiently low level of the inter-group externality.

We also highlight that, in terms of the social welfare point of view, a restrictive policy is welfare enhancing relatively to an unrestricted policy, either under delegation or under no delegation.

Then, we also observe the relationship between both policies and tipping.

Firstly, suppose the owners never delegate the long run decision. According to Proposition (1) and Proposition (6), both policies are comparable for \(\alpha < \frac{1}{3} t\). From the platform’s point of view, we find that the unrestricted policy strictly dominates the restrictive policy for this level of the inter-group externality. However, for \(\alpha \in \left[\frac{1}{3} t, \frac{5}{16} t\right]\), tipping strictly dominates the restrictive policy but both strategies are strictly dominated by the unrestricted policy. For \(\alpha \geq \frac{1}{2} t\), tipping is the optimal strategy whatever the policy considered.

Now, suppose the owners always delegate the long run decision. According to Proposition (2) and Proposition (5), both policies are comparable for \(\alpha < \frac{1}{2} t\). In this case, the restrictive policy strictly dominates the unrestricted policy for \(\alpha \in \left[0, \frac{11}{16} t\right]\) and the unrestricted policy strictly dominates the restrictive policy for \(\alpha \in \left[\frac{11}{16} t, \frac{1}{2} t\right]\). For \(\alpha \in \left[\frac{1}{2} t, \frac{3}{2} t\right]\), tipping strictly dominates the restrictive policy but both strategies are strictly dominated by the unrestricted policy. For \(\alpha \geq \frac{3}{2} t\), tipping is the optimal strategy whatever the regime considered.

**Lemma 11** From the platform’s point of view: (i) a restrictive policy is an optimal strat-
egy only under delegation for sufficiently low inter-group externalities; (ii) an unrestrictive policy is more likely to appear under the delegation strategy and (iii) tipping is more likely to appear under the no delegation strategy.

7 Owners establish unrestrictive long run decisions without managers

Consider the case where the owners’ choose the long run decisions since no manager is hired. The difference relatively to subgame 4 (unrestrictive policy with delegation strategy) is that here the owner and the manager are the same individual (one man show business model with an unrestricted policy on the long run decision).

We start by establishing the profit concavity condition.

Assumption 5 $\alpha < \frac{5}{4}t$.

However, the unique interior equilibrium holds under a more restrictive condition.

Proposition 12 If $\alpha < \frac{5}{4}t$, maximum differentiation, i.e., $(a, b) = \left(-\frac{3}{4}, \frac{7}{4}\right)$ is a location equilibrium.

Proof. see Appendix 10.6. ■

The timing of this game considers: in stage one, the owners simultaneously decide the long run decision (platforms’ location), while in stage two the owners simultaneously decide their incentive parameters (since are also the platform’s managers) in order to maximize their correspondent profits. The third stage (simultaneously price decision by the owners of both platforms) has been already derived. The profit function of platforms $A$ and $B$ are given by:

\[
\begin{align*}
\pi^A(\lambda, a, b, \alpha) &= \frac{\left[ ((b-a)(2+b+a)-\lambda B-2\lambda A^2-3\alpha) \left[ ((b-a)(2+b+a)+\lambda A^2-\lambda B-3\alpha) \right] \right]}{9((b-a)-\alpha)} \\
\pi^B(\lambda, a, b, \alpha) &= \frac{\left[ ((b-a)(2+b+a)-\lambda A^2-2\lambda B^2-3\alpha) \left[ ((b-a)(2+b+a)+\lambda B^2-\lambda A^2-3\alpha) \right] \right]}{9((b-a)-\alpha)}.
\end{align*}
\]
At stage two, both owners decide over incentive parameters. Deriving expressions (10) relatively to the correspondent incentive parameters yields:

$$\lambda^A(a, b, \alpha) = \frac{t(a-b)(4+b+a)}{5} + \alpha$$

$$\lambda^B(a, b, \alpha) = \frac{t(a-b)(6-b-a)}{5} + \alpha.$$  \hspace{1cm} (11)

After substituting $\lambda^A(a, b, \alpha)$ and $\lambda^B(a, b, \alpha)$ from expression (11) into expression (10), we obtain that the profit of platform $A$ and $B$ are given by:

$$\pi^A(\lambda^A(a, b, \alpha), a, b, \alpha) = \frac{t(b-a)(2+b+a) - \lambda^B(a, b, \alpha) - 2\lambda^A(a, b, \alpha) - 3\alpha}{g(t(b-a)-\alpha)}$$

$$\pi^B(\lambda^B(a, b, \alpha), a, b, \alpha) = \frac{t(b-a)(2+b+a) - \lambda^A(a, b, \alpha) - 2\lambda^B(a, b, \alpha) - 3\alpha}{g(t(b-a)-\alpha)}$$  \hspace{1cm} (12)

In stage one, the platforms’ owners decide simultaneously their optimal long run decisions (locations). Analytically, this is equivalent to:

$$\begin{align*}
\frac{\partial \pi^A(\lambda^A(a, b, \alpha), a, b, \alpha)}{\partial a} &= 0 \\
\frac{\partial \pi^B(\lambda^B(a, b, \alpha), a, b, \alpha)}{\partial b} &= 0
\end{align*}$$

Combining both derivatives simultaneously, leads to the equilibrium locations:

$$\begin{align*}
a^* &= \frac{-3}{4} \\
b^* &= \frac{7}{4}
\end{align*}$$  \hspace{1cm} (13)

Then, the first stage equilibrium outcomes are the following:

$$\begin{align*}
a^* &= \frac{-3}{4} & b^* &= \frac{7}{4} \\
p^{A0^*} &= 5t - 2\alpha & p^{B0^*} &= 5t - 2\alpha \\
D^{A0^*} &= \frac{1}{2} & D^{B0^*} &= \frac{1}{2} \\
\pi^{A0^*} &= 5t - 2\alpha & \pi^{B0^*} &= 5t - 2\alpha \\
CS^{O^*} &= 2v + 5\alpha - \frac{295}{24}t & TSW^{O^*} &= 2v + \alpha - \frac{49}{24}t \\
\overline{x}_j &= \frac{1}{2}
\end{align*}$$

where the superscript “$O$” denotes unrestricted policy on the long run decision taken by an owner without manager.
7.1 Comparison of the regimes

The strategic delegation of price decisions to managers increases the incentives of one platform to locate farther from the rival. Those platforms are able to charge higher prices, relaxing product market competition and obtaining higher profits. If they do not hire managers (no delegation) we would obtain $a^* = -\frac{1}{4} \cap b^* = \frac{5}{4}$ and each platform earns a profit of $\frac{3}{4}t + 2\alpha$. The results collapse with Lambertini [13] for $\alpha = 0$.

While in the unconstrained Hotelling game the distance between the two rivals is $\frac{3}{2}$, here it is $\frac{5}{2}$. Thus, when platforms are able to establish themselves outside the city limits, the one man show business model with an unrestricted policy increases the degree of product differentiation.

We perform a comparative-static analysis of the strategy "owners decide without manager" with an unrestricted policy on the long run decision relatively to the previous perfect subgames.

Proposition 13 (i) The owners without manager vs delegation with unrestric-
tive policy. Let $\alpha \in \left[0, \frac{5}{4}t\right)$. The equilibrium profit is higher when the owners have no managers relatively to the situation where the platforms' have managers that establish unrestricted long run decisions for sufficiently low level of the inter-group externality ($\alpha \leq \frac{17}{16}t$); the equilibrium consumer surplus is higher when the owners have no managers relatively to the situation where the platforms' have managers that establish unrestricted long run decisions for sufficiently high level of the inter-group externality ($\alpha \in \left[\frac{17}{16}t, \frac{5}{4}t\right)$); the social welfare is always lower when the owners have no managers relatively to the situation where the platforms' have managers that establish unrestricted long run decisions.

(ii) The owners without manager vs delegation with restrictive policy. Let $\alpha \in \left[0, \frac{1}{2}t\right)$. The equilibrium profit is always higher and the consumer surplus and total welfare are always lower when the owners have no managers relatively to the situation where the platforms' have managers that establish restrictive long run decisions.

(iii) The owners without manager vs no delegation with unrestricted policy. Let $\alpha \in \left[0, \frac{1}{2}t\right)$. The equilibrium profit is always higher and the consumer surplus and total welfare are always lower when the owners have no managers relatively to the situa-
tion where the owners establish restrictive long run decisions. (iv) The owners without manager vs no delegation with restrictive policy. Let $\alpha \in \left[0, \frac{1}{3} t\right)$. The equilibrium profit is always higher and the total welfare and the consumer surplus are always lower when the owners have no managers relatively to the situation where the owners establish restrictive long run decisions.

**Proof.** see Appendix 10.6.

This result reinforces the relevance of delegating the long run decision to the managers for an intermediate level of the inter-group externality ($\alpha \in \left[\frac{17}{16} t, \frac{5}{4} t\right)$). Also notice that from the platform's point of view and for $\alpha \in \left[\frac{5}{4} t, \frac{3}{2} t\right)$, tipping strictly dominates the strategy "owner without manager" but both cases are strictly dominated by the strategy "delegation with an unrestricted policy". Summing up, we conclude the following.

**Lemma 14** For sufficiently low level of the inter-group externality ($\alpha \leq \frac{17}{16} t$), the strategy "owner without manager" is the optimal strategy; for an intermediate level of the inter-group externality ($\alpha \in \left[\frac{5}{4} t, \frac{3}{2} t\right)$), delegation with an unrestricted policy on the long run decision is the optimal strategy and for sufficiently high level of the inter-group externality ($\alpha \geq \frac{3}{2} t$), tipping is the optimal strategy.

Lemma 14 details the role of the inter-group externalities. Clearly, for an intermediate level of the inter-group externality, the owners of symmetric platforms should delegate the long run decision to their managers. Our result is in line with Bárcena-Ruiz and Cazado-Izaga [4] for sufficiently low level of the inter-group externality and finally, for sufficiently high level of the inter-group externality, only tipping occurs.

### 8 Asymmetric choices

In the spirit of Bárcena-Ruiz and Cazado-Izaga [4], we intend to understand the role of the inter-group externality under asymmetric regimes between platforms. Suppose that the
owner of platform A decides the platform location but in platform B, the owner delegates the decision to their manager.

We start by establishing the profit concavity condition.

**Assumption 6** \( \alpha < 2t \).

However, the unique interior equilibrium holds under a more restrictive condition.

**Proposition 15** If \( \alpha < \frac{2}{3}t \), maximum differentiation, i.e., \((a, b) = (1, 3)\) is a location equilibrium.

**Proof.** see Appendix 10.7. ■

The timing of the game is the following: in stage one, the owner of platform B chooses the incentive parameter \( \lambda_B \); in stage two, the owner of platform A and the manager of platform B choose locations simultaneously; in stage three, the owner of platform A chooses \( \lambda_A \), and in stage four the managers of both platforms decide simultaneously about prices. The fourth stage has been already solved. In stage three, the owner of platform A chooses \( \lambda_A \) that maximizes their profits. From the first order condition we obtain:

\[
\frac{\partial \pi^A}{\partial \lambda_A} = 0 \Leftrightarrow \lambda^A(a, b, \lambda^B) = \frac{t}{4}(a - b)(2 + b + a) + \frac{3a + \lambda^B}{4}.
\]  

(14)

In stage two, the owner of platform A and the manager of platform B decide simultaneously the locations of the platforms. Given that:

\[
\pi^A = \frac{t(b - a)(2b + a) + 3a + \lambda^B}{4(2b + a)} \left[ \frac{t(b - a) - \lambda^B}{(2b + a)} \right] - 3a - \lambda^B
\]

the combination of the first order conditions imply:

\[
\begin{align*}
\{ a(\lambda^B) &= \frac{\alpha - \lambda^B}{4(t - \alpha)} \\
 b(\lambda^B) &= \frac{8t - 7\alpha - \lambda^B}{4(t - \alpha)}
\}
\]  

(15)
In the first stage, the owner of platform $B$ chooses $\lambda^B$. After substituting expressions (14), and the equations from expression (15) we obtain $\pi^B(\lambda^A(a, b, \lambda^B), \lambda^B, a(\lambda^B), a(\lambda^B), \alpha)$. Deriving the outcome in order to $\lambda^B$, we find the optimal incentive parameter that maximizes the profits of platform $B$. It corresponds to:

$$\lambda^{B^*} = -4t + 5\alpha.$$  \hfill (16)

Then, the first stage equilibrium outcomes are given by:

<table>
<thead>
<tr>
<th></th>
<th>$a^*$</th>
<th>$b^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda^A^*$</td>
<td>$-4t + 2\alpha$</td>
<td>$-4t + 5\alpha$</td>
</tr>
<tr>
<td>$p^A^C^*$</td>
<td>$8t - 4\alpha$</td>
<td>$4t - 5\alpha$</td>
</tr>
<tr>
<td>$D^A^C^*$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\pi^A^C^*$</td>
<td>$8(2t - \alpha)$</td>
<td>0</td>
</tr>
<tr>
<td>$U^A^C^*$</td>
<td>$4(2t - \alpha)$</td>
<td>$U^B^C^*$</td>
</tr>
<tr>
<td>$CS^C^*$</td>
<td>$2v + 10\alpha - \frac{50}{3} t$</td>
<td>$TSW^C^*$</td>
</tr>
<tr>
<td>$\overline{x}_j^C^*$</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

where the superscript “$C$” denotes the equilibrium outcome in the case where the long run decision of platform $A$ is decided by the owner and the long run decision of platform $B$ is decided by the manager.

### 8.1 Asymmetric choices: should owners delegate or not?

Bárcena-Ruiz and Cazado-Izaga [4] conclude that under asymmetric choices the firms owners’ should take by themselves the long run decision. With the incorporation of the inter-group externalities, we also conclude that for sufficiently low level of the inter-group externality the platform that delegates the long run decision will get no market share and the whole market is conquered by the platform with the owner long run management.

The intuition behind the result is the following. The owner who does not delegate the location decision (the owner of platform $A$) gets higher profits than the owner who delegates this decision (the owner of platform $B$). In equilibrium, the platform $A$ is
located at the city’s right boundary ($a^* = 1$) while platform $B$ locates outside the city limits ($b^* = 3$) and the distance between the two rival platforms, $b - a$, does not depend on the value of $\lambda^B$. Indeed, the distance between the two rivals is constant and equal to 2.

The owner of platform $B$ is the leader in incentive decisions. When chooses the incentive parameter of his manager, $\lambda^B$, he takes into account that if the incentive parameter $\lambda^B$ increases, given the strategic complementarity, the incentive parameter chosen by the follower, $\lambda^A$, also increases. By setting $\lambda^B > \lambda^A$ the owner of platform $B$ manages to get the other owner to choose a higher $\lambda^A$, which would relax competition in the product market by increasing prices. But, when the incentive parameter $\lambda^B$ increases, this not only guarantees a better location of platform A towards the middle of the market but also pushes platform B far from platform A, which is harmful for platform B.

Since the reduction in the demand of platform B has a greater weight relatively to the price effect on both platforms, the owner that does not delegate the long run decision conquers the whole market and not delegating the long run decision dominates delegation. Thus, platforms avoid becoming leaders in incentive parameters and guarantee less competition in the product market in the future.

However, even under asymmetric choices and for sufficiently high inter-group externalities, minimum differentiation stands and tipping equilibria are the optimal strategies.

**Proposition 16 (Asymmetric choices: delegate long run decisions or not?)** Let $\alpha \in [0, \frac{8}{3} t)$. The game in which platforms’ owners choose whether to delegate the long run decision to a manager or whether to choose it by themselves has a unique subgame perfect Nash equilibrium in which both owners choose the long run decisions of the platform by themselves. Let $\alpha \geq \frac{8}{3} t$. The game in which platforms’ owners choose whether to delegate the long run decision to a manager or whether to choose it by themselves implies tipping equilibria in favor of one of the platforms.

**Proof.** see Appendix 10.7.

As a final remark, it should be clear that this result does not contradict Lemma 9
since on the previous perfect subgames, both platforms take the same strategy while in this subgame, the strategies differ between platforms.

9 Conclusions

We study a two-sided market with utilities à la Armstrong [2] normalizing the marginal costs to zero on both platforms and assuming the same transportation costs and intergroup network externalities on both sides of the market as in Zacarias and Serfes [19].

Firstly, in equilibrium, both platforms charge the same price to both sides. However, prices charged are distinct between both platforms. Under tipping, only one platform survives but both will be placed on the same extreme of the linear city.

Secondly, we investigate the effects of the location restrictions on social welfare. We find that the ownership delegation of restrictive long run decisions (location) generates lower consumer surplus and higher profits, whatever the inter-group externalities level and has no welfare impacts relatively to no delegation of long run decisions (location). However, the ownership delegation of unrestricted long run decisions (location) generates lower consumer surplus and higher profits only for sufficiently high inter-group externalities and has no welfare impacts relatively to no delegation of long run decisions (location).

When the owner never delegates long run decisions (location), the equilibrium profits and total welfare with unrestricted long run decisions (location) are always higher than with restrictive long run decisions (location). However, the consumer surplus is always lower. When the owner always delegates long run decisions (location), the equilibrium profits with unrestricted long run decisions (location) is always higher than with restrictive long run decisions (location) and the consumer surplus with unrestricted long run decisions (location) is always lower than with restrictive long run decisions (location) only for sufficiently high inter-group externalities. Total welfare with unrestricted long run decisions (location) is always higher, whatever the inter-group externalities level.

D’Aspremont et al. [7] and Tabuchi and Thisse [21] show that the profits are higher when locations are unrestricted since it accelerates the competition between the firms.
because of the demand-enhancing effect resulting from the stronger incentive contracts. However, when the locations are unrestricted the consumer surplus is lower. The above conclusions indicate that incorporating the separation of ownership and management and the reward contracts of the managers into the models of d’Aspremont et al. [7] and Tabuchi and Thisse [21] may disrupt their results, as Matsumura and Matsushima [16, 17] point out.

Our paper also sheds light on the value of strategic commitment in a two-sided markets model. Under unrestrictive long run decisions, the platforms have strong incentives for exhibiting belligerent behavior in the subsequent stages. In contrast, under restrictive long run decisions, the platforms have hard incentives for exhibiting less aggressive behavior. Thus, our result indicates that the strategic value of commitment crucially depends on the model formulation of the location choices (long run decisions).

We also show that under asymmetric choices the owners have incentives to keep the long run decisions to them. Here, less aggressive behavior is exhibited in both cases - with and without the restriction of long run decisions. However, no restrictions on the locations always reduces consumer surplus. By taking long run decisions, an owner can avoid becoming a leader in incentives and thus insure a better location in the market and a gain a lower product market competition because if an owner delegates the long run decision to his manager, it may become a leader in incentives and thus getting a worse location in the market.

Summing up, our manuscript finds that for sufficiently low intensity of the inter-group externality, the owners of symmetric platforms should take the long run decisions by themselves. However, and contrary to what traditional literature (that does not consider the presence of inter-group externalities) argues (e.g., Bárcena-Ruiz and Cazado-Izaga [4]), for an intermediate level of the inter-group externality, the owners of symmetric platforms should delegate the long run decision to their managers. Finally, for sufficiently high level of the inter-group externality, only tipping equilibria occur. In line with markets without network effects, under an asymmetric environment, that is, when one platform owner establishes long run decisions and the other delegates long run decisions to their manager the long run decisions should be taken by the platform’s owners.
For direct future research, it would be important to understand the impact of sequential decisions (short and long run) in delegation cases of online platforms with the incorporation of partial (or total) multihoming. The intuition in two-sided markets is that under the presence of pure singlehoming agents the market is more likely to tip and as the multihoming tendency increases, the probability that a smaller competitor can survive increases. Thus, linking multihoming and delegation is a major field of innovation since this characteristic is a normal tendency in most two-sided markets. Our prior intuition relies that strategic delegation will be harder to be sustained when multihoming occurs.
10 Appendix

10.1 Tipping

Tipping in favor of a platform A corresponds to a situation where platform A gets the whole market on both sides while platform B has no market share and charges a zero price on both sides. In such a situation:

\[ D_1^A = D_2^A = 1, \]
\[ D_1^B = D_2^B = 0, \]
\[ p_1^B = p_2^B = 0. \]

To attract the whole market, the maximum price that platform A can charge is such that the customers located at \( x = 1 \) are indifferent between the two platforms:

\[ u_1^A(1) = u_1^B(1) \iff \alpha - p_1^A - t(1 - a)^2 = -t(1 - b)^2 \iff p_1^A = \alpha - t(1 - a)^2 + t(1 - b)^2, \]
\[ u_2^A(1) = u_2^B(1) \iff \alpha - p_2^A - t(1 - a)^2 = -t(1 - b)^2 \iff p_2^A = \alpha - t(1 - a)^2 + t(1 - b)^2. \]

The resulting profit of platform A is:

\[ \pi^{A(T)} = 2\alpha - 2t(1 - a)^2 + 2t(1 - b)^2, \]

while the profit of platform B is obviously null.

Platform A can never obtain a profit that is greater than \( 2\alpha \). This is due to the fact that:

\[
\begin{array}{ll}
\text{Platform’s A action: } & \max_a \pi^A & \text{Platform’s B action: } \min_b \pi^A \\
\frac{\partial \pi^{A(T)}}{\partial a} = 0 & \iff a^* = 1 & \frac{\partial \pi^{A(T)}}{\partial b} = 0 & \iff b^* = 1 \\
\frac{\partial^2 \pi^{A(T)}}{\partial a^2} = -4t < 0 & & \frac{\partial^2 \pi^{A(T)}}{\partial b^2} = 4t > 0 &
\end{array}
\]

Thus, tipping in favor of platform A implies minimum differentiation at the right extreme of the linear city and the maximum profit that platform A obtains is given by:

\[ \pi^{A(T)*}(1,1) = 2\alpha. \] (17)
The argument standing for platform B is similar. The only difference is that tipping in favor of platform B implies minimum differentiation at the left extreme of the linear city and the maximum profit that platform B obtains is given by:

\[ \pi^{B(T)}(0, 0) = 2\alpha. \]

It should be clear that since consumers on both sides of the market are within the city boundaries \( x_j \in [0, 1], j = 1, 2 \), these equilibria stand whatever the policy (restrictive or unrestrictive) or strategy (delegation or no delegation) adopted.

### 10.2 Profit concavity

The second order conditions to ensure that the first order conditions find a maximum on the profit function on each one of the perfect subgames are given by:

\[
(i) \quad \frac{\partial^2 \pi^A}{\partial p_1^A \partial p_2^A} = \frac{\partial^2 \pi^A}{\partial p_2^A \partial p_1^A} = -2t(b - a) < 0
\]

\[
(ii) \quad \frac{\partial^2 \pi^A}{\partial p_1^A \partial p_2^A} - \left( \frac{\partial^2 \pi^A}{\partial p_1^A \partial p_2^A} \right)^2 = 4t(b - a) - 4\alpha \implies \implies t(b - a) > \alpha \iff t(b - a) - \alpha > 0 \iff \alpha < t(b - a)
\]

Then, to obtain the profit concavity condition for each one of the perfect subgames, we simply substitute \( a \) and \( b \) by the correspondent equilibrium location of platform A and platform B, respectively.

### 10.3 Restrictive policy

**Proof of Proposition 1**

Our purpose is to investigate whether maximum differentiation (i.e., a situation in which platforms locate at the extremes of the linear city) is a location equilibrium. We will assume that platform B locates at the right extreme and seek to find whether locating at the left extreme is a best response for platform A. Maximum differentiation is a location equilibrium if and only if that is the case.
Given that the resolution of this problem requires using backward induction, we start by solving the second stage. In this stage, given their locations, the platforms simultaneously set prices, with the objective of maximizing their individual profits.

We start by naively considering the system of equations of demands, that only applies in an interior equilibrium, and set \( b = 1 \). This implies that:

\[
\begin{align*}
D_A^1 &= \frac{1}{2(1-a)} \left[ \alpha (2D_A^2 - 1) + p_B^1 - p_A^1 + t (1 - a^2) \right] \\
D_A^2 &= \frac{1}{2(1-a)} \left[ \alpha (2D_A^1 - 1) + p_B^2 - p_A^2 + t (1 - a^2) \right] \\
D_B^1 &= \frac{2(1-a)}{4(1-a)^2 - 4a^2} \left[ t (1 - a^2) + p_B^1 - p_A^1 + \alpha a + \frac{\alpha}{t(1-a)} (p_B^2 - p_A^2 - \alpha) \right] \\
D_B^2 &= \frac{2(1-a)}{4(1-a)^2 - 4a^2} \left[ t (1 - a^2) + p_B^2 - p_A^2 + \alpha a + \frac{\alpha}{t(1-a)} (p_B^1 - p_A^1 - \alpha) \right].
\end{align*}
\]

It should be clear that the profit of a platform is a concave function of the prices it sets if and only if \( t(1-a) > \alpha \). If this condition is not verified, then there can only exist tipping equilibria.

**Lemma 17** If \( t(1-a) > \alpha \), there exists a unique equilibrium that is interior, with prices and market shares given by:

\[
\begin{align*}
p_A^1(a, 1) &= p_A^2(a, 1) = t(1-a) \left( 1 - \frac{a}{3} \right) - \alpha, \\
p_B^1(a, 1) &= p_B^2(a, 1) = t(1-a) \left( 1 + \frac{a}{3} \right) - \alpha, \\
D_A^1(a, 1) &= D_A^2(a, 1) = \frac{2(1-a)}{4(1-a)^2 - 4a^2} \left[ t (1 - a^2) + \frac{2\alpha(1-a)}{3} + \frac{5\alpha a}{3} - \frac{\alpha^2}{t(1-a)} \right].
\end{align*}
\]

By locating at the left extreme of the city \( a = 0 \), platform A obtains a profit that is equal to:

\[
\Pi_A^1(0, 1) = 2p_A^A(0, 1)D_A^1(0, 1) = t - \alpha.
\]

Solving the inequality resulting from expression (18) and expression (17), Proposition 1 is, now, straightforward.

**Proof of Proposition 2**

\[\text{This result is the same as the one obtained by Armstrong (2006) for } t_1 = t_2 = t \text{ and } \alpha_1 = \alpha \text{ and } \alpha_2 = \alpha.\]
The proof is similar to Proposition 1. Thus, we simplify the proof by incorporating the prominent aspects\footnote{The proof is done assuming tipping in favor to platform A but results are the same if we assume tipping in favor of platform B.}. Tipping in favor of platform A implies $\pi^{A(T)*}(1, 1) = 2\alpha$. The profit of this platform resulting from the unique interior equilibrium is $\pi^{A(Interior)*}(0, 1) = 2t - 2\alpha$. Then, maximum differentiation is a location equilibrium if and only if $\pi^{A(Interior)*}(0, 1) \geq \pi^{A(T)*}(1, 1)$ and Proposition 2 is, now, straightforward.

**Proof of Proposition 3**

Let $\alpha \in \left[0, \frac{1}{3}\right]$. (i) Prices: $p_{i}^{R-D}* \geq p_{i}^{R-ND*} \iff 2(t - \alpha) \geq t - \alpha \iff 2 > 1$. (ii) Market shares: $D_{i}^{R-D*} = D_{i}^{R-ND*} = \frac{1}{2}$. (iii) Consumer surplus: $CS_{i}^{R-D} \geq CS_{i}^{R-ND} \iff 2v + 5\alpha - \frac{25}{6}t \geq 2v + 3\alpha - \frac{13}{6}t \iff 2\alpha \geq 2t \iff \alpha \geq t$. (iv) Profits: $\pi^{(R-D)*} \geq \pi^{(R-ND)*} \iff 2(t - \alpha) \geq t - \alpha \iff 2 \geq 1$. (v) Total welfare: $TSW_{i}^{R-D} \geq TSW_{i}^{R-ND} \iff 2v + \alpha - \frac{t}{6} \equiv 2v + \alpha - \frac{t}{6}.$

**10.4 Unrestrictive policy**

**Proof of Proposition 5**

Tipping in favor of platform A implies $\pi^{A(T)*}(1, 1) = 2\alpha$. The profit of this platform resulting from the unique interior equilibrium is $\pi^{A(Interior)*}(\frac{1}{4}, \frac{3}{4}) = \frac{3}{4}t + 2\alpha$. Then, maximum differentiation is a location equilibrium if and only if $\pi^{A(Interior)*}(\frac{1}{4}, \frac{3}{4}) \geq \pi^{A(T)*}(1, 1)$ and Proposition 5 is, now, straightforward.

**Proof of Proposition 6**

Tipping in favor of platform A implies $\pi^{A(T)*}(1, 1) = 2\alpha$. The profit of this platform resulting from the unique interior equilibrium is $\pi^{A(Interior)*}(\frac{1}{4}, \frac{3}{4}) = \frac{3}{4}t - \alpha$. Then, maximum differentiation is a location equilibrium if and only if $\pi^{A(Interior)*}(\frac{1}{4}, \frac{3}{4}) \geq \pi^{A(T)*}(1, 1)$ and Proposition 6 is, now, straightforward.

**Proof of Proposition 7**

Let $\alpha \in \left[0, \frac{1}{3}\right]$. (i) Prices and Profits: $p_{i}^{U-D*} \geq p_{i}^{U-ND*} \iff \frac{3}{4}t + 2\alpha \geq \frac{3}{4}t - \alpha \iff \alpha \geq \frac{1}{4}.$
(ii) Market shares: $D^{U-D} = D^{U-ND} = \frac{1}{2}$. (iii) Consumer surplus: $CS^{U-D} \geq CS^{U-ND}$
$\Leftrightarrow 2v - 3\alpha - \frac{49}{24}t \geq 2v + 5\alpha - \frac{85}{24}t \Leftrightarrow \alpha \leq \frac{4}{6}$. (iv) Total welfare: $TSW^{U-D} \geq TSW^{U-ND}$
$\Leftrightarrow 2v + \alpha - \frac{13}{24}t \equiv 2v + \alpha - \frac{13}{24}t$.

10.5 Restrictive vs unrestrictive policy: what is better for the managers?

Proof of Proposition 10

(i) Let $\alpha \in \left[0, \frac{1}{2}t\right)$ and suppose the owners never delegate the long run decision.
Profits: $\pi^{U-ND} \geq \pi^{R-ND} \Leftrightarrow \frac{3}{2}t - \alpha \geq t - \alpha \Leftrightarrow t \geq 0$. Consumer surplus: $CS^{U-ND} \geq CS^{R-ND} \Leftrightarrow 2v + 5\alpha - \frac{85}{24}t \geq 2v + 3\alpha - \frac{55}{6}t \Leftrightarrow \alpha \geq \frac{11}{18}t$, thus, never holds for $\alpha < \frac{4}{6}$. Social welfare: $TSW^{U-ND} \geq TSW^{R-ND} \Leftrightarrow 2v + \alpha - \frac{13}{24}t \geq 2v + \alpha - \frac{4}{6} \Leftrightarrow t \leq 0$.

(ii) Let $\alpha \in \left[0, \frac{1}{2}t\right]$ and suppose the owners always delegate the long run decision.
Profits: $\pi^{U-D} \geq \pi^{R-D} \Leftrightarrow \frac{3}{4}t + 2\alpha \geq 2(t - \alpha) \Leftrightarrow \alpha \geq \frac{5}{16}t$. Consumer surplus: $CS^{U-D} \geq CS^{R-D} \Leftrightarrow 2v - 3\alpha - \frac{49}{24}t \geq 2v + 5\alpha - \frac{25}{6}t \Leftrightarrow \alpha \leq \frac{51}{192}t$, thus never holds for $\alpha \in \left(\frac{51}{192}t, \frac{t}{2}\right)$. Social welfare: $TSW^{U-D} \geq TSW^{R-D} \Leftrightarrow 2v + \alpha - \frac{13}{24}t \geq 2v + \alpha - \frac{5}{6} \Leftrightarrow t \leq 0$.

10.6 Owners establish unrestrictive long run decisions without managers

Proof of Proposition 12

Tipping in favor of platform A implies $\pi^{A(\mathcal{T})\star}(1, 1) = 2\alpha$. The profit of this platform resulting from the unique interior equilibrium is $\pi^{A(\text{Interior})\star} \left(-\frac{2}{3}, \frac{2}{3}\right) = 5t - 2\alpha$. Then, maximum differentiation is a location equilibrium if and only if $\pi^{A(\text{Interior})\star} \left(-\frac{2}{3}, \frac{2}{3}\right) \geq \pi^{A(\mathcal{T})\star}(1, 1)$ and Proposition 12 is, now, straightforward.

Proof of Proposition 13

(i) Let $\alpha \in \left[0, \frac{2}{5}t\right)$ and consider the comparison: the owners without manager versus delegation with unrestrictive policy. Profits: $\pi^{O} \geq \pi^{U-D} \Leftrightarrow 5t - 2\alpha \geq \frac{3}{4}t + 2\alpha \Leftrightarrow \alpha \leq \frac{17}{16}t$. 34
Consumer surplus: \( CS^O \geq CS^{U-D} \iff 2v + 5\alpha - \frac{295}{24} t \geq 2v - 3\alpha - \frac{49}{24} t \iff \alpha \geq \frac{256}{192} t \). Social welfare: \( TSW^O \geq TSW^{U-D} \iff 2v + \alpha - \frac{49}{24} t \geq 2v + \alpha - \frac{13}{24} t \iff t \leq 0 \).

(ii) Let \( \alpha \in \left[0, \frac{1}{2} t\right] \) and consider the comparison: the owners without manager vs delegation with restrictive policy. Profits: \( \pi^O \geq \pi^{R-D} \iff 5t - 2\alpha \geq 2(t - \alpha) \iff t \geq 0 \).
Consumer surplus: \( CS^O \geq CS^{R-D} \iff 2v + 5\alpha - \frac{295}{24} t \geq 2v + 5\alpha - \frac{25}{6} t \iff t \leq 0 \).
Social welfare: \( TSW^O \geq TSW^{R-D} \iff 2v + \alpha - \frac{49}{24} t \geq 2v + \alpha - \frac{13}{6} t \iff t \leq 0 \).

(iii) Let \( \alpha \in \left[0, \frac{1}{3} t\right] \) and consider the comparison: the owners without manager vs no delegation with unrestrictive policy. Profits: \( \pi^O \geq \pi^{U-ND} \iff 5t - 2\alpha \geq \frac{3}{2} t - \alpha \iff \alpha \leq \frac{t}{2} \).
Consumer surplus: \( CS^O \geq CS^{U-ND} \iff 2v + 5\alpha - \frac{295}{24} t \geq 2v + 5\alpha - \frac{85}{24} t \iff t \leq 0 \).
Social welfare: \( TSW^O \geq TSW^{U-ND} \iff 2v + \alpha - \frac{49}{24} t \geq 2v + \alpha - \frac{13}{24} t \iff t \leq 0 \).

(iv) Let \( \alpha \in \left[0, \frac{1}{3} t\right] \) and consider the comparison: the owners without manager vs no delegation with restrictive policy. Profits: \( \pi^O \geq \pi^{R-ND} \iff 5t - 2\alpha \geq t - \alpha \iff \alpha \leq \frac{4t}{3} \).
Consumer surplus: \( CS^O \geq CS^{R-ND} \iff 2v + 5\alpha - \frac{295}{24} t \geq 2v + 5\alpha - \frac{13}{6} t \iff \alpha \geq \frac{243}{48} t \).
Social welfare: \( TSW^O \geq TSW^{R-ND} \iff 2v + \alpha - \frac{49}{24} t \geq 2v + \alpha - \frac{t}{6} \iff t \leq 0 \).

10.7 Asymmetric choices

Proof of Proposition 15

Tipping in favor of platform A implies \( \pi^{A(T)*}(1,1) = 2\alpha \). The profit of this platform resulting from the unique interior equilibrium under asymmetric choices between platforms is \( \pi^{A(Interior)*}(1,3) = 8(2t - \alpha) \). Then, maximum differentiation is a location equilibrium if and only if \( \pi^{A(Interior)*}(1,3) \geq \pi^{A(T)*}(1,1) \) and Proposition 15 is, now, straightforward.

Proof of Proposition 16

Let \( \alpha \in \left[0, \frac{2}{5} t\right] \). Then, \( \pi^{A-D} \geq \pi^{R-ND} \iff 8(2t - \alpha) \geq 0 \iff t \geq 0 \). For \( \alpha \geq \frac{2}{5} t \), minimum differentiation at \((a,b) = (1,1)\) is an equilibrium according to Proposition 15, which completes the proof of Proposition 16.
References


