Public Regulatory Intervention in Consumer-Friendly Firms

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November 6th, 2014.

Abstract. We study a duopoly with differentiated and substitutable goods composed of one consumer-friendly firm and one pure-profit maximizing firm. In such a duopoly, a regulatory authority intervenes to control the degree of altruism of the consumer-friendly firm. We conclude that under quantity competition, if firms sell goods that are too homogeneous the policymaker should impose a ceiling on the level of benevolence of the consumer-friendly firm. However, under price competition, the policymaker never imposes a ceiling on the level of kindness of the consumer-friendly firm. Our results also show that, whatever the degree of product differentiation, the social welfare under price competition is always higher than the social welfare under quantity competition, which restores the arguments pointed out by the traditional literature and constitutes a sharp contrast with Nakamura (2013).

Keywords: Consumer-Friendly Firm, Product Differentiation, Public Intervention.

JEL Classification Numbers: D43, L11, L13, R12, R32, R52.

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1 Introduction

The increasing number of social actions by private companies, usually defined as "Corporate Social Responsibility", is confirmed as a new trend of the XXI century (see, among others, Economist Intelligence Unit (2007), McKinsey and Company (2007), Ernst and Young (2010), UN and Accenture (2010) and KPMG (2011)). Currently, the debate on firms’ social responsibility is discussed more frequently in the economic mainstream.

Several reasons are pointed out for such dispute. Hillman and Keim (2001) argue that "globalization has increased calls for corporations to use firm’s resources to help alleviate a wide variety of social problems". As the authors point out, a remarkable example is the pharmaceutical industry. This industry is asked frequently to donate free drugs and vaccines to third world countries that are not able to pay their corresponding intrinsic value. Another example relies in manufacturing, where the firms are encouraged to apply developed nation’s laws to issues such as child labor and environmental pollution once they operate in less developed countries. Jensen (2001) shows that firm’s stakeholders should additionally be considered in firm’s decision making process by redefining the firm’s objective as an "Enlightened Value Maximization" and Porter and Kramer (2011) indicate the need to incorporate in firm’s responsibility the societal care by introducing the definition of a "shared value creation".

Starting in Goering (2007), the idea that moral responsibilities of directors have to be considered alongside the role of the corporation in meeting the legitimate expectations of its investors and shareholders gained weight. In this sense, a stream of economic literature begins to deal with mixed oligopolies where a pure profit-maximizing firm competes against a socially concerned firm. The social concern arises by the fact that, in addition to its own profit, the consumer-friendly firm maximizes an overall utility that also depends on a share of the consumer surplus. Goering (2008a) shows that a movement away from pure profit maximizing behavior by a socially concerned firm may be detrimental to social welfare.

Thereafter, researchers introduce additional worries to set clearer how should markets operate under philanthropy. One relevant feature is the simultaneous combination of cor-
porate social responsibility and strategic delegation, whose formal goal is to understand how a managerial strategic incentive counterbalances (or not) the social concern of the firm (see, among others, Goering (2007), Kopel and Brand (2012) and Brand and Kopel (2013)). The intuition, as detailed in Brand and Kopel (2013), is that a socially concerned firm pays low-powered incentives to their executives. Although intuitive that the managerial incentives mechanism may lead to a sharp contrast with the consumer-friendly firms goodwill dynamics, corporate social responsibility keeps generating a large amount of interest in the literature. Dahlsrud (2008) argues that at least 37 different definitions for corporate social responsibility have been used or proposed. Nonetheless, Goering (2013) indicates that corporate social responsibility is most often defined as "the firm conduct that indicates a responsible attitude or business approach". Examples of social responsibility activities include alcohol and tobacco manufacturers ‘responsible drinking and smoking’ campaigns, support of charities and implementation of non-profit foundations and environmental initiatives used by private firms and by governments (recycling, decreasing the production waste and, more recently, the "green fiscal policy" concern, as well).\(^1\) Carroll and Shabana (2010) confirm that the firms will engage in consumer-friendly activities as long as it enhances their profits or corporate financial performance. Manasakis et al. (2007), Kopel and Brand (2012) and Brand and Kopel (2013) in the case of oligopolies and Brand and Grothe (2013) and Goering (2013) in the case of a bilateral monopoly show that firms’ social responsibility can be chosen strategically. However, note that the link between corporate social responsibility and corporate financial performance was already under the scope of literature review and theoretical analysis (see Donaldson and Fafaliou (2003), Vogel (2006), Amalric (2006) and Besley and Ghatak (2007)) and by empirical contributions (see Bolvig (2005), Husted and Allen (2006) and Brammer and Millington (2008)).

All of the mentioned manuscripts highlight the prominence of corporate social responsibility, however, none of them introduce the intervention of a public body in regulating the level of altruism that a consumer-friendly firm can attain. Bitcha (2003) highlights

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\(^1\)For the portuguese case please check the following link:

the corporate social responsibility as an alternative method to the usually regulatory policy punishments. The author proclaims that "corporate social responsibility may have a role to play as one of the ‘instruments’ of, or surrogates for, regulatory policy but that depends not only on its theoretical potential but whether that potential can be demonstrated in practice by companies".

In the context of international trade, Wang et al. (2011) explore how strategic tariff policy and welfare are affected by the consumer-friendly initiative of foreign exporting firms. They define a firm that is consumer-friendly or non-profit-based if it considers both its own profit and consumer surplus and examine the tariff policy and welfare. The consumer-friendly initiative that leads to trade liberalization is a ‘Win-Win-Win’ solution in the sense that it is not only beneficial for foreign exporting firms but also for the government and consumers of the importing country.

However, real-world evidence can give us a different appealing. In the sociology literature, Costa (2005) argues that there are situations in which the consumer-friendly firms act themselves as "welfare companies", functioning even as substitutes of Laissez Governments. In some other cases, firms intend to increase the corporate social responsibility but policymakers restrict the level of philanthropy that companies desire to reach. A motivating example relies on the regulation of the Internet and the quantity competition that the search engines Baidu and Google engage to get additional end-users on board. China’s Great Firewall is a well-known example that adopts technological handles to retain the control of online content. On the one hand, the national com-

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2For a full description of her argument check the following document (in portuguese): http://www.ces.uc.pt/publicacoes/rccs/artigos/73/RCCS73-067-089-Maria%20Alice%20Costa.pdf

3Whereas Google operates worldwide, the core business of Baidu is essentially based in China cyberspace. Nonetheless, Baidu is one of the largest search engines in the world in terms of end-user basis.

4The Government requires state-licensed Internet service providers (ISP), which operate the gateways through which Internet content travels into the country, to filter certain identified URL domains and, thus, prevents them from being accessed by end-users in China. The resulting bottleneck creates a delay and makes pages difficult to load for websites hosted outside the Great Firewall, including Google main search site, google.com. For more information see the American Society of International Law (ASIL) website in the following link: http://www.asil.org/insights/volume/14/issue/25/google-china-and-search
pany benevolent actions are systematically controlled by the Chinese Government. On the other hand, Google is a company committed to active philanthropy and, since 2007, started a series of social initiatives in China that include Google’s China social innovation cup for college students, supporting earthquake relief efforts and the provision of several grants and donations across the country. In 2010, Google decided to stop censoring its search results in China.\(^5\) In retaliation, the Chinese Government restricted some of the Google’s benevolent initiatives. Another motivating example relies on food waste and price competition between supermarkets in Portugal. A recent project study developed by CESTRAS documents that Portugal loses or wastes nearly 1 million tons of food per year, 324,000 of them in customers’ homes.\(^6\) To prevent food waste, the Spanish Government has proposed in 2013 to increase the shelf life of some products (as an example, in the case of yogurts the validity is likely to be extended to 35 days instead of the current 28 days). In Portugal, the trend is to adopt cutting-price strategies in perishable goods.\(^7\) As the Portuguese Association for Consumer Protection (DECO) reveals, this strategy is conducted by "most hypermarkets and for approximately 20 supermarkets in Lisbon and Porto visited last March, 2013". It is a way for businesses to eliminate waste, but the onus of consumption is given to the customer. The non-consumed food is, then, destroyed. Since corporate social responsibility is a hot topic in Portuguese food retail companies\(^8\), seems arguable why policymakers do not make any \textit{ex-ante} social benevolent decision (such as to introduce a requirement to firms provide wasted food for free to some

\(^5\)Given that this measure enhances the life quality of consumers, it is plausible to consider that the importance of the consumer surplus has increased from Google’s standpoint.

\(^6\)CESTRAS is a Portuguese center for studies and strategies for sustainability. In Portuguese we are talking about the Projeto de Estudo e Reflexão sobre Desperdício Alimentar (PERDA), developed by the Centro de Estudos e Estratégias para a Sustentabilidade (CESTRAS). The news can be found online here: [http://www.deco.proteste.pt/alimentacao/produtos-alimentares/noticia/alargar-prazos-validade-nao-resolve-desperdicio-alimentar](http://www.deco.proteste.pt/alimentacao/produtos-alimentares/noticia/alargar-prazos-validade-nao-resolve-desperdicio-alimentar)

\(^7\)A price reduction of 25% to 50% over the regular price and the supply of additional quantity of the product without price change by promoting the strategy "take 2, pay 1" are the most common modes.

\(^8\)See, for instance, the case of the Portuguese food retail chain Continente (from the Sonae Group) that embraces measures that include commitment to health benefits and the improvement of career skills for their employees and at the societal level promotes several voluntary activities and every year, since 2009, the organization of a mega picnic that agglomerates thousands of individuals in the city of Lisbon just to let people listening the same famous singer Tony Carreira.
disadvantaged consumers or to implement restrictions on the level of benevolence of such firms since they cause negative externalities on society by, objectively, incurring in waste costs rather than fully internalizing a social concern that could be welfare enhancing).

The public intervention on the level of consumer-friendly activities is difficult to evaluate not only because of the existence of a degree of subjectivity intrinsic to prohibited and permitted conducts but also because, at a first glance, it seems irrational to urge for a public intervention in such markets because, ceteris paribus, all the social concern seems to promote positive impacts in our living standards. If not, at least social corporate responsibility gives us the subjective idea that the poorest are better off with the help of the richest.

However, given Goering (2008a) conclusion that a movement away from pure profit-maximizing behavior by a socially concerned firm may be detrimental to social welfare, it is certainly a challenge for policymakers to understand until which socially desirable level should a consumer-friendly firm conduct its benevolent actions.

In this sense, the research question we ask ourselves relies on understanding whether policymakers should impose a ceiling on the level of firms social benevolence. Should regulatory authorities let consumer-friendly firms operate their benevolent activities in a disorderly manner or should somehow restrict their business concerns with welfare? If so, under which circumstances: quantity competition or price competition? If it is true that firms' social responsibility can be chosen strategically to provide firms with a better social status by strengthening the brand and image of the organization it should also be verified, as Bitcha (2003) argues, that "governments have to take overall responsibility for ensuring that conduct failures - whether market or non-market failures - are regulated appropriately, taking into account each one of the three pillars of sustainable development: the economic, social and environmental pillars". Summing up, our framework intends to provide the circumstances under which the role of Government in corporate social responsibility is

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9As Land (2010) point out, decisions about such limit conducts are intimately related with domestic policies in each of the various countries. For instance, any mention to Nazi paraphernalia and Holocaust denial is banned in Germany but not in other countries.
Departing from a mixed duopoly with a pure profit-maximizing firm and a consumer-friendly firm, we find that under quantity competition, if firms sell goods that are too homogeneous the policymaker should impose a ceiling on the level of benevolence of the consumer-friendly firm. If not, this firm incurs in negative profits since quantity competition becomes too intense. On the other hand, under price competition, the policymaker never imposes a ceiling on the level of kindness of the consumer-friendly firm. Applying such conclusions into our examples, we clarify the Chinese Government decision to restrict the benevolent actions of Google and Baidu (that operate in a quantity competition basis) but also justify why supermarkets (that embrace price competition in perishable goods) are not restricted to policymakers on the level of social awareness.

The remainder of the paper is organized as it follows. Section 2 presents the model. Sections 3 and 4 study quantity and price competition, respectively. The main result is disposed in section 5. Finally, conclusions are drawn in section 6. Appendix for details is attached in section 7.

2 Model

We study a mixed duopolistic model with differentiated and substitutable goods. We consider the following version of Singh and Vives (1984) among others thereafter where, on the demand side of the market, the representative consumer’s utility is a symmetric-quadratic function of two products, $q_1$ and $q_2$, and a linear function of a numeraire good $q$:

$$U(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2} (q_1^2 - 2bq_1q_2 + q_2^2) + q,$$

where $a > 0$ is a demand parameter corresponding to the market size and $b$ represents the degree of product differentiation. Since we consider the case of substitute goods yields: $0 < b < 1$. Note that $b = 0$ and $b = 1$ set the maximum (independent or perfect complement goods) and the minimum (homogeneous or perfect substitute goods) degree of product differentiation, respectively.
The above utility function generates the system of linear demand functions:
\[ q_i = \frac{1}{1 - b^2} \left[ a(1 - b) - p_i + bp_j \right], \]  
with \( b \in (0, 1) \) and \( i, j = \{1, 2\}, i \neq j \) and \( p_i \) represents the price of good \( i \) \( (i = \{1, 2\}) \).

Then, the inverse demand functions can be inverted to obtain:
\[ p_i = a - q_i - bq_j, \]  
with \( b \in (0, 1) \) and \( i, j = \{1, 2\}, i \neq j \).

The profit of firm \( i \) is given by:
\[ \pi_i = [p_i - C(q_i)]q_i. \]

The production cost function \( C(q_i) \) is given by:
\[ C(q_i) = c_i q_i + F_i, \]
where \( c_i \) denotes the marginal cost of firm \( i \) and \( F_i \) is the fixed cost of production of firm \( i \). Assume that \( c_i = 0 \) and \( F_i = 0 \).  

Following Goering (2007), Goering (2008a), Goering (2008b), Kopel and Brand (2012) and Nakamura (2013), the objective function of firm 1 and firm 2 are, respectively, given by:
\[ V_1 = \pi_1 + \lambda CS; \]
\[ V_2 = \pi_2, \]
where \( \lambda \in \left[ 0, \frac{2-b}{2-b^2} \right] \) measures the degree of appreciation of the consumer surplus by the firm 1.  
Thus, the market is composed by two firms: a consumer-friendly private firm \((firm \ 1)\) which maximizes the weighted sum of its absolute profit with a fraction of the consumer surplus and a private firm \((firm \ 2)\) which is a pure profit-maximizing firm.

\[ ^{10}\text{The results remain qualitatively the same if we consider a positive (non-discriminatory) marginal and fixed cost between firms. Also, note that we do not consider the free entry problem and we focus on the degree to which firm 1 emphasizes consumer surplus in its objective function.} \]

\[ ^{11}\text{As explained in Nakamura (2013), the domain of social concern } \lambda \text{ is a restricted open interval } \lambda \in \left[ 0, \frac{2-b}{2-b^2} \right] \text{ and not the interval } \lambda \in [0,1] \text{ to keep the absolute profit of firm 1 positive in equilibrium.} \]
The social welfare, denoted by $W$, is measured as the sum of consumer surplus ($CS$) and producer surplus ($PS$):

$$W = CS + PS,$$

where $PS$ is the sum of firms’ profits:

$$PS = \pi_1 + \pi_2,$$

and $CS$ is given by:

$$CS = \frac{1}{2} (q_1^2 + 2bq_1q_2 + q_2^2).$$

Alternatively, using (1), the $CS$ can be defined as a function of prices:

$$CS = \frac{2a^2(1-b) + p_1^2 - 2bp_1p_2 + p_2^2 - 2a(1-b)(p_1 + p_2)}{2(1-b^2)}.$$

We intend to derive the equilibrium market outcomes: (i) in quantity competition and (ii) in the price competition. Under this setting, we consider that a policymaker defines the optimal level of social contribution of the consumer-friendly firm.\(^{12}\) Then, the timing of the game is the following: in the first stage, the policymaker defines the philanthropic optimal level $\lambda$ of the consumer-friendly firm that maximizes social welfare and, in the second stage, both firms compete in quantities or prices.\(^{13}\) The game is solved by the method of backward induction.

## 3 Quantity competition

We discuss the quantity competition game where firms 1 and 2 simultaneously choose their output levels.

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\(^{12}\)Our setting captures one of the following interpretations: (i) or the consumer-friendly firm fully internalizes the maximization of social welfare, (ii) or an external public authority defines the maximum altruism level that the consumer-friendly firm can attain. In the first interpretation, the firm acts itself as a policymaker. Then, in the first stage of the game, the firm 1 chooses the level $\lambda^*$ that maximizes social welfare and, in the second stage, both firms engage in quantity (or price) competition.

\(^{13}\)Our model can be considered an extension of Nakamura (2013) since his model only captures the second stage of our game by deriving the equilibrium market outcomes both in the quantity competition and in the price competition.
3.1 Stage 2

Given the level of social concern \(\lambda\) of the consumer friendly-firm, the degree of product differentiation \(b\) and using (2), (3) and (6), the objective function of firm 1 is given by:

\[
V_1(q_1, q_2, \lambda, b) = (a - q_1 - bq_2)q_1 + \frac{\lambda}{2} \left( q_1^2 + 2bq_1q_2 + q_2^2 \right).
\]  

Using (2) and (4), the absolute profit of firm 2 is given by:

\[
V_2(q_1, q_2, b) = (a - q_2 - bq_1)q_2.
\]  

Deriving (8) relatively to \(q_1\) and (9) relatively to \(q_2\), the reaction functions of the firms are given by:

\[
q_1(q_2, \lambda, b) = \frac{a - bq_2(1 - \lambda)}{2 - \lambda};
\]

\[
q_2(q_1, b) = \frac{a - bq_1}{2}.
\]

Then, the outcomes of the second stage of the game come as follows:

\[
q_1^*(\lambda, b) = \frac{a \left[ 2 - b(1 - \lambda) \right]}{4 - b^2(1 - \lambda) - 2\lambda};
\]

\[
q_2^*(\lambda, b) = \frac{a \left( 2 - b - 2\lambda + b^2\lambda \right)}{4 - b^2(1 - \lambda) - 2\lambda};
\]

\[
\pi_1^*(\lambda, b) = \frac{a^2 \left[ 2 - b(1 - \lambda) \right] \left( 2 - b - 2\lambda + b^2\lambda \right)}{4 - b^2(1 - \lambda) - 2\lambda}^2;
\]

\[
\pi_2^*(\lambda, b) = \left[ \frac{a \left( 2 - b - 2\lambda + b^2\lambda \right)}{4 - b^2(1 - \lambda) - 2\lambda} \right]^2;
\]

\[
CS^q(\lambda, b) = \frac{a^2 \left[ 24 - 2b^2(1 - \lambda)^2 - 20\lambda + 3\lambda^2 - b^2(2 - 6\lambda + \lambda^2) - 2b(8 - 7\lambda + 2\lambda^2) \right]}{2 \left[ 4 - b^2(1 - \lambda) - 2\lambda \right]^2} + q;
\]

\[
W^q(\lambda, b) = \frac{a^2 \left[ 24 - 2b^2(1 - \lambda)^2 - 20\lambda + 3\lambda^2 - b^2(2 - 6\lambda + \lambda^2) - 2b(8 - 7\lambda + 2\lambda^2) \right]}{2 \left[ 4 - b^2(1 - \lambda) - 2\lambda \right]^2} + q;
\]

3.2 Stage 1

Now, we solve the first stage of the game. Our problem requires to find the level of altruism \(\lambda^*\) on the consumer-friendly firm that maximizes equation (16). The optimal \(\lambda^*\) is given by:

\[
\lambda^*(b) = \frac{-b^4 + 13b^3 - 6b^2 + 4(2 - b)}{2b^4 + 7b^3 - 9b^2 + 2(4 + b)}.
\]
Substituting (17) into expressions (12) to (16) implies the following equilibrium outcomes:

\[
q_1^* = \frac{a (3b^5 - 10b^4 - 17b^3 + 24b^2 - 4b - 16)}{3b^5 - 6b^4 - 13b^3 + 4b^2 + 24b^2 - 16(b + 1)}; \tag{18}
\]

\[
q_2^* = \frac{2a (b^5 + b^4 - 5b^3 + 7b^2 - 4)}{3b^5 - 6b^4 - 13b^3 + 4b^2 + 24b^2 - 16(b + 1)}; \tag{19}
\]

\[
p_1^* = \frac{a (b^5 - 11b^4 + 7b^3 + 7b^2 - 4b)}{3b^5 - 6b^4 - 13b^3 + 4b^2 + 24b^2 - 16(b + 1)}; \tag{20}
\]

\[
p_2^* = \frac{2a (b^5 + b^4 - 5b^3 + 7b^2 - 4)}{3b^5 - 6b^4 - 13b^3 + 4b^2 + 24b^2 - 16(b + 1)}; \tag{21}
\]

\[
\pi_1^* = \frac{a^2 (b^5 - 11b^4 + 7b^3 + 7b^2 - 4b) (3b^5 - 10b^4 - 17b^3 + 24b^2 - 4b - 16)}{[3b^5 - 6b^4 - 13b^3 + 4b^2 + 24b^2 - 16(b + 1)]^2}; \tag{22}
\]

\[
\pi_2^* = \frac{4a^2 (b^5 + b^4 - 5b^3 + 7b^2 - 4)^2}{[3b^5 - 6b^4 - 13b^3 + 4b^2 + 24b^2 - 16(b + 1)]^2}; \tag{23}
\]

\[
\text{CS}^* = \frac{2a^2 (12b^{11} - 15b^{10} - 220b^9 + 274b^8 + 640b^7 - 1143b^6 - 296b^5)}{4 [-3b^5 + 6b^4 + 13b^3 - 4b^2 + 24b^2 + 16(b + 1)]^2}
+ \frac{2a^2 (1676b^4 - 320b^3 - 912b^2 + 384b + 320)}{4 [-3b^5 + 6b^4 + 13b^3 - 4b^2 + 24b^2 + 16(b + 1)]^2}; \tag{24}
\]

\[
W^* = \frac{a^2 [3b^6 - 6b^5 - 13b^3 + 4b^2 + 24b^2 - 16(b + 1)]^2}{16 [3b^6 - 6b^5 - 13b^3 + 4b^2 + 24b^2 - 16(b + 1)]^2}; \tag{25}
\]

The following Proposition summarizes.

**Proposition 1 (Quantity Competition)** Let \( \lambda \in \left[0, \frac{2 - b}{2 - b^3}\right] \) and \( b \in (0, \bar{b}) \), with \( \bar{b} \approx 0.83 \).

In a duopoly with differentiated and substitutable goods, composed by a consumer-friendly firm and a pure profit-maximizing firm that engage in quantity competition, the equilibrium quantities, prices and profits of firms 1 and 2 are given by expressions (18), (19), (20), (21), (22) and (23), respectively. The consumer surplus is given by (24) and the social welfare is given by (25).

**Proof.** See Appendix 7.1. □
3.3 Comparative statics

Recall that the parameter $b$ measures the degree of product differentiation such that as $b$ goes from 0 to 1 the independent goods become homogeneous goods. In this subsection, we perform a comparative-static analysis to understand how the equilibrium variables vary according to the degree of product differentiation between the two goods, under quantity competition. The following Lemma summarizes.

**Lemma 2** Let $b \in (0, \bar{b})$, with $\bar{b} \approx 0.83$. As the degree of product differentiation increases:

**Prices:** (i) The equilibrium price of the pure profit-maximizing firm is strictly decreasing; (ii) The equilibrium price of the consumer-friendly firm is increasing for $b \in (0, b^{q^*})$, reaches a local maximum in $b^{q^*} \approx 0.417$ and is decreasing in $b \in (b^{q^*}, \bar{b})$; (iii) The equilibrium price of the pure profit-maximizing firm is always higher than the equilibrium price of the consumer-friendly firm.

**Quantities:** (i) The equilibrium quantity of the pure profit-maximizing firm is strictly decreasing; (ii) The equilibrium quantity of the consumer-friendly firm is decreasing for $b \in (0, b^{q^*})$, reaches a local minimum in $b^{q^*} \approx 0.519$ and is increasing in $(b^{q^*}, \bar{b})$; (iii) The equilibrium quantity of the pure profit-maximizing firm is always lower than the equilibrium price of the consumer-friendly firm.

**Profits:** (i) The equilibrium profit of the pure profit-maximizing firm is strictly decreasing; (ii) The equilibrium profit of the consumer-friendly firm is increasing for $b \in (0, b^{q^*})$, reaches a local maximum in $b^{q^*} \approx 0.399$ and is decreasing in $(b^{q^*}, \bar{b})$; (iii) The equilibrium profit of the pure profit-maximizing firm is always higher than the equilibrium price of the consumer-friendly firm.

**Consumer Surplus:** (i) The equilibrium consumer surplus is decreasing for $b \in (0, b^{CS^*})$, reaches a local minimum in $b^{CS^*} \approx 0.566$ and is increasing in $(b^{CS^*}, \bar{b})$.

**Social Welfare:** (i) The equilibrium social welfare is strictly decreasing.

**Proof.** See Appendix 7.1. □
The intuition behind the results comes from two distinct effects: (i) the effect of product differentiation on the relative importance of consumer surplus, $\frac{d\lambda^* (b)}{db}$ (let us call this product differentiation effect) and (ii) the effect of the relative importance of consumer surplus on equilibrium prices and market shares, $\frac{\partial p^*_2(\lambda, b)}{\partial \lambda^* (b)}$ and $\frac{\partial q^*_2(\lambda, b)}{\partial \lambda^* (b)}$ (let us call this benevolence effect).

Given the level of altruism $\lambda^* (b)$ defined by the policymaker, follows that the minimum altruism level is attained when the product differentiation is intermediate and as the goods become independent ($b \rightarrow 0$) or homogeneous ($b \rightarrow \overline{b}$), the altruism levels become local maxima ($\frac{d\lambda^* (b)}{db} < 0 \iff b^{\lambda^*} \geq 0.5096$ and $\frac{d^2 \lambda^* (b)}{db^2} \bigg|_{b=0.5096} = 2.20783 > 0$).

Relatively to firm 2, we observe that both the equilibrium price and quantity of firm 2 decrease (in the same magnitude) with an increment on the level of altruism $\lambda$ such that

$$\frac{\partial q^*_2(\lambda, b)}{\partial \lambda} = \frac{\partial q^*_2(\lambda, b)}{\partial \lambda} = -\frac{ab(2-b)(1+b)}{4-b^2(1-\lambda)-2\lambda^2} < 0, \forall b \in (0, \overline{b}).$$

For $b > b^{\lambda^*}$, as the value of $b$ becomes sufficiently high (i.e., product differentiation softens), the goods become homogeneous and the market competition turns to be more intense. Then, the equilibrium price and quantity of the pure profit-maximizing firm decrease because the negative benevolence effect more than surpasses the positive product differentiation effect. Analytically, and for $b > b^{\lambda^*}$:

$$\frac{\partial q^*_2(\lambda, b)}{\partial b} = \frac{\partial q^*_2(\lambda, b)}{\partial b} = \left. \frac{\partial p^*_2(\lambda, b)}{\partial \lambda^* (b)} \right|_{\lambda^*} < 0.$$ 

For $b < b^{\lambda^*}$, as the value of $b$ becomes sufficiently low (i.e., product differentiation intensifies), the goods become independent and, thus, the market competition tends to soften. The equilibrium price and quantity of the pure profit-maximizing firm increase because both the benevolence effect and the product differentiation effect are negative. Analytically, follows that for $b < b^{\lambda^*}$:

$$\frac{\partial q^*_2(\lambda, b)}{\partial b} = \frac{\partial q^*_2(\lambda, b)}{\partial b} = \left. \frac{\partial p^*_2(\lambda, b)}{\partial \lambda^* (b)} \right|_{\lambda^*} < 0.$$ 

As a result, the equilibrium profit of the pure profit-maximizing firm is strictly decreasing in $b$. 

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On the other hand, we observe two different impacts on the consumer-friendly firm: 
\[
\frac{\partial p_1^*}{\partial \lambda} = \frac{-a(2-b^2)[2+b(1+b)]}{[4-b^2(1-\lambda)-2\lambda]} < 0 \quad \text{and} \quad \frac{\partial q_1^*}{\partial \lambda} = \frac{2a[2+b(1+b)]}{[4-b^2(1-\lambda)-2\lambda]} > 0, \quad \forall b \in (0, \bar{b}).
\]
The higher the relative importance that firm 1 attaches to the consumer surplus, the higher is the equilibrium price and the lower is the equilibrium quantity of firm 1.

For \(b > b^{\lambda*}\), as the value of \(b\) becomes sufficiently high, the goods become homogeneous and the market competition turns to be more intense. Then, the equilibrium price of the consumer-friendly firm decreases because the negative benevolence effect more than exceeds the positive product differentiation effect. However, the equilibrium quantity of the consumer-friendly firm increases given the positive benevolence and product differentiation effects. Analytically implies:
\[
\left. \frac{\partial p_1^*}{\partial b} \right|_{b^{\lambda*}} < 0 \quad \text{and} \quad \left. \frac{\partial q_1^*}{\partial b} \right|_{b^{\lambda*}} > 0,
\]
respectively.

For \(b < b^{\lambda*}\), as the value of \(b\) increases the equilibrium quantity of the consumer-friendly firm decreases as long as the negative product differentiation effect more than outweighs the positive benevolence effect. Analytically, follows that for \(b < b^{\lambda*}\):
\[
\left. \frac{\partial q_1^*}{\partial b} \right|_{b^{\lambda*}} > 0.
\]
The equilibrium quantity reduces until a minimum level \(b^{q\ast} \approx 0.519\), value above which an increment on the degree of product differentiation increases the equilibrium quantity since the positive benevolence effect more than outperforms the negative product differentiation effect.

Finally, given the strategic substitutability of the output levels between firms 1 and 2, the equilibrium price of the consumer-friendly firm increases but just until the maximum level of product differentiation \(b^{pp*} \approx 0.417\). Above this value and, therefore, for \((b^{pp*}, b^{\lambda*})\) both the negative pressure of the benevolence and product differentiation effects dictate that an increment on the degree of product differentiation leads to a reduction on the equilibrium price of the consumer-friendly firm.
As a result, the equilibrium profit of the consumer-friendly firm depends on the combination of all the above mentioned effects such that is increasing in $b$ until it reaches a local maximum in $b^{\pi*} \approx 0.399$ and is decreasing in $(b^{\pi*}, b)$.

### 4 Price competition

Consider also the price competition game where firms 1 and 2 simultaneously choose their output levels.

#### 4.1 Stage 2

Given the level of social concern $\lambda$ of the consumer friendly-firm, the degree of product differentiation $b$ and using (1), (3) and (7), the objective function of firm 1 is given by:

$$V_1(p_1, p_2, \lambda, b) = \frac{a(1-b) - p_1 + bp_2}{1-b^2}p_1 + \lambda \left[ \frac{2a^2(1-b) + p_1^2 - 2bp_1p_2 + p_2^2 - 2a(1-b)(p_1 + p_2)}{2(1-b^2)} \right]. \quad (26)$$

Using (1) and (4), the absolute profit of firm 2 is given by:

$$V_2(p_1, p_2, b) = \frac{a(1-b) - p_2 + bp_1}{1-b^2}p_2. \quad (27)$$

Deriving (26) relatively to $p_1$ and (27) relatively to $p_2$, the reaction functions of the firms are given by:

$$p_1(p_2, \lambda, b) = \frac{a(1-b) + bp_2}{2} (1-\lambda); \quad (28)$$

$$p_2(p_1, \lambda) = \frac{a(1-b) + bp_1}{2}. \quad (29)$$

Then, the outcomes of the second stage of the game come as follows:

$$p_1^e(\lambda, b) = \frac{a(1-b)(2+b)}{4-b^2(1-\lambda)-2\lambda}; \quad p_2^e(\lambda, b) = \frac{a(1-b)(2+b(1-\lambda)-\lambda)}{4-b^2(1-\lambda)-2\lambda}; \quad (30)$$

---

14 In Appendix, we provide a graphical illustration on the different equilibrium outcomes as a function of the degree of product differentiation, for both firms.
\[ q_1^p(\lambda, b) = \frac{a(2 + b)}{(1 + b)(4 - b^2(1 - \lambda) - 2\lambda)}; \quad q_2^p(\lambda, b) = \frac{a[2 + b - \lambda(1 + b)]}{(1 + b)[4 - b^2(1 - \lambda) - 2\lambda]}; \quad (31) \]

\[ \pi_1^p(\lambda, b) = \frac{a^2(1 - b)(2 + b)^2(1 - \lambda)}{(1 + b)[4 - b^2(1 - \lambda) - 2\lambda]^2}; \quad \pi_2^p(\lambda, b) = \frac{a^2(1 - b)[2 + b(1 - \lambda) - \lambda]^2}{(1 + b)[4 - b^2(1 - \lambda) - 2\lambda]^2}; \quad (32) \]

\[ CS^p(\lambda, b) = \frac{a^2[8 + 2b^2(1 - \lambda) - 4\lambda + \lambda^2 + b(8 - 6\lambda + \lambda^2)]}{2(1 + b)[4 - b^2(1 - \lambda) - 2\lambda]^2} + q; \quad (33) \]

\[ W^p(\lambda, b) = \frac{a^2[24 + 2b^3(1 - \lambda)^2 - 20\lambda + 3\lambda^2 - b^2(2 - 6\lambda + \lambda^2) - 2b(8 - 7\lambda + 2\lambda^2)]}{2[4 - b^2(1 - \lambda) - 2\lambda]^2} + q; \quad (34) \]

where the superscript "p" denotes the outcomes under price competition.

### 4.2 Stage 1

Now, we solve the first stage of the game. Our problem requires to find the level of altruism \( \lambda^* \) on the consumer-friendly firm that maximizes equation (34). The optimal \( \lambda^* \) is given by:

\[ \lambda^*_p(b) = \frac{(2 - b)^2}{4 - 2b^2 - b}. \quad (35) \]

Substituting (35) into expressions (30) to (34) implies the following equilibrium outcomes:

\[ p_1^p = \frac{3ab(2 + b)(1 - b)^2}{3b^4 - 3b^3 - 10b^2 + 4b + 8}; \quad (36) \]

\[ p_2^p = \frac{a(3b^3 - 4b^2 - 2b + 4)}{3b^4 - 3b^3 - 10b^2 + 4b + 8}; \quad (37) \]

\[ q_1^p = \frac{a(-2b^3 - 5b^2 + 2b + 8)}{3b^5 - 13b^3 - 6b^2 + 12b + 8}; \quad (38) \]

\[ q_2^p = \frac{a(-3b^3 - 2b^2 + 2b + 4)}{3b^5 - 13b^3 - 6b^2 + 12b + 8}; \quad (39) \]

\[ \pi_1^p = \frac{3a^2b(b^2 + b - 2)^2(-2b^2 - b - 4)}{(1 + b)^3(2 - b)^2(4 - 3b^2)^2}; \quad (40) \]

\[ \pi_2^p = \frac{a^2(1 - b)(3b^3 + 2b^2 - 2b - 4)^2}{(1 + b)^3(2 - b)^2(4 - 3b^2)^2}; \quad (41) \]

\[ CS^p = \frac{a^2(-4b^4 - 13b^3 - 3b^2 + 8b + 20)}{2(1 + b)^3(2 - b)^2(4 - 3b^2)}; \quad (42) \]
The following Proposition summarizes.

Proposition 3 (Price Competition) Let \( \lambda \in \left[ 0, \frac{2-b}{2-b^2} \right) \) and \( b \in (0, 1) \).

In a duopoly with differentiated and substitutable goods, composed by a consumer-friendly firm and a pure profit-maximizing firm that engage in price competition, the equilibrium prices, quantities and profits of firms 1 and 2 are given by expressions (36), (37), (38), (39), (40) and (41), respectively. The consumer surplus is given by (42) and the social welfare is given by (43).

Proof. See Appendix 7.2. ■

4.3 Comparative statics

Similar to the case of quantity competition, we perform a comparative-static analysis to understand how the equilibrium variables vary according to the degree of product differentiation, but now under price competition. The following Lemma summarizes.

Lemma 4 Let \( b \in (0, 1) \). As the degree of product differentiation increases:

Prices: (i) The equilibrium price of the pure profit-maximizing firm is strictly decreasing; (ii) The equilibrium price of the consumer-friendly firm is increasing for \( b \in (0, b^{pp*}) \), reaches a local maximum in \( b^{pp*} \approx 0.42 \) and is decreasing in \( (b^{pp*}, 1) \); (iii) The equilibrium price of the pure profit-maximizing firm is always higher than the equilibrium price of the consumer-friendly firm for \( b \in (0, 1) \) and both are equal to zero for \( b = 1 \).

Quantities: (i) The equilibrium quantity of the pure profit-maximizing firm is strictly decreasing; (ii) The equilibrium quantity of the consumer-friendly firm is decreasing for \( b \in (0, b^{pq*}) \), reaches a local minimum in \( b^{pq*} \approx 0.708 \) and is increasing in \( (b^{pq*}, 1) \); (iii) The equilibrium quantity of the pure profit-maximizing firm is always lower than the equilibrium price of the consumer-friendly firm.
**Profits:** (i) The equilibrium profit of the pure profit-maximizing firm is strictly decreasing; (ii) The equilibrium profit of the consumer-friendly firm is increasing for $b \in (0, b^\pi*)$, reaches a local maximum in $b^\pi* \approx 0.359$ and is decreasing in $(b^\pi*, 1)$; (iii) The equilibrium profit of the pure profit-maximizing firm is higher than the equilibrium profit of the consumer-friendly firm for $b \in (0, 1)$ and both are equal to zero for $b = 1$.

**Consumer Surplus:** (i) The equilibrium consumer surplus is decreasing for $b \in (0, b^{CS*})$, reaches a local minimum in $b^{CS*} \approx 0.538$ and is increasing in $(b^{CS*}, 1)$.

**Social Welfare:** (i) The equilibrium social welfare is strictly decreasing.

**Proof.** See Appendix 7.2. ■

The intuition of Lemma 4 under price competition is similar to the intuition regarding quantity competition. Interestingly, the results on the ranking order of the equilibrium social welfare between the quantity competition and the price competition lead us to the following Corollary.

**Corollary 5 (Social welfare and product differentiation)**

(i) The social welfare under price competition is higher than the social welfare under quantity competition, $\forall b \in (0, \bar{b})$;

(ii) As goods become nearly independent, the difference in social welfare between price competition and quantity competition increases.

**Proof.** See Appendix 7.2. ■

Nakamura (2013) concludes that the equilibrium social welfare can be larger in the quantity competition than in the price competition when the degree of product differentiation is sufficiently low and when the extent of the importance of consumer surplus to the consumer-friendly firm is relatively high. The trivial result of standard duopoly with substitutable goods is that the social welfare is always larger in the price competition than in the quantity competition because the market competition is relatively intense owing to the strategic complementarity of the price levels between the two rival firms.
We find that the results of Nakamura (2013) can be reversed to the traditional evidence argued in the literature once the parameter that measures the importance of consumer surplus to the consumer-friendly firm is endogenous. Firstly, the equilibrium social welfare is always higher under price competition since both profits and consumer surplus are higher under price competition. Secondly, as in Nakamura (2013), we verify that when goods are nearly homogeneous \((b \to 1)\), the difference in social welfare between price competition and quantity competition is positive. However, the novelty here relies on the fact that, contrary to Nakamura (2013), as the goods become nearly independent \((b \to 0)\), the difference in the equilibrium social welfare between the price competition and the quantity competition slightly increases more such that the maximum social welfare difference is attained at \(b = 0\). This conclusion holds because the positive impact of goods becoming nearly independents \((b \to 0)\) in the consumer surplus more than compensates the positive difference in firm’s profits and consumer surplus as the degree of product differentiation increases \((b \to 1)\). Figure 1 clearly illustrates the above finding.

5 Main result

Given Propositions 1 and 3, we now show the core message of the manuscript, summarized as it follows.

**Proposition 6 (Public intervention on the consumer-friendly firm)**

(ii) Under quantity competition, if firms sell goods that are too homogeneous the policymaker should impose a ceiling on the level of benevolence of the consumer-friendly firm.

(ii) Under price competition, the policymaker never imposes a ceiling on the level of kindness of the consumer-friendly firm.

**Proof.** The proof is straightforward given the proof of Propositions 1 and 3 discussed in Appendix 7.1 and 7.2, respectively. ■
Our conclusion shows that a regulatory intervention to restrain the benevolence of the consumer-friendly firm depends: (i) on the type of competition that firms are dealing with and (ii) in the case of quantity competition, on the degree of product differentiation between the market goods. Mathematically and in the quantity-setting duopoly, we find that for $b < b^* \approx 0.83$, the policymaker allows the company with social concerns to be as worried as much as the firm desires. However, for $b > b^* \approx 0.83$, the policymaker must restrict the benevolence of the firm with social concerns, otherwise, due to market competition intensification, the positive absolute profit of the consumer-friendly firm is no longer secure, which harms the social welfare. On the other hand, under price competition, a regulatory intervention to block philanthropy is never required, $\forall b \in (0, 1)$.

The finding is relevant for policymaking decision, so that the public authorities are able to better understand when to act in a market dealing with quantity competition and, on the other hand, to allow for a relaxation in the definition of the degree of altruism of a consumer-friendly firm under price competition.

6 Conclusions

Companies have a new role of both internal and external social responsibility and to become competitive need to do it well. The new consumer is more aware of the issue of corporate social responsibility and influences to a better performance of both companies and regulatory bodies. The management and governance within the context of ethical norms is now a guideline for the balance of the markets. In this sense, we model a duopoly with differentiated and substitutable goods composed of one consumer-friendly firm and one pure-profit maximizing firm. We consider that the consumer-friendly firm maximizes the weighted sum of its absolute profit and consumer surplus. In such a duopoly, a regulatory authority intervenes to control the degree of altruism of the consumer-friendly firm in order to guarantee the maximization of social welfare.

Our manuscript finds that under quantity competition, if firms sell goods that are too homogeneous the policymaker should impose a ceiling on the level of benevolence
of the consumer-friendly firm. However, under price competition, the policymaker never imposes a ceiling on the level of kindness of the consumer-friendly firm. Our results also conclude that, whatever the degree of product differentiation, the social welfare under price competition is always higher than the social welfare under quantity competition, which is in line with the arguments pointed out by traditional literature and establishes a contrast with Nakamura (2013).

For direct future research, it would be important to understand more about competition between firms incorporating social corporate responsibility. In particular, it is important to understand what is the optimal level of social corporate responsibility in markets where all firms embrace both profitable and social concerns. Other important extension would be to understand the role of sequential decisions between firms and the introduction of segmentation in business objectives or, by other words, to split in a theoretical model the so called long-term plans relatively to short-term decisions. We predict that the firm with a huge fraction of short-run targets may overlook social corporate responsibility in contrast to the company with greater long-term commitments.
7 Appendix

7.1 Quantity competition

Proof of Proposition 1

We must prove that $q(b) \leq \frac{2 - b}{2 - b^2}$. For these boundaries, the consumer-friendly firm maintains its social concern and simultaneously does not incur in negative profits. Given (17), follows that:

\[ q(b) \geq 0 \iff -b^4 + 13b^3 - 6b^2 + 4(2 - b) \geq 0, \forall b \in [0, 1]. \]

On the other hand:

\[ q^*(b) < \frac{2 - b}{2 - b^2} \iff \frac{b^5 - 11b^4 + 7b^3 + 7b^2 - 4}{(2 - b^2)[2b^4 + 7b^3 - 9b^2 + 4(2 - b)]} < 0. \]

This inequality holds for:

\[ b \in (0, \bar{b}) \]

with $\bar{b} \approx 0.83$. For the open interval $b \in (0, \bar{b})$, the inequality $b^5 - 11b^4 + 7b^3 + 7b^2 - 4 < 0$ is always satisfied and, thus, all the equilibrium variables hold a positive value. Proposition 1 is, now, straightforward.

Proof of Lemma 2

To prove the Lemma, we simply compute the first and second order derivatives for each one of the equilibrium outcomes.

Prices

Relatively to the consumer-friendly firm 1, the equilibrium price function $p_1^q(b)$ is continuous in $b \in [0, \bar{b})$ and $p_1^q(0) = 0$. Then, the first order derivative is given by:

\[ \frac{\partial p_1^q(b)}{\partial b} = \frac{a(27b^{10} + 688b^9 + 134b^8 + 925b^7 - 693b^6 + 848b^5 + 556b^4 - 64b^3 + 64)}{[3b^6 - 9b^4 - 4b^3 + 24b^2 - 16(b+1)]^2}. \]  

(44)

Solving (44) follows that the level $b$ that secures the highest possible price level is:

\[ \frac{\partial p_1^q(b)}{\partial b} = 0 \iff b^* \approx 0.417, \]
since the second order derivative is given by:

$$\frac{\partial^2 p^*_q(b)}{\partial b^2} = \frac{2a}{[35b^6-65b^5-135b^4+4b^3+245b^2-16(b+1)]^2} < 0,$$

$$\forall b \in (0, \bar{b}) \cap \forall a > 0.$$ Evaluating the second order derivative at \(b^*\) we obtain:

$$\frac{\partial^2 p^*_q(b)}{\partial b^2} \bigg|_{b^*=0.417} = -0.837677a < 0,$$

$$\forall a > 0.$$ Finally:

$$\left\{ \begin{array}{ll}
\frac{\partial p^*_q(b)}{\partial b} > 0 \text{ for any } [0, b^*] & \Rightarrow p^*_q(b) \text{ is increasing and} \\
\frac{\partial^2 p^*_q(b)}{\partial b^2} < 0 \text{ for any } [0, b^*] & \Rightarrow \text{ with the concavity facing downwards in } [0, b^*]\end{array} \right.$$

$$\left\{ \begin{array}{ll}
\frac{\partial p^*_q(b)}{\partial b} < 0 \text{ for any } [b^*, \bar{b}] & \Rightarrow p^*_q(b) \text{ is decreasing and} \\
\frac{\partial^2 p^*_q(b)}{\partial b^2} < 0 \text{ for any } [b^*, \bar{b}] & \Rightarrow \text{ with the concavity facing downwards in } [b^*, \bar{b}]\end{array} \right.$$

Relatively to the pure profit-maximizing firm 2, the equilibrium price function \(p^*_2(b)\) is continuous in \(b \in [0, \bar{b})\) and \(p^*_2(0) = \frac{a}{2}\). The first order derivative is given by:

$$\frac{\partial p^*_2(b)}{\partial b} = \frac{2a(-3b^{10} - 6b^9 + 3b^8 + 13b^6 + 137b^5 + 238b^4 - 396b^3 - 112b^2 + 176b^2 - 32b - 64)}{[35b^6 - 65b^5 - 135b^4 + 4b^3 + 245b^2 - 16(b+1)]^2} < 0,$$

\(\forall b \in (0, \bar{b}) \cap \forall a > 0\). We provide a graphic illustration for robustness (c.f. Figure 2). \(^{15}\)

\[ \text{[Insert Figure 2 here]} \]

Quantities

Relatively to the consumer-friendly firm 1, the equilibrium quantity function \(q^*_1(b)\) is continuous in \(b \in [0, \bar{b})\) and \(q^*_1(0) = a\). Then, the first order derivative is given by:

$$\frac{\partial q^*_1(b)}{\partial b} = \frac{2a(-3b^{10} + 20b^9 + 18b^8 - 156b^7 + 1496b^6 + 486b^5 - 300b^4 + 128b^3 + 24b^2 - 64)}{[35b^6 - 65b^5 - 135b^4 + 4b^3 + 245b^2 - 16(b+1)]^2}. \quad (46)$$

\(^{15}\)As proved, the results hold for any \(a > 0\) but we set \(a = 1\) for all further simulations.
Solving (46) follows that the level $b$ that secures the lowest possible quantity level is:

$$
\frac{\partial q^*_1(b)}{\partial b} = 0 \iff b^* \approx 0.5187,
$$

since the second order derivative is given by:

$$
\frac{\partial^2 q^*_1(b)}{\partial b^2} = \frac{6a(99^{15} - 906^{14} + 99^{13} + 11248^{12} - 28418^{11} + 17340^{10} + 59296^{9} - 14112^{8} - 144^{7})}{(36^6 - 6b^6 - 13^{6} + 4b^3 + 24b^2 - 16(b+1))^3} + \frac{6a(15664b^6 + 1440b^5 - 2496b^4 - 512b^3 - 2338b^2 - 768b - 1024)}{(36^6 - 6b^6 - 13^{6} + 4b^3 + 24b^2 - 16(b+1))^3} > 0.
$$

$\forall b \in (0, b) \cap \forall a > 0$. Evaluating the second order derivative at $b^*$ we obtain:

$$
\frac{\partial^2 q^*_1(b^*)}{\partial b^2} \bigg|_{b^* \approx 0.5187} = 1.92301a > 0,
$$

$\forall a > 0$. Finally:

$$
\left\{ \begin{array}{ll}
\frac{\partial q^*_1(b)}{\partial b} & < 0 \text{ for any } ]0, b^*[ \iff q^*_1(b) \text{ is decreasing and} \\
\frac{\partial^2 q^*_1(b)}{\partial b^2} & > 0 \text{ for any } ]0, b^*[ \Rightarrow \text{ with the concavity facing upwards in } ]0, b^*[ \\
\end{array} \right.
$$

$$
\left\{ \begin{array}{ll}
\frac{\partial q^*_2(b)}{\partial b} & > 0 \text{ for any } ]b^*, \bar{b}[ \iff q^*_2(b) \text{ is increasing and} \\
\frac{\partial^2 q^*_2(b)}{\partial b^2} & > 0 \text{ for any } ]b^*, \bar{b}[ \Rightarrow \text{ with the concavity facing upwards in } ]b^*, \bar{b}[ \\
\end{array} \right.
$$

Relatively to the pure profit-maximizing firm 2, the equilibrium quantity function $q^*_2(b)$ is continuous in $b \in [0, \bar{b})$ and $q^*_2(0) = \frac{a}{2}$. The first order derivative is given by:

$$
\frac{\partial q^*_2(b)}{\partial b} = \frac{\partial p^*_2(b)}{\partial b} < 0,
$$

$\forall b \in (0, \bar{b}) \cap \forall a > 0$. We provide a graphic illustration for robustness (c.f. Figure 3).

[Insert Figure 3 here]

Profits
Relatively to the consumer-friendly firm 1, the equilibrium profit function $\pi_1^*(b)$ is continuous in $b \in [0, \overline{b})$ and $\pi_1^*(0) = 0$. Then, the first order derivative is given by:

$$
\frac{\partial \pi_1^*(b)}{\partial b} = a^2(-9b^{16} + 240b^{15} - 1143b^{14} + 829b^{13} + 7133b^{12} + 14652b^{11} + 8905b^{10} + 21934b^9 - 47964b^8) \\
+ a^2(4560b^7 + 41280b^6 - 161280b^5 - 156160b^4 + 58880b^3 + 53760b^2 - 5120 - 1024)
$$

(47)

Solving (47) follows that the level $a > 0$ b that secures the highest possible profit level is:

$$
\frac{\partial \pi_1^*(b)}{\partial b} = 0 \iff b^* \approx 0.3989,
$$

since the second order derivative is given by:

$$
\frac{\partial^2 \pi_1^*(b)}{\partial b^2} = a^2(27b^{21} - 1053b^{20} + 7092b^{19} - 7620b^{18} - 4803b^{17} + 23400b^{16} - 40523b^{15} - 162228b^{14}) \\
+ 2a^2(-180905b^{13} - 292264b^{12} - 1239144b^{11} + 329884b^{10} - 415792b^9 - 1235280b^8 + 2337125) \\
+ 2a^2(-35328b^5 - 1020672b^6 + 473334b^7 + 153600b^8 - 110592b^9 - 4096b - 28672) \\
\left[3b^6 - 6b^5 - 13b^4 + 4b^3 + 24b^2 - 16(b + 1)^4 \right] < 0,
$$

$\forall b \in (0, \overline{b}) \cap \forall a > 0$. Evaluating the second order derivative at $b^*$ we obtain:

$$
\left. \frac{\partial^2 \pi_1^*(b)}{\partial b^2} \right|_{b^* \approx 0.3989} = -0.575066a^2 < 0,
$$

$\forall a > 0$. Finally:

$$
\begin{cases}
\frac{\partial \pi_1^*(b)}{\partial b} > 0 \text{ for any }]0, b^*[ & \implies \pi_1^*(b) \text{ is increasing and} \\
\frac{\partial^2 \pi_1^*(b)}{\partial b^2} < 0 \text{ for any }]0, b^*[ & \implies \text{with the concavity facing downwards in }]0, b^*[ \\
\frac{\partial \pi_1^*(b)}{\partial b} < 0 \text{ for any }]b^*, \overline{b}[ & \implies \pi_1^*(b) \text{ is decreasing and} \\
\frac{\partial^2 \pi_1^*(b)}{\partial b^2} < 0 \text{ for any }]b^*, \overline{b}[ & \implies \text{with the concavity facing downwards in }]b^*, \overline{b}[
\end{cases}
$$

Relatively to the pure profit-maximizing firm 2, the equilibrium price function $\pi_2^*(b)$ is continuous in $b \in [0, \overline{b})$ and $\pi_2^*(0) = \frac{a^2}{4}$. The first order derivative is given by:

$$
\frac{\partial \pi_2^*(b)}{\partial b} = \frac{8a^2(b^5 + b^4 - 5b^3 + 7b^2 - 4)}{3b^6 - 6b^5 - 13b^4 + 4b^3 + 24b^2 - 16(b + 1)} \left( \frac{\partial \pi_1^*(b)}{\partial b} \right) < 0,
$$

25
\( \forall b \in (0, \bar{b}) \cap \forall a > 0. \) We provide a graphic illustration for robustness (c.f. Figure 4).

\[ \text{[Insert Figure 4 here]} \]

**Consumer Surplus**

Relatively to the consumer surplus, the function \( CS^{a}(b) \) is continuous in \( b \in [0, \bar{b}] \) and \( CS^{a}(0) = \frac{5a^2}{16} \). Then, the first order derivative is given by:

\[
\frac{\partial CS^{a}(b)}{\partial b} = \frac{\sigma^2 \left( 18b^{16} - 9b^{15} - 756b^{14} + 19896b^{13} + 8388d^{12} - 127836b^{11} + 18648b^{10} + 11459b^9 - 54444b^8 + 19440b^7 \right)}{36b^6 - 6b^5 - 13b^4 + 4b^3 + 24b^2 - 16(b+1)^4} + \frac{a^2 \left( 48144b^6 - 32928b^5 - 23232b^4 + 16384b^3 + 9984b^2 - 2304b - 2048 \right)}{2 \left( 36b^6 - 6b^5 - 13b^4 + 4b^3 + 24b^2 - 16(b+1)^4 \right)}, \quad (48)
\]

Solving (48) follows that the level \( b \) that secures the lowest possible consumer surplus level is:

\[
\frac{\partial CS^{a}(b)}{\partial b} = 0 \iff b^* \approx 0.566,
\]

since the second order derivative is given by:

\[
\frac{\partial^2 CS^{a}(b)}{\partial b^2} = \frac{3a^2 \left( 36b^{21} + 9b^{20} - 27120b^{19} + 111726b^{18} - 108486b^{17} - 78954b^{16} + 290184b^{15} - 187764b^{14} + 36b^6 - 6b^5 - 13b^4 + 4b^3 + 24b^2 - 16(b+1)^4 \right)}{2 \left( 36b^6 - 6b^5 - 13b^4 + 4b^3 + 24b^2 - 16(b+1)^4 \right)} + \frac{3a^2 \left( -80068b^{13} + 1491673b^{12} - 1211720b^{11} + 1365760b^{10} + 890928b^9 - 1511936b^8 \right)}{2 \left( 36b^6 - 6b^5 - 13b^4 + 4b^3 + 24b^2 - 16(b+1)^4 \right)} + \frac{3a^2 \left( 140032b^6 + 543744b^5 - 341320b^4 - 94208b^3 + 92160b^2 + 32768b + 20480 \right)}{2 \left( 36b^6 - 6b^5 - 13b^4 + 4b^3 + 24b^2 - 16(b+1)^4 \right)} > 0,
\]

\( \forall b \in (0, \bar{b}) \cap \forall a > 0. \) Evaluating the second order derivative at \( b^* \) we obtain:

\[
\frac{\partial^2 CS^{a}(b)}{\partial b^2} \bigg| _{b^* \approx 0.566} = 0.488055a^2 > 0,
\]

\( \forall a > 0. \) Finally:

\[
\begin{cases}
\frac{\partial CS^{a}(b)}{\partial b} < 0 \text{ for any } 0, b^*[ & \text{C}S^{a}(b) \text{ is decreasing and} \\
\frac{\partial^2 CS^{a}(b)}{\partial b^2} > 0 \text{ for any } 0, b^*[ & \text{with the concavity facing upwards in } 0, b^*[ \\
\frac{\partial CS^{a}(b)}{\partial b} > 0 \text{ for any } b^*, \bar{b}[ & \text{C}S^{a}(b) \text{ is increasing and} \\
\frac{\partial^2 CS^{a}(b)}{\partial b^2} > 0 \text{ for any } b^*, \bar{b}[ & \text{with the concavity facing upwards in } b^*, \bar{b}[ 
\end{cases}
\]
Social Welfare

Relatively to the social welfare function $W_q^*(b)$ is continuous in $b \in [0, \tilde{b})$ and $W_q^*(0) = \frac{7a^2}{16}$. The first order derivative is given by:

$$\frac{\partial W_q^*(b)}{\partial b} = \frac{3a^2[3b^{10}-5b^8+66b^7+7b^6+367b^5-284b^4-120b^3+208b^2-16(b+4)]}{2[3b^8+6b^6+13b^4-4b^3-4b^2+16b+1]^2} < 0,$$

$\forall b \in (0, \tilde{b}) \cap \forall a > 0$, which completes the proof of Lemma 2. \hfill \Box

### 7.2 Price competition

**Proof of Proposition 3**

We must prove that $\lambda^*(b) \in \left[0, \frac{2-b}{2-b^2}\right)$. For these boundaries, the consumer-friendly firm maintains its social concern and simultaneously does not incur in negative profits. Given (35), follows that:

$$\lambda^*(b) \geq 0 \iff (2 - b)^2 > 0, \forall b \in [0, 1].$$

On the other hand:

$$\lambda^*(b) < \frac{2-b}{2-b^2} \iff -\frac{(2-b)b(1-b)^2}{(2-b^2)(4-b-2b^2)} < 0, \forall b \in (0, 1).$$

Then, for the open interval $b \in (0, 1)$, the inequality is always satisfied and, thus, all the equilibrium variables hold with positive values. Proposition 3 is, now, straightforward. \hfill \Box

**Proof of Lemma 4**

The proof of Lemma 4 is similar to the proof of Lemma 2. We compute the first and second order derivatives for each one of the equilibrium outcomes.

**Prices**

Relatively to the consumer-friendly firm 1, the equilibrium price function $p_1^{eq}(b)$ is continuous in $b \in [0, 1]$ and $p_1^{eq}(0) = 0$. Then, the first order derivative is given by:

$$\frac{\partial p_1^{eq}(b)}{\partial b} = -\frac{3a[36b^5+2b^3+15b^5+44b^3-38b^2+16(3b-1)]}{(b-2)^2(1-b^2)(4-3b^2)^2}. \quad (49)$$
Solving (49) follows that the level $b$ that secures the highest possible price level is:

$$\frac{\partial p_1^{*}\,(b)}{\partial b} = 0 \iff b^* \approx 0.42,$$

since the second order derivative is given by:

$$\frac{\partial^2 p_1^{*}\,(b)}{\partial b^2} = \frac{6a\left(3b^8 + 18b^5 - 26b^4 - 14b^3 + 12b^2 + 8(2 - b)\right)}{(b - 2)^3(1 - b)^3(3b^2 - 4)^2} < 0,$$

$\forall b \in (0, 1) \cap \forall a > 0$. Evaluating the second order derivative at $b^*$ we obtain:

$$\left.\frac{\partial^2 p_1^{*}\,(b)}{\partial b^2}\right|_{b^*=0.42} = -1.13702a < 0,$$

$\forall a > 0$. Finally:

$$\begin{cases} \frac{\partial p_1^{*}\,(b)}{\partial b} > 0 \text{ for any } \, ]0, b^*[ & \Rightarrow \ p_1^{*}\,(b) \text{ is increasing and} \\ \frac{\partial^2 p_1^{*}\,(b)}{\partial b^2} < 0 \text{ for any } \, ]0, b^*[ & \Rightarrow \text{ with the concavity facing downwards in } \, ]0, b^*[ \end{cases}$$

$$\begin{cases} \frac{\partial p_1^{*}\,(b)}{\partial b} < 0 \text{ for any } \, ]b^*, 1[ & \Rightarrow \ p_1^{*}\,(b) \text{ is decreasing and} \\ \frac{\partial^2 p_1^{*}\,(b)}{\partial b^2} < 0 \text{ for any } \, ]b^*, 1[ & \Rightarrow \text{ with the concavity facing downwards in } \, ]b^*, 1[ \end{cases}$$

Relatively to the pure profit-maximizing firm 2, the equilibrium price function $p_2^{*}\,(b)$ is continuous in $b \in [0, 1]$, and $p_2^{*}\,(0) = \frac{a}{2}$. The first order derivative is given by:

$$\frac{\partial p_2^{*}\,(b)}{\partial b} = -\frac{2a\left(3b^8 + 18b^5 - 26b^4 - 14b^3 + 12b^2 + 8(2 - b)\right)}{(3b^2 - 1)b^2 + 4(b + 2)} < 0,$$

$\forall b \in (0, 1) \cap \forall a > 0$. We provide a graphic illustration for robustness (c.f. Figure 5).

[Insert Figure 5 here]

Quantities

Relatively to the consumer-friendly firm 1, the equilibrium quantity function $q_1^{*}\,(b)$ is continuous in $b \in [0, 1]$ and $q_1^{*}\,(0) = a$. Then, the first order derivative is given by:

$$\frac{\partial q_1^{*}\,(b)}{\partial b} = \frac{a\left(12b^8 + 33b^5 - 57b^4 - 116b^3 + 12b^2 + 96b - 80\right)}{(b - 2)^2(1 + b)^3(4 - 3b^2)^2}.$$  \hspace{1cm} (51)
Solving (51) follows that the level $b$ that secures the lowest possible quantity level is: $$\frac{\partial q^*_1(b)}{\partial b} = 0 \iff b^* \approx 0.708,$$

since the second order derivative is given by:

$$\frac{\partial^2 q^*_1(b)}{\partial b^2} = -\frac{2a(54b^8+162b^9-651b^7+1890b^6+224b^5-2848b^4+1200b^3+1376b^2-1024)}{(b-2)^2(1+b)^2(3b^2-4)^2} > 0$$

\forall b \in (0, 1) \cap \forall a > 0. Evaluating the second order derivative at $b^*$ we obtain:

$$\left.\frac{\partial^2 q^*_1(b)}{\partial b^2}\right|_{b^* \approx 0.708} = 1.25116a > 0,$$

\forall a > 0. Finally:

$$\left\{\begin{array}{ll}
\frac{\partial q^*_1(b)}{\partial b} < 0 \text{ for any } ]0, b^*[
\quad & q^*_1(b) \text{ is decreasing and} \\
\frac{\partial^2 q^*_1(b)}{\partial b^2} > 0 \text{ for any } ]0, b^*[
\quad & \text{with the concavity facing upwards in } ]0, b^*[ \n\end{array}\right.$$

$$\left\{\begin{array}{ll}
\frac{\partial q^*_2(b)}{\partial b} > 0 \text{ for any } ]b^*, 1[
\quad & q^*_2(b) \text{ is increasing and} \\
\frac{\partial^2 q^*_2(b)}{\partial b^2} > 0 \text{ for any } ]b^*, 1[
\quad & \text{with the concavity facing upwards in } ]b^*, 1[ \n\end{array}\right.$$

Relatively to the pure profit-maximizing firm 2, the equilibrium quantity function $q^*_2(b)$ is continuous in $b \in [0, 1]$ and $q^*_2(0) = \frac{a}{2}$. The first order derivative is given by:

$$\frac{\partial q^*_2(b)}{\partial b} = \frac{2a(9b^6-126b^5-224b^4+128b^3-8(2-3b))}{(b-2)^2(1+b)^2(4-3b^2)^2} < 0,$$

\forall b \in (0, 1) \cap \forall a > 0. We provide a graphic illustration for robustness (c.f. Figure 6).

Relatively to the consumer-friendly firm 1, the equilibrium profit function $\pi^*_1(b)$ is continuous in $b \in [0, 1]$ and $\pi^*_1(0) = 0$. Then, the first order derivative is given by:

$$\frac{\partial \pi^*_1(b)}{\partial b} = \frac{3a^2(1-b)(2+b)(-128-2757+776+26b^5-128b+192b^3+80b^2-224b+64)}{(b-2)^3(1+b)^3(3b^2-4)^3}.$$  \hspace{1cm} (52)
Solving (52) follows that the level $b$ that secures the highest possible profit level is:
\[
\frac{\partial \pi_1^{**}(b)}{\partial b} = 0 \iff b^* \approx 0.3586,
\]

since the second order derivative is given by:
\[
\frac{\partial^2 \pi_1^{**}(b)}{\partial b^2} = \frac{6a^2 (54b^{13} + 216b^{12} - 456b^{11} - 693b^{10} - 2876b^9 - 3225b^8 - 6816b^7 + 18152b^6)}{(b - 2)^4(1 + b)^4(4 - 3b^2)^4} < 0,
\]

$\forall b \in (0, 1) \cap \forall a > 0$. Evaluating the second order derivative at $b^*$ we obtain:
\[
\frac{\partial^2 \pi_1^{**}(b)}{\partial b^2} \bigg|_{b^* \approx 0.3586} = -0.783626a^2 < 0,
\]

$\forall a > 0$. Finally:
\[
\begin{aligned}
&\left\{ \begin{array}{l}
\frac{\partial \pi_1^{**}(b)}{\partial b} > 0 \text{ for any } ]0, b^*[ \quad \Rightarrow \quad \pi_1^{**}(b) \text{ is increasing and} \\
\frac{\partial^2 \pi_1^{**}(b)}{\partial b^2} < 0 \text{ for any } ]0, b^*[ \quad \Rightarrow \quad \text{with the concavity facing downwards in } ]0, b^*[ \\
\end{array} \right.
\end{aligned}
\]

\[
\begin{aligned}
&\left\{ \begin{array}{l}
\frac{\partial \pi_1^{**}(b)}{\partial b} < 0 \text{ for any } ]b^*, 1[ \quad \Rightarrow \quad \pi_1^{**}(b) \text{ is decreasing and} \\
\frac{\partial^2 \pi_1^{**}(b)}{\partial b^2} > 0 \text{ for any } ]b^*, 1[ \quad \Rightarrow \quad \text{with the concavity facing downwards in } ]b^*, 1[ \\
\end{array} \right.
\end{aligned}
\]

Relatively to the pure profit-maximizing firm 2, the equilibrium price function $\pi_2^{**}(b)$ is continuous in $b \in [0, 1]$ and $\pi_2^{**}(0) = \frac{a^2}{4}$. The first order derivative is given by:
\[
\frac{\partial \pi_2^{**}(b)}{\partial b} = \frac{-2a^2 (27b^{10} - 12b^9 - 120b^8 + 160b^7 + 80b^5 - 128b^3 + 64b^2 - 128(1-b))}{(b - 2)^3(1 + b)^2(4 - 3b^2)^3} < 0,
\]

$\forall b \in (0, 1) \cap \forall a > 0$. We provide a graphic illustration for robustness (c.f. Figure 7).

[Insert Figure 7 here]

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Consumer Surplus
Relatively to the consumer surplus, the function $CS^p(b)$ is continuous in $b \in [0, 1]$ and $CS^p(0) = \frac{5a^2}{8}$. Then, the first order derivative is given by:

$$\frac{\partial CS^p(b)}{\partial b} = -\frac{a^2(18b^7 + 66b^6 - 20b^5 - 103b^4 - 209b^3 + 124b^2 + 248b - 128)}{(b - 2)^2(1 + b)^4(4 - 3b)^2}. \quad (53)$$

Solving (53) follows that the level $b$ that secures the lowest possible consumer surplus level is:

$$\frac{\partial CS^p(b)}{\partial b} = 0 \iff b^* \approx 0.538,$$

since the second order derivative is given by:

$$\frac{\partial^2 CS^p(b)}{\partial b^2} = \frac{a^2(216b^{10} + 882b^9 - 630b^8 - 1539b^7 - 4631b^6 + 5124b^5 + 9756b^4 - 12496b^3 - 528b^2 + 8640b - 4544)}{(b - 2)^2(1 + b)^4(3b^2 - 4)^3} > 0,$$

$\forall b \in (0, 1) \cap \forall a > 0$. Evaluating the second order derivative at $b^*$ we obtain:

$$\left.\frac{\partial^2 CS^p(b)}{\partial b^2}\right|_{b^* \approx 0.538} = 0.903469a^2 > 0,$$

$\forall a > 0$. Finally:

$$\begin{cases} 
\frac{\partial CS^p(b)}{\partial b} < 0 \text{ for any } ]0, b^*[ \quad \Rightarrow \quad CS^p(b) \text{ is decreasing and} \\
\frac{\partial^2 CS^p(b)}{\partial b^2} > 0 \text{ for any } ]0, b^*[ \quad \Rightarrow \quad \text{with the concavity facing upwards in } ]0, b^*[ 
\end{cases}$$

$$\begin{cases} 
\frac{\partial CS^p}{\partial b} > 0 \text{ for any } ]b^*, 1[ \quad \Rightarrow \quad CS^q(b) \text{ is increasing and} \\
\frac{\partial^2 CS^p}{\partial b^2} > 0 \text{ for any } ]b^*, 1[ \quad \Rightarrow \quad \text{with the concavity facing upwards in } ]b^*, 1[ 
\end{cases}$$

**Social Welfare**

Relatively to the social welfare function $W^p(b)$ is continuous in $b \in [0, 1]$ and $W^p(0) = \frac{7a^2}{8}$. The first order derivative is given by:

$$\frac{\partial W^p(b)}{\partial b} = -\frac{3a^2(1-b)(4-3b)}{(4-3b)^2} < 0,$$

$\forall b \in (0, 1) \cap \forall a > 0$. The first derivative equals zero for $b^* = 1.6369 \notin (0, 1)$, which completes the proof of Lemma 4. \(\square\)
Proof of Corollary 5

(i) Let \( b \in (0, \tilde{b}) \), with \( \tilde{b} \approx 0.83 \). Using (25) and (43), suppose by contradiction that:

\[
W^{qs} > W^{ps} \iff \frac{a^2 \left[-6b^5 + 5b^4 - 32b^3 + 45b^2 - 4(b + 7)\right]}{16(3b^9 - 6b^8 - 13b^6 + 4b^5 + 24b^2 - 16(b + 1))^2} > \frac{a^2(7 - 6b)}{8 - 6b^2}
\] (54)

Rearranging (54), we obtain:

\[
\frac{a^2}{4} \left[\frac{2(7 - 6b)}{3b^2 - 4} \cdot \frac{6b^5 - 5b^4 + 32b^3 - 45b^2 + 4(b + 7)}{3b^6 - 6b^5 - 13b^4 + 4b^3 + 24b^2 - 16(b + 1)}\right] > 0.
\] (55)

The polynomial secures real roots in \( b_{c_1}^* \approx -0.576 \) and \( b_{c_2}^* \approx 1.85 \). Since \((0, \tilde{b}) \subset (b_{c_1}^*, b_{c_2}^*)\) and given that for \( b \in (b_{c_1}^*, b_{c_2}^*) \) the inequality (55) is never satisfied, then \( W^{qs} > W^{ps} \) can never hold for \( b \in (0, \tilde{b}) \).

(ii) Now, considering \( \Delta W = W^{ps} - W^{qs} \), follows that:

\[
\Delta W = \frac{a^2}{4} \left[\frac{2(7 - 6b)}{3b^2 - 4} + \frac{6b^5 - 5b^4 + 32b^3 - 45b^2 + 4(b + 7)}{3b^6 - 6b^5 - 13b^4 + 4b^3 + 24b^2 - 16(b + 1)}\right].
\]

The first derivative relatively to \( b \) equals:

\[
\frac{\partial \Delta W}{\partial b} = \frac{3a^2 \left(81b^{14} + 387613b^{12} + 446612b^{10} - 237910 + 1913b^9 - 4212b^8\right)}{2(4 - 3b^2)^2 \left[-3b^5 + 6b^4 + 13b^3 - 4b^2 - 24b^2 + 16(b + 1)\right]^2} - \frac{3a^2 \left(-441b^7 - 1312b^6 - 246b^5 + 6272b^4 + 1536b^3 - 486b^2 + 256b + 1024\right)}{2(4 - 3b^2)^2 \left[-3b^5 + 6b^4 + 13b^3 - 4b^2 - 24b^2 + 16(b + 1)\right]^2},
\]

while the second derivative is given by:

\[
\frac{\partial^2 \Delta W}{\partial b^2} = \frac{3a^2 \left(145821 - 104499b^2 + 456845b^4 - 59967b^6 + 168642b^8 + 536085b^{10} - 67392b^{12}\right)}{2(4 - 3b^2)^2 \left[-3b^5 + 6b^4 + 13b^3 - 4b^2 - 24b^2 + 16(b + 1)\right]^2} + \frac{3a^2 \left(-135980b^{11} + 1515132b^{10} + 258820b^9 - 1672032b^8 + 248318b^7 + 1205440b^6\right)}{2(4 - 3b^2)^2 \left[-3b^5 + 6b^4 + 13b^3 - 4b^2 - 24b^2 + 16(b + 1)\right]^3} + \frac{3a^2 \left(-308070b^8 + 510259b^7 + 306432b^6 + 348057b^5 + 160512b^4 + 630784b^3 - 798720b^2 + 491528 + 114688\right)}{2(4 - 3b^2)^2 \left[-3b^5 + 6b^4 + 13b^3 - 4b^2 - 24b^2 + 16(b + 1)\right]^3}.
\]

Then, it is straightforward to check that \( \frac{\partial \Delta W}{\partial b} = 0, \forall b \in (0, \tilde{b}) \). Indeed: \( \frac{\partial \Delta W}{\partial b} < 0 \land \frac{\partial^2 \Delta W}{\partial b^2} > 0, \forall b \in (0, \tilde{b}) \), and the proof is completed. \( \square \)
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Profit of the consumer-friendly firm under Price Competition
Profit of the pure profit-maximization firm under Price Competition
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