Long-Term Regulatory Orientation and the Ideal Timing of Quality Investment

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Long-term regulatory orientation and the ideal timing of quality investment

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Abstract. This paper builds on a duopoly with horizontally differentiated firms, where firms simultaneously decide the long-term plan (location) in addition to the short-term issue (price). As in Bárcena-Ruiz and Casado-Izaga (2014), we introduce a third entity in the city by considering the presence of a policymaker that targets the long-run ideology (location) of the regulated sector. While Bárcena-Ruiz and Casado-Izaga (2014) relies on a non-discriminatory setting relatively to firms’ quality, here we introduce quality distortion (a high-quality firm versus a low-quality firm). Our aim is to study the relationship between the long-term regulatory guidance provided by a policymaker and the ideal timing of the quality investment conducted by the high-quality firm. We find that it is irrelevant to the firm invest before or after the long-term decision of the policymaker. In this sense, we show that the long-term strategic guideline conducted by a regulatory authority is not the motivating source of firms’ improvement-quality investments. Finally, we conclude that the presence of an asymmetric quality environment between firms leads to a movement to the right on firms’ location, creates an ambiguous (an enhancing) effect on the equilibrium profit of the low-quality (high-quality) firm and generates a reduction of the equilibrium consumer surplus and equilibrium social welfare as well, relatively to a situation where no quality discrimination exists.

Keywords: Spatial competition, Long-run decision, Policymaker location, Quality asymmetry, Price competition.

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1 Introduction

Especially nowadays, market regulation is a major concern. As OECD (2011) reports, the objective of regulatory policy is "to ensure that regulation works effectively and is in the public interest. Regulatory policy (...) is taking shape in different ways across OECD members and beyond (...) many OECD countries did not have a regulatory policy ten years ago; nearly all do now". The economic literature has also shown that the great majority of the markets not only embrace horizontal product differentiation but vertical differentiation as well, i.e, asymmetric qualities between active firms operating (see, among many others, Gabszewicz and Wauthy (2012)). In this sense, given (i) the existence of asymmetric duopolies (regarding quality) and (ii) that a high-quality firm intends to provide quality into the market, our goal is to secure an answer to the following research question: is it relevant for the investor (high-quality firm) the timing of the long-run orientation conducted by a policymaker? If so, should the high-quality firm invest in quality provision before or after the long-term decision of the policymaker? The economic literature lacks this contribution for two main reasons. First, there is no agreement relatively to firms investment activities and the regulator’s long-term guideline. In the context of environmental economics, Gray and Shadbegian (1998) show that technology choice is influenced by regulation in the sense that new mills in states with strict environmental regulations tend not to employ more polluting technologies involving pulping. However, state regulatory stringency have little or no effect on annual investment spending at existing plants. Jaffe et al. (2003) conclude that environmental policy interventions create new constraints and incentives that affect the process of technological change. Regarding the energy sector, Marques et al. (2014) conclude that strong incentive regulation mechanisms are able to stimulate the adoption of smart grid (SG) technologies. Moreover, the authors also empathize that the regulation should not jeopardize conventional investments that are unable to be substituted by these new technologies. Secondly, very few researchers deal with our issue from a mathematical point of view. Smith (2000) focuses on a relatively narrow set of issues related with the location of a regulatory authority within the broader institutional arrangement. However, the author did not explore the issue mathematically. Guthrie (2006) confirms that competition is actively promoted by many regulators and
both the regulator and regulated firms must often confront rapid technological progress, which requires an additional investment effort. To the best of our knowledge, Bárcena-Ruiz and Casado-Izaga (2014) is the first attempt closer to ours, where the authors study the topic of waste management regulation under spatial competition. The authors find that the optimal decisions have an impact on firms’ competition and welfare and depend on the relationship between waste and product transportation costs. However, the model is assumed to be fully symmetric in terms of the quality served to consumers by the active competitors. Our research builds on a duopoly with horizontally differentiated firms, where firms decide the long-term plan (location) in addition to short-term issues (prices). As in Bárcena-Ruiz and Casado-Izaga (2014), we introduce a third entity in the city by considering the presence of a policymaker that targets the long-run ideology (location) to the regulated sector. Whilst in Bárcena-Ruiz and Casado-Izaga (2014) there is no quality discrimination, here we introduce a quality distortion between firms. Our aim is to understand whether the ideal timing to the quality investment of the high-quality firm is before or after the long-term decision set by the policymaker. Our manuscript concludes that the long-term normative emerging from the policymaker is not the driving force that incentives the high-quality firm to provide quality into the market. The remainder of the paper comes as it follows. Section 2 presents the model. Sections 3 and 4 provide the analysis emerging from the presented model. Section 5 encompasses the main results of the manuscript and section 6 concludes. An Appendix in section 7 contains the key technical details.

2 Model

We consider a duopoly with horizontally differentiated firms that deal with two types of decisions: short-term and long-term issues. On the demand side of the market, consumers are located at $x$ and are distributed uniformly and with unitary density along the interval $[0, 1]$. They incur in quadratic transportation costs relatively to distance $td^2$, where $t > 0$ measures the degree of horizontal differentiation between firms. There is a continuum of singlehoming consumers that inelastically demand one unit of the good. Each consumer
derives a surplus from consumption, gross of price and transportation costs, denoted by $v$, that is sufficiently large such that the whole market is fully covered. On the supply side of the market, two private firms indexed by $i$ ($i \in \{A, B\}$) endogenously choose locations $a$ and $b$, respectively ($\{a, b\} \subseteq \mathbb{R}$). Therefore, firms may locate outside the unitary interval. We consider that firm $A$ is located to the left of or at the same point as firm $B$ such that $a \leq b$. Our aim is to introduce quality asymmetry between firms. Therefore, we consider that only firm $B$ invests in quality provision.\(^1\) In this sense, we assume the presence of a low-quality firm $A$ and a high-quality firm $B$ that provides a quality level $q^B$ to customers and each consumer chooses the preferred firm based on price, transportation cost and quality. Then, the indirect utility of an agent that is located at $x \in [0, 1]$ that chooses firm $A$ is given by:

$$u^A(x) = v - p^A - t(x - a)^2, \tag{1}$$

while the indirect utility of an agent that is located at $x \in [0, 1]$ that chooses firm $B$ is given by:

$$u^B(x) = v + q^B - p^B - t(b - x)^2. \tag{2}$$

Note that, as in Bárcena-Ruiz and Casado-Izaga (2005) and Matsumura and Matsushima (2012), the location decision is considered to be the long-term ideological plan of firms. Without loss of generality, we normalize the firms’ production costs to zero. The firm’s $B$ quality provision cost function is given by:\(^2\)

$$CT(q^B) = \frac{k}{2} (q^B)^2. \tag{3}$$

As in Bárcena-Ruiz and Casado-Izaga (2014), we introduce a third entity in the city by considering the presence of a policymaker located at $r \in \mathbb{R}$. The production of the good involves to follow a certain long-term ideology. When the long-run ideology of firm

\(^1\)By other words, the marginal cost of quality provision to firm $A$ is so high such that this firm does not to invest in quality provision. Another reinterpretation of such analysis is to consider the presence of a quality differential $q \equiv q^B - q^A > 0$.

\(^2\)The results remain qualitatively the same if we assume the same positive (and non-discriminatory) marginal cost. The separation in quality and quantity of firms’ costs implies that quality has the characteristics of a public good for the consumers, a widely used assumption in the literature (e.g., Economides (1989), Calem and Rizzo (1995) and Sanjo (2007)). Also $\frac{d^2CT(q^B)}{dq^B} = kq^B$ and $\frac{d^3CT(q^B)}{dq^B dq^2} = k > 0$ meaning that the quality provision cost per unit is increasing with the quality of the good $i$. 

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i mismatches the long-term guidance of the policymaker, firm i incurs in a cost. The ideological mismatch cost of firm A ($M^A$) and the ideology mismatch cost of firm B ($M^B$) are given by the following equations, respectively:

$$M^A = g(a - r)^2 D^A; \quad M^B = g(b - r)^2 D^B,$$

where $g > 0$ measures the degree of mismatch between the long-term ideology of each firm relatively to the regulator’s long-term guidance and $D^i$ is the demand of firm $i$. Thus, the ideological inconsistency costs are quadratic with respect to the distance travelled from each firm to the regulator’s location and proportional to the amount of firm’s market share. We also consider that the long-run decision of the firms and of the regulator cannot be changed in the future. The definition of the regulator’s long-run standpoint is decided only by the regulator whose purpose is social welfare maximization. As usually, the social welfare ($W$) consists in the sum of the profits of firms and consumer surplus:

$$W = \pi^A + \pi^B + CS.$$  

Then, the profit of firm A and the profit of firm B are given by:

$$\pi^A = (p^A - M^A) D^A; \quad \pi^B = (p^B - M^B) D^B - \frac{k}{2} (q^B)^2,$$

respectively, where $p^i$ denotes the price set by firm $i$ ($i = \{1, 2\}$) and $k > 0$ denotes the marginal quality provision cost of firm $B$. The consumer surplus ($CS$) is given by:

$$CS = v - p^A D^A - p^B D^B - \int_0^{\bar{x}} t(x - a)^2 dx - \int_{\bar{x}}^1 t(b - x)^2 dx.$$  

Rearranging (7), the $CS$ equals:

$$CS = v - p^A D^A - p^B D^B + \bar{x} \bar{x}^2 (a - b) + t \bar{x}(b^2 - a^2) + tb - \frac{t}{3} - tb^2.$$  

Using (1) and (2), the location of the indifferent consumer between attending to firm A or B is given by:

$$\bar{x} = \frac{a + b}{2} + \frac{p^B - p^A}{2t(b - a)} - \frac{q^B}{2t(b - a)}.$$  

Thus, the respective demands of firm A and B are given by:

$$D^A = \begin{cases} 1, & \text{if } \bar{x} > 1 \\ \bar{x}, & \text{if } 0 \leq \bar{x} \leq 1 \\ 0, & \text{if } \bar{x} < 0 \end{cases} \quad D^B = \begin{cases} 1, & \text{if } \bar{x} < 0 \\ 1 - \bar{x}, & \text{if } 0 \leq \bar{x} \leq 1, \\ 0, & \text{if } \bar{x} > 1 \end{cases}$$  

(10)
since demand is assumed to be totally inelastic. Our model is equivalent to Bárcena-Ruiz
and Casado-Izaga (2014). However, we introduce quality asymmetry and the timing of
the game differs substantially. Then, we study two models and compare the respective
results to drive conclusions. In the first (called analysis of an accommodated high-quality
firm), the timing of the game is the following: in the first stage, the policymaker defines
the long-term guidance of the regulated sector, location \( r \); in the second stage, the (ac-
commodated) high-quality firm \( B \) decides the quality level, \( q^B \); in the third stage, both
firms simultaneously define their long-term plans, locations \( a \) and \( b \); finally, in the last
stage of the game, both firms engage in price competition. In the second (called analysis
of a proactive high-quality firm), the timing of the game is the following: in the first stage,
the (proactive) high-quality firm \( B \) anticipates the long-term guidance of the policymaker
and decides the amount of quality provision, \( q^B \); in the second stage, the policymaker
defines the long-term guidance of the regulated sector, location \( r \); in the third stage, both
firms simultaneously define their long-term plans, locations \( a \) and \( b \); finally, in the last
stage of the game, both firms engage in price competition. We rely our analysis on the
backward induction method.

3 Analysis of an accommodated high-quality firm

**Price competition.** Using (6), (4) and given the quality provision of the high-quality
firm \( q^B \) and long run decisions \( a, r \) and \( b \), the equilibrium profits of firms \( A \) and \( B \) are
given by:

\[
\pi^A = \left[ p^A - g(a - r)^2 \right] \left( \frac{a + b}{2} + \frac{p^B - p^A}{2t(b - a)} - \frac{q^B}{2t(b - a)} \right); \quad (11)
\]

\[
\pi^B = \left[ p^B - g(b - r)^2 \right] \left[ 1 - \left( \frac{a + b}{2} + \frac{p^B - p^A}{2t(b - a)} - \frac{q^B}{2t(b - a)} \right) \right] - \frac{k}{2} (q^B)^2. \quad (12)
\]

Deriving (11) and (12) relatively to \( p^A \) and \( p^B \) we obtain the first order condition for each
firm. Solving both simultaneously, follows that the equilibrium prices are given by:

\[
p^A(r, q^B, a, b) = \frac{t(b - a)(2 + b + a) + g[2a(a - 2r) + b(b - 2r) + 3r^2]}{3} - \frac{q^B}{3}; \quad (13)
\]

\[
p^B(r, q^B, a, b) = \frac{t(b - a)(4 - b - a) + g[2b(b - 2r) + a(a - 2r) + 3r^2]}{3} + \frac{q^B}{3}, \quad (14)
\]
respectively. Substituting (13) and (14) into (9), (6) (8) and (5) follows that the demands and profits of both firms and social welfare are given by\(^3\):

\[
D_A(r, q^B, a, b) = \frac{t(2 + b + a) + g(a + b - 2r)}{6t} - \frac{q^B}{6t(b - a)}; \tag{15}
\]

\[
D_B(r, q^B, a, b) = \frac{t(4 - b - a) - g(a + b - 2r)}{6t} + \frac{q^B}{6t(b - a)}; \tag{16}
\]

\[
\pi_A(r, q^B, a, b) = \frac{\{b - a\} [t(2 + b + a) + g(a + b - 2r)] - q^B}{18t(b - a)}; \tag{17}
\]

\[
\pi_B(r, q^B, a, b) = \frac{\{b - a\} [t(4 - b - a) - g(a + b - 2r)] + q^B}{18t(b - a)} - \frac{k}{2} (q^B)^2; \tag{18}
\]

\[
W(\cdot) = v - \frac{4q^B(b-a)|a+b+2r-t(1-a-b)|+(q^B)^2[12t(3gr^2+t)+4t^2(5gr+7t)+a^2(5g(t)+5gr+4t)+5a^2(5g(t)+5gr+4t)]}{36t(b-a)}
\]

\[
- \frac{(b-a)\{12t(6gr^2+t)+4t^2(5ggr+7t)+a^2(5g(t)+5gr+4t)+5a^2(5g(t)+5gr+4t)\}}{36t(b-a)}; \tag{19}
\]

**Long-run decision of firms.** Given the long run decision of the policymaker \(r\) and the quality provision level of the high-quality firm \(q^B\), using (17) and (18) and deriving both relatively to \(a\) and \(b\) respectively, follows that the equilibrium locations of both firms are given by:\(^4\)

\[
a(r, q^B) = \frac{1}{12} \left[ -3 - \frac{4q^B}{t} + \frac{3(g + 4gr)}{g + t} \right]; \tag{20}
\]

\[
b(r, q^B) = -\frac{q^B}{3t} + \frac{4gr + 5t}{4(g + t)}. \tag{21}
\]

Substituting (20) and (21) into (18) and (19), follows that the profit of the high-quality firm and the social welfare are given by, respectively:

\[
\pi_B(r, q^B) = \frac{81t^4 + 72q^Bt^2(g + t) - 2(q^B)^2(5g + 27kt^2 - 8(g + t))}{108t^2(g + t)}; \tag{22}
\]

\[
W(r, q^B) = v - \frac{9t^3[16g(1 - 3r + 3r^2) + 13t] + 8(q^B)^2(5g + 27kt - 42g + 2t)}{432t^2(g + t)}. \tag{23}
\]

\(^3\)The second order conditions are always satisfied. See Appendix 7 for technical details.

\(^4\)The second order conditions are always satisfied. See Appendix 7 for technical details.
Quality decision of the accommodated high-quality firm. Given the policy-maker long-run decision \( r \), the (accommodated) high-quality firm maximizes profits with respect to its quality level \( q^B \). Deriving (22) relatively to \( q^B \) follows that:

\[
q^B_* = \frac{18t^2}{27kt^2 - 8(g + t)}.
\] (24)

Substituting (24) into (19), follows that the social welfare is given by:

\[
W(r) = s - \frac{t \left\{ 16g(1 - 3r + 3r^2) + 13t + \frac{288(g + t)[27kt^2 - 2(g + t)]}{[27kt^2 - 8(g + t)]^2} \right\}}{48(g + t)}.
\] (25)

Long-run guidance of the policymaker. Finally, the policymaker defines the location \( r \) that maximizes social welfare. Deriving (25) relatively to \( r \), we obtain that the optimal long run decision of the policymaker is given by:

\[
r^* = \frac{1}{2}.
\] (26)

The policymaker establishes a central long-run position in the unconstrained city. Substituting (26) and (24) into (20), (21), (13), (14), (15), (16), (17), (18), (8) and (5) follows that the equilibrium outcomes are given as it follows:

\[
q^B_* = \frac{18t^2}{27kt^2 - 8(g + t)},
\] (27)

\[
a_* = -\frac{1}{4} + \frac{3g}{4(g + t)} - \frac{6t}{27kt^2 - 8(g + t)}, \quad b_* = \frac{5}{4} - \frac{3g}{4(g + t)} - \frac{6t}{27kt^2 - 8(g + t)};
\] (28)

\[
p^{A*} = \frac{t \left\{ 33g + 24t + \frac{576g(g + t)^2}{27kt^2 - 8(g + t)^2} - \frac{48(g + t)(g + 4t)}{27kt^2 - 8(g + t)^2} \right\}}{16(g + t)^2},
\] (29)

\[
p^{B*} = \frac{t \left\{ 33g + 24t + \frac{576g(g + t)^2}{27kt^2 - 8(g + t)^2} + \frac{48(g + t)(g + 4t)}{27kt^2 - 8(g + t)^2} \right\}}{16(g + t)^2};
\] (30)

\[
D^{A*} = \frac{1}{2} - \frac{4(g + t)}{27kt^2 - 8(g + t)}, \quad D^{B*} = \frac{1}{2} + \frac{4(g + t)}{27kt^2 - 8(g + t)};
\] (31)

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5The second order condition is always satisfied as long as \( k > \frac{8(g + t)}{27t^2} \equiv \bar{k} \). See Appendix 7 for technical details.

6The second order condition is always satisfied. See Appendix 7 for technical details.
\[ \pi^A = \frac{3t^2 [27kt^2 - 16(g + t)]^2}{4(g + t) [27kt^2 - 8(g + t)]^2}, \quad \pi^B = \frac{8kt^4}{4(g + t) [27kt^2 - 8(g + t)]^2}; \]
\[ CS^* = v - \frac{t \left\{ 4g + 85t + \frac{4032t(g+t)^2}{27kt^2-8(g+t)} \right\}}{48(g+t)}, \quad W^* = v - \frac{t \left\{ 4g + 13t + \frac{288t(g+t)[27kt^2-2(g+t)]}{(27kt^2-8(g+t))^2} \right\}}{48(g+t)}. \]

Proposition 1 (Market equilibrium with an (accommodated) high-quality firm)

In the game where, in the first stage, the policymaker defines the long-term guidance of the regulated sector, in the second stage, the (accommodated) high-quality firm B decides its quality level, in the third stage, both firms simultaneously define their long-term plans and, finally, in the fourth stage of the game, both firms engage in price competition: (i) the equilibrium locations of the policymaker and of firms A and B are given by (26) and (28), respectively; (ii) the equilibrium prices and market shares of firms A and B are given by (29), (30) and (31), respectively; (iii) the equilibrium profits of firms A and B are given by (32) and (iv) the equilibrium consumer surplus and equilibrium social welfare are given by (33).

Proof. The proof is straightforward from the above methodological description.

Proposition 1 lacks economic intuition. In equilibrium, the policymaker locates in the central position of the linear city and his decision is irresponsive to any other parameter, which defines, as Stern (1997) points out, the independence principle of regulatory policy relatively to the private firms’ investments. Apart from (i) the demand effect that pushes both firms towards the middle of the market in order to locate them closer to consumers, (ii) beyond the strategic price effect that pushes both firms to locate further from the rival to mitigate price competition and (iii) aside the ideological cost reducing effect that pushes both firms to locate closer to the regulator’s long-term point of view, another significant impact emerges: (iv) the quality distortion effect. Note that, the equilibrium quality provision level is negatively affected by the cost of quality provision \( k \) and by the degree of product differentiation \( t \) and positively influenced by the degree of ideological mismatch \( g \), respectively \( \frac{d\pi^B}{dk} < 0, \frac{d\pi^B}{dt} < 0 \) and \( \frac{d\pi^B}{dg} > 0, \forall g \in \mathbb{R}^+ \cap t \in \mathbb{R}^+ \). Given that, in the third stage, the location of each firm is negatively influenced by quality...
provision (because $\frac{\partial a(r,q^B)}{\partial q^B} < 0$ and $\frac{\partial b(r,q^B)}{\partial q^B} < 0$) and, in the fourth stage, the quality distortion has a positive (negative) impact over the equilibrium price and market share of firm $B$ ($A$) (since $\frac{\partial D^A(r,q^B,a,b)}{\partial q^B} < 0$, $\frac{\partial D^B(r,q^B,a,b)}{\partial q^B} > 0$, $\frac{\partial p^A(r,q^B,a,b)}{\partial q^B} < 0$ and $\frac{\partial p^B(r,q^B,a,b)}{\partial q^B} > 0$, respectively), we can counteract some of the impacts observed in Bárcena-Ruiz and Casado-Izaga (2014). As an example, note that the long run position of the low-quality firm is, now, negatively influenced by $g$ since:

$$\frac{\partial (aoq^B)}{\partial g} = \frac{\partial a^*}{\partial q} \frac{dq^*}{dg} < 0 + \frac{\partial a^*}{\partial g} = \frac{3g\{27kt^2-8(g+t)[1+8(g+t)]\}}{4(g+t)^2[27kt^2-8(g+t)]^2} < 0, \forall g > 0 \cap t > 0.$$  

In this case, although the ideological cost reducing effect pushes firm $A$ to locate closer to the policymaker, the quality distortion effect pushes the firm far away from its rival. Since the later is stronger than the former, the overall impact $\frac{\partial (aoq^B)}{\partial g} < 0$ constitutes a sharp contrast relatively to Bárcena-Ruiz and Casado-Izaga (2014). Summing up, the overall magnitude of these four effects determines the response of equilibrium variables to each one of the relevant parameters $k$, $g$ and $t$, respectively.

### 4 Analysis of a proactive high-quality firm

Let us now analyze the presence of a (proactive) high-quality firm that defines its quality investment before the long-term guidance of the policymaker. The stages concerning firms’ location and price competition are equivalent to the previous section and, thus, omitted.

**Long-term guidance of the policymaker.** Given the anticipatory quality investment $q^B$, the policymaker maximizes social welfare relatively to its sectorial ideological long-term plan, $r$. Deriving (23) relatively to $r$ follows that:

$$r^* = \frac{1}{2}, \quad (34)$$

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7 Note that the function $aoq^B$ stands for "decision of the long run position $a$ by firm $A$, after observing the quality choice $q^B$ of firm $B". 
which is precisely (26). Thus, the *independence principle of regulatory policy* relatively to private firms’ investments still holds.

**Quality decision of the proactive high-quality firm.** In this stage, the (proactive) high-quality firm maximizes its profits with respect to the quality level $q^B$. Deriving (22) relatively to $q^B$, we obtain that the optimal quality provision of firm $B$ is given by:

$$q^{B*} = \frac{18t^2}{27kt^2 - 8(g + t)};$$  \tag{35}

which is exactly (24). Then, the equilibrium outcomes are given by the same expressions as in the case of an accommodated high-quality firm.

## 5 Main results

The main finding of this manuscript is, now, straightforward.

**Lemma 2 (Ideal timing of quality provision)** *It is irrelevant for the high-quality the timing of the long-run orientation conducted by the policymaker.*

Whatever the situation considered, $q^{B*}$ is irresponsive to any long-run guidance change conducted by the policymaker, $r$. Then, according to (15) and (16), the market shares of both firms are also unchangeable with respect to $r$. Finally, it is vital to observe that, once we substitute (20) and (21) into (18) and (17), both profits no longer depend on the long-run decision of the policymaker, $r$. Finally, we compare our results with the case of a non-discriminatory quality analyzed in Bárcena-Ruiz and Casado-Izaga (2014). First, we restore their results and, then, Lemma 3 Concludes. The intuition is provided in the Appendix.

**Remark 1 (Inexistence of quality distortion)** *In a game where, in the first stage the policymaker chooses the long-term orientation of the regulated sector, in the second stage the firms simultaneously decide their long-term plans and, in the third stage, both firms*
engage in price competition the equilibrium outcomes are given by:

\[ q^B = 0, \quad r^* = \frac{1}{2}, \quad a^* = \frac{1}{4} + \frac{3g}{4(g + t)}, \quad b^* = \frac{5}{4} - \frac{3g}{4(g + t)}; \]

\[ \pi^{A*} = \pi^{B*} = \frac{3t^2}{4(g + t)}; \]

\[ CS^* = v - \frac{t(3g + 85t)}{48(g + t)}, \quad W^* = v - \frac{t(4g + 13t)}{48(g + t)}. \]

**Lemma 3 (Impacts of asymmetric quality)** Let \( k > \frac{8(g + t)}{27t^2} \equiv \overline{k} \). The presence of an asymmetric quality between firms implies: (i) a movement to the right of the equilibrium location of both firms; (ii) an increment on the equilibrium profit of the high-quality firm; (iii) an ambiguous effect on the equilibrium profit of the low-quality firm, depending on the cost of quality provision, \( k \); (iv) a reduction of the of the equilibrium consumer surplus and (v) a reduction of the equilibrium social welfare.

**Proof.** See Appendix 7. ■

### 6 Conclusions

In this paper, we study the relationship between the long-term regulatory guidance provided by a policymaker and the ideal timing of quality investment in a duopoly embracing a high-quality firm and a low-quality firm. We find that it is irrelevant for the high-quality firm to decide the quality provision before or after the policymaker long run decision. In this sense, the *ex-ante* investment of high-quality regulated companies near the beginning of a new regulatory long-term guideline has no rational economic support since the firm is able to reach the same profit if the investment is conducted after the entrance of the long-run public regulation. By other words, the long-term normative emerging from the policymaker is not the driving force that incentives the high-quality firm to provide quality into the market. We let for future research a deeper analysis of this issue to understand what could be the critical driving force. We predict that the equilibrium outcomes are distorted if the quality stage is conducted after the long-term decision of firms and before firms engage in price competition.
7 Appendix

Price competition. The second order conditions that secure that the equilibrium prices in the fourth stage are local maxima are given by:

\[
\frac{\partial^2 \pi^A}{\partial (p^A)^2} = \frac{\partial^2 \pi^B}{\partial (p^B)^2} = -\frac{1}{t(b-a)} < 0, \forall a \leq b.
\]

Long-run decision of firms. The second order conditions that secure that the equilibrium locations \(a\) and \(b\) are local maxima are given by:

\[
\frac{\partial^2 \pi^A}{\partial a^2} = \frac{(q^B)^2 - (b-a)^3(g + t) [(b + 3a)(g + t) - 4(r g - t)]}{9t(b-a)^3},
\]

\[
\frac{\partial^2 \pi^B}{\partial b^2} = \frac{(q^B)^2 - (b-a)^3(g + t) [4(2t + rg) - (3b + a)(g + t)]}{9t(b-a)^3}.
\]

To secure profit maximization, we need to accomplish:

\[
\frac{\partial^2 \pi^A}{\partial a^2} < 0 \iff (q^B)^2 < (b-a)^3(g + t) [(b + 3a)(g + t) - 4(r g - t)]; \tag{36}
\]

\[
\frac{\partial^2 \pi^B}{\partial b^2} < 0 \iff (q^B)^2 < (b-a)^3(g + t) [4(2t + rg) - (3b + a)(g + t)]. \tag{37}
\]

Then, it is straightforward to check that the reaction functions described in expressions (20) and (21) are the unique conditions always satisfied under (36) and (37) simultaneously. Also note that if \(q^B = 0\), our model coincides with the non-quality discriminatory model of Bárcena-Ruiz and Casado-Izaga (2014). Thus, substituting \(q^B\) by 0 implies that conditions (20) and (21) would correspond to the same reaction functions of their framework.

Quality decision of the accommodated high-quality firm. The second order condition that secures that the equilibrium quality level \(r\) is a local maximum is given by:

\[
\frac{\partial^2 \pi^B}{\partial (q^B)^2} = \frac{-27kt^2 + 8(g + t)}{27t^2}.
\]
To secure profit maximization, we need to satisfy the following condition:

\[
\frac{\partial^2 \pi^B}{\partial (q^B)^2} < 0 \iff 27kt^2 - 8(g + t) > 0 \iff k > \frac{8(g + t)}{27t^2} \equiv \widetilde{k}, \tag{38}
\]

Below the threshold \(\widetilde{k}\), the high-quality firm tips the market. Thus, under condition (38) the equilibrium quality level (24) holds.

**Long-term guidance of the policymaker.** The second order condition that secure that the equilibrium location \(r\) is a local maximum is given by:

\[
\frac{\partial^2 W}{\partial r^2} = -\frac{2gt}{g + t} < 0, \quad \forall g \in \mathbb{R}^+ \cap t \in \mathbb{R}^+.
\]

All the other substitutions are, then, straightforward. We also let the note that the second order condition of stage two (one) of the analysis of a (proactive) high-quality firm coincides with the second order condition of stage one (two) of the analysis of an (accommodated) high-quality firm, respectively. \(\square\)

**Proof of Lemma 3**

Let us generically consider a variable \(z\) with \(\Delta z = z^{D*} - z^{ND*}\), where the superscript "\(D\)" denotes the equilibrium variable \(z\) with quality distortion and the superscript "\(ND\)" denotes an environment with no quality distortion. Regarding the equilibrium location of the policymaker, it is immediate that \(\Delta r = 0\) and we obtain a positive equilibrium quality of the high-quality firm in accordance with condition (38); (i) In terms of equilibrium location of firms, both move to the right since:

\[
\Delta a = \Delta b = -\frac{6t}{27kt - 8(g + t)} < 0; \quad b^{D*} - a^{D*} = b^{ND*} - a^{ND*} = \frac{3t}{2(g + t)}.
\]

(ii) Relatively to the equilibrium profit of the high-quality firm \(B\) follows:

\[
\Delta \pi^B = \frac{6t^2}{27kt^2 - 8(g + t)} > 0, \quad \forall g \in \mathbb{R}^+ \cap t \in \mathbb{R}^+ \cap k \in \left[\widetilde{k}, +\infty\right].
\]

(iii) The following consequence emerges on the equilibrium profit of firm \(A\):

\[
\Delta \pi^A = \frac{36t^2 \left[-9kt^2 + 4(g + t)\right]}{[27kt^2 - 8(g + t)]^2} \leq 0.
\]
The threshold $\tilde{k} = \frac{4(g+t)}{9t^2}$ appears to be vital. Then, and accordingly to (38): $\Delta \pi^A > 0$ if $k \in \left[\frac{8(g+t)}{27t^2}, \frac{4(g+t)}{9t^2}\right]$ and $\Delta \pi^A < 0$ if $k \in \left[\frac{4(g+t)}{9t^2}, +\infty\right[$. Interestingly, the low-quality platform $A$ secures positive profits with the introduction of a quality gap as long as the cost of quality provision of firm $B$ is sufficiently low.

(iv) We also analyze the effects on the equilibrium consumer surplus and on the equilibrium social welfare:

$$\Delta CS = -\frac{t}{48(g+t)} \left( g + \frac{4032t(g+t)^2}{27kt^2 - 8(g+t)^2} \right) < 0, \forall g \in \mathbb{R}^+ \cap t \in \mathbb{R}^+ \cap k \in \left[\tilde{k}, +\infty\right[.$$  

$$\Delta W = -\frac{6t^2}{27kt^2 - 8(g+t)} \left[27kt^2 - 2(g+t)\right].$$

Given (38) follows that $k < \frac{2(g+t)}{27t^2} \equiv \tilde{k}$ is impossible. Thus, we obtain a negative impact over the equilibrium social welfare, $\forall g \in \mathbb{R}^+ \cap t \in \mathbb{R}^+ \cap k \in \left[\tilde{k}, +\infty\right[$. Therefore, the negative impact on the equilibrium consumer surplus and on the equilibrium profit of the low-quality firm overcomes the positive impact on the profit of the high-quality firm. □

References


