A Dynamic Aggregate Supply and Aggregate Demand Model with Matlab

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Abstract

We use the framework implicit in the model of inflation by Shone (1997) to address the analytical properties of a simple dynamic aggregate supply and aggregate demand (AS-AD) model and solve it numerically. The model undergoes a bifurcation as its steady state smoothly interchanges stability depending on the relation between the sensitivity of the demand for liquidity to variations in the interest rate and the way expectations on inflation are formed based on real output fluctuations. Using code embedded into a unique function in Matlab, we plot the numerical solutions of the model and simulate different dynamic adjustments using different parameter values. The same function also accommodates for the implementation of different policy shocks: monetary policy shocks through changes in the growth rate of money supply, fiscal policy shocks due to variations in public spending and in the exogenous tax rate, and supply side shocks as given by changes in the level of natural output.

Keywords: business cycles; local dynamics; computational economics; policy shocks

JEL codes: C62, C63, E32

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1 Introduction

The simple aggregate demand and aggregate demand (AS-AD) model is one of the bulwarks used in economic theory to explain economic fluctuations and business cycles. Its dynamic version presented here can be used to assess the dynamic adjustments of output and inflation after different macroeconomic shocks. Due to the specification of the model, it is possible to verify that the equilibria of the model interchange stability depending on the relation between the sensitivity of the demand for liquidity to variations in the interest rate and the way expectations on inflation are formed based on real output fluctuations. In fact, we can characterize the model’s fixed points as stable or unstable spirals, or as stable or unstable nodes.\(^1\) This qualitative change in the local flow near the equilibrium is explained by the existence of a Hopf bifurcation as the pair of eigenvalues yielded by the corresponding two dimensional dynamic system crosses the imaginary axis for a specific value of the bifurcation parameter. This bifurcation parameter corresponds to the sensitivity of agents demand for real money balances (liquidity) to variations in the nominal interest rate. Contrary to many business cycle studies, however (e.g. Goodwin, 1951; Torre, 1977; Chang and Smyth, 1981), the absence of non-linearity in our very simple set-up rules out the possibility of limit cycles and other more complex economic behaviour.\(^2\)

Using a function developed in Matlab, we portray the dynamics of the model using different parameter values by means of an illustration that includes, not only the evolution of real output and inflation throughout time, but also a phase diagram that fully describes the qualitative properties of its equilibria.

We consider three types of shocks: monetary supply shocks, fiscal policy shocks, and supply side shocks. The first shocks operate through changes in the growth rate of money supply. Fiscal policy shocks are consequences of changes in either the level of public spending or the exogenous tax rate. The latter shocks are technological shocks that exogenously affect the level of natural output. The Matlab function also allows for the combination between the different shocks.

The organization of the rest of this paper is as follows. The second section derives the model and studies its analytical properties. The third section contains simulations for

\(^1\)We shall see in section 4 that unstable nodes turn out to be numerically ruled out.

\(^2\)Generally, the Hopf bifurcation requires the branching from an equilibrium into a periodic orbit. Here, we use the term “Hopf” just to highlight the fact that a fixed point looses stability as the eigenvalues of the Jacobian at the fixed point cross the imaginary axis of the complex plain.
different parameter values, each specification pertaining to a different phase portrait. The fourth section addresses the simulation of different shock types. The last section concludes.

2 Model and analysis

Most of the construction of the model follows Shone (1997, chap. 9). I begin by deriving the aggregate demand side, the “investment-saving” and “liquidity preference-money supply” (IS-LM) model. Starting with the goods market (IS curve), consumption is given by:

\[ c = c_0 + b(1 - \tau)y, \quad 0 < b, \tau < 1, \]

where \( \tau \) is the exogenous tax rate. Investment is equal to:

\[ i = i_0 - h(r - \pi^e), \quad h > 0, \]

where \( r \) stands for the nominal interest rate and \( \pi^e \) is the expected inflation rate. Hence, investment depends negatively on the real interest rate. Real output is given by:

\[ y = c + i + g, \]

where \( g \) corresponds to the exogenous level of government spending. The previous equations allow us to derive the IS curve.

The money market (LM curve) is described by the following equations:

\[
\begin{align*}
md &= ky - dr \\
ms &= m - p \\
m_d &= m_s, 
\end{align*}
\]

where \( m_d \) represents real money demand in logs, \( m_s \) is the real money supply in logs, and \( p \) is the logarithm of the price level. Combining the results for the goods and money market, we can solve for \( y^* \) and \( r^* \) to obtain:

\[
\begin{align*}
y^* &= \frac{(c_0 + i_0 + g) + \frac{h}{d}(m - p) + h\pi^e}{1 - b(1 - \tau) + \frac{hk}{d}} \\
r^* &= \frac{ky^* - (m - p)}{d}.
\end{align*}
\]
Hereinafter, I shall focus solely on $y^*$, the equilibrium level of real output. Notice that it is a linear equation in terms of the real money supply and expected inflation, i.e.,

$$y = a_0 + a_1(m - p) + a_2\pi^e,$$  \hspace{1cm} (1)$$

$$a_0 = \frac{c_0 + i_0 + g}{1 - b(1 - \tau) + \frac{hk}{d}},$$

$$a_1 = \frac{h}{d}\frac{1}{1 - b(1 - \tau) + \frac{hk}{d}},$$

$$a_2 = \frac{h}{1 - b(1 - \tau) + \frac{hk}{d}}.$$  

Equation (1) represents our aggregate demand (AD) curve, since it denotes equilibrium in both the goods and money market.

Turning to the supply side, I assume that the rate of inflation is proportional to the output gap and adjusted for expected inflation as follows:

$$\pi = \alpha(y - y_n) + \pi^e, \hspace{1cm} \alpha > 0.$$  

This is our aggregate supply (AS) curve. It stems from the combination between an augmented Phillips curve $\pi = \pi^e - a(U - U_n)$ and Okun’s law, whereby $U - U_n = -b(y - y_n)$, with $a, b > 0$. The AS curve represents a situation where prices are completely flexible. Thus, in equilibrium, actual inflation equals expected inflation and output equals its natural level whatever the price level $p$.

Introducing a dynamic adjustment for inflationary expectation and taking the derivative of (1) with respect to time, we get the full model:

$$\dot{y} = a_1(\dot{m} - \pi) + a_2\dot{\pi}^e, \hspace{1cm} a_1, a_2 > 0$$  \hspace{1cm} (2)$$

$$\pi = \alpha(y - y_n) + \pi^e,$$  \hspace{1cm} (3)$$

$$\dot{\pi}^e = \beta(\pi - \pi^e), \hspace{1cm} \beta > 0.$$  \hspace{1cm} (4)$$

The adaptive expectations scheme for expected inflation in the last equation shows that agents revise their expectations upwards whenever inflation at any given time is higher than the expected inflation at that same time. Equation (2) is the demand pressure curve. To consider the dynamics of the model, we shall reduce it to 2 differential equations. Using (3)
and (4) together, we obtain:
\[ \dot{\pi}^e = \alpha \beta (y - y_n). \]  
(5)

Next, we take equations (3) and (5) and plug them into (2) to get:
\[ \begin{align*}
\dot{y} &= a_1 (\dot{m} - \alpha (y - y_n) + \pi^e) + a_2 \alpha \beta (y - y_n) 
\Leftrightarrow \\
\dot{y} &= a_1 \dot{m} - \alpha (a_1 - a_2 \beta) (y - y_n) - a_1 \pi^e.
\end{align*} \]

Thus, the dynamics of the model are fully described by the following two differential equations:
\[ \begin{align*}
\dot{y} &= a_1 \dot{m} - \alpha (a_1 - a_2 \beta) (y - y_n) - a_1 \pi^e \\
\dot{\pi}^e &= \alpha \beta (y - y_n).
\end{align*} \]  
(6)

Notice that the two state variables are real output and expected inflation. However, actual inflation can be readily obtained from the AS curve in (3), hence:
\[ \pi(t) = \alpha (y(t) - y_n) + \pi^e(t). \]

A steady state implies that \( \dot{\pi}^e = 0 \) and \( \dot{y} = 0 \). From the first condition we get \( y = y_n \).
Combining this result with the second condition we end up with \( \pi^e = \dot{m} \).
From the AS curve it immediately follows that \( \pi = \pi^e = \dot{m} \) at steady state. That is, in equilibrium, real output is given by the natural level of output and actual inflation is equal to the growth rate of the money supply.

From the \( \dot{\pi}^e = 0 \) locus we can see that if \( y > y_n \), \( \pi^e \) is rising. Conversely, if \( y < y_n \), expected inflation \( \pi^e \) is declining.
Considering the \( \dot{y} = 0 \) locus we have:
\[ \pi^e = \dot{m} - \left(1 - \frac{a_2 \beta}{a_1}\right) (y - y_n). \]

Thus, what happens to \( y \) above or below the \( \dot{y} = 0 \) locus depends on the slope of the previous equation. If \( 1 - a_2 \beta / a_1 > 0 \), the previous equation is negatively sloped. Hence, to the left or below the \( \dot{y} = 0 \) locus, \( y \) is increasing. To the right, there are forces decreasing the level of output \( y \).

\[ ^3 \text{This condition is assumed in Shone (1997). As we shall see further ahead, this condition is sufficient for stability of the equilibrium of the model. For the sake of numerical presentation, I do not assume this holds } \text{a priori.} \]

5
Now, consider again the dynamical system in (6). The Jacobian matrix of the system at equilibrium is given by:

\[
J = \begin{bmatrix}
-\alpha(a_1 - a_2\beta) & -a_1 \\
\alpha\beta & 0
\end{bmatrix}.
\] (7)

It is straightforward to check that the determinant of the matrix is equal to \( \det(J) = a_1\alpha\beta > 0 \). Since it is positive, there are no eigenvalues with real parts of opposite sign. Hence there are no saddle points.

The trace of the matrix is given by:

\[
tr(J) = -\alpha(a_1 - a_2\beta) \iff tr(J) = -\alpha \left[ \frac{h\left(\frac{1}{d} - \beta\right)}{1 - b(1 - \tau) + \frac{bk}{d}} \right].
\]

Since \( b(1 - \tau) < 1 \), the denominator of \( tr(J) \) is positive. Moreover, since \( \alpha > 0 \), the trace is negative if and only if \( 1/d > \beta \). Hence, we can say that the equilibrium is stable if \( 1/d > \beta \) and unstable if \( 1/d < \beta \). The condition that \( 1/d > \beta \) is exactly the same as requiring \( 1 - a_2\beta/a_1 > 0 \). This means that if the \( \dot{y} = 0 \) locus is negatively sloped, the equilibrium \((y^*, \pi^*) = (y_n, \dot{m})\) is stable. This is assumed \textit{ex-ante} in Shone (1997), but not here.

A necessary and sufficient condition for the existence of complex eigenvalues requires \( 4\det(J) - tr(J)^2 < 0 \), that is:

\[
4a_1\alpha\beta > \alpha^2 \left[ \frac{h\left(\frac{1}{d} - \beta\right)}{1 - b(1 - \tau) + \frac{bk}{d}} \right]^2 \iff
\]

\[
\frac{4\beta}{d} > \frac{\alpha h\left(\frac{1}{d} - \beta\right)^2}{1 - b(1 - \tau) + \frac{bk}{d}}.
\]

(8)

A low enough \( \alpha \) favours oscillatory solutions. Finally, if \( d = 1/\beta \) we get a null trace and complex eigenvalues because \( 4\beta/d > 0 \). Moreover, the equilibrium is a stable centre, because the eigenvalues are purely imaginary. Thus, the equilibrium is stable if and only if \( 1/d \geq \beta \).

On the account of the previous analysis, we can claim that the dynamic AS-AD model accounts for the possibility of stable or unstable nodes or focuses (spirals), or a stable centre, depending on the parameters chosen.

We now formalize the following result.
Proposition 1. The system described by (6) undergoes a Hopf bifurcation at $d = d_0 = 1/\beta$; however, no periodic orbit branches from the steady-state.

Proof. See Appendix A.

The existence of a Hopf bifurcation explains the transition from a stable focus to an unstable focus, as the eigenvalues cross the imaginary axis at non-zero speed. On the other hand, the linearity of the system (6) in its state variables forcibly precludes the existence of small amplitude limit cycles branching from the steady state.\footnote{The latter would be a common prerequisite for Hopf bifurcations, which may occur in systems of non-linear differential equations, as already mentioned in section 1.}

As the model undergoes the bifurcation, the value of the money balances demand’s sensitivity to variations in the nominal interest rate $d$ is inversely proportional to the way expectations on inflation are formed based on deviations of real output $y$ from its natural level $y_n$, as captured by the parameter $\beta$ (see equation (4) from the full model).

3 Numerical evaluation

In this section we present some numerical simulations with different parameter values, illustrating the four different types of equilibria and dynamic adjustments that may arise in the AS-AD model.\footnote{Hereinafter, all presented output is generated from the Matlab function ASADdynamic.m. The function code is available upon request.} For the moment, I shall refrain from the introduction of shocks to the model, which is left for the next section. The program’s output is twofold. First, it presents the values of the parameters used, as well as the steady state values for real output, expected and actual inflation. For the sake of presentation, the output shown in Matlab’s command window is presented in Appendix B. Second, it provides illustrations for the evolution of the solutions, along with an intuitive phase portrait.

The first simulation, which uses the default parameter values in the program, depicts the transition dynamics for the solution when the equilibrium is a stable focus. The parameters used are $c_0 = 10; i_0 = 5; G = 5; b = 0.8; k = 0.05; d = 0.05; h = 0.1; \tau = 0.3; \alpha = 0.1; \beta = 1; \dot{m} = 0.01; y_n = 1$. The initial conditions are $y(t_0) = 0.05$ and $\pi^e(t_0) = 0.15$. 

\begin{thebibliography}{99}
  \bibitem{1} The former would be a common prerequisite for Hopf bifurcations, which may occur in systems of non-linear differential equations, as already mentioned in section 1.
  \bibitem{2} Hereinafter, all presented output is generated from the Matlab function ASADdynamic.m. The function code is available upon request.
\end{thebibliography}
Figure 1 – Simulation (1). To the left: solution for $y(t)$. To the right: $\pi(t)$ and $\pi^e(t)$.

We have $1/d > \beta$, hence the steady state $(y^*, \pi^{e*}) = (y_n, \dot{m})$ is stable, as we can see from figure 1. Also, the solutions are clearly oscillating. One can check that actual inflation oscillates more than the expected inflation and, naturally, the latter follows the first in its adjustment. Figure 2 illustrates the phase diagram which depicts the stable spiral.

Figure 2 – Simulation (1). Phase diagram depicts a stable focus.

The second simulation presents a different dynamic adjustment. We set $d = 0.5$ and $\beta = 2$, all other parameter values remaining equal. Since $1/d = \beta$ the equilibrium is now stable centre. The evolution of output, expected and actual inflation, and the phase diagram are depicted in figures 3 and 4, respectively.
Both figures show that there is no convergence or divergence towards the steady state, as the latter is only stable in the Lyapunov sense. All solutions are periodic orbits with period $T = 2\pi/\omega$, where $\omega$ is the imaginary part of the corresponding complex eigenvalues.

Figure 4 presents no $\dot{y} = 0$ locus, since its slope is equal to zero, as we can infer from the analysis in the previous section.

For the third simulation, we now set $d = 1$ and $\beta = 1.2$, with all other parameter values equal to the default set-up. Inasmuch as $\beta > 1/d$ we know that the equilibrium will be unstable. Moreover, because of this, the $\dot{y} = 0$ is negatively sloped. Furthermore, with this set of parameters, the equilibrium can be shown to be an unstable focus (recall also that $\alpha$ is very low), so the solution oscillates away from the steady state. This is confirmed by figures 5 and 6.
Once again, we can see that expected inflation follows up closely after actual inflation, reflecting the way by which expectations are formed.

In the fourth simulation we set $d = 0.025$ and dramatically decrease $\beta$ to 0.1. Now the eigenvalues are real and, since $1/d > \beta$, the equilibrium is a stable node.
By inspection of figure 7 one can observe that, contrary to the previous cases, expectations are now formed correctly, as expected inflation now decreases steadily towards the steady state level, whereas actual inflation overshoots its equilibrium level, following the evolution of real output.

Figure 8 – Simulation (4). Phase diagram illustrates a stable node.

Figure 8 shows the phase diagram where we can observe that the steady state is a stable node in the fourth simulation.

The parametrization required for the equilibrium to become an unstable node seems implausible as it imposes either exceedingly high values for $\alpha$ or implies trajectories assuming negative values for output. Therefore, we rule out this possibility.
4 Modeling shocks

In this section we introduce three different types of shocks to the dynamic AS-AD model. A combination between any of the shocks is possible, but for the sake of presentation we simulate each separately and report the results. The baseline set for parameter values presented here is the same across all simulations and is equal to the default (first) set-up presented in section 3.

First, we run an expansionary fiscal policy shock, setting new values for public spending and the tax rate at \( G = 5.5 \) and \( \tau = 0.25 \), respectively, after the program asks for the implementation of policy shocks. Previous to the shock, the economy is at equilibrium with \( \pi = \pi^e = 0.01 \) and \( y = 1 \). After a permanent increase in the government spending and permanent decrease in the tax rate at time \( t_0 \), the economy must jump up to a higher level output. This is given by the AD curve in (1). At \( t_0 \), real output \( y \) jumps to a new level \( y_0 = 2.0799 \). The economy is now farther away from its equilibrium level. Since the level of natural output is unchanged, the economy will converge to equilibrium and will do so in an oscillatory fashion, after a very sharp decrease in output in the first periods. Conversely, inflation increases sharply after \( t_0 \) (however, it does not change at \( t_0 \)) but then starts to converge to its initial value, also oscillating. This situation is described in figure 9.

![Figure 9 - Fiscal policy shock. To the left: solution for \( y(t) \). To the right: solutions for \( \pi(t) \) and \( \pi^e(t) \).](image)

The \( \dot{\pi}^e = 0 \) and \( \dot{y} = 0 \) loci are unaffected after the shock and the steady state lies at their intersection. Figure 10 shows the phase diagram, with the economy slowly returning

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\(^6\)This result is given by the new initial condition \( y_0 \) from the output in the command window from ASADdynamic.m.
to the steady state after its deviation at $t_0$ due to the fiscal policy shock.

![Phase diagram](image)

**Figure 10** – Phase diagram portraying the dynamic adjustment after a fiscal policy shock.

The second shock is an expansionary monetary policy shock. For it, we set the money supply growth rate $\dot{m} = 0.04$ once the function asks to set new parameter values. Once again, previous to the shock, the economy is at equilibrium with $y = y_n = 1$ and $\pi^e = \pi = \dot{m} = 0.01$. After the shock occurring at $t_0$, the $\dot{y} = 0$ locus (2) shifts upwards. Hence, the economy is out of equilibrium, and will oscillate towards the new steady state, where $y = 1$ and $\pi^e = 0.04$. This situation is reported in figure 11.

![Output](image)

**Figure 11** – Monetary policy shock. To the left: solution for $y(t)$. To the right: solutions for $\pi(t)$ and $\pi^e(t)$.

Initially, output will rise as will inflation. After $y$ crosses the $\dot{y} = 0$ locus, it will start to decrease while inflation continues to rise. Only when the solution crosses the $\dot{\pi}^e = 0$ locus will inflation start to fall. At this point, expected inflation will lie above its steady
state value and output will be lower than \( y_n \). The oscillating process continues as output converges to its natural level \( y_n = 1 \) and expected and actual inflation converge to their new equilibrium value \( \dot{m} = 0.04 \). This process is illustrated in figure 12.

![Phase diagram](image)

**Figure 12** – Phase diagram portraying the dynamic adjustment after a monetary policy shock.

The third and final shock we consider is a supply side shock. It is given by a positive technological shock which, for exogenous reasons, affects the natural level of output, such that \( y_n = 2 \). At the time of the shock, both the \( \dot{\pi}^e = 0 \) and \( \dot{y} = 0 \) loci shift rightwards, which is straightforward from (3) and (2). Hence, the economy will adjust towards the new steady state given by the intersection between the two new curves. Since the growth rate of money supply is unchanged, we can anticipate that the steady state levels of actual and expected inflation are the same and given by \( \pi = \pi^e = 0.01 \). However, real output at the new steady state is now given by \( y = y_n = 2 \). Again, convergence towards the new equilibrium is oscillatory, with a decrease in both actual and expected inflation and an increase in real output in the first periods. This case is depicted in pictures 13 and 14.
Figure 13 – Supply side shock. To the left: solution for $y(t)$. To the right: solutions for $\pi(t)$ and $\pi^e(t)$.

Figure 14 – Phase diagram portraying the dynamic adjustment after a supply side shock.

Conclusions

We studied the dynamic properties of a simple dynamic version of the well known AS-AD model and used code, embedded in a single Matlab function, to simulate the model in order to numerically analyse the dynamic adjustments in the economy under a wide array of different applications concerning policy shocks.

By standard analytical inspection of the linearized system of the resulting two differential equations, we find that the inflation model may have either stable or unstable spirals (focuses), or nodes, but no saddle point equilibria. Unstable nodes are ruled out numerically. A Hopf bifurcation occurs when the value of the demand for money balance’s sensitivity to
variations in the nominal interest rate is inversely proportional to the way expectations on inflation are formed based on real output deviations, which explains the smooth transition between unstable and stable focuses. However, the model’s linearity naturally rules out any possibility for occurrence of limit cycles.

Numerical simulations of the model with recurrence to the function developed in Matlab allow for the illustration of different cases of dynamic adjustments, with a complete description of the results. Furthermore, the function conveniently accounts for the possibility of implementing different shocks after the initial simulation. These shocks pertain to monetary and fiscal policy shocks, as well as supply side shocks. The function also shows how the transition to new steady state levels may occur, with appropriate illustrations and output. Admittedly, the AS-AD model is oversimplifying in its assumptions and limited in accounting for complex business cycles and other economic phenomena. It is in the author’s view, however, that its simplicity and perceptiveness renders it as very tractable candidate that contributes to further connect the bridge between economic model’s analytical and numerical assessment.

References


Appendix A

Here we provide the full analytical proof that the AS-AD model undergoes a Hopf bifurcation at \( d = d_0 \).

Proof of Proposition 1. Let \( \lambda_j = v(d) \pm i\omega(d) \) be the pair of eigenvalues obtained from the Jacobian matrix \( J \) in (7), where \( d \) is the bifurcation parameter. The conditions for existence of a Hopf bifurcation (Marsden and McCracken, 1976; Guckenheimer and Holmes, 2002; Th. 3.4.2 in p. 151) are given by:

(i) \( \omega(d_0) \neq 0 \); (ii) \( v(d_0) = 0 \); and (iii) \( v'(d_0) \neq 0 \);

When \( d = d_0 = 1/\beta \), the condition for existence of complex eigenvalues in (8) is satisfied, since \( 4\beta^2 > 0 \). Therefore, we have \( \omega(d_0) \neq 0 \), which satisfies condition (i).

Next, \( v(d_0) \) gives us \( \text{tr}(J) \) evaluated at the bifurcation point \( d_0 = 1/\beta \), thus yielding:

\[
v(d_0) = -\alpha \left[ \frac{h(\beta - \beta)}{1 - b(1 - \tau) + \frac{bk}{d}} \right] = 0,
\]

which satisfies condition (ii).

Finally, taking the first order derivative of \( v(d) \), we get:

\[
v'(d) = \frac{\alpha h(b(\beta k + \tau - 1) + 1)}{(d(b(\tau - 1) + 1) + bk)^2}.
\]

Evaluating at \( d_0 = 1/\beta \) yields:

\[
v'(d_0) = \frac{\alpha \beta^2 h}{b(\beta k + \tau - 1) + 1} \neq 0.
\]

Hence, condition (iii) holds and the model undergoes a Hopf bifurcation at \( d = d_0 \).

The additional genericity condition that \( a \neq 0 \) (Hassard and Wan, 1978; Guckenheimer and Holmes, 2002; Th. 3.4.2 (H2) p. 151), where:

\[
a = \frac{1}{16} (f_{xxx} + f_{xyy} + g_{xx} + g_{yy}) + \frac{1}{16\omega} [f_{xy}(f_{xx} + f_{yy}) - g_{xy}(g_{xx} + g_{yy}) - f_{xx}g_{xx} + f_{yy}g_{yy}],
\]

with \( f = \dot{y} \) and \( g = \dot{\pi}^x \) in (4) and subscripts denoting different order derivatives evaluated at equilibrium and at \( d = d_0 \), is not verified. Since all the derivatives of order higher or equal to 2 are zero, we have \( a = 0 \). Therefore, no periodic solutions exist for \( d \neq d_0 \), which concludes the proof. □
Appendix B

In this Appendix we provide the Matlab function’s reports on some of the simulations and shocks exemplified in the previous sections.

Output for first simulation

The following first example is given after the first simulation, using the default parameter values set by the program (see section 3).

------------------------

Dynamic AS-AD model
------------------------

AS-AD dynamic model: dy/dt=a1*dm/dt-alpha(a1-a2*beta)*(y-yn)-a1*pie  
dpie/dt=beta*alpha(y-yn)

AS and AD curves: (AD)y=dm/dt-alpha(a1-a2*beta*pie  (Long run AS) y=yn

y: Real output
pie: Expected inflation

a0=(c0+i0+G)/(1-b*(1-tau)+((h*k)/d) = 37.037
a1: (h/d)/(1-b*(1-tau)+((h*k)/d)) = 3.7037
a2: h/(1-b*(1-tau)+((h*k)/d)) = 0.185185

Parameter values: c0 = 10; i0 = 5; G = 5;b = 0.8; k = 0.05; d = 0.05;
               h = 0.1; tau = 0.3; alpha = 0.1; beta = 1; mdot = 0.01; yn = 1;
Initial conditions: y0 = 0.05; pie0 = 0.15

Steady-State:

Steady-state real ouput  Steady state expected inflation
1.0000000  0.010000

Steady state actual inflation
0.0100

Eigenvalues

-0.1759 + 0.5826i  -0.1759 - 0.5826i

Stable spiral.
**Output for first shock**

After the expansionary fiscal shock (recall section 4), the function’s reported output in Matlab’s command window was as follows:

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Policy and Technology shocks
-----------------------------

\[
\begin{align*}
a_0 &= 41 \\
a_1 &= 4 \\
a_2 &= 0.2 \\
\end{align*}
\]

New parameter values: \( c_0 = 10; \ i_0 = 5; \ G = 5.5; \ b = 0.8; \ k = 0.05; \ d = 0.05; \ h = 0.1; \ tau = 0.25; \ alpha = 0.1; \ beta = 1; \ mdot = 0.01; \ yn = 1; \)

New initial conditions: \( y_0 = 2.07998; \ pie_0 = 0.0100036 \)

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New Steady-State:
Steady-state real output Steady state expected inflation
\[ 
\begin{array}{cc}
1.000000 & 0.010000 \\
\end{array}
\]
Steady state actual inflation
0.0100
Editorial Board \{wps@fep.up.pt\}

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