Full Disclosure of Knowledge between Rivals

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ABSTRACT

We develop a symmetric duopoly model with strategic R&D spillovers where a specific type of innovation, recombinant innovation is introduced. Two major factors of effective knowledge spillovers, the technological learning parameters of the recombinant generation of new knowledge and the absorptive capacity of firms, are assumed to be exogenously determined. However, the third principal factor of the effectiveness of learning from rivals is endogenous: it is assumed that firms have control over the two individual spillover coefficients of the model. It is shown that identical firms operating in the same industry choose the highest level for the two spillover variables under plausible constellations of learning parameters. Furthermore, the realistic set of learning parameters is enlarged in the case where firms are able to commit to knowledge sharing strategies at the outset, thereby increasing the possibility of firms fully disclosing their knowledge in equilibrium.

KEYWORDS: Endogenous spillovers, knowledge sharing, absorptive capacity, recombinant innovation.

JEL Classification: O30
1. Introduction

We develop a symmetric duopoly model of R&D and endogenous knowledge spillovers where each firm is assumed to choose independently the level of knowledge it shares with its rival. In a three-stage model, firms undertake R&D efforts to lower unit production costs, choose the two R&D spillover variables simultaneously, and compete in the final product market. We conceive two variants of this three-stage duopoly model, and they uniquely differ as to the order of the first two moves taken by the two firms, R&D investment and knowledge sharing strategies. Strategic variables are chosen independently at all stages of either version of the game.

The focus of the paper is on the strategic sharing of knowledge with rival firms in a duopoly setting. The effectiveness of learning from others is a function of the extent of these endogenous R&D spillovers and the exogenously determined learning parameters of recombinant innovation and absorptive capacity. We show that identical firms choose non-cooperatively to fully disclose their knowledge to rivals when the exogenous technological opportunities for lowering unit costs of production are sufficiently high. Furthermore, the ability of firms to pre-commit to knowledge sharing strategies increases the possibility of free revelation of knowledge in equilibrium.

Full sharing of information is assumed in the analysis of cooperative R&D. The role of cooperation to overcome several sorts of market failures associated with R&D and technological innovation has been addressed in a number of papers. There are appropriability problems, often referred to as externalities problems, which are unique to R&D activities and make R&D a complex subject. Virtually all this literature on R&D and spillovers treat the R&D spillovers as exogenous. R&D activities are characterized by spillovers, and these externalities are modeled as an automatic result of firms’ investment in R&D. For example, d’Aspremont and Jacquemin (1988) assume that the same spillover parameter applies with and without cooperation. The spillover is not perfect and so the spillover parameter takes values between zero and unity. While Motta (1992) and Beath, Poyago-Theotoky and Ulph (1998) assume that when firms cooperate they achieve full information.

In contrast to virtually all the literature on R&D and spillovers, we assume that spillovers are endogenous in the absence of cooperation in R&D. There is a range of recent papers on endogenous spillovers related to this paper. Katsoulacos and Ulph (1998a, 1998b) show that full revelation of information is possible even in the absence of cooperation in R&D when firms operate in different but complementary industries. When spillovers are endogenous it is important to distinguish between substitute and complementary research path, and between firms located in the same industry or in different industries. De Fraja (1993) finds that it may in a firm’s best private interest to disclose scientific knowledge to product market competitors. In a model of patent race, the rate of spillover is a strategic variable endogenously determined by the optimizing behavior of the players. Under plausible conditions, it is shown that the only non-cooperative equilibrium of the two-firm disclosure game considered is such that both firms do disclose their knowledge. Harhoff et al. (2003) explore the incentives that users might have to openly reveal their proprietary innovations to other competing users and manufactures. End users of products and processes frequently are the developers of many
important innovations. A game-theoretical model is developed to explore the effects of these incentives on users’ decisions to reveal or hide their proprietary information. It is found that, under realistic parameter configurations, free revealing is profitable.

The paper is structured as follows. In Section 2, we describe our model. We Section 3, we solve for the equilibrium values of final output and R&D investment. In Section 4, we derive the main results of the model. Concluding remarks close the paper in Section 5.

2. The model

A simple duopoly model can be set up to highlight the role of strategic voluntary exchange of knowledge for R&D rivalry. We propose a three-stage game where each firm chooses how much to spend on R&D and how of its own knowledge to share with its rival non-cooperatively in the first two stages. Each firm’s output level is chosen in the game’s third stage.

Two firms produce a homogenous good and face a linear demand, \( P = A - (Q_i + Q_j) \), with \( P \) the market price and \( Q_i \) and \( Q_j \) quantities, \( i, j = 1,2, i \neq j \), and \( A > Q_i + Q_j > 0 \). Firms incur a given unit production cost \( C \) if they do not make any efforts in innovation. Firms can however improve their profitability by performing R&D investments \( x_i \) that reduce unit costs. The resource commitments in innovation allow to lower the unit cost of firm \( i \) to

\[
C - \gamma_i(\beta_i, \beta_j),
\]

where \( \gamma_i \) is the effective level of unit-cost reduction of firm \( i \), which depends not only on own R&D effort, but also on the other firm’s R&D effort, \( x_j \), via knowledge sharing or involuntary spillovers. R&D activities are typically associated with positive spillovers. We define \( \beta_i \) as the degree of firm \( i \)'s knowledge that is transferred or leaks to its rival, \( \beta_i \leq \beta_i \leq 1 \), and assume that it is a function of the strategy of knowledge sharing adopted by firm \( i \) and the spillover potential determined by environmental aspects such as protection of intellectual property rights or labor mobility in an industry. The lower bound of \( \beta_i \) is the exogenous rate of knowledge spillover \( \beta_i \in [0,1) \). This minimum admissible value reflects the extent to which knowledge involuntarily leaks to rivals. There is voluntary knowledge sharing by firm \( i \) only if \( \beta_i > \beta_i \).

Specifically, the effective level of R&D effort of firm \( i \) is given by

\[
\gamma_i = x_i + \beta_j p_j \left( y_j - q_i \beta_i p_i x_i \right),
\]

where \( p_i \) and \( q_i \) are technological learning parameters, \( 0 \leq p_i < 1, 0 \leq q_i \leq 1 \). Effective knowledge sharing and involuntary spillovers are determined by technological and organizational characteristics. Parameter \( p_i \) indicates the amount of firm \( i \)'s transferred knowledge that is actually acquired by its rival and is a function of its rival’s absorptive capacity. The concept of absorptive capacity of firms was developed by Cohen and Levinthal (1989, 1990), and identifies the ability of a firm to exploit spillovers of knowledge. The benefits of spillovers can be realized only by incurring the costs of maintaining absorptive capacity. R&D
investments provide the firm with an in-house technical capability that facilitates the acquisition and assimilation of new knowledge developed elsewhere.

Parameter \( q_i \) reflects a new perspective on learning to be incorporated in the representation of a firm’s effective R&D effort, and its complement \( 1 - q_i \) indicates the extent to which the initial investment of \( i \) is reconfigured in new ways with the external contribution of the rival firm. The notion of recombinant innovation as developed by Weitzman (1996, 1998), referring to the way old ideas can be reconfigured in new ways to make new ideas, can be used as a foundation for the introduction of such a parameter \( q_i \) that may take values less than unity. Knowledge can build upon itself in a combinatoric feedback process and may have important implications for economic growth.

The effective level of knowledge of firm \( i \) can be expressed as a function of own investment and external knowledge. The expression of knowledge generated with the contribution of the firm \( j \) and new to its rival, denoted by \( \Delta y_j \), is given by

\[
\Delta y_j = y_j - q_i \beta_i p_j x_i. \tag{3}
\]

This expression of external knowledge is sufficiently general to accommodate the conventional representation of effective level of knowledge in the literature as a particular case. For \( q_i = 1, i = 1,2 \), we obtain external knowledge to firm \( i \) as a fraction less than one of its rival’s investment in R&D, \( \Delta y_j = x_j \). But then equations (3), \( j = 1,2 \), for this special case are equivalent to equations (2), \( i = 1,2 \). Thus, \( y_i = x_i + \beta_j p_j x_j, i = 1,2 \), and if \( p_i = 1, i = 1,2 \), then \( y_i = x_i + \beta_j x_j \).

Solving the system of equations \([y_j], i = 1,2 \) as the given by (2) for the levels of R&D investment, yields the reduced-form representation of firm’s \( i = 1,2 \) effective cost reduction

\[
y_i(\beta_i, \beta_j) = \frac{1-\beta_i \beta_j p_j q_i}{1-\beta_i \beta_j p_j p_i} x_i + \frac{1-\beta_i \beta_j p_j q_j}{1-\beta_i \beta_j p_j p_i} \beta_j p_j x_j. \tag{4}
\]

Firms fully anticipate the learning effects of their R&D investments on industry costs and naturally incorporate spillovers in their investment decisions. R&D spillovers increase the efficiency of the cost reduction process at the industry level. Dynamic technical efficiency rises if the R&D investment required to achieve a given cost reduction lowers. By contrast, Spence (1984) finds that when firms in a high spillover environment ignore the feedback of spillovers on industry costs, industry performance is improved significantly. There are industries that have high spillovers yet seem to perform very well in terms of dynamic technical efficiency. In industries like certain electronics in which spillovers are high enough one might expect a problem with incentives for product development. The failure to anticipate spillovers partially solve the incentive problem created by spillovers as the effect of ignoring spillovers is to make the investment decisions of firms more aggressive.

The main results of this paper are derived for the case where the technological environment where firms operate and learn is symmetric:

\[
p = p_i, \text{ and } q = q_i, i = 1,2. \tag{5}
\]

Let us define
Thus, firm’s $i = 1, 2$ reduction of unit cost due to R&D efforts reduces to

$$B_{ii}(\beta_i, \beta_j) = \frac{1 - \beta_i \beta_j p^2}{1 - \beta_i \beta_j p^2}$$
$$B_{ij}(\beta_i, \beta_j) = \frac{1 - \beta_i \beta_j p^2}{1 - \beta_i \beta_j p^2} \beta_j p, \ i = 1, 2, \ i \neq j. \quad (6)$$

The common factor of expressions $B_{ii}$ and $B_{ij}$ in (6) or (7) reflects the contribution of external knowledge to the innovation capacity of each firm. Social interactions in an innovation system give rise to external knowledge and magnify the initial investment in R&D of firms through a multiplier effect. Social interactions and a knowledge multiplier play a central role in determining the rate of technological change of each firm. The knowledge multiplier, denoted by $M$, is given by

$$M(\beta_i, \beta_j) = \frac{1 - \beta_i \beta_j p^2}{1 - \beta_i \beta_j p^2}. \quad (8)$$

Social interactions multiply the effects of the actions of each firm under mild conditions imposed on the learning parameters. Specifically, $M > 1$ so long as $p > 0$ and $q < 1$. Antonelli and Scellato (2013) argue that social interactions play a crucial role in the access to external knowledge and the generation of new technological knowledge. External knowledge is viewed as a necessary input, complementary to internal knowledge actively used in the recombinant generation of new technological knowledge. The concept of a knowledge multiplier is developed by Antonelli and Scellato. In their paper, knowledge multipliers take the form of localized pecuniary externalities that make increasing returns in innovation systems possible. Empirical results seem to provide evidence in the support of the existence of knowledge multipliers deriving from three layers of knowledge interactions: intra-industry nationwide, inter-industrial within region, and the localized intra-industrial within region.

We have assumed that $p \in [0, 1)$ to account for the equilibrium outcome where both firms choose full disclosure strategies $\beta_i = 1, \ i = 1, 2$. This is a slightly more stringent restriction imposed on parameter $p$, as compared to $q$ to avoid that the denominator of expressions $B_{ii}$ and $B_{ij}$ ever be equal to zero in equilibrium, otherwise such expressions and resulting equilibrium outcomes are not well defined.

In order to ensure that marginal costs are non-negative, we must assume that $y_i \leq C$. There are two active firms in the final product market, and so we assume that $A - C > 0$. In addition, we assume that each firm’s R&D costs are quadratic:

$$\frac{\gamma}{2} x_i^2, \quad (9)$$

Parameter $\gamma$ determines the extent of diminishing returns in undertaking R&D, and is inversely related to the efficiency of R&D. The value of a firm’s profit function can therefore be written as

$$\pi_i = (A - Q_i - Q_j - (C - y_i)) Q_i - \frac{\gamma}{2} x_i^2. \quad (10)$$
We need to complete the timeline of the model, in particular, representing the order of choices of R&D effort and knowledge sharing by firms. We consider two possibilities in this respect.

**Version 1:** No commitment to knowledge sharing.

The complete timing of the game under the first variant is as follows. In the first stage firms select their R&D investment strategies, then how much of their knowledge to share, and in the final stage how much to produce. In this variant of the model, the \( \beta_i \)'s have not yet been selected when firms choose R&D efforts, although they can be correctly conjectured by firms.

**Version 2:** Commitment to knowledge sharing.

In the second version of the model, such order of the first two moves is reversed, and so the first competitive moves by which firms induce rivals to certain responses is the choice of the extent of knowledge sharing. In this variant of the model, the \( \beta_i \)'s were chosen by the time firms decide their investment levels in R&D.

Firms can either select full knowledge sharing strategies or bear the inevitable involuntary spillovers of own knowledge to rivals. These strategies of full disclosure once established may involve commitments that to a large degree are sunk and so very costly to reverse.

Institutions represent societal rules which facilitate coordination among people by helping them form expectations. Institutions play an important role in the process of induced innovation, and institutional innovations are at least partially endogenous (Hayami and Ruttan, 1971). Institutions can be made endogenous within a model of individual choice based upon individual maximization through the use of some concepts of game theory (Grabowski, 1995). Individuals may choose alternative strategies representing different rules of governing behavior. In a two-person game, strategies must be accepted by both individuals to become institutions. Once the rules are established, no individual has an incentive to cheat by disobeying.

In any variant of our model, however, each \( \beta_i \) is a variable, not parameter, so the level of knowledge spillover is endogenized through the choices of \( \beta_i \), \( i = 1,2 \). The case of endogenous spillovers is also dealt with other models. In Poyago-Theotoky’s (1999) model, firms first choose their R&D budgets, then how much to of their information to share, and in the last stage their Cournot output level. In another three-stage game, proposed by Kultti and Takalo (1998), firms choose their R&D expenditures levels in the first stage, and choose whether or not to share information in the second stage.

3. **Equilibrium output and R&D strategies**

The plausible approach to find the equilibrium values of the model is to work back starting at the third stage (output stage). In the third stage of the model, the firms engage in Cournot competition. In this period, quantity competition prevails given the previously chosen investment and disclosure strategies. The solution of the output stage is given by:
\[ Q_i^* (\beta_i, \beta_j) = \frac{A - C + 2y_i(\beta_i \beta_j) - y_j(\beta_i \beta_j)}{2}, \]  

and so firm’s \( i \) payoff function reduces to

\[ \pi_i (\beta_i, \beta_j) = \left( \frac{A - C + 2y_i(\beta_i \beta_j) - y_j(\beta_i \beta_j)}{2} \right)^2 - \frac{\gamma}{2} \left( x_i (\beta_i, \beta_j) \right)^2. \] (12)

Let us now define

\[ a_i (\beta_i, \beta_j) = \frac{2}{9} \left( 2B_{ii}(\beta_i, \beta_j) - B_{ij}(\beta_i, \beta_j) \right)^2 - \gamma, \]

\[ b_i (\beta_i, \beta_j) = \frac{2}{9} \left( 2B_{ii}(\beta_i, \beta_j) - B_{ji}(\beta_i, \beta_j) \right) \left( 2B_{ij}(\beta_i, \beta_j) - B_{jj}(\beta_i, \beta_j) \right), \]

\[ d_i (\beta_i, \beta_j) = \frac{2}{9} (A - C) \left( 2B_{ii}(\beta_i, \beta_j) - B_{ij}(\beta_i, \beta_j) \right). \] (13)

The computation of the simultaneous move equilibrium at the R&D stage is straightforward (see Appendix A1). The equilibrium values \( x_i, i = 1, 2 \), which solve the system of equations consisting of two first-order conditions for the individual R&D efforts, are:

\[ x_i (\beta_i, \beta_j) = \frac{-a_j(\beta_i, \beta_j)d_i(\beta_i, \beta_j) + b_i(\beta_i, \beta_j)d_i(\beta_i, \beta_j)}{a_i(\beta_i, \beta_j)a_j(\beta_i, \beta_j) - b_i(\beta_i, \beta_j)b_j(\beta_i, \beta_j)}, \quad i \neq j. \] (14)

In a setting with quadratic objective functions explicit equilibrium solutions \( x_i \) can be computed after which relevant economic implications arising from firms’ strategies can be more easily obtained. In what follows, we proceed this way with regard to Version 1, but the equilibrium values in Version 2 will be only obtained through numerical simulations.

4. Main results of the model

Our analysis of this three-stage game framework where firms choose R&D investment and knowledge sharing strategies non-cooperatively discloses a number of results. The first one concerns the strategic interaction game between rival firms taking place at the knowledge revelation stage.

4.1 The possibility of full disclosure

The strategy of full disclosure of knowledge may well prove the more profitable one for firms at the revelation stage of the game under certain conditions. This case is where the knowledge multiplier taking high enough values is a real possibility. In what follows we establish an equilibrium condition defining the frontier line between a region of full disclosure and a region of no knowledge sharing. Since each firm \( i \)'s payoff function (12) is convex over the range of possible values of variable \( \beta_i \), equilibrium values at the stage of knowledge revelation are obtained by looking for corner solutions. Therefore, the solution of the model is of type bang-bang: the optimum can be chosen so that \( \beta_i \in (\bar{\beta}_i, 1) \).

At the stage of knowledge revelation, a rational firm compares its profit when voluntarily shares its knowledge and conjectures that the other firm also makes full disclosure of its
knowledge to the profit arising from an alternative option of not wanting to fully share knowledge. In this second option, the only knowledge that is diffused to the rival is the one that leaks out involuntarily and in such case it is the competitively and institutionally determined technological environment – i.e. the strength of IPR – that determines the degree of knowledge spillover, $\beta_i$. Henceforth, we assume a symmetric technological environment such that $\beta_i = \beta$, $i = 1,2$.

Given that firms face by hypothesis a common competitive environment defined by learning parameters $(q, p)$, it is reasonable to expect that the equilibrium values of R&D effort as given by (14) and knowledge sharing decisions be symmetric. Thus, we consider the case of symmetry in behavior in what follows.

The frontier line of a region of full disclosure of knowledge referred to above is defined by the condition

$$\pi_i(1, 1) = \pi_i(\beta, 1).$$

Points in the $(q, p)$ space such that $\pi_i(1, 1) > \pi_i(\beta, 1)$, for all $\beta_i \in [\beta, 1]$, are to the left and above the boundary defined by this equilibrium condition. Conversely, points in the $(q, p)$ space such that $\pi_i(1, 1) \leq \pi_i(\beta, 1)$, for all $\beta_i \in [\beta, 1]$, are to the right and below this frontier line. Of course, equilibrium condition (15) is only met for points $(q, p)$ on the boundary line.

Suppose now that $\beta = 0$, the minimum lower bound admissible for knowledge leakages. When $\beta_i = 0$, $i = 1,2$, in the region of knowledge leakages, we have the duopoly R&D game of Spencer and Brander (1983). We refer the reader to this paper with regard to the analysis of equilibrium and stability conditions of this particular setting.

In the first version of the model, the one of no commitment of knowledge sharing, each level of R&D effort $x_i$, $i = 1,2$, is chosen before firms select the extent of the knowledge diffusion $\beta_i$. We assume consistent and symmetric conjectures on knowledge sharing. Set then $x = x_i$, $i = 1,2$. Once firms reach the stage of knowledge revelation, these values are taken as given. Thus, solving equilibrium equation (15) for $q$ (as shown in Appendix A2), we obtain the expression

$$q = 2 - \frac{1}{p}.$$  

This equation corresponds to a frontier line of the region of voluntary disclosure of knowledge. From equation (16) it is possible to draw the corresponding boundary line in the $(q, p)$ space, which has a positive slope. On a first reading, we get to know in which parts of the space firms will not adopt strategies of full disclosure: below and to the right of the boundary, therefore accounting for the most part of the $(q, p)$ space. A region of free revelation of knowledge emerges only if $p > \frac{1}{2}$ and $q < 1$. Thus, we may expect freely revealed knowledge in equilibrium under plausible combinations of parameters.

(INsert Figure 1 around here)
The table below is also obtained from equation (16) and can be useful for making comparisons with an analogous frontier line of full disclosure of the second variant of the model obtained using numerical simulations.

\[ \text{(INSERT TABLE 1 AROUND HERE)} \]

We have just established a first important result for the Version 1 of the model when $\beta = 0$, and which can be generalized to other lower bounds of $\beta_i$ as shown in Section 4.2, and to the second version of our model as we will see in Section 4.6 below. In cases where the second-order and stability conditions in R&D stage are satisfied, the result of the voluntary sharing of knowledge becomes an equilibrium result. We will show below in Section 4.4 that such conditions are met for meaningful sets of learning parameters $q$, $p$ and cost parameter $\gamma$.

\[ \text{Result 1: Full disclosure of knowledge between direct rivals is a possible outcome of a non-cooperative game under plausible configurations of learning parameters.} \]

Complete sharing of knowledge is a cooperative outcome obtained in a non-cooperative game between two competitors in the market for the final product. What drives this important result is the knowledge multiplier $M(\beta_i, \beta_j)$ taking high values in the region of full disclosure of knowledge. In fact, $M(1,1)$ can take high or very high values for relatively low high $p$’s and relatively low $q$’s.

In order for a firm to share its knowledge with its rival, the economic incentive or net benefit for doing it must be positive. A given point in the $(q, p)$ space belongs to the region of full knowledge disclosure in equilibrium only if the payoff of knowledge sharing is the greatest for each firm $i = 1,2$. In this case, a profile of strategies of full disclosure will be a well-behaved equilibrium as soon as the second-order and stability conditions defined at the R&D stage are met. Otherwise, a point $(q, p)$ does not belong to the region of complete knowledge sharing. In the latter case, firms will not be interested in the full disclosure of their knowledge, and what is expected is that firms endure involuntary knowledge leakages in equilibrium, $\beta_i^* = \underline{\beta}$, $i = 1,2$.

\[ \text{4.2 The impact of involuntary leakage on full disclosure} \]

The value of the exogenously determined parameter $\underline{\beta}$ affects the size of the region of full disclosure. In generic terms, the domain of $\beta_i$ in the optimization problem of firm $i = 1,2$ at the knowledge revelation stage is given by the interval $[\underline{\beta}, 1]$. Changing this domain by
truncating the admissible values of $\beta_i$, one changes the assumptions of the problem and so its solution.

On points $(q, p)$ of a frontier line between the region of full disclosure and the region of no knowledge sharing, each firm is indifferent between choosing the maximum amount of knowledge sharing, $\beta_i = 1$, and accepting the minimum acceptable level, $\beta$, whatever it might be. The payoff for each firm obtained in any option is the same by definition.

With changes of parameter $\beta$, equilibrium condition (15) that establishes the boundary line of a full disclosure region is changed as one term of this equation changes, that is, the profit that one firm receives if it chooses not to share knowledge with his rival.

In Version 1 of the model, by solving equation (15) for $q$ while considering this time a generic lower bound $\beta$, one obtains (as shown in Appendix A3) the expression for the frontier line of a full disclosure region

$$q = \frac{-1+2p}{p(1-\beta_p(1-p))}.$$  

(17)

Thus we establish a second relevant result.

Result 2: The frontier line of a full disclosure region shifts to the right and downwards with increases in the extent of involuntary leakage of knowledge.

The possibilities of finding full disclosure equilibria in the $(q, p)$ space are increasing with increases in the degree of exogenous knowledge leakage. Nevertheless, as we see from inspection of equation (17), all such frontier lines have two points in common in the $(q, p)$ space: $(0, \frac{1}{2})$ and $(1, 1)$.

4.3 The impact of market size on full disclosure

Market size as defined by the difference $A - C$ plays no role on establishing the location of the frontier line of a full disclosure region in any version of the model.

Equation (15) corresponding to the boundary line between the region of full disclosure and the region of involuntary leakage of knowledge does not depend on the difference $A - C$. This frontier line is established by comparing individual payoffs obtained with different individual strategies of knowledge exchange, and individual payoffs as given by equations (12) are directly proportional to the square of $A - C$ (as shown in Appendix A4). In the first version of the model, this comparison can be made just between profits gross of R&D efforts arising from different knowledge sharing strategies, as individual R&D efforts are given by the time firms enter the knowledge revelation stage.
Here we take an additional result with implications on the conduct of the remaining investigation.

**Result 3**: The frontier line of the region of full disclosure on any version of the model does not depend on the size of the market.

Thus, the numerical simulations that will be presented below assume always that $A - C = 100$. It will not be necessary, according to this result, to do additional computations with other values for the size of the market.

In any version of our model, a region of full disclosure is bounded by frontier lines set by equilibrium condition (15) and the second-order and stability conditions at the R&D stage, as it will be shown in the next sections. The second-order and stability conditions do not depend on the level of R&D effort as well, nor on the level of final output, as the cost undertaking R&D is by hypothesis a quadratic function of the level of effort of I&D as given by equation (9), implying that the individual payoff functions (12) in any version of the model are quadratic expressions of the individual levels of R&D effort.

### 4.4 Satisfaction of second-order conditions

Let us analyze the second-order conditions in R&D stage. The equation corresponding to the boundary line between the region where the second-order conditions (SOC) are satisfied and the region in which the SOC are not satisfied for a given $\gamma$ is (as shown in Appendix A5)

$$ q = \frac{4 - 2p + \sqrt{2\gamma(p^2 - 1)}}{2(2-p)p^2}. $$ (18)

This mathematical expression holds for both versions of the model. In the first version of the model, the SOC are satisfied in a part of the full disclosure region for $\gamma = 8/9$. This limit-value of $\gamma$ is known from literature where it is established when the exogenously determined spillover parameter takes the value 0. From (18), the equation corresponding to the boundary line between the region where the SOC are satisfied and the region in which the SOC are not satisfied when $\gamma = 8/9$ is

$$ q = \frac{2p - 1}{p(2-p)}. $$ (19)

We depict the corresponding boundary line dividing the region where the SOC are met and the region where the SOC are not in the following figure.

(INSET FIGURE 2 AROUND HERE)
This boundary and the frontier line of the region of full disclosure given by equilibrium condition (15) coincide in only two points in the \((q, p)\) space. The interception of the two frontier lines are at \((0, \frac{1}{2})\) and \((1, 1)\) (see Appendix A5).

Points in the \((q, p)\) space above and to the left of the boundary line defined by equation (18) do not satisfy the SOC for a given \(\gamma\). As one would expect, this region vanishes as \(\gamma \to \infty\). High enough values of \(\gamma\) guarantee that the SOC are satisfied for any point \((p, q)\) in the inner region of full disclosure (see Appendix A5).

The following result sets the lower bound for \(\gamma\) above which we find points in the \((q, p)\) space which lie between the frontier line of full disclosure and the frontier line of the SOC (see proof in Appendix A5).

**Result 4:** A part of the region of full disclosure where the second-order conditions hold emerge for \(\gamma > \frac{1}{2}\).

This sub-region where firms choose full sharing strategies and the SOC are satisfied begins to show up in the upper right corner of the \((q, p)\) space around point \((1, 1)\).

### 4.5 Satisfaction of stability conditions

Let us now analyze the stability conditions in R&D stage. Though stability requirements in duopoly must apply for choice variables, namely, output and R&D levels, we do not analyze the reaction functions in output space. Such output reaction functions will always cross “correctly” as we have assumed that \(A – C > 0\), \(y_i \leq C\), and R&D levels as given.

The equation corresponding to the boundary line between the region where the stability conditions are satisfied and the region in which the stability conditions are not satisfied for a given \(\gamma\) is (as shown in Appendix A6)

\[
q = \frac{1}{p^2} \left(1 + \frac{\sqrt{2}}{3} (-1 + p) \sqrt{\frac{p+1}{2-p}} \right).
\]  

(20)

This mathematical expression holds for both versions of the model. Stability imposes more stringent restrictions on the values of \(\gamma\) that one can employ to satisfy the non-cooperative equilibrium concept of our three-stage model.

In the first version of the model, although the second-order conditions are satisfied in a part of the \((q,p)\) space when \(\gamma = \frac{8}{9}\), stability is not assured for any of these points. The equation corresponding to the boundary line between the region where the stability conditions are satisfied and the region in which the stability conditions are not satisfied when \(\gamma = \frac{8}{9}\) is (as shown in appendix A6)
In the following figure, we depict the corresponding boundary line dividing the region where the stability conditions are met and the region where the stability conditions are not met. It can be shown that this boundary lies to the left and above the boundary line associated with the SOC and which was derived in the last section (see Appendix A6).

\[
q = \frac{1}{p^2} \left( 1 + 2(-1 + p) \frac{\sqrt[21]{p}}{2-3} \right).
\] (21)

This boundary line and the frontier line corresponding to equation (15) coincide in just two points in the \((q, p)\) space. The interception of the two boundary lines are at \((0, \frac{1}{2})\) and \((1, 1)\); otherwise, the former boundary lies below and to the right the latter in the \((q, p)\) space.

Hence, any increase in \(\gamma\), by shifting upwards and to the left the boundary of the stability conditions, implies that a part of the region of full disclosure that simultaneously satisfy the stability conditions emerges.

The portion of the region of full disclosure where the stability conditions do not hold collapses for large enough \(\gamma\) (see Appendix A6). For sufficiently large values of \(\gamma\), the region of points \((q, p)\) where the conditions of stability are not satisfied disappears, implying that the region where the SOC are not satisfied virtually collapses too.

We derive another result regarding the lower bounds of parameter \(\gamma\) this time imposed by the stability requirements.

**Result 5**: A portion of the region of full disclosure where the stability conditions hold emerge for \(\gamma > 8/9\).

For points \((q, p)\) between the frontier lines defined by equations (15) and (20), there are profiles of full disclosure strategies that are Nash equilibria at the stage of knowledge revelation.

4.6 Impact of R&D efficiency on full disclosure

We know from the existing literature that the extent of diminishing returns in undertaking R&D change the amount of R&D efforts chosen by firms. Specifically, the optimal amount of R&D decreases with increases in \(\gamma\). Now will show that \(\gamma\) parameter also affects the likelihood of firms selecting strategies of full disclosure of knowledge in points of the \((q, p)\) space and therefore the amounts of R&D later chosen by them.
In the second version of the model, the boundary line of the full disclosure region defined by equation (15) shifts up and to the left with increases of \( \gamma \). We conduct numerical simulations assuming that \( \beta = 0 \) that show the evolution in the \((q, p)\) space of the region where firms choose full disclosure strategies non-cooperatively for an increase in \( \gamma \). Some of their results are presented in the following table.

**Result 6:** The boundary line of the full disclosure region shifts upward and to the left with increases in \( \gamma \) in the second version of model, whereas the analogous boundary remains unchanged in the first version of the model.

The reason for this difference lies in the timing to choose R&D efforts \( x_i, i = 1,2, \) and knowledge sharing levels \( \beta_i \) assumed in each of the two versions of the model. In the model without commitment, firms choose \( x_i \) first and \( \beta_i \) later. Once \( x_i \) is selected, the choice of \( \beta_i \) by firm \( i \) does not change its marginal cost of doing R&D, \( \gamma x_i \) as implied by (9). In contrast, in the model with commitment, the initial choice of \( \beta_i \) will influence the cost of doing R&D at the margin, \( \gamma \frac{\partial x_i(\beta_i)}{\partial \beta_i} \), and therefore the incentive at the margin to choose \( \beta_i = 1 \) in equilibrium.

A study of comparative statics for the R&D stage of the duopoly game in Section 4.9 allows us to find the sign of \( \frac{\partial x_i}{\partial \beta_i} \) as a function of \( \beta_i, i = 1,2, \) as well as model parameters \( q \) and \( p \).

4.7 Effect of commitment on of full disclosure

There is a second important difference between the two versions of our model. In the second version of the model, the boundary line of the full disclosure region given by equation (15) ceases to shift upward and to the left of the \((q, p)\) space for quite high values of \( \gamma \) and eventually stabilizes, remaining located below and to the right of the analogous boundary defined for the first version of the model. In common the two boundaries always have the same extremes \((0, \frac{1}{2})\) and \((1, 1)\) in the \((q, p)\) space.
The important implication is that the region of full disclosure defined in the case of knowledge compromise and for large \( \gamma \) is more extensive than the corresponding full disclosure region defined for the case of no compromise.

Numerical simulations assuming that \( \beta = 0 \) allow us to identify in the \((q, p)\) space the location of the boundary line established by equilibrium condition (15) when \( \gamma \) is large or very large for the second version of the model. The next table will be useful for comparisons with analogous situation in the first version of the model where explicit solutions are possible. So reference is made to previous Table 1 on the equilibrium condition (16) of the first version of the model.

(INsert Table 3 Around Here)

Thus, we can state an important result on the comparison of full disclosure regions with and without commitment of knowledge.

Result 7: The region of full disclosure under knowledge commitment is larger and contains the region of full disclosure without knowledge commitment.

The following two sections deal with the strategic nature of R&D efforts under full disclosure of knowledge and its implications for market outcomes.

4.8 Sources of strategic complementarity and full disclosure

In this section, we show that the actions of firms at the stage of R&D undertaking are strategic complements when firms decide to completely share their knowledge. The nature of strategic interaction in R&D depends on the choices of firms at the stage of knowledge revelation and is thus determined endogenously. One must keep in mind that in our model each \( \beta_i, i = 1,2 \), is a variable not a parameter.

The strategic nature of R&D actions is established by the slope of the firms’ reaction functions in R&D space. Investments that lower production costs have a strategic complement (substitute) nature in Bulow et al.’s (1985) terminology if investment reactions functions are upward (downward) sloping.

In our model, the slope of firm’s \( i \) reaction function is \( \frac{dx_i}{dx_j} = -\frac{b_i}{a_i}, i = 1,2, i \neq j \) (see Appendix A7). The denominator of this equation is the second-order condition in the R&D stage that must be less than 0 for an equilibrium. Thus, the slope of the reaction function is determined by the sign of numerator, specifically the sign of \( 2B_{ij} - B_{jj} \) in \( b_i \) as given by (13), as it is the
case that the other difference $2B_{ii} - B_{ji}$ in $b_i$ as given (13) is always positive. The reaction function is upward sloping if $2B_{ij} - B_{jj} > 0$ and downward sloping if $2B_{ij} - B_{jj} < 0$.

The sign and magnitude of $2B_{ij} - B_{jj}$ depend crucially on the interaction between knowledge exchange variables $\beta_i$, $i = 1, 2$, and technology parameters $q$ and $p$. We set this difference equal to zero to determine the region of points in the $(q, p)$ space where R&D actions are strategic complements. The equation corresponding to the frontier line in the $(q, p)$ space between a region of strategic complementarity and a region of strategic substitutability is given by (as shown in Appendix A7)

$$p = \frac{1}{2\beta_j}$$

(22)

We want to show that the region of full disclosure is contained in a larger region where $2B_{ij}(1,1) - B_{jj}(1,1) > 0$. Suppose that both firms share completely their knowledge, $\beta_i = 1$, $i = 1, 2$. Substituting these full disclosure strategies into equation (22), yields

$$p = \frac{1}{2}$$

(23)

Hence, for points $(q, p)$ above and to the left of the boundary line corresponding to this equation where firms select $\beta_i = 1$, $i = 1, 2$, R&D actions are definitely strategic complements.

A comparison with Tables 1 and 2 allows us to immediately conclude that the region of full disclosure completely lies to the left and above the boundary line dividing the $(q, p)$ space between the region of strategic complementarity and the region of strategic substitutability.

Result 8: The amounts of R&D efforts chosen by firms are strategic complements under full disclosure of knowledge.

In De Bondt and Veugelers (1991), the nature of the investment that reduces costs depends crucially on the spillovers parameter and on the demand specifications. In the particular case of perfect substitutes in consumption, the reduced-from investment reaction curves are upward sloping for values of the exogenously given spillover parameter larger than $\frac{1}{2}$.

In our model, when firms do not voluntarily share knowledge and the only knowledge they receive from their rivals is what leaks out involuntarily in reduced amounts, R&D actions may become strategic substitutes in a part of the $(q, p)$ space. After substituting $\beta_j$ for the common spillover parameter $\beta$ in equation (22), we immediately see that the region of strategic substitutability, located below the boundary line corresponding to this equation increases in size as the exogenous spillover parameter decreases. For $\beta < \frac{1}{2}$, R&D actions are strategic substitutes at every point $(q, p)$ and the region of strategic substitutability encompasses all the $(q, p)$ space.
4.9 Effects of strategic complementarity under full disclosure

The magnitude of incentives for R&D efforts and the sign of changes in R&D investments as a result of changes in knowledge sharing strategies are dependent upon the nature of the same investments.

Consider the first version of the model. Suppose that firms correctly anticipate full disclosure of knowledge when they first choose their R&D efforts. At the knowledge revelation stage, R&D efforts are given and symmetric: \( x_i = x, \ i = 1,2 \). The following result assumes that equilibrium and stability conditions at the R&D stage are met in the region of full disclosure where firms choose to act. Stability conditions subsume second-order conditions (Dixit, 1986). Thus, given that investment actions are strategic complements in the full disclosure region as shown in Section 4.8, the signs of \( \frac{d x_{i(1,1)}}{d \beta_i} \) and \( \frac{d x_{j(1,1)}}{d \beta_j} \), \( i \neq j \), are the same as the sign of

\[
\mu_{i(1,1)}^{i} = \frac{d}{d \beta_i} \left( a_i(1,1)x + b_i(1,1)x + d_i(1,1) \right),
\]

which can be positive or negative. The proof of this result is given in Appendix A8.

Marginal increases in the degree of knowledge shared by a firm in the vicinity of full disclosure increases both the level of R&D effort undertaken by the other firm and the own level of R&D effort of the same firm under a certain condition.

Result 9: We have both \( \frac{d x_{i(1,1)}}{d \beta_i} > 0 \) and \( \frac{d x_{j(1,1)}}{d \beta_j} > 0 \), \( i = 1,2, i \neq j \), if \( \mu_{i(1,1)}^{i} > 0 \).

By contrast, in the case of the noncooperative model developed by d’Aspremont and Jacquemin (1988), firms tend to free ride on the other firm’s knowledge, given that their R&D levels fall as the spillover parameter increases (Henriques, 1990). In this symmetric duopoly model of R&D and spillovers, the levels of R&D \( x_i \) and \( x_j \), \( i \neq j \), are positively related under large spillovers.

5. Conclusion

In this paper the results of the economic analysis undertaken were conditional on the assumption of a symmetric technological learning environment thus ensuring the emergence of symmetric solutions at all stages of the game. A natural development of the model is to generalize it by considering the broader class of ex ante asymmetric learning environments in order to study the emergence of equilibrium solutions with symmetric degrees of spillovers. A second extension of the model is to endogenize spillovers in a different way from the one followed in the paper by assuming that the recombinant innovation and absorptive capacity parameters depend on say the technology diversification strategies previously adopted by large, multiproduct firms.
APPENDICES

A1. Satisfaction of the first-order conditions

We want to find the solution to the optimization problem of each firm \(i = 1,2\) at the R&D stage in either variant of our model. To begin with, making use of equations (7), the first-order condition (FOC) for firm \(i\)'s problem (12) with respect to \(x_i\) is

\[
\frac{2}{9} (A - C + 2(B_{ii}x_i + B_{ij}x_j) - (B_{ji}x_i + B_{jj}x_j))(2B_{ii} - B_{jj}) - \gamma x_i = 0,
\]

or equivalently after using (13),

\[
a_i x_i + b_i x_j = -d_i, \quad i = 1,2, \quad i \neq j.
\]

Solving this system of simultaneous equations consisting of FOC for the two firms with respect to their individual R&D efforts, yields

\[
x_i = \frac{-a_i d_i + b_i d_j}{a_i a_j - b_i b_j}.
\]

The envisaged parameter values \(q, p, A, C\) and \(\gamma\) for any equilibrium of the model are those that allow for positive profits and additionally satisfy the second-order conditions for a regular maximum as well as the stability conditions. These additional conditions are dealt with in other Appendices below.

A2. Boundary of the full disclosure region

In the first version of the model, from equations (7), we

\[
y_i(\beta_i, \beta_j) = B_{ii}(\beta_i, \beta_j)x + B_{ij}(\beta_i, \beta_j)x, \quad i = 1,2, \quad i \neq j.
\]

Terms \(B_{ii}\) and \(B_{ij}\) are expressions of \(q\) and \(p\) according to equations (6). Condition (15) simplifies to

\[
2y_i(1,1) - y_j(1,1) = 2y_i(0,1) - y_j(0,1),
\]

or equivalently to

\[
2(B_{ii}(1,1) + B_{ij}(1,1)) - (B_{ji}(0,1) + B_{jj}(0,1)) = 2(B_{ii}(0,1) + B_{ij}(0,1)) - (B_{ji}(0,1) + B_{jj}(0,1)).
\]

Solving this equation for \(q\), we get

\[
q = 2 - \frac{1}{p}.
\]

A3. Involuntary knowledge and full disclosure

In the first version of the model, for a generic lower bound \(\underline{p}\), condition (15) simplifies to

\[
2y_i(1,1) - y_j(1,1) = 2y_i(\underline{p},1) - y_j(\underline{p},1).
\]

Solving this equation for \(q\), one obtains

\[
q = \frac{-1 + 2p}{p(1 - \beta_i p(1 - p))}.
\]

A4. Market size and full disclosure

Firms’ payoffs as given by equations (12) can be shown to be directly proportional to \((A - C)^2\).

In a Cournot game, the profit of each firm gross of R&D cost is a quadratic function of the individual output level. In equilibrium, the individual output level given by (11) is a multiplicative function of \(A - C\). In fact, in any version of our model, \(A - C\) enter solely as multiplicative factor in equations (14) of the optimal levels of R&D effort \(x_i, \quad i = 1,2\), which in turns makes the effective exchange of knowledge \(2y_i - y_j\), a difference expressed in terms of the individual levels of R&D efforts of firms \(i = 1,2\) as implied by equations (7), itself a
multiplicative function of $A - C$. On the other hand, R&D costs are a quadratic function of each firm’s own R&D effort as given by equations (9).

A5. Boundary of the second-order conditions region

The equation corresponding to the frontier line between the region in the $(q, p)$ space where the second-order conditions (SOC) at R&D stage are satisfied and the region in which the SOC are not satisfied is $\frac{\partial^2 \pi_i}{\partial x_i^2} = 0$, or from (13) as $a_i = \frac{\partial^2 \pi_i}{\partial x_i^2}$, $a_i(\beta_i, \beta_j) = 0$. Suppose that these conditions are satisfied when firms voluntarily share knowledge: $\beta_i = 1$, $i = 1, 2$. Hence this equation becomes $\frac{2}{9} \left(2B_{ii}(1,1) - B_{jj}(1,1)\right)^2 - \gamma = 0$. According to equations (6), terms $B_{ii}$ and $B_{jj}$, $i \neq j$, are functions of $q$ and $p$. Solving this equation for $q$, we obtain the expression (of one of the roots of quadratic equation above) $q = \frac{4 - 2p + 3\sqrt{2}p\sqrt{p^2 - 1}}{2(2-p)p^2}$.

Now set $\frac{4 - 2p + 3\sqrt{2}p\sqrt{p^2 - 1}}{2(2-p)p^2} = 2 - \frac{1}{p}$, where the right-hand side is taken from equation (15), and solve for $\gamma$. For $\gamma = 8/9$, we get $p = \frac{3}{5}$ and $p = 1$, whereas for $\gamma = \frac{3}{5}$, we get $p = 1$. For $\gamma = \frac{3}{5}$, the boundary line defined by (15) lies above the frontier line of the SOC for all $q \neq 1$ in the $(q, p)$ space, whereas for $\gamma = 8/9$, the boundary line defined by equation (15) lies below the frontier line of the SOC for all $q \neq 0$ and 1.

From inspection of equation $q = \frac{4 - 2p + 3\sqrt{2}p\sqrt{p^2 - 1}}{2(2-p)p^2}$, the region in the $(q, p)$ space where the SOC are not satisfied gets smaller and smaller as $\gamma$ gets larger and larger. From this equation, we set $q = 0$ and solve for $\gamma$, to obtain $\gamma = \left(\frac{2(2-p)}{3\sqrt{2}(1-p^2)}\right)^2$. Hence, $\gamma \to \infty$ as $p \to \infty$. Appealing to continuity arguments, we conclude that the SOC are satisfied for points in the $(q, p)$ space in the neighborhood of $(0, 1)$ only if $\gamma$ is high enough.

A6. Boundary of the stability conditions region

Let us define $\Delta = \frac{\partial^2 \pi_i}{\partial x_i^2} - \frac{\partial^2 \pi_j}{\partial x_j^2} = \frac{\partial^2 \pi_i}{\partial x_i x_j}$, $i = 1, 2$, $i \neq j$, or from (13) as $a_i(\beta_i, \beta_j) = \frac{\partial^2 \pi_i}{\partial x_i}$ and $b_i(\beta_i, \beta_j) = \frac{\partial^2 \pi_i}{\partial x_i x_j}$. A line defined by equation $\Delta = 0$ and delimiting the region in the $(q, p)$ space where the stability conditions in R&D stage are satisfied lies to the right and below the frontier line defined by equation $\frac{\partial^2 \pi_i}{\partial x_i^2} = 0$.

Suppose that, for a given point $(q, p)$, we have $a = \frac{\partial^2 \pi_i}{\partial x_i} = 0$ (the symmetric case, with $a = a_1 = a_2$). Thus, $a = 0 < b = \frac{\partial^2 \pi_i}{\partial x_i x_j}$ (the symmetric case, with $b = b_1 = b_2$) and so $\Delta = a^2 - b^2 < 0$. 

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Suppose that firms voluntarily share knowledge: $\beta_i = 1, i = 1,2$. It is known that $\frac{\partial^2 \pi_i}{\partial x_i^2} + \gamma > 0$ and that $\frac{\partial^2 \pi_i}{\partial q \partial x_i} < 0$. The equation corresponding to the frontier line between the region in the $(q, p)$ space where the stability conditions at R&D stage are satisfied and the region in which the stability conditions are not satisfied is $a = -b$. Suppose that this condition is satisfied when firms voluntarily share knowledge: $\beta_i = 1, i = 1,2$. In our model, this equation becomes $\frac{2}{\gamma} \left( 2B_{ii}(1,1) - B_{ii}(1,1) \right)^2 - \gamma = -\frac{2}{\gamma} \left( 2B_{ii}(1,1) - B_{ij}(1,1) \right) \left( 2B_{ij}(1,1) - B_{jj}(1,1) \right)$. According to equations (6), terms $B_{ii}$ and $B_{ij}, i \neq j$, are functions of $q$ and $p$. Solving this equation for $q$, we obtain the expression (of one of the roots of quadratic equation above) $q = \frac{1}{p^2} \left( 1 + 3 \sqrt{\frac{2}{\gamma} (-1 + p) \frac{\sqrt{1+p}}{2-p}} \right).$

Now set $\frac{1}{p^2} \left( 1 + 3 \sqrt{\frac{2}{\gamma} (-1 + p) \frac{\sqrt{1+p}}{2-p}} \right) = 2 - \frac{1}{p}$, where the right-hand side is taken from equation (15), and solve for $\gamma$. For $\gamma = 8/9$, we get $p = \frac{7}{6}$ and $p = 1$, and the boundary line defined by (15) lies above the frontier line of the stability conditions $q = \frac{1}{p^2} \left( 1 + 2(-1 + p) \right) \frac{\sqrt{1+p}}{2-p}$ for all $q \neq 0$ and 1 in the $(q, p)$ space.

From inspection of equation $q = \frac{1}{p^2} \left( 1 + 3 \sqrt{\frac{2}{\gamma} (-1 + p) \frac{\sqrt{1+p}}{2-p}} \right)$, the region in the $(q, p)$ space where the stability conditions are not satisfied gets smaller and smaller as $\gamma$ gets larger and larger. From this equation, we set $q = 0$ and solve for $\gamma$, to obtain $\gamma = \frac{4 - 2p}{9(1-p-\sqrt{1+p})^2}$. Hence, $\gamma \to \infty$ as $p \to \infty$. Using continuity arguments, we conclude that the stability conditions are satisfied for points in the $(q, p)$ space in the neighborhood of $(0, 1)$ only if $\gamma$ is high enough.
A5.1 Shifts of frontier line defined by $a = 0$

The region where the SOC do not hold gets smaller with increases in $\gamma$. The equation corresponding to the line dividing the regions where the SOC are satisfied and the region where such conditions do not hold is given by $\frac{\partial^2 \pi_i}{\partial x_i^2} = 0$. Thus, for a given $p$, one obtains $\frac{da}{dy} = -\left(\frac{\partial}{\partial y} \left(\frac{\partial^2 \pi_i}{\partial x_i^2}\right) + \frac{\partial^2 \pi_i}{\partial q \partial x_i}\right) < 0$, as $\frac{\partial}{\partial y} \frac{\partial^2 \pi_i}{\partial x_i^2} = -1$ and $\frac{\partial^2 \pi_i}{\partial q \partial x_i} < 0$, which means that, with successive increases in $y$, the area where the SOC are not satisfied, located in the upper-left corner of the $(q, p)$ space, gets smaller and smaller.

A6.1 Shifts of the lower frontier line defined by $\Delta = 0$

The frontier line defined by $\Delta = 0$ located to the right of frontier line defined by $a = 0$ shifts to the left in the $(q, p)$ space with increases in $\gamma$. In this case, $\Delta = 0$ implies that $a = -b$. The equation corresponding to the lower frontier line between the region where the stability condition are satisfied and the region where such conditions do not hold is given by $\frac{\partial^2 \pi_i}{\partial x_i \partial x_j} = 0$. Thus, for a given $p$, one gets $\frac{da}{dy} = -\left(\frac{\partial}{\partial y} \left(\frac{\partial^2 \pi_i}{\partial x_i^2} + \frac{\partial^2 \pi_i}{\partial x_i \partial x_j}\right) \right) < 0$, as $\frac{\partial}{\partial y} \left(\frac{\partial^2 \pi_i}{\partial x_i^2} + \frac{\partial^2 \pi_i}{\partial x_i \partial x_j}\right) = -1$ and $\frac{\partial}{\partial q} \left(\frac{\partial^2 \pi_i}{\partial x_i^2} + \frac{\partial^2 \pi_i}{\partial x_i \partial x_j}\right) < 0$, which means that, with successive increases in $y$, the area where the stability conditions are not satisfied, located in the upper side of the $(q, p)$ space, gets smaller and smaller.

A7. Slope of R&D reaction functions

From the FOC for firm’s $i = 1,2$ maximization problem (12) with respect to $x_i$, $\frac{2}{9}(A - C + 2(B_{ii}x_i + B_{ij}x_j) - (B_{ji}x_i + B_{jj}x_j))(2B_{ii} - B_{jj}) - \gamma x_i = 0$, or equivalently after using (13), $a_i x_i + b_i x_j = -d_i$, $i \neq j$, it follows the reduced-form reaction function of firm $i$ in R&D space, $x_i = -\frac{d_i}{a_i} - \frac{b_i}{a_i} x_j$. From here it immediately follows the slope of the reaction function, $\frac{dx_i}{dx_j} = -\frac{b_i}{a_i}$. As the satisfaction of the second-order condition requires that $a_i < 0$, the slope of the reaction function is established by the sign of $b_i$. This sign is positive and so the reaction function is upward sloping if $2B_{ij} - B_{jj}$ and $B_{ij}$ in $b_i$ as given by (13) is positive, and negative if $2B_{ij} - B_{jj} < 0$. The other difference in $b_i$ as given by (13), $2B_{ii} - B_{jj}$ is always positive.

Thus, setting $2B_{ij} - B_{jj} = 0$, which simplifies to $2\beta_i p - 1 = 0$, and solving for $p$, yields the equation corresponding to the frontier line between the region where R&D actions are strategic complements and the region of strategic substitutability: $p = \frac{1}{2\beta_i}$. For the case of full disclosure of knowledge with $\beta_i = 1$, $i = 1,2$, we get $p = \frac{1}{2}$.
A8. Comparative statics for the duopoly game

Following Dixit (1986), we derive comparative statics results for the case where a parameter \( \beta_i, i = 1,2 \), affects both firms’ profits at the knowledge revelation stage in the first version of our model. Take the symmetric case, where \( x_i = x, i = 1,2 \), is the given R&D effort at the beginning of the second stage. Let us define \( K \) and \( I \), or from (13) as 
\[
\Delta \left( \beta_i, \beta_j \right) = \frac{\partial^2 \pi_i}{\partial x_i \partial x_j} - \frac{\partial^2 \pi_i}{\partial x_i \partial x_j}, \quad i = 1,2, \quad i \neq j, \quad \text{or from (13) as } a_i(\beta_i, \beta_j) = \frac{\partial^2 \pi_i}{\partial x_i^2} 
\]
and \( b_i(\beta_i, \beta_j) = \frac{\partial^2 \pi_i}{\partial x_i \partial x_j} \Delta(\beta_i, \beta_j) = a_1(\beta_i, \beta_j) a_2(\beta_i, \beta_j) - b_1(\beta_i, \beta_j) b_2(\beta_i, \beta_j) \).

Totally differentiating the first-order conditions for firms’ \( i = 1,2 \) profit maximization problems (12) at the R&D stage, we get
\[
\begin{bmatrix} a_i & b_i \end{bmatrix} \begin{bmatrix} dx_i \\ dx_j \end{bmatrix} = - \begin{bmatrix} \mu_{\beta_i} & d\beta_i \\ \mu_{\beta_j} & d\beta_j \end{bmatrix} \].
\]
The solution to this system of simultaneous equations after simple rearrangement is
\[
\begin{bmatrix} dx_i \\ dx_j \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} -a_j & b_i \\ b_j & -a_i \end{bmatrix} \begin{bmatrix} \mu_{\beta_i} \\ \mu_{\beta_j} \end{bmatrix}.
\]

We assume that the stability conditions are met in the full region disclosure, and so \( a_i(1,1) < 0, i = 1,2, \) and \( \Delta(1,1) > 0 \). As shown in Appendix A7, R&D actions are strategic complements in the region of full disclosure, and so \( b_i(1,1) > 0 \). As expected, \( \mu_{\beta_i}^{(1,1)} > 0 \). Hence, the signs of both \( \frac{dx_i(1,1)}{d\beta_i} \) and \( \frac{dx_j(1,1)}{d\beta_i} \) are positive if \( \mu_{\beta_i}^{(1,1)} > 0 \).

REFERENCES


FIGURE 1: Frontier line of the region of full disclosure – Version 1
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TABLE 1: Frontier line of the equilibrium condition – Version 1
FIGURE 2: Frontier line of the SOC region for $\gamma = 8/9$ – Version 1
FIGURE 3: Frontier line of the stability region for $\gamma = 8/9$ – Version 1
<table>
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<th>$p$</th>
<th>$q$ (= 10)</th>
<th>$q$ (= 100)</th>
</tr>
</thead>
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<td>0.</td>
<td>0.</td>
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<tr>
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<td>0.122092</td>
<td>0.117077</td>
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<td>0.627991</td>
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<td>0.686081</td>
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**Table 2:** Frontier line of the equilibrium condition for two values of $\gamma$ – Version 2
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<th>q (γ = 10000)</th>
<th>q (γ = 100000)</th>
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<td>0.</td>
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</tbody>
</table>

TABLE 3: Frontier line of the equilibrium condition for high values of γ – Version 2
Editorial Board \{wps@fep.up.pt\}

also in [http://ideas.repec.org/PaperSeries.html](http://ideas.repec.org/PaperSeries.html)