The Two Revolutions, Landed Elites and Education during the Industrial Revolution

Duarte N. Leite\textsuperscript{1,2}  
Óscar Afonso\textsuperscript{2,3}  
Sandra T. Silva\textsuperscript{2,3}

\textsuperscript{1} Munich Center for Economics of Aging, Max Planck Institute for Social Law and Policy  
\textsuperscript{2} CEF.UP, Research Center in Economics and Finance, University of Porto  
\textsuperscript{3} FEP-UP, School of Economics and Management, University of Porto
The Two Revolutions, Landed Elites and Education during the Industrial Revolution

Duarte N. Leite
Munich Center for Economics of Aging and CEFUP¹
Max Planck Institute for Social Law and Policy, Amalienstrasse 33, Munich, Germany.
Corresponding author: semedo-leite@mea.mpisoc.mpg.de

Óscar Afonso
University of Porto, Faculty of Economics and CEFUP²
University of Porto, Faculty of Economics, Rua Dr. Roberto Frias, Porto.
E-mail: oafonso@fep.up.pt

Sandra T. Silva
University of Porto, Faculty of Economics and CEFUP²
University of Porto, Faculty of Economics, Rua Dr. Roberto Frias, Porto.
E-mail: sandras@fep.up.pt

Abstract: How we are to understand the Industrial Revolution, the process of transition from a Malthusian equilibrium to today’s Modern Economic Growth, has been the subject of passionate debate. This paper adds more insights to the process of industrialization and the demographic transition that followed this period. By applying the theory of interest groups to landownership and by analyzing landed elites incentives to allow education, it is shown that their political power is important for an understanding of the main events that marked the Industrial Revolution. Contributions are also made to the existence and role of the Agricultural Revolution. It is advanced that it played a significant role in hastening the process of industrialization. A model and numerical simulations are presented to demonstrate these results.

Keywords: Industrial and Agricultural Revolution; Demographic Transition; Education; Interest Groups.

Jel Classification: N53, O13, O14, O43, O50

¹² We particularly thank Oded Galor and Pedro Mazeda Gil for all the very helpful suggestions. We also benefited from the comments of participants of several seminars and conferences. Duarte N. Leite gratefully acknowledges financial support from FCT Portugal.
1. Introduction

The Great Divergence, which started two centuries ago, has been one of the main research challenges economists have been facing in the fields of Growth and Development Economics. The understanding of the Industrial Revolution has been the subject of a passionate debate. Many hypotheses have been put forward to explain the process of transition from a Malthusian to a Post Malthusian era, and thence to today’s Modern Economic Growth era. The Unified Growth Theory has attempted to understand and proffer explanations as to the behavior of economies in this particular time period. Comparative economic development has considered factors such as geographic, institutional, ethnic, religious, human capital formation and colonization as main explanatory elements. Meanwhile, the processes of declining fertility, educational and human capital formation, and the agricultural transformation were intimately related to the onset of the Industrial Revolution, as the Unified Growth Theory has consistently shown (Galor and Weil, 2000; Galor and Moav, 2004; 2006; Voigtländer and Voth, 2006).

In this paper, by applying tools from the Unified Growth Theory and the theory of interest groups, it is sustained that the way landed elites (landowners) observe their gains and losses from the process of education and decide whether to support education or not changes the ability and willingness of workers to educate their children, and hence the timing of the provision of education among the population. This force preventing the rise of education will delay both the process of demographic transition and the real take off of the industrial sector, which cannot entirely achieve its full potential without human capital. Indeed, the lack of institutions that promote human capital reduces the rate and the timing of the transition from an agricultural to an industrial economy. Along with this argument, it is also proposed that the improvement in agricultural processes, which it is claimed to have happened in the century prior to the Industrial Revolution, contributed to accelerating the process of industrialization and to landed elites being more favorably disposed towards education, since the risk of losing rents and their share in the economy was lower due to the prevailing higher productivity of land (Engerman and Sokoloff, 2000; Galor et al, 2009; Litina, 2012).

\(^2\) Although there are theories with rather different approaches explaining these issues - Strulik et al (2013), Desmet and Parente (2012) and Peretto (2013) – this paper will focus on the contributions in the line of Unified Growth Theory referred above.
The Industrial Revolution as a process of transformation of an agricultural economy into an industrial one occurred at different rates in different countries. This different pace concerning both the take-off and the demographic transition led to the so called “Great Divergence” in income per capita as well as in population growth across regions. Although by the end of the first millennium Asia had become the world leader in both wealth and knowledge, during the 19th century Europe overtook those societies (Pomeranz, 2000; Galor, 2011). Empirical analysis on this period and beyond shows that besides England, where the Industrial Revolution first took place, most countries in continental Europe followed the trend and underwent their own process of industrialization. France, Belgium, Prussia and the Netherlands are some examples of western countries that witnessed this revolution just after England where this process had started earlier (Bairoch, 1982). Along with these countries, the western offshoots, such as the US, Canada and Australia also witnessed the development of their economies, which very soon surpassed those of European countries. As for the other countries in the world, most of their economies remained stagnant for almost the next two centuries (Landes, 1998; Maddison, 2003).

There are three outstanding features worth noting in this period of transition from a Malthusian era to the Modern Growth Era: first, the Agricultural Revolution, which had an impact on eighteenth century economies, particularly on England; second, educational dependence on the willing support of the elites, state and government; and finally, the demographic transition which derived from the quantity - quality trade-off. These features, which will be addressed in the next section, are essential to reconcile the events which occurred both during and after the Industrial Revolution and to understand how they are interconnected. Why did education lag behind for so long? Did the Agricultural Revolution contribute to the onset of the Industrial Revolution and education of the population? If so, to what extent? Can the forces behind these developments be identified? Did they hasten or delay the spread of education?

This paper aims to advance the proposition that, as regards the political process that set capitalists against landowners in the nineteenth century, not only capitalists were prone to demand education but also landowners did not oppose to it after some point in time. Landed elites, despite the competition against capitalists, have at some point in time incentives to allow education of the masses to occur, and, by consequence, allow the fostering of the process of industrialization. As gains on rents derived from industrial sector’s spillovers as well as from education itself surpass the costs of education, elites
will have economic incentives to support education. While other studies suggest that education would mostly harm the landed elites by minimizing their revenues (Bourguignon and Verdier, 2000; Galor et al., 2009), it is suggested that although this may be the case initially, nonetheless, as time goes by, it will be profitable for them to support education. Therefore, the proposed theory differs but complements the existent literature (Mokyr, 1990; Bourguignon and Verdier, 2000; Acemoglu and Robinson, 2006; Galor et al., 2009), as it explores the power of elites to prevent education but at the same time shows how economic incentives shift elites’ decisions. The resulting analysis is consistent with the delayed process of education verified in history: the Industrial Revolution took place in the late 1700s and education only spread in the mid to late 1800s (Flora et al., 1983; Brown, 1991). This paper does not deny the emergence of industrialization in the late 18th century, but instead suggests that the political and financial power of elites could only delay the boom of the industrialization process through the delay of one its most important fostering elements: human capital. The focus on elites is justified in order to account for the incentives explaining the initial delay on the industrialization process and the smoother and quick process of education and rise on human capital that occurred afterwards in some countries as well. This is usually not emphasized in the discussions of the conflict between capitalists and landed elites.

In addition, following the patterns during the transition from the Malthusian era to the Modern Growth era, it is argued that the process of the Agricultural Revolution has had an impact on the way landowners agreed with the process of education of the population. In fact, another novelty of this paper is how it shows that continuous advances in technology cause regions, and even countries, to allow education to emerge sooner. Therefore, this paper contributes to the debate on the positive and negative impacts of the Agricultural Revolution on the Industrial Revolution, and argues that, since it occurred in the 1700s in some countries such as England and the Netherlands the take-off of the industrial sector and education took place earlier there. Indeed, it must be asked how the rise in land productivity may have had both a negative and a positive effect on the effectiveness of the Industrial Revolution. The former (negative) effect relates to the higher marginal gains for farm workers compared to urban workers, contributing to a lower pace of migration from the countryside to urban areas. The latter (positive effect) relates to the willingness of elites to provide education. Since their rents increase due to a more productive labor force, the loss caused by the financial support provided to educate the population and the loss of workers to the industrial sector become less significant.
The risk of being overtaken by industry disappears sooner, since the benefits of externalities from industry outstrips the costs of allowing industry to complete its take-off at an early phase\(^3\).

To sum up, the proposed argument aims to add to the extant literature a distinctive explanation for the later rise of education and consequent demographic transition and differences between countries. Nonetheless, being always in line with the Malthusian, Post-Malthusian and Modern Growth trends inherent to this time period, it proposes a complementary perspective of landed elites according to the Unified Growth Theory. Finally, it contributes to the discussion on the positive and negative impacts of the Agricultural Revolution on landed elites’ decisions, and consequently to education.

The paper is organized as follows. In Section 2, we present a historical overview of the periods before and during the Industrial Revolution. Some related literature is presented in Section 3. In Section 4, the set-up of the model is defined, together with the main assumptions. Section 5 provides the analysis of the main predictions of the model and an empirical discussion of these same predictions. In the final section some concluding remarks are made.

2. **Historical Overview**

The divergence on income that began during the Industrial Revolution marked the end of the Malthusian era and pointed towards the Modern Growth regime. During this period some forces were said to be influencing the economy, mainly in England. What later came to be known as the Agricultural Revolution is supposed to have led to the improvement of agricultural productivity, and hence standards of living. The importance of the Agricultural Revolution in the creation of the modern world economy is, for some, greater than the Industrial Revolution itself. Indeed, between 1700 and 1850, it made it possible for output per acre and output per worker to increase to levels far above those verified for the Middle Ages (Clark, 1993). There is considerable controversy concerning the real dimension of the Agricultural Revolution and whether it actually occurred in England. It is argued that long before the Industrial Revolution English farmers were already quite productive (Mokyr, 2009).

---

\(^3\) Besides these effects, we have also the positive effects highlighted in the literature on the provision of more and cheaper goods as well as migration of labor to the urban population, which would sustain and expand it (Overton, 1996; Allen, 2009).
Looking at the estimate of rents in different studies for England, we observe that the majority of the results present an upsurge of rents at the beginning of the 17th century and that this is followed by a slower growth in rents until the beginning of the 19th century (Allen, 1988; Clark, 2002). Nevertheless, there were some significant changes in the English process of farming that amounted to a rise in output and productivity of land. Two levels are considered to allow for the increase in output: intensity and efficiency in the usage of land (Brown, 1991; Clark, 1993; Mokyr, 2009). The enclosure movement, which took place mostly in England, is a very controversial subject, since there is an ongoing discussion regarding its real effects prior to the Industrial Revolution (McCloskey, 1972; Clark, 1993; Allen, 2009). The goal was to achieve higher productivity due to better organization of lands, easier agreements on new production techniques, and an increase in the size of the average agricultural holding (Mokyr, 2009).

Education was already regarded as an asset in the eighteenth century, although it played a minor role during the first phase of industrialization. Only after the mid-1800s, when demand for education was reaching fever pitch, did education start to grow and become essential for the definite take-off of the industrial sector. In the first phase of industrialization demand for skilled workers was small, because the requirements for work in industry were still very simple, illiteracy still being very common among workers. As industrialization grew apace, industrial work became more and more demanding and a higher level of education was required. Despite some educational reforms taking place during the eighteenth and nineteenth centuries, the most important ones, those leading to a real increase in the educational level of workers, only emerged in the late nineteenth century. This was pernicious for the economy, since despite the high demand for education and capital formation, each country went at its own pace in instituting educational reforms (Cubberley, 1920; Galor, 2011). In Prussia, the first education laws came into force in the early 1700s. But they met with resistance everywhere, since there was no willingness by the population in general, and landowners in particular, to cope with the financial burden. This led to a slow advance in the measures that it would be necessary to implement (Cubberley, 1920). The same happened in France and Italy, where the influence of the French revolution and the new tendencies in education changed the way in which people envisaged it. But soon after the end of the revolution, education regressed as a result of the imposition of restrictions on state schools and the enhancement of church schools and private schools (Cubberley, 1920; Green, 1990). As for England,
educational progress began only in the 1850s, when several reforms were effective in promoting education among children (Flora et al., 1983; Green, 1990; Mokyr, 1990).

Besides education, another key trend emerged during this period: the decline in fertility rates. This decline has characterized the demographic transition in most countries throughout the last two centuries. In Western countries this reduction in population growth started in the late nineteenth century, while in Latin America and Asia the phenomenon began much later, in the mid-20th century. This transition continued throughout the last century and contributed to fertility reaching the limit of its replacement level (Lee, 2003). Of more interest is how several studies show that education and fertility are interconnected. The decline in fertility was dominated by investment in education, so that there was a negative correlation between both factors (Flora et al., 1983). In several studies this negative correlation is associated with the trade-off between child quantity and quality. Becker et al. (2010) and Becker et al. (2012) found evidence of this trade-off in nineteenth century Prussia, while Murphy (2010) found evidence for it in France in the late nineteenth century. If this is true, any explanation of the transition during the Industrial Revolution must account for this phenomenon.

Some authors have shown that small interest groups promote an obstruction of new technologies and better institutions in order to maintain their own power and their rent income. As Mancur Olson stated: “…small groups in a society will usually have more lobbying and cartelistic power per capita…” (pp. 41, Olson, 1982). Indeed, small groups organize to pursue their own interest, disregarding society’s interests as a whole, obstructing and delaying any process of development and the shift of institutional or technological environment when it harms their interests (Olson, 1982; Acemoglu and Robinson, 2000; Lizzeti and Persico, 2004; Acemoglu and Robinson, 2008).

The period of the Industrial Revolution was no exception regarding the rise of these groups. Landed elites were a small group in pre-industrial societies. They were extremely powerful and their initial incentives were to halt any process of education, and hence the complete take-off of the Industrial Revolution (Galor et al., 2009). As mentioned above, this group was the one the state had recourse to in order to finance education. This power and unwillingness to support education was the main reason for the conflict between the emerging capitalist class and the old landowners. In fact, the transition from an agricultural to an industrial economy caused the pervasive agrarian economic conflict to mutate into the industrial conflict. While the former featured the landowners and the masses as the main protagonists, in the latter the struggle for power was fought out
between these landed elites and the emerging industrialist elites. The fight for more education in these centuries was one of the main points of divergence between these two groups. While industrialists wanted more educated masses to boost their production, landed elites would perceive the loss of land workers to cities, and so were staunchly opposed to their education. The power of these elites at this period was strong enough to prevent the dissemination of education. As they were the largest and richest group at that time, their financial influence implied that most monarchs’ and rulers’ decisions depended on the advantages elites had (Ekelund and Tollison, 1997; Lizzeri and Persico, 2004). In fact, the dependence of rulers on landowners’ money for warfare and other expenses made it easy for the latter to extract from the former the concession of monopolies, private businesses, patents, and other advantageous deals, whereby a less widely disseminated form of public education could be included. For instance, it is known that, stemming from the mercantilist era, the powers-that-be were at that time inundated with private interests and interest groups that were able to conduct policy-making in the way that best suited them (Ekelund and Tollison, 1997).

3. Related Literature

The role of institutional factors has been studied as a main determinant of economic performance, and hence of the Great Divergence. Having been accorded greater significance in the last decades, modern institutional theories had their historical birth with North and Thomas (1973), North (1981), Greif (1989), and North (1990), being followed in a more empirical fashion by La Porta et al. (1999), Rodrik et al. (2004), Banerjee and Iyer (2005), Hall and Jones (2010) and others. Institutions are singled out as having the force to shape and guide agents in their actions. Authors argue that institutions such as property rights, informal laws, culture, contracts, and so on can facilitate the enhancement of economic growth, since they constitute a myriad of rules, both formal and informal, which constitute the framework in which agents behave. For instance, environmental forces can influence the evolution of the economy indirectly by determining how societies construct their rules and institutions (Engerman and Sokoloff, 2000; Acemoglu et al, 2001; Acemoglu and Johnson, 2005; Nugent and Robinson, 2010). According to this theory, the natural land endowments were a source of more or less inequality, which eventually triggered extractive institutions. The kind of land determined the type of institutions generated in each region.
Under the same institutional branch, the processes of political and social conflict were examined by many authors - Mancur Olson (1982), Mokyr (1990), Acemoglu and Robinson (2000), Bourguignon and Verdier (2000), Grossman and Kim (2003), Lizzeri and Persico (2004), Acemoglu and Robinson (2006) among others. Some argue that small interest groups, the elites, have the power to influence the distribution of income among the population, and may collude to defend their own private interests. Others consider this effect but highlight that elites may support reforms and redistribution to the masses to avoid socio-political instability and predation, stimulating investment and economic growth. For example, regarding the effect of elites on education, Bourguignon and Verdier (2000) argue that if education determines political participation, elites may not find it beneficial to subsidize universal public education, although there are positive externalities from human capital. While Grossman and Kim (2003) show that predation is mitigated by education, Lizzeri and Persico (2004) argue that elites use the provision of public services in their own interests, so that the extension of franchise redirects resources from wasteful redistribution to public goods. Doepke and Zilibotti (2005) show that child labor regulation may benefit capitalists by increasing the human capital of the workforce through children’s education.

Moreover the role of human-capital formation is highlighted as a key element in the transition from stagnation to growth. This line of research is amply supported by the Unified Growth Theory (Galor, 2011). This theory links the rise in the demand for human capital in the emergence of industrialization with technological progress, and the onset of demographic transition, leading to the transition from stagnation to growth (Galor and Weil, 2000). Further research has studied human capital in the light of the dispute between elites. Galor et al. (2009) suggest that the importance of human capital in production increased the incentives for capitalists to support the provision of public education, triggering the demise of the existing class structure, whereas Galor and Moav (2006) argued that inequality in the distribution of landownership negatively affected the emergence of human-capital promoting institutions. It is to this line of research that this paper contributes with an analysis of the willingness of landed elites to support education during the Industrial Revolution, also taking into account the previous process of the Agricultural Revolution.

Other factors have been emphasized as determinants for economic growth and the Great Divergence. For instance, geographical factors suggest that more favorable geographical conditions made Europe less vulnerable to the risk associated with climate
and disease, leading to the early European take-off (Jones, 1981; Diamond, 1997), while cultural/institutional factors imply that societies in which norms and ethics enhance the "entrepreneurial spirit" and openness to new ideas and advances in other societies (cultural assimilation) are those that attained industrialization in the 1800s, during the Industrial Revolution (Hall, 1986; Landes, 1998; Landes, 2006; Ashraf and Galor, 2011). Nevertheless, geographical and cultural factors are beyond the scope of this paper.

4. Model Setup

Galor et al. (2009), Ashraf and Galor (2011) and Litina (2012), consider an overlapping-generations economy operating over infinite discrete time. In the pre-industrial era, the economy produces a single homogeneous good, using land and labor as inputs. After the emergence of the industrial sector, the economy produces agricultural and manufactured goods, using land and labor as inputs. The supply of land is exogenous and fixed over time. The number of efficiency units of labor is determined by households' decisions in the preceding period regarding the number and human-capital level of their children.

The model comprises two types of individuals: workers and elites. Workers reproduce themselves asexually. The number of offspring relies on workers’ decisions. Elites have one child each and do not take part of the productive process. Thus, in each period $t$, a generation of a continuum of $L_t$ identical workers enters the labor force. Individuals of generation $t$ live for two periods.

4.1. Production

To produce a good, each worker supplies inelastically one unit of labor in each period. The aggregate supply of workers evolves over time at the endogenously determined rate of population growth. In the early Malthusian phase, the agricultural sector is the only one operating, since the industrial sector is not yet economically viable. As technology in the industrial sector evolves over time, at some point the productivity threshold is reached, the industrial sector emerges, and both sectors operate in the economy.

4.1.1. Production in both sectors

The output produced in the agricultural sector occurs according to a constant-returns-to-scale technology. In period $t$, $Y_t^A$ is determined by land, $X_t$; labor employed in the agricultural sector, $L_t^A$; agricultural technology $A_t^A$, determined endogenously.
\[ Y_t^A = (A_t^A X_t)^\alpha (L_t^A)^{1-\alpha} \quad \text{for} \quad 0 < \alpha < 1, \]

where \( L_t^A = (1 - \lambda_t) L_t \) where \((1 - \lambda_t)\) is the share of workers in the agricultural sector. \((1 - \lambda_t) \in (0,1)\) but is set to one until the emergence of the industrial sector.

To allow for the characterization of the periods before and after the emergence of the industrial sector, the specifications in (2) and (3) were adopted to avoid any nuisance with the performance and movements of workers to the industrial sector\(^4\). From this, the output of the industrial sector has two structures: before and after the emergence of industry.

In the pre-industrial era, we have a linear production function relying on technology \(A_t^I\) and on efficient labor \(H_t\), at each \(t\):\(^5\)

\[ Y_t^I = A_t^I H_t, \]

with \(H_t = \lambda_t h_t L_t\), where \(h_t\) is the human-capital level and again \(\lambda_t\) is the share of workers in the industrial sector.

After the emergence of the industrial sector, constant returns to scale are assumed in the production function. The same happens to technology gains. This implies that now both sectors are always open with \(\lambda_t \in (0,1)\). The elements of technology and efficient labor are maintained:

\[ Y_t^I = (A_t^I)^{1-\theta} (H_t)^\theta \quad \text{for} \quad 0 < \theta < 1, \]

Finally, the total labor force is given by the sum of workers in each sector:

\[ L_t = L_t^A + L_t^I, \]

where \(L_t^I = \lambda_t L_t\) and \(L_t > 0\) in each period \(t\).

### 4.1.2. Factor prices, labor market and the technology threshold

The economy has two types of agents: workers and elites. Workers receive their wages according to their productivity in the sector they are working in. Elites receive rents from land, since they own their property rights. Thus, returns from land are not zero. Property rights are not transmissible to other elite members or workers. They are inherited by the child of each member of the elite.

---

\(^4\) With this specification, marginal gains before industrialization are not infinite, and so industrialization does not take place immediately. Furthermore, with (3), marginal gains of industry are decreasing, with the result that after industrialization we always have an equilibrium in the labor market so that both sectors are open.

\(^5\) Capital is important in the production process. Nevertheless, due to the focus on human capital and to simplify the analysis we drop it from the production function.
Rents are determined as the marginal gains for each unit of land held by a member of the elite. We assume that all members have the same share of land. Thus, the rent received by \( i \) is:

\[
\rho_t = \alpha (A_t^A)^\alpha (X_t)^{\alpha-1} (L_t^A)^{1-\alpha},
\]

(5)

We are going to assume a fixed value for land \( X_t = 1 \). From the above, rents are positively related to technology, land fertility and the number of workers allocated to the agricultural sector: \( \rho_A(A_t^A, L_t^A) > 0 \), \( \rho_Y(A_t^A, L_t^A) > 0 \) and \( \rho_L(A_t^A, L_t^A) > 0 \) for any \( A_t^A, L_t^A > 0 \).

Depending on the era, before or after the emergence of the industrial sector, wages can be earned either in the agricultural sector or in the agricultural and industrial sector. The market for labor is perfectly competitive, and thus wages are given by the marginal productivity of labor in each sector. Given (1), the marginal product and hence the inverse demand of labor in the agricultural sector is:

\[
w_t^A = (1 - \alpha) (A_t^A)^\alpha (X_t)^{\alpha} ((1 - \lambda_t) L_t)^{-\alpha},
\]

(6)

where \( w_t^A \) is the wages of agriculture workers.

From (2), before the industrial sector takes off, workers could supply \( h_t \) efficient units to the industrial sector and earn the potential wage:

\[
w_t^I = A_t^I h_t,
\]

(7)

In the second phase, marginal productivity is in turn determined using (3):

\[
w_t^I = \theta (A_t^I)^{1-\theta} (H_t)^{\theta-1} h_t = \theta (A_t^I)^{1-\theta} (\lambda_t L_t)^{\theta-1} (h_t)^{\theta}.
\]

(8)

From (6) and (7), the productivity of the industrial sector is finite and initially low (if we consider initial low technology values for industrial technology), whereas productivity in the agricultural sector tends to infinity for low initial levels of employment. Thus, the agricultural sector is open in every period, and the industrial sector emerges only when its labor productivity exceeds the marginal productivity of labor in the agricultural sector, considering that the entire labor force is employed in the agricultural sector. When the emergence takes place, the new production structure is also applied in the industrial sector. Therefore, (6) and (8) must be equal to guarantee the perfect labor mobility assumption and hence determine the share of workers in each sector. To establish the necessary conditions for the emergence of the industrial sector we set Lemma 1.
Lemma 1: If wages are determined by (6) and (7), there is a threshold value for industrial technology $\hat{A}_t^I$ from which the industrial sector is economically viable$^6$:

$$ \hat{A}_t^I > \frac{(1 - \alpha)(YA_t^A X_t)^\alpha}{L_t^a h_t^\theta} $$

See proof in the appendix.

When the threshold is exceeded, the industrial sector emerges. From Lemma 1, if $A_t^I < \hat{A}_t^I$, wages are set equal to the marginal product of the agricultural sector $w_t = w_t^A$. Otherwise, $w_t = w_t^I = w_t^I$ and wages are set to be equal to the marginal product of the industrial sector (8). The equilibrium share of labor between the two sectors at period $t$ is given by:$^7$

$$ \lambda_t = \left\{ \begin{array}{ll} 0 & \text{if } A_t^I < \hat{A}_t^I \\ \frac{1}{\theta^\alpha A_t^I (h_t)^\theta} & \text{if } A_t^I \geq \hat{A}_t^I, \end{array} \right. \quad (9) $$

And,

$$ w_t = \left\{ \begin{array}{ll} (1 - \alpha)(YA_t^A)^\alpha (X_t)^\alpha ((1 - \lambda_t) L_t)^{-\alpha} & \text{if } A_t^I < \hat{A}_t^I \\ \theta (A_t^I)^{1-\theta} (\lambda_t L_t)^{\theta-1} (h_t)^\theta & \text{if } A_t^I \geq \hat{A}_t^I, \end{array} \right. \quad (10) $$

4.2. Workers

As for workers, they are raised by their parents in the first period of their lives (childhood) and may be educated, acquiring human capital. In the second period of their lives (adulthood), individuals supply their efficiency units of labor and allocate the resulting wage income. The preferences of members of generation $t$ (those born in $t - 1$) are defined over consumption above a subsistence level $\bar{c} > 0$ in and over the potential aggregate income of their children. i.e. the number of their children, their acquired human capital and their correspondent wages (observed in $t + 1$). They are represented by the utility function:

$$ u_t = c_t^\gamma (h_{t+1} n_t)^{1-\gamma} \quad \text{for } 0 < \gamma < 1, $$

where $c_t$ is consumption, $h_{t+1}$ is the human-capital level of each child, and $n_t$ is the number of children of members of generation $t$. Following Galor and Weil (2000), the individual’s function is strictly monotonically increasing and strictly quasi-concave, satisfying the conventional boundary conditions that ensure that, for sufficiently high

---

$^6$ Equation (7) is used since workers, when only the agricultural sector is open, compare the gains in this sector with the supposed gains in the industrial sector, given by (7).

$^7$ Note that for the easy tractability of the equilibrium we will assume that $\theta = 1 - \alpha$. 
income, there exists an interior solution for the utility maximization problem. For a sufficiently low level of income the subsistence consumption constraint is binding. Let $z_t = w_t h_t$ (where for $e_t = 0, h_{t+1} = 1$) be the level of potential income per worker, which is divided between expenditure on child-rearing (quantity as well as quality) and consumption. We will define $\bar{z}$ as the level of potential income below which subsistence consumption is binding.

Let $\tau^e > 0$ be the time endowment cost faced by a member of generation $t$ for raising a child, regardless of quality, and let $g(\tau^e, T_t) > 0$ be the time endowment cost necessary for each unit of education per child. The function $g(.)$ depends positively on $\tau^e > 0$, a fixed cost of educating children, and negatively on $T_t \geq 0$. It depicts the intervention of elites in the process of education. $T_t$ is the amount of resources raised by elites among themselves which are transferred to workers to reduce the cost of education to motivate parents (workers) to educate their children. $T_t = f(t_t, b_t)$, which relies positively on both variables, tax rate ($t_t$) and the elite’s bequest ($b_t$) — see Section 4.3 below.

Human capital in the second period of life is determined by the units of education received during childhood. Human capital is an increasing and concave function of education

$$h_{t+1} = h(e_{t+1}),$$

where $h(0) = 1, \lim_{e \to \infty} h'(e_{t+1}) = 0, \lim_{e \to 0} h'(e_{t+1}) = \chi < \infty$. In the absence of education, individuals possess basic skills — one efficiency unit of human capital.

We can now sketch the budget constraint faced by parents in the second period:

$$c_t + w_t h_t n_t (\tau^e + g(\tau^e, T_t) e_{t+1}) \leq w_t h_t,$$

(13)

4.2.1. Optimization

The members of generation $t$ maximize utility subject to the budget constraint. They choose the number of children and the level of education of each child and their own consumption. Substituting (13) by (11), the optimization problem for a member of generation $t$ reduces to:

$$(n_t, e_{t+1}) = \arg\max w_t h_t (1 - n_t (\tau^e + g(\tau^e, T_t) e_{t+1})))^\gamma ((h_{t+1} n_t))^{1-\gamma},$$

subject to

---

8 We follow, for instance, Galor et al. (2009) and Galor and Moav (2006), although there are other interesting approaches, such as linking human capital to the growth rate of technology or linking it to teachers’ wages, albeit this beyond the scope of this paper.
\[ w_t h_t \left(1 - n_t (\tau^r + g(\tau^e, T_t) e_{t+1}) \right) \geq \hat{c} \]
\[ n_t, e_{t+1} \geq 0 \]

It follows from the optimization process that:

\[
n_t = \begin{cases} 
\frac{1 - \frac{\hat{c}}{w_t}}{\left(\tau^r + g(\tau^e, T_t)e_{t+1}\right)} & \text{if } z_t < \bar{z} \\
\frac{1 - \gamma}{1 - \frac{\hat{c}}{w_t}} & \text{if } z_t \geq \bar{z}
\end{cases}
\]

For a binding consumption constraint \( z_t < \bar{z} \), the optimal number of children for a member of generation \( t \) is an increasing function of individual \( t \)'s income. This mimics one of the fundamental features of the Malthusian era. The individual consumes the subsistence level \( \hat{c} \), and uses the rest of the time endowment for child-rearing. The higher the wage he earns, the lower the time he allocates to labor, so that the time spent rearing his children increases.

Independently of the division between time devoted to consumption and child rearing, the units of education for each child only depend on the relative weight of raising costs and educating costs. While the raising costs are constant, the educating costs depend on the willingness of elites to devote resources to fostering education. The higher the resources devoted to education by elites, the higher the units of education given to children. Using (14) and (15), the optimization with respect to \( e_{t+1} \) shows how the implicit function \( G(.) \) only depends on \( e_{t+1} \) and \( T_t \):

\[
E(e_{t+1}, T_t) = h'_{t+1}(\tau^r + g(\tau^e, T_t)e_{t+1}) - h_{t+1}g(\tau^e, T_t),
\]

where \( E_e(e_{t+1}, T_t) < 0 \). \( E_T(e_{t+1}, T_t) = g'(\tau^e, T_t)\left[h'_{t+1}e_{t+1} - h_{t+1}\right] > 0 \) for a specific set of equations. To guarantee that for a positive level of \( T_t \) the chosen level of education is higher than zero, it is assumed that:

\[
E(0,0) = h'_{t+1}(0)\tau^r - h_{t+1}(0)g(\tau^e) = 0,
\]

**Lemma 2:** If (A 1) is satisfied, then, for the specific set of equations referred to above, the level of education of generation \( t \) is a non-decreasing function of \( T_t \).

\[
e_{t+1} = \begin{cases} 
0 & \text{if } T_t \leq 0 \\
> 0 & \text{if } T_t > 0
\end{cases}
\]

and \( e'_{t+1}(T_t) > 0 \) for \( T_t > 0 \) for \( T_t > 0 \)

See [the] proof in the appendix.

From the above information and (15) we can draw some conclusions regarding the behavior of education and the number of offspring.

**Proposition 1:** From Lemma 2, (15), (16) and (A 1):
(A) The number of offspring and level of education are affected by the level of $T_t$. An increase in $T_t$ results in a decline in the number of offspring and in an increase in their level of education: $\frac{\partial n_t}{\partial T_t} < 0$ and $\frac{\partial e_{t+1}}{\partial T_t} > 0$.

(B) The number of offspring is affected by changes in the potential income of parents if the subsistence consumption constraint is binding, while the level of education is not affected. Otherwise, none of the two variables are affected:

$$\begin{align*}
\frac{\partial n_t}{\partial z_t} &> 0 \text{ and } \frac{\partial e_{t+1}}{\partial z_t} = 0 \quad \text{if } z_t < \bar{z} \\
\frac{\partial n_t}{\partial z_t} & = \frac{\partial e_{t+1}}{\partial z_t} = 0 \quad \text{if } z_t \geq \bar{z}
\end{align*}$$

4.3. Elites

As already stated, elites have one child each. These elites make no decisions regarding the quantity or quality of children they have. In the first period (childhood) of their lives, these children are raised by their parents. The children receive their bequest and decide how to spend it. They can use part of it to support education or to consume and leave a bequest in adulthood to their own children. Hence, in the second period of their lives (adulthood), elites divide the value of rent from land and the bequest left from period one into consumption and the bequest to their children. The preferences of members of generation $t$ (those born in period $t-1$) are defined over consumption as well as over the bequest left to their children. They are represented by:

$$u_t = c_t^{\mu} (b_{t+1})^{1-\mu} \quad \text{for } 0 < \mu < 1,$$

where $c_{t+1}$ is consumption in period two, $b_{t+1}$ is the bequest for the child. The elites function is again strictly monotonically increasing and strictly quasi-concave, satisfying the conventional boundary conditions. We will assume that $\mu = \gamma$.

The available income to use as consumption and bequest is defined by the period two rent and the remains of the bequest after deducting the amount to support education. This amount ($T_t$) is deducted in the first period, when the bequest is received. The bequest keeps the same value through periods – the interest rate equals zero. $T_t = f(t_t, b_t)$ where $f_t(t_t, b_t) > 0$ and $f_b(t_t, b_t) > 0$. We will assume $f_{t_t}(t_t, b_t) < 0$ and $f_{b_b}(t_t, b_t) < 0$, since a concave reaction to the amount spent in decreasing the cost of education appears to be reasonable. A boundary to the gains of more spending seems to be plausible to avoid infinite gains. The rent depends on the amount of land each elite member has. $\tilde{x}^i$
estimates how land is divided among elites; the higher is the value, the less dispersed is
the land.

We can now sketch the budget constraint faced by elites in the first period:
\[ c_{t+1} + b_{t+1} \leq \rho_{t+1} + (1 - t_t)b_t, \]  
(18)

4.3.1. Optimization

Members of generation \( t \) maximize the utility, subject to the budget constraint. They
choose the tax level, the consumption in the following period and the next period’s
bequest for their children. Substituting (18)(13) into (17), the optimization problem for
a member of generation \( t \) reduces to:
\[(t_t, b_{t+1}) = \text{argmax}\{\rho_{t+1} + (1 - t_t)b_t - b_{t+1}\}^{\gamma}\left\{ \left(h_{t+1}n_t\right)^{-1}, \right\}
(19)
subject to
\[ b_{t+1} \geq 0 \]
and \( t_t \in [0,1] \)

It follows from the optimization process that:
\[ b_{t+1} = (1 - \gamma)\left( \rho_{t+1} + (1 - t_t)b_t \right), \]
(20)
Elites spend \((1 - \gamma)\) of their income in providing a bequest for their children.

Using (5) and (19), the optimization with respect to \( t_t \) shows this as an implicit
function \( G(. \) which depends on \( t_t, A^A_t, A^I_t \) and \( L_t \):
\[ G(A^A_t, A^I_t, L_t, t_t) = \frac{d\rho_{t+1}}{dt_t} - b_t = 0, \]
(21)
The implicit function has two different characterizations depending on whether it is
before or after the structural break (see below). There are different effects that influence
the decision of elites and are reflected in \( G(.) \). The primary effect is the “bequest effect”.
The higher the bequest, the higher is the amount transferred to support education. This
effect is always negative. The “rent effect” is the other effect that can be divided between
two main effects: “technology effect” and “workers’ effect”. The technology effect is
positive, since a higher amount spent in supporting education increases the externality of
the industrial technology on the agricultural technology. The sign of the worker effect is
ambiguous. With more education, the agricultural share can increase due to the
externalities that increase productivity in this sector, while the industrial sector can benefit
from more human capital, which also increases marginal productivities in this sector.
Thus, depending on the strength of forces, the share of workers either in industry or in
agriculture may increase. Now, since the elites’ decision relies on an implicit function
and \( t_t \in [0,1], \) we can draw some conclusions about the decision, depending on whether \( G(.) \leq 0. \)

**Lemma 3:** The decision to set taxes at a higher rate than zero depends on the value of the implicit function. Since \( G(.) \) can take different values in \([0,1],\) then:

If for the entire interval \([0,1],\):

\[
\begin{align*}
G(.) < 0, & \quad \Rightarrow \quad t_t = 0 \\
G(.) > 0, & \quad \Rightarrow \quad t_t = 1
\end{align*}
\]

If in the interval \([0,1],\):

\[
G(.) = 0, \quad \Rightarrow \quad t_t \in [0,1]
\]

See proof in the appendix.

Thus, when \( G(.) \) does not equal zero then either an increase in \( t_t \) increases the utility at a maximum of \( t_t = 1 \) or it decreases the utility so that the best choice is \( t_t = 0. \) Otherwise, \( t_t \in [0,1], \) so that the best choice is an interior solution.\(^9\)

From (5), (6), (7) and (21) we can determine the implicit function before the structural break:

\[
G(.) = \alpha \bar{x}^i (L_t)^\varepsilon (A_t^l)^\delta \left[ \frac{(1 - \alpha)^{1/2}}{(1 + e_{t+1})^\alpha} \left( \frac{1 - \alpha}{(1 + h_t L_t^d)^\varepsilon A_t^l)^{1/2}} \right)^{1-\alpha} \frac{de_{t+1}}{dt_t} \right] \left( A_t^l \right)^b - \beta (1 - \alpha) \left[ \frac{1 + e_{t+1}}{(1 + e_{t+1})^\alpha} \right] (1 + e_{t+1})^b \quad (22)
\]

where only \( e_{t+1} \) depends on \( t_t. \) Before the break, the decision of elites regarding education relies on the technology effect, which is higher than the workers’ effect and the bequest effect.

**Lemma 4:** Before the structural break, for \( G(.) \geq 0 \) the condition \((A_t^l)^b - \frac{\beta (1 - \alpha) (1 + e_{t+1})^b}{\alpha} \frac{(1 + e_{t+1})^b}{(1 + e_{t+1})^\alpha} \) must be positive.

Proof: this follows directly from (22).

After the structural break, when the Industrial Revolution takes off, the decision rule differs, due to the new industrial structure. The decision continues to contemplate the same three effects, plus one more. In contrast, with the previous decision rule, the “population effect” does not vanish, and thus the rent effect now has three effects within it. The population effect relates to education, since the more education they receive, the less time endowment do workers have to raise their children, so that the quantity–quality

\(^9\) We show by simulation that \( G(.) \) is a decreasing function with respect to \( t_t. \)
trade-off is instrumental. The implicit function after the structural break is derived from (5), (6), (8) and (21) – see the appendix for the full equation.

**Lemma 5:** After the structural break, for \( G(.) \geq 0 \) it must be true that:

\[
1 - (1 - \alpha) \left( \frac{1}{(1 + e_{t+1})^{\beta \theta}} \right) \times \left( \frac{1}{(1 + h_{t+1}L_t^\gamma)} \right) \]

is positive.

Proof: This follows from the implicit function condition - see the appendix.

Regarding the decision rules, the elites define when to support and allow education in the economy. Decisions depend on the macroeconomic environment and main variables, such as agricultural technology. These implications are analyzed in Section 4.2.

### 4.4. Dynamic Paths

The economy is governed by three main macroeconomic variables: agricultural productivity \( A_t^A \), industrial productivity \( A_t^I \) and the working population \( L_t \).

#### 4.4.1. Population Dynamics

From (15), the size of the labor force in period \( t + 1 \), \( L_{t+1} \), is determined by:

\[
L_{t+1} = L_t n_t = \begin{cases} 
1 - \frac{\bar{e}}{w_t} & \text{if } z_t < \bar{z} \\
1 - \gamma & \text{if } z_t \geq \bar{z} 
\end{cases} \frac{L_t}{(\tau^r + g(\tau^e, t_t)e_{t+1})} \)

\[
(\tau^r + g(\tau^e, t_t)e_{t+1}) \]

where the initial historical size of the adult population, \( L_0 > 0 \), is given.

#### 4.4.2. Technology Dynamics

The level of each technology is affected by its level in the previous period\(^{10}\). Agricultural technology at time \( t + 1 \) is affected by two elements: the externality of the “learning by doing effect” and the general knowledge effect of the population on technology, and the external effect from the gains from educating youngsters in the period along with the existing level of industrial technology. This latter effect allows for interconnections between technology and education and the current working population

---

\(^{10}\) The dynamic paths are inspired by Litina (2012) and Ashraf and Galor (2011).
and level of agricultural technology. The law of motion of agricultural technology is such that:

$$A_{t+1}^A = (1 + e_{t+1}(A_t^I)^b)(L_t)^\epsilon (A_t^A)^\delta,$$  \hspace{1cm} (24)

where \((L_t)^\epsilon (A_t^A)^\delta\) captures the “learning by doing effect” and general externalities of the growing population in agricultural technology. The factor \(e_{t+1}(A_t^I)^b(L_t)^\epsilon (A_t^A)^\delta\) is the external effect of industrial technology and education. These dynamics are dependent on the imperfect intergenerational transmission of knowledge.

We assume that \(\epsilon > 0\) and \(\delta > 0\) and \(\epsilon + \delta < 1\), which implies that the population has a decreasing effect on knowledge creation, and it also implies a "fishing out" effect, namely the negative effect of past discoveries on making discoveries today. In addition, \(b > 0\), so that when people are educated, externalities of industrial technologies spill over to technology in agriculture.

Evolution in industrial technology is given by the past period level of technology and the improvement in knowledge driven by the working population measured by its human capital. The greater the human capital and the more the number of workers in the economy, the greater the gains to industrial technology driven by learning by doing and externalities associated with human capital.

$$A_{t+1}^I = (1 + h_tL_t^\Delta A_t^I)^\zeta A_t^I,$$  \hspace{1cm} (25)

where \(\zeta \in (0,1)\) as well as \(\Delta \in (0,1)\). Equation (25) shows that industrial technology advances according to the expansion of the existent level of technology due to increasing population and human capital, but in a diminishing returns fashion.

5. Dynamics of the Development Process

In this section, we examine how the structure of the economy and agents’ decisions shape the path of the development process. It is shown how the economy can evolve from a pre-industrial equilibrium to a state of sustained economic growth and how agricultural technology and elites’ decisions affect the economic equilibrium during different states. It is argued that more positive shocks in agricultural technology due to a presumable Agricultural Revolution also cause the early emergence of education.

5.1. Before the Industrial Revolution

This section presents the evolution from a Malthusian era and the endogenous transition to industrialization. It is shown that the transitional process will depend on agricultural technology and on the elites’ decisions relating to supporting education.
Hence, these causes affect the rise of education in the economy, delaying the process of deep industrialization.

During the Malthusian era, the economy is governed by the dynamic system given by equations (23), (24) and (25), which, given the initial values \((A^A_0, A^I_0, L_0)\), yield the sequence of state variables \(\{A^A_t, A^I_t, L_t\}_{t=0}^{\infty}\).

Following Ashraf and Galor (2011), the pre-industrial equilibrium can be analyzed by the behavior of the two variables \(\{A^A_t, L_t\}\) and the distance to the Industrialization frontier. The industrial technology variable does not affect the pre-industrial equilibrium, since until the emergence of the industrial sector it is just a latent variable. It must also be stressed that in the pre-industrial era the economy is under the Malthusian regime, i.e. it evolves under the assumption that the subsistence consumption constraint is binding, and so fertility depends on the income of workers. Thus, as is shown, under a steady state equilibrium the economy is trapped in the Malthusian regime and a binding consumption constraint.

### 5.1.1. The Industrialization Frontier

The Conditional Industrialization Frontier (CIF) gives the frontier between the agricultural economy and the industrial economy. Once the economy’s trajectory crosses the frontier, the industrial sector becomes operative. The CIF is then given by:

\[
CIF|A^I_t \equiv \{(A^A_t, L^I): L^I = L(A^A_t, A^I_t)\}.
\]  

and we can establish the following lemma:

**Lemma 6:** If \(\{A^A_t, L^I_t\}\) belongs to the CIF, then, for a given \(A^I_t\),

\[
L^I_t = \frac{(1 - \alpha)A^A_tX_t}{(h^I_tA^I_t)^{\frac{1}{\alpha}}}.
\]

where \(\frac{\partial L(A^A_t, A^I_t)}{\partial A^A_t} > 0\) and \(\frac{\partial L(A^A_t, A^I_t)}{\partial A^I_t} < 0\).

Proof: This follows directly from Lemma 1, (6), (7) and (26).

The CIF is upward sloping. In the region strictly below the frontier, agriculture is the only open sector, whereas in the region above, both sectors are open. The higher is \(A^I_t\), the closer we are to the trigger and to surpassing the CIF.

The agricultural technology locus is set, for all the pairs \(\{A^A_t, L^I_t\}\), such that \(A^A_t\) is in steady state.
Lemma 7: If \( \{A_t^A, L_t\} \) belongs to \( \text{AA} \), then
\[
L_t = \left( A_t^A \right)^{\frac{1-\delta}{\varepsilon}} \equiv L^{AA}(A_t^A)
\]
where \( \frac{\partial L^{AA}(A_t^A)}{\partial A_t^A} > 0 \) and \( \frac{\partial^2 L^{AA}(A_t^A)}{\partial (A_t^A)^2} > 0 \).

Proof: This follows from (24) and (6), using the steady state condition, and (27).

The \( \text{AA} \) locus is a convex, upward sloping curve. Above \( L^{AA} \) the number of workers is large enough to ensure the expansion of the technology frontier, overcoming the erosion effects of imperfect intergenerational transmission of knowledge. Below the \( L^{AA} \) workers are too few to overcome the latter effect, shrinking the technology level.

The population locus (\( LL \)) is the set of all pairs \( \{A_t^A, L_t\} \) such that \( L_t \) is in steady state\(^{11}\).
\[
LL \equiv \{(A_t^A, L_t) : L_{t+1} = L_t = 0 \mid L_t < \bar{L}; z_t < \bar{z}\},
\]

Lemma 8: If \( \{A_t^A, L_t\} \) belongs to \( L \), then
\[
L_t = \left[ \frac{(1 - r^\gamma)(1 - \alpha)}{\bar{c}} \right]^{\frac{1}{\alpha}} A_t^A \equiv L^{LL}(A_t^A)
\]
where \( \frac{\partial L^{LL}(A_t^A)}{\partial A_t^A} > 0 \) and \( \frac{\partial^2 L^{LL}(A_t^A)}{\partial (A_t^A)^2} = 0 \).

Proof: This follows from (23), using the steady state equilibrium condition and (28).

Hence, the \( LL \) locus is an upward sloping linear function. \( L_t \) grows over time below the \( LL \) locus (\( L_{t+1} > L_t \)), when for a lower population size wages increase, and hence allow for fertility above replacement. Otherwise, wages are lower, and due to the consumption constraint, resources available for fertility are reduced (\( L_{t+1} < L_t \)). The relationship between the \( LL \) locus in Lemma 8 and the CIF in Lemma 6 is:

Lemma 9: For \( A_t^A > 0 \) and for all \( A_t^A \) such that \( (A_t^A, \hat{L}(A_t^A, A_t^A)) \in CIF(A_t^A) \) and \( (A_t^A, L^{LL}(A_t^A)) \in LL \)
\[
\hat{L}(A_t^A, A_t^A) \geq L^{LL}(A_t^A) \in LL
\]
if and only if
\[
A_t^A \leq \frac{c}{(1-r^\gamma)h_t}
\]

Proof: This follows from comparing \( \hat{L}(A_t^A, A_t^A) \) and \( L^{LL}(A_t^A) \) in Lemma 6 and Lemma 8, respectively.

5.1.2. Equilibrium and Global Dynamics

\(^{11}\) Although population increased during some periods of the Malthusian era (Maddison, 2003), the model regards this period on a steady state level.
If we consider the pre-industrial Malthusian equilibrium, we have to ensure that the condition $A_t^I < \frac{\bar{c}}{(1-\tau^t)h_t}$ in Lemma 9 is verified and the subsistence consumption constraint is binding $z_t < \bar{z}$. Following these conditions, the Malthusian steady state is characterized by a globally stable steady state equilibrium $\{A_{ss}^A, L_{ss}\}$ – see appendix. By ruling out the unstable equilibrium at the origin ($L_0 > 0$ and $A_0^A > 0$), the globally stable equilibrium $\{A_{ss}^A, L_{ss}\}$ is maintained. At initial stages of development, agriculture is the pervasive sector, since the latent industrial sector has a very low level of productivity and thus is not sufficiently attractive. The economy operates exclusively in the agricultural sector. So the CIF locus is located above the LL locus and the above mentioned dynamics of $L_t$ and $A_t^I$ are valid. To guarantee that the discrete dynamic system is globally stable and that the convergence to the steady state takes place monotonically over time, the following lemma is considered:

**Lemma 10**: If $A_t^I < \frac{\bar{c}}{(1-\tau^t)h_t}$, then the equilibrium in the dynamical system:

1. is globally stable if the Jacobian matrix $J(A_{ss}^A, L_{ss})$ has real eigenvalues with modulus less than 1; and
2. the convergence to the steady state is monotonically stable.

See proof in the appendix.

We must also ensure that the subsistence consumption constraint remains binding during this regime, so that:

$$z_t|_{A_{ss}^A, L_{ss}} = w_{ss}h_t < \bar{z} \quad \text{(A 2)}$$

for an initial $h_t = 1$. With only the agricultural sector operative, all workers are employed in this sector. Thus, it follows from (1) and the globally stable steady state values that, in line with the dynamics in the Malthusian era - in the long-run, the level of income is independent of the level of technology and is constant - $\tilde{y}_{ss} = 0$.

As the economy evolves during the Malthusian era within the pre-industrial steady state the latent and endogenous process of industrialization implies that sooner the take-off to a state of sustained economic growth will take place. $A_t^I$ continues to increase and gets closer of the trigger of the industrial sector. When $A_t^I > \frac{\bar{c}}{(1-\tau^t)h_t}$, the industrial sector emerges.
5.2. The Industrial Revolution

As referred above, it is assumed that in the pre-industrial period the subsistence consumption constraint is binding and the economy is trapped in a steady-state. However, after the emergence of the industrial sector two possible regimes emerge. In the first regime the subsistence consumption constraint is still binding. Nevertheless, as the previous steady state equilibrium vanishes, wages increase and so the subsistence constraint will vanish in time.

In this first regime the economy is governed by a four-dimensional non-linear first-order autonomous system:

\[
\begin{align*}
A^A_{t+1} &= (1 + e_{t+1}(A^I_t)^b)(L_t)^e(A^A_t)^\delta \\
A^I_{t+1} &= (1 + h_tL_d^d)^\xi A^I_t \\
e_{t+1} &= e(T_t(A^A_t, A^I_t, L_t)) \\nL_{t+1} &= \frac{1 - \frac{c}{w_t}}{\tau r + g(\tau^e, \tau_t)e_{t+1}} L_t
\end{align*}
\]  

(29) 

In the second regime, the subsistence constraint is no longer binding. Population grows at a constant level that will only be affected by choices of workers regarding education. Again, the regime is governed by the same four-dimensional system, although population growth does not depend on income of workers:

\[
\begin{align*}
A^A_{t+1} &= (1 + e_{t+1}(A^I_t)^b)(L_t)^e(A^A_t)^\delta \\
A^I_{t+1} &= (1 + h_tL_d^d)^\xi A^I_t \\
e_{t+1} &= e(T_t(A^A_t, A^I_t, L_t)) \\nL_{t+1} &= \frac{1 - \gamma}{\tau r + g(\tau^e, \tau_t)e_{t+1}} L_t
\end{align*}
\]  

(30)

Despite the four dimensional system, the transition between the two regimes is given by the distance to the Malthusian Frontier (MF). As explained previously in (29) and (30), the economy departs from the first regime when potential income \( z_t \) exceeds that level. So, \( \{A^A_t, A^I_t, L_t, e_{t+1}\} \) belongs to \( MF \) if

\[
MF \equiv \left\{ (A^A_t, A^I_t, L_t, e_{t+1}) : \theta(A^I_t)^{1-\theta}(\lambda_t(A^A_t, A^I_t, L_t)L_t)^{\theta-1}(h_t)^\theta = \frac{c}{1-\gamma} \right\},
\]

(31)

Lemma 11: The economy surpasses the Malthusian regimes if:

\[
w_t^I = \theta(A^I_t)^{1-\theta}(\lambda_t(A^A_t, A^I_t, L_t)L_t)^{\theta-1}(h_t)^\theta \geq \frac{c}{1-\gamma}
\]
Proof: this follows from (8), definition of $z_e$ and $\bar{z}$, and (31).

6. From the Malthusian era to the Modern Growth era

The economy evolves from the Malthusian era to the Modern Economic Growth era, passing through the Post Malthusian era and the demographic transition. This path derives from Section 4.1 and the two regimes explained above.

As the economy is trapped in the pre-industrial equilibrium, population is small and agricultural technology is stagnant. Initially, education is not supported by elites and thus workers do not provide it. When the economy changes to the Post-Malthusian regime workers split between the two sectors. Population starts to grow over time and, from (24) and (25), it has a scale effect on both technologies and there is an interconnection between variables, since an increased population and more technology lead to higher wages. As income increases, and the economy is still under a Post-Malthusian regime, it positively affects fertility. More income means higher fertility, and there is a boost in population. The three state macroeconomic variables grow over time. Therefore, the more income available, the less restrictive is the budget constraint, so that consumption increases over time. In reaction to increasing disposable income, the subsistence consumption constraint vanishes. As this occurs, the economy moves to the second regime. Here, population grows at a steady rate (given in (30)) and income does not affect fertility. Thus fertility is only dependent on the quality-quantity trade-off. Elites again determine whether workers should provide education or not (see Lemma 2 and Proposition 1). Since industrial technology is growing and the share of workers is mostly in the industrial sector, the marginal gains from the technology effect at some point exceed the workers’ effect and the population effect. As this condition applies, the overcoming of the bequest effect, which is negatively affected by growing industrial technology, soon follows. At this point elites have an incentive to promote education, since they gain more from technology improvements than they lose from transfers of a share of their bequest and from population movements to the industrial sector. Besides the gains in technology resulting from education, the process of demographic transition is also an effect. As explained above, from the quality-quantity trade-off, the more educated children are, the lower is the number of offspring, which causes the fall on population growth rates.

The growth of the industrial sector and the posterior rise in education have a virtuous effect. Higher earnings of workers means more quality of children which will also lead to cumulative effects on increasing technology, and hence increasing productivity levels.
of workers in both sectors. This increases earnings, implying more income available for consumption and for raising children.

From simulations of the model, as the economy evolves, the main macroeconomic variables assume a constant pattern: while the population continues to grow at a slow rate, productivity in both sectors increases over time, with industrial productivity growing more than agricultural productivity. As for the share of workers, a shift of most of the population to the industrial sector is observed. Education increases over time but remains almost stable after the initial boost.

Now the interaction between the features referred to in the introduction can be fully understood. Firstly, it is demonstrated how the model behaves and how elites behave in their willingness to allow the provision of education for their children and consequently cause the demographic transition. Furthermore, the possible role of the Agricultural Revolution is discussed, i.e. how it can account for the onset and continuous process of industrialization and education.

From here we can draw the two main hypotheses advanced in this paper:

H1 - The emergence of education, and hence the demographic transition, depend on the decision of elites: they delay the emergence of education even after the onset of the Industrial Revolution;

H2 – The Agricultural Revolution has a positive effect on the emergence of education;

6.1. Model Calibration: Education and Demographic Transition

This section begins with the simulation of the model and its properties. Galor (2011), Lagerlöf (2006) and Voigtländer and Voth (2006) provide quantitative analyses of Unified Growth models. Their calibration of parameters will be closely followed.

Firstly, the functional forms for human capital and the cost of education function are specified to conform to Lemma 2. From (12):

\[ h_{t+1} = (1 + e_{t+1})^{\beta}, \]

with \( 0 < \beta < 1 \). It is an increasing, strictly concave function, of the investment in education, \( e_{t+1} \). The time endowment cost for each unit of education and for each child, \( g(\tau^e, T_t) \), is given by:

\[ g(\tau^e, T_t) = (\tau^e + T(t_t, b_t))^\phi, \]
where \(-1 < \phi < 0\) and \(T(t, b_t) = \frac{t_{b_t}}{1 + t_{b_t}}\) is an increasing, concave function, in \(t\) and \(b_t\). And so, \(g(\cdot)\) is decreasing in \(t\) and \(b_t\).

The baseline parameters are depicted in Table 1:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land share ((\alpha)/Labor share ((1 - \alpha))</td>
<td>0.4 / 0.6</td>
</tr>
<tr>
<td>Human Capital share ((\theta))</td>
<td>0.6</td>
</tr>
<tr>
<td>Land ((X))</td>
<td>1</td>
</tr>
<tr>
<td>Weight on children in utility function ((\gamma = \mu))</td>
<td>0.645</td>
</tr>
<tr>
<td>Fixed time cost of raising children ((\tau^r))</td>
<td>0.34</td>
</tr>
<tr>
<td>Time cost of educating children ((g(\tau^e)))</td>
<td>0.119</td>
</tr>
<tr>
<td>Subsistence consumption ((\bar{c}))</td>
<td>1</td>
</tr>
<tr>
<td>Human capital ((\beta))</td>
<td>0.35</td>
</tr>
<tr>
<td>Time endowment cost concavity ((\phi))</td>
<td>-0.9</td>
</tr>
<tr>
<td>Weight of population on agricultural “learning by doing effect” ((\varepsilon))</td>
<td>0.05</td>
</tr>
<tr>
<td>Weight of agricultural technology on agricultural “learning by doing effect” ((\delta)</td>
<td>0.07</td>
</tr>
<tr>
<td>Externality of industrial technology ((b))</td>
<td>0.80</td>
</tr>
<tr>
<td>Weight of population on industrial “learning by doing effect” ((\Delta))</td>
<td>0.05</td>
</tr>
<tr>
<td>Diminishing returns effect on [the] industrial dynamic path ((\zeta))</td>
<td>0.06</td>
</tr>
</tbody>
</table>

The initial conditions of the model are given by the equilibrium values for the pre-industrial period of \(A_0^A\) and \(L_0\) as well as for fertility, education, industrial productivity, share of workers and bequest (Table 2):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population ((L_0))</td>
<td>0.0861</td>
</tr>
<tr>
<td>Agricultural productivity ((A_0^A))</td>
<td>0.8726</td>
</tr>
<tr>
<td>Industrial productivity ((A_0^I))</td>
<td>0.6</td>
</tr>
<tr>
<td>Fertility ((n_0))</td>
<td>1</td>
</tr>
<tr>
<td>Education ((e_0))</td>
<td>0</td>
</tr>
<tr>
<td>Share of workers ((\lambda_0))</td>
<td>0</td>
</tr>
<tr>
<td>Bequest ((b_0))</td>
<td>1</td>
</tr>
</tbody>
</table>

Using these parameterization and initial values, the patterns of the economy’s behavior resembles the expected patterns observed in modern history. As depicted in Figure 1 and Figure 2, initially the economy is in a pre-industrial Malthusian equilibrium with
population, agricultural productivity and income per capita constant over time, while industrial productivity keeps increasing. From both figures, it is possible to observe the take-off of the industrial sector.

In Figure 1, the pre-industrial Malthusian regime vanishes after the beginning of the industrial phase, so that population growth rates are higher than before until the demographic transition takes place. Consistent with the empirical evidence, this occurs after about 100 periods, when education is permitted by elites and starts increasing over time. Prior to that, education levels were always zero. There is a clear path from the pre-industrial Malthusian regime, in which fertility depends on workers’ income and has a positive correlation with it, and where education is not provided, to a Modern Growth regime, under which fertility no longer depends on income, education emerges after elites allow for it, and the demographic transition begins. The Post Malthusian regime continues to form part of the transition process. Here, fertility is higher than in the previous pre-industrial era but still depends on income and education is still not provided. These features are shown in Figure 1.

![Figure 1: Quantitative analysis of calibrated model – education and fertility](image)

In Figure 2, after the pre-industrial Malthusian equilibrium, agricultural and industrial productivities increase continuously along with population. In this process both sectors’ productivities receive a boost due to the effect that education has on them. That is, agricultural productivity reaches a peak after education starts to be provided, with high growth rates in this period, while industrial productivity undergoes a small increase but keeps its ascending path almost constant. As time goes by, the industrial sector and
income per capita continue to grow at increasing rates, although agricultural productivity grows at a slower pace than industrial. Figure 2 presents these features:

![Figure 2: Quantitative analysis of calibrated model – productivity rates and income per capita](image)

From the analysis above it is possible to derive the following proposition.

**Proposition 2:** The aversion of the elite to education does not persist over time. The elites’ decision to support education by themselves occurs at some point in time. Elites may delay education but they do not prevent its growth indefinitely.

Proof: This follows from the numerical simulation of the model and can be derived from Lemma 4, Lemma 5, (22), and elites’ decision after the structural break, (32) and (33).

What must be borne in mind from this proposition is that, in contrast with many theoretical contributions (Mokyr, 1990; Bourguignon and Verdier, 2000; Acemoglu and Robinson, 2006; Galor *et al.*, 2009), it is possible that elites had an incentive to allow education to progress. Nevertheless, in line with those same theories, elites also had the power to prevent education emerging sooner, due to their own incentives. This means that although they decided to support education they only made that decision later in time, which is historically consistent with the delayed process of education: the Industrial Revolution took place in the late 1700s and education only spread in the mid to late 1800s (Flora *et al.*, 1983; Brown, 1991). This delay can be traced to the power and willingness of elites to support it. If, as is argued in the initial sections, elites were a small interest group which had decision-making powers over society and were the only group with sufficient economic resources to provide the means to educate people, they thus held the
power over whether to impede education or not. Hence, they had incentives to block 
education during industrial emergence, but also had incentives to support it afterwards, 
when it was economically beneficial to them. In line with the literature, it can be shown 
that elites are not always against education and industrial enhancement.

6.2. The Agricultural Revolution

The hypothesis advanced in this paper suggests that increasing technology in 
agriculture leads to the elites being more disposed to support education.

Higher levels of agricultural productivity mean higher initial rents being available for 
the elite during the process of industrialization. Although there is the bequest effect, 
which increases, the technology effect now becomes greater than the bequest effect since 
gains from externalities of industry boost an already higher level in the recipient 
technology. In other words, the higher level of agricultural technology enhances the effect 
of the industrial externalities on the technology of agriculture.

Moreover, besides the effects of the Agricultural Revolution, we should also debate 
on whether it really occurred, and if so, when it took place. We must voice our intuition 
that experiencing shocks relating to agricultural productivity would trigger a speedier 
positive decision of the elite as regards education, and exert a negative effect on the time 
of industrialization. It is possible to advance the following proposition:

**Proposition 3:** The Agricultural Revolution has a positive impact on elites’ decision 
to educate the population. The higher agricultural productivity during the process of 
industrialization, the more prone are the elites to support education.

Proof: It follows from the numerical simulation of the model and can be derived from 
Lemma 1, Lemma 4, Lemma 5, (22), elites’ decision after the structural break, (32) and 
(33). 

Using in the model a positive random shock with a uniform distribution to simulate an 
exogenous increase in agricultural technology, it is clear that higher shocks have a 
positive impact on the early onset of education. Overall, shocks in agriculture affect the 
decision of elites to support education as well as the take-off of the industrial sector with 
higher agricultural technology shocks.
Figure 3: Period of take-off of education for different scenarios of shocks in technology of agriculture

From the model, in a Malthusian equilibrium the gains of a single shock vanish over time - equilibrium is globally stable - and so there is no effective impact on the outcomes in the economy. However, if there are shocks in the economy, so that the level of population and agricultural technology increase consistently above the equilibrium levels at the time of take-off, this implies that there is an effect on rents as well as on bequests, and hence on the willingness of the elites to provide education. There is a virtuous cycle in the economy that will then imply a faster economic boost due to more education and therefore higher industrial and agricultural technology growth rates. This means that countries that underwent an Agricultural Revolution, mainly England and some continental countries benefited from an earlier take off of education and an earlier economic boom. The other countries lagged behind, which may have contributed to the divergence process in industrialization which was verified in this period (Galor, 2011).

From the ongoing debate, it is possible to argue that there must have been agricultural technology shocks in the 1700s, so that the take-off of the industrial sector and education took place earlier in countries such as England and the Netherlands. Shocks in the late 1600s or in the mid to late 1700s may not have had the necessary impact because they occurred too early. The argument followed here points to the fact that a consistent level of ongoing shocks during the 1700s was essential to the occurrence of a stronger industrial revolution and an early escape to Modern Economic Growth.
7. Concluding Remarks

The results presented in the previous sections show how interest groups have a role in determining the pace of the economy. These results show on the one hand the contribution of the Unified Growth Theory to the study of the Industrial Revolution, while on the other they demonstrate the current contribution of policy-makers to analyzing how developing economies face delays in their development processes due to these political forces. Substantial research has been carried out on the interconnections and willingness of landed elites to promote education. For the most part its conclusion is that elites’ interest was minimal (Bourguignon and Verdier, 2000; Galor et al., 2009). Nevertheless, the present paper shows that, with the right incentives, even landed elites ultimately agree with the promotion of education. This result does not negate other theories, but instead complements them. In fact, this paper aims to advance the proposition that, as regards the political process that set capitalists against landowners in the nineteenth century, due to the rising power of the former and the increasing willingness of the latter to allow education, both may have reached a consensus whereby capitalists demanded education and landlords did not oppose it. As for today, the lesson to learn is that it is important to be aware how interest groups react and what incentives they have, in order to intervene in the best manner possible and reach an agreement not only which is fully acceptable to each group, but also which benefits the population in general in terms of gains from education or any other element that may be a source of conflict between groups in the same society.

Furthermore, the paper contributes to a deeper understanding of the role of the Agricultural Revolution and whether there was a continuous sequence of technology shocks during the eighteenth century. This is a novelty and an important contribution to the literature. The conclusion that the Agricultural Revolution may have contributed to the early onset of the Industrial Revolution, and what is more, to a quicker process of education of the masses, is an important new contribution to the debate currently taking place in the literature. It may further help to explain why England developed earlier than other countries in Europe. As for soil fertility, it was possible to find this positive force underlying land endowments, which also confirms that there are positive effects deriving from land fertility rather than those commonly referred to in the literature, which tends to indicate a relationship between better and more poorly endowed countries whereby there is a reversal of fortunes. (Mokyr, 1990; Litina, 2012).
Finally, the numerical simulation presents the main insights of the model and shows the main conclusions referred to above. In line with the Unified Growth Theory it is possible to conclude that interest groups had a role to play in the main events during the period of industrialization. Given their decisions, the rise of education was initially halted until their objections were overcome, and as a consequence, the process of demographic transition occurred later in the nineteenth century. Furthermore, the occurrence of the Agricultural Revolution in the previous century positively influenced the onset of both industrialization and education.

References


33

Cubberley, E. P. (1920) *The History of Education: educational practice and progress considered as a phase of the development and spread of western civilization* Cambridge, Massachussets The Riverside Press.


Appendix

Proof Lemma 1:

If we equalize (6) and (7), using the assumption of perfect labor mobility, we know that workers are also employed in the industrial sector if the marginal productivity in the industrial sector $A_l$ is equal to or exceeds the marginal productivity in the agricultural sector.

Proof Lemma 2:

Take (16), (A1) and the properties of $g(t^e, T_t)$. We know that $E(0, T_t)$ is increasing in $T_t$. Also, the $\lim_{T \to \infty} E(0, T_t)$ is higher than $(0,0)$, so that from (A1) it is positive. Therefore, for $T_t > 0$, and from (6), $e_{t+1} > 0$. Also, from the Implicit Function Theorem and (6), we can show that $e_{t+1}$ is a single valued function of $T_t$ and $e_{t+1} = e_{t+1}(T_t)$ so that $e_{t+1}'(T_t) = -\frac{\partial E/\partial e}{\partial E/\partial T} > 0$.

Proof Lemma 3:

Since we are maximizing the utility we want the values of $t_t$ which for the interval [0,1] yield that maximum. It must be also added also that from the numerical simulations the function $G(.)$ is always decreasing in $t_t$. So,

When $\frac{du}{dt_t} \neq 0$ for the interval of $t_t \in [0,1]$;

If $\frac{du}{dt_t} > 0$ ⇒ $G(.) = \tilde{x}^t \frac{d\rho_{t+1}}{dt_t} - b_t > 0$ ⇒ $t_t = 1$

If $\frac{du}{dt_t} < 0$ ⇒ $G(.) = \tilde{x}^t \frac{d\rho_{t+1}}{dt_t} - b_t < 0$ ⇒ $t_t = 0$

Since $\frac{du}{dt_t}$ is a decreasing function, from numerical simulations, these are the only valid cases, and $\frac{du}{dt_t}|_{t_t = 0} < 0$ and $\frac{du}{dt_t}|_{t_t = 1} > 0$ does not apply.

After-structural-break implicit function and lemma 5 explanation:
$$G(.) = \frac{d\rho_{t+1}}{dt}$$

$$= \frac{\bar{\gamma}^i \alpha Y(1 - \alpha)^{1-a} (L_t)^{1+\epsilon-a} (A_t^t)^{\delta}}{\left[ (1 + e_{t+1}(A^t_t)^b)(L_t)^{\epsilon}(A_t^t)^{\delta} YX_{t+1}(1 - \alpha)^{1-a} + \theta \alpha(1 + h_t L_t^\delta) \xi A_t^\xi(1 + e_{t+1})^{\beta \alpha} \right]^{1-a}}$$

$$\left( \frac{de_{t+1}}{dt} n_t^{1-a} (A_t^t)^b \right) (1$$

$$- \bar{\gamma} \frac{Y(L_t)^{\epsilon}(A_t^t)^{\delta} X_{t+1}(1 - \alpha)^{1-a} + \theta \alpha(1 + h_t L_t^\delta) \xi A_t^\xi(1 + e_{t+1})^{\beta \alpha}}{Y(L_t)^{\epsilon}(A_t^t)^{\delta} X_{t+1}(1 - \alpha)^{1-a} + \theta \alpha(1 + h_t L_t^\delta) \xi A_t^\xi(1 + e_{t+1})^{\beta \alpha}}$$

$$- \alpha) \left( \frac{d(\cdot)}{dt} e_{t+1} + \frac{de_{t+1}}{dt} g(.) \right)$$

$$- b_t$$

Since $b_t$ is always positive, and the derivative with respect to $n_t$ is also always positive - these are the two latter parts of $G(.)$ - then, if Lemma 5’s condition is negative, $G(.) < 0$. So, only when Lemma 5’s condition is positive will we at some point have $G(.) \geq 0$.

More explicitly, as $\beta \frac{\theta}{\alpha}$ is constant, $\frac{1}{(1+e_{t+1})(A^t_t)^b} > \frac{1}{(1+e_{t+1})(A^t_t)^b} \iff 1 > (A^t_t)^b$ which happens only when the economy is almost rural. When the economy starts to industrialize and $(A^t_t)^b > 1$ then we can certainly guarantee that:

$$0$$

$$< (1$$

$$\frac{Y(L_t)^{\epsilon}(A_t^t)^{\delta} X_{t+1}(1 - \alpha)^{1-a} + \theta \alpha(1 + h_t L_t^\delta) \xi A_t^\xi(1 + e_{t+1})^{\beta \alpha}}{Y(L_t)^{\epsilon}(A_t^t)^{\delta} X_{t+1}(1 - \alpha)^{1-a} + \theta \alpha(1 + h_t L_t^\delta) \xi A_t^\xi(1 + e_{t+1})^{\beta \alpha}}$$

$$- \alpha) \left( \frac{d(\cdot)}{dt} e_{t+1} + \frac{de_{t+1}}{dt} g(.) \right)$$

$$< 1$$
And hence,

\[
\begin{pmatrix}
Y(L_t)^r(A_t^A)^s X_{t+1}(1-\alpha)^{\frac{1}{\alpha}} + \theta^s(1 + h_t L_t^2)^s A_t^s(1 + e_{t+1})^s \left( \frac{\beta^{\alpha} (1 + e_{t+1})^{\alpha}}{(1 + e_{t+1})^{\alpha}} \right)
\end{pmatrix} > 1
\]

So, for \(A_t^1\) to be sufficiently large we will have a positive condition. So, and at some point in time \(G(.) \geq 0\) - we will observe it in the numerical simulations.

**Globally Stable Steady State:**

Using Lemma 7 and Lemma 8, the pre-industrial steady-state values of productivity in the agricultural sector \(A_{ss}\), and the size of the working population \(L_{ss}\), are given by:

\[
A_{ss}^A = \left[ \frac{(1-\tau r)(1-\alpha)}{\tilde{c}} \right]^{\frac{\epsilon}{\alpha(1-\delta-\epsilon)}} \left[ \frac{1-\delta}{\alpha(1-\delta-\epsilon)} \right] \frac{1}{X^{1-\alpha(1-\delta-\epsilon)}},
\]

\[
L_{ss} = \left[ \frac{(1-\tau r)(1-\alpha)}{\tilde{c}} \right]^{\frac{1-\delta}{\alpha(1-\delta-\epsilon)}} \left[ \frac{1-\delta}{\alpha(1-\delta-\epsilon)} \right] \frac{1}{X^{1-\alpha(1-\delta-\epsilon)}},
\]

**Proof Lemma 10:**

Given the Jacobian matrix:

\[J(A_{ss}^A, L_{ss}) = \begin{bmatrix}
\frac{dA^A(A_{ss}^A, L_{ss})}{dA_t^A} & \frac{dA^A(A_{ss}^A, L_{ss})}{dL_t} \\
\frac{dL(A_{ss}^A, L_{ss})}{dA_t^A} & \frac{dL(A_{ss}^A, L_{ss})}{dL_t}
\end{bmatrix}\]

\[= \begin{bmatrix}
\delta & \epsilon \left[ \frac{(1-\tau r)(1-\alpha)}{\tilde{c}} \right]^{\frac{1}{\alpha}} \\
\frac{\alpha \tilde{c}}{1-\alpha} \left[ \frac{(1-\tau r)(1-\alpha)}{\tilde{c}} \right]^{\frac{1-\delta}{\alpha(1-\delta-\epsilon)}} & 1 - (1 + \alpha)(1-\tau r)
\end{bmatrix}\]

The eigenvalues are given by \(\{\lambda_1, \lambda_2\}\). We know that: \(det(A_{ss}^A, L_{ss}) = \lambda_1 \lambda_2\) and \(tr(A_{ss}^A, L_{ss}) = \lambda_1 + \lambda_2\)
\[ \text{tr}(A_{ss}^A, L_{ss}) = \delta + \frac{1-(1+\alpha)(1-\tau^r)}{\tau^r} > 0 \text{ for } (1 + \alpha)(1 - \tau^r) < 1 \iff \tau^r > \frac{\alpha}{1+\alpha} \]

\[ \text{det}(A_{ss}^A, L_{ss}) = \delta \frac{1 - (1 + \alpha)(1 - \tau^r)}{\tau^r} - \varepsilon \frac{1}{\tau^r} \left( \frac{(1 - \tau^r)(1 - \alpha)}{\bar{c}} \frac{\varepsilon}{\alpha(1-\delta-\varepsilon)} + \alpha(1-\tau^r) \right) \]

so that the equilibrium is globally stable if: \( \lambda_1, \lambda_2 \in (-1,1) \)

(1) To guarantee that the convergence to the steady state is monotonically stable:

i. \( \text{Det}(A_{ss}^A, L_{ss}) > 0; \)

ii. and \( \text{Tr}(A_{ss}^A, L_{ss}) > 0. \)

For (i):

\[ \delta \frac{1 - (1 + \alpha)(1 - \tau^r)}{\tau^r} - \varepsilon \frac{1}{\tau^r} \left( \frac{(1 - \tau^r)(1 - \alpha)}{\bar{c}} \frac{\varepsilon}{\alpha(1-\delta-\varepsilon)} + \alpha(1-\tau^r) \right) > 0 \]

\[ \iff \delta(1 - (1 + \alpha)(1 - \tau^r)) > \varepsilon \left( \frac{(1 - \tau^r)(1 - \alpha)}{\bar{c}} \frac{\varepsilon}{\alpha(1-\delta-\varepsilon)} + \alpha(1-\tau^r) \right) \]

(condition 1)

For (ii): \( \text{Tr}(A_{ss}^A, L_{ss}) \) always higher than zero from the above inequality

(2) To guarantee that the equilibrium is globally stable:

i. \(-2 < \text{Tr}(A_{ss}^A, L_{ss}) < 2; \)

ii. \(-1 < \text{Det}(A_{ss}^A, L_{ss}) < 1; \)

iii. \( \text{Det}(A_{ss}^A, L_{ss}) - \text{Tr}(A_{ss}^A, L_{ss}) \geq -1; \)

iv. and \( \text{Det}(A_{ss}^A, L_{ss}) + \text{Tr}(A_{ss}^A, L_{ss}) \geq -1. \)

For (i): from before we know that \( \text{Tr}(A_{ss}^A, L_{ss}) > 0 > -2 \)
\[ \text{Tr}(A_{ss}^A, L_{ss}) < 2 \]

\[ \Rightarrow \delta + \frac{1-(1+\alpha)(1-\tau^r)}{\tau^r} < 2 \Leftrightarrow 1 - (1 + \alpha)(1 - \tau^r) < (2 - \delta)\tau^r \Leftrightarrow \tau^r(1 - \delta - \alpha) + 1 + \alpha > 1 \quad \text{P.V.} \]

\[ \Rightarrow \text{Tr}(A_{ss}^A, L_{ss}) \in (-2,2) \]

For (ii):

\[ \text{Det}(A_{ss}^A, L_{ss}) > -1: \]

\[ \delta \frac{1-(1+\alpha)(1-\tau^r)}{\tau^r} - \varepsilon \frac{1}{\tau^r} \left( \frac{(1-\tau^r)(1-\alpha)}{c} \frac{\varepsilon}{\alpha(1-\delta-\varepsilon)} + \alpha(1-\tau^r) \right) > -1 \quad \text{P.V.} \]

From condition 1 we know this inequality holds.

\[ \text{Det}(A_{ss}^A, L_{ss}) < 1: \text{ (by contradiction)} \]

\[ \delta \frac{1-(1+\alpha)(1-\tau^r)}{\tau^r} - \varepsilon \frac{1}{\tau^r} \left( \frac{(1-\tau^r)(1-\alpha)}{c} \frac{\varepsilon}{\alpha(1-\delta-\varepsilon)} + \alpha(1-\tau^r) \right) > 1 \]

\[ \Leftrightarrow (\delta - 1)\tau^r - \alpha(1 - \tau^r) > \varepsilon \left( \frac{(1-\tau^r)(1-\alpha)}{c} \frac{\varepsilon}{\alpha(1-\delta-\varepsilon)} + \alpha(1-\tau^r) \right) \Rightarrow \text{P.F.} \Rightarrow \]

\[ \text{Det}(A_{ss}^A, L_{ss}) < 1 \]

For (iii):

\[ \delta \frac{1-(1+\alpha)(1-\tau^r)}{\tau^r} - \varepsilon \frac{1}{\tau^r} \left( \frac{(1-\tau^r)(1-\alpha)}{c} \frac{\varepsilon}{\alpha(1-\delta-\varepsilon)} + \alpha(1-\tau^r) \right) - \delta \]

\[ - \frac{1-(1+\alpha)(1-\tau^r)}{\tau^r} \geq -1 \]

\[ \Leftrightarrow \alpha(1 - \delta)(1 - \tau^r) \geq \varepsilon \left( \frac{(1-\tau^r)(1-\alpha)}{c} \frac{\varepsilon}{\alpha(1-\delta-\varepsilon)} + \alpha(1-\tau^r) \right) \]

(condition 2)
For (iv):

$$
\delta \frac{1 - (1 + \alpha)(1 - \tau^r)}{\tau^r} - \epsilon \frac{1}{\tau^r} \left( \frac{(1 - \tau^r)(1 - \alpha)}{\tilde{c}} \right) \frac{\epsilon}{\alpha(1 - \delta - \epsilon)} + \alpha(1 - \tau^r) + \delta \\
+ \frac{1 - (1 + \alpha)(1 - \tau^r)}{\tau^r} \geq -1
$$

If condition 2 holds, then, since the trace is positive, this inequality will also hold.

**Proof proposition 3:**

To ascertain the impact of agricultural technology on the decisions of elites we need to apply the Implicit Function Theorem in (21) so that we can derive the impact of $A_t^A$ on $t_t$. Computing $\frac{dG}{dt_t}$ and $\frac{dG}{dA_t^A}$ and since it is not possible to analytically reach a definite signal we must use a numerical simulation. What we find is that $\frac{dG}{dt_t} < 0$ for the entire period of time, whereas $\frac{dG}{dA_t^A}$ is oscillatory. Nevertheless, for the period previous to the growth in education $\frac{dG}{dA_t^A} < 0$. Therefore:

$$
\frac{dt_t}{dA_t^A} = -\frac{dG}{dt_t} \begin{cases} < 0 \quad & \text{before onset of Industrial Revolution} \\
> 0 \quad & \text{after onset of Industrial Revolution}
\end{cases}
$$

Therefore, we can infer that the higher the value of $A_t^A$ on the onset of industrialization the more likely is it that this onset will follow sooner. Consequently, previous improvements in agriculture (during the eighteenth century) exert a positive influence on the early rise in education.