Contingent payment mechanisms and entrepreneurial financing decisions

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CONTINGENT PAYMENT MECHANISMS AND
ENTREPRENEURIAL FINANCING DECISIONS

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Abstract

We discuss how Contingent Payment Mechanisms (also known as Contingent Earn-Outs) enable of Entrepreneurial Financing decisions. First, we introduce a taxonomy of contingent payment mechanisms, by combining features regarding their term and amount. Second, we introduce each of these alternative mechanisms on a previously developed real options framework for analyzing Entrepreneurial Financing decisions, in which one wealth constrained Entrepreneur is looking for an external equity provider – taken as a Venture Capitalist – to support a given growth strategy. We conclude that different contingent payment mechanisms are equivalent in obtaining joint support from Entrepreneurs and Venture Capitalists regarding optimum investment timing and, therefore, that the choice on the optimum mechanism to use depends on variables which are exogenous to the model, such as liquidity preferences or constraints, timing requirements, post-deal integration or overall deal terms.

Keywords: Venture Capital, Entrepreneurial Finance, Real Options, Growth Options, Entrepreneurship, Earn-Outs, Contingent Payments

JEL Codes: G24, G31, G34, L26, M13

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1. INTRODUCTION

Entrepreneurial Financing decisions cover a distinct range of financial and non-financial terms that Entrepreneurs and Venture Capitalists (VCs) negotiate, which should as a whole trigger their willingness to forego firm ownership, provide funds to support a given growth strategy or get access to a range of financial and managerial skills (Croce et al., 2013; Hsu, 2004; Kaplan and Strömberg, 2001).

From a financial perspective, and leaving aside post-deal compensation and interest alignment mechanisms which may be set between the parties, Entrepreneurial Financing decisions usually involve discussions on valuing the Entrepreneurial Firm, and on how parties will split firm ownership. Considering how uncertainty surrounds the prospects of Entrepreneurial Firms, parties may choose to solve discussions on valuation and firm ownership by engaging into an up-front share or cash premium (or discount) or, alternatively, by setting a deferred and Contingent Payment Mechanism (CPM) subject to a given performance benchmark or strategic milestone of the Entrepreneurial Firm.

While up-front cash and shares largely dominate as deal currency mechanisms, accounting for 80.8% and 24.9% of the total transactions that took place between 2000 and 30th June, 2015 according to Zephyr3, deals involving CPMs – also known on the literature as Earn-Outs or Contingent Earn-Outs – stood for 7.9% of total deal volumes during this period, and are being increasingly used, standing for 11.3% of total deal volumes from January to June, 2015 against 4.5% in 2000. CPMs seem to be more popular in industries especially reliant on intangible assets, such as “Computer, IT and Internet Services” (where 14.2% of deal volumes between 2000 and 30th June, 2015 used CPMs) or “Biotechnology, Pharmaceuticals and Life Sciences” (11.4%), or

2 Bear in mind that one given deal may have more than one deal currency mechanism (for example, a combination of cash and shares). Therefore, summing up the share of different deal currency mechanisms on total deal volumes leads to over 100.0%.

3 Estimates based on a sample extracted from Zephyr comprising completed deals from 1st January, 2000 to 30th June, 2015, including acquisitions, institutional buy-outs, capital increases, management buy-ins, management and buy-outs, involving targets located in the Baltic States, Eastern Europe, North America, Oceania, Scandinavia, and Western Europe and acquisitions with, at least, 15.0% stakes on target firms, totaling 331,419 transactions. Deals for which no payment terms are available are excluded from the statistics of deal payment terms mentioned throughout the paper.
industries featuring significant volatility on cash flow generation, such as “Construction” (10.3%). As they introduce additional complexity on deal terms, CPMs are more popular on professional investors, such as deals involving Private Equity or VCs divestment (12.6%) or on deals involving Sovereign Wealth Funds (6.7%). In addition, CPMs are more frequent on smaller deals in terms of deal value, as only 23.7% of the deals including CPMs involve deal values on the top quartile of our sample (i.e., deals above € 34 M).

Such evidence on CPMs is broadly consistent with previous literature findings. Cain et al. (2011) posited that higher contingent payments are observed when targets possess high growth opportunities and are exposed to greater uncertainty, while Datar et al. (2001) found that acquisitions of high technology, service intensive or small private companies are more prone to use CPMs. Barbopoulos and Sudarsanam (2012) points out that contingent payments are more likely in the “Media and Entertainment”, “Consumer Products”, “High Technology”, “Healthcare” and “Telecommunications” industries, which hold large intangible assets, and are surrounded by greater volatility on their cash flow generation, taking prospective bidders to higher value at risk alongside information asymmetries.

Overall, both literature and empirical findings support the idea that CPMs should be particularly relevant within an Entrepreneurial Financing context, in which Entrepreneurial Firms also face valuable growth opportunities, and major uncertainties on future cash flow generation and business prospects.

In spite of such conceptual argument, literature on decision-making models for Entrepreneurial Financing decisions involving CPMs – or, even more broadly, within a Mergers and Acquisitions (M&A) context – is scarce (Lukas et al., 2012) and has not still, at the moment, comparatively discussed the design of different types of CPMs. Therefore, we expect to provide a contribution to fill this gap, by presenting a taxonomy for classifying different CPMs based on their payment term and amount, by introducing an options-based approach to value each of the four major different CPMs we identified, and by demonstrating how the key terms on each of the four major CPMs should be computed so that Entrepreneurs and VCs would be jointly willing to support a

\[4^{a}\text{ Industry taxonomy as provided by Zephyr database according to Zephus classification.}\]
given Entrepreneurial Firm and its growth strategy. With this purpose, we introduce a real options based framework for analysing Entrepreneurial Financing decisions.

This paper is structured as follows. In Section 2 we go through the existing literature on the topic. In Section 3, we propose a taxonomy for classifying CPMs. In Section 4 we value each of the four major CPMs we identified and derive optimum CPM terms that enable Entrepreneurs and VCs to jointly support a given Entrepreneurial Firm with a growth opportunity. In Section 5 we illustrate the different CPMs with a numerical example. In Section 6 we discuss our findings and conclude in Section 7.

2. LITERATURE REVIEW

The concept of Contingent Payment Mechanisms is often defined on the literature. For example, Bruner and Stiegler (2014) highlight that CPMs are “contingent on achievement of financial or other performance targets after the deal close”5, while Reuer et al. (2004) underline that they are “deferred variable payments (...) within a certain time frame after the deal has been consummated”6. We understand that a more general definition is required for a better understanding of CPMs, as these (i) may either comprise fixed or variable amounts (for example, when it equals a given multiple over revenues, EBITDA or profits above a certain threshold, measured at a given date), (ii) may either be paid on a pre-determined date or at any date within a given period (i.e., when a certain financial or business milestone is met) or (iii) may either require or not require a pre-specified goal to be met (for example, when a contingent payment equals a given multiple of all incremental EBITDA generated post-deal against the one on the deal completion accounts). Therefore, we follow the more general approach introduced by Datar et al. (2001) in which CPMs are defined as “a method of acquisition where the final consideration received by the seller is based on the future performance of his business”7.

5 Bruner and Stiegler (2014) define an Earn-Out as “an arrangement under which a portion of the purchase price in an acquisition is contingent on achievement of financial or other performance targets after the deal closes”.

6 Reuer et al. (2004) define Contingent Earn-Outs as “deferred variable payments tied to the target's ability to meet pre-specified performance goals within a certain time frame after the deal has been consummated”.

7 Usually CPMs do not require vendors to return part of the initial consideration to acquirers whether a certain future performance is not achieved by the target firm. This would lead to the introduction of “Contingent Earn-Ins”, which
Some general insights on the pros and cons of CPMs from the M&A literature are extendable to an Entrepreneurial Financing context, even though, differently from an M&A process, Entrepreneurial Financing decisions do not involve a sale and purchase agreement of part of the whole firm ownership\(^8\).

On the one hand, CPMs may reduce the risk of adverse selection and overpayment on the existence of private information on the business of the target firm (Datar et al., 2001; Kohers and Ang, 2000; Ragozzino and Reuer, 2009) or, conversely, reduce the risk of inverse adverse selection and underpayment, where bidders are potentially more informed than vendors, by allowing the latter to benefit from post-deal value creation (Ragozzino and Reuer, 2009). In addition, CPMs mitigate potential moral hazard risks on the post-deal stage, by providing incentives for vendors and/or target management to adjust their behavior with the purpose of maximizing the probability of obtaining a contingent payment in the future (Kohers and Ang, 2000; Krug and Hegarty, 2001). Finally, CPMs may also be regarded as a financial leverage enhancer, by providing the acquirer with an option to fund total deal consideration with the underlying cash flow of the target firm through a deferred payment, or by providing the acquirer with the benefits of a staged investment process, given that it may be required to commit additional capital to support the growth opportunities of the target firm (Del Roccili and Fuhr Jr, 2001).

On the other hand, CPMs may incentivize acquirers, vendors or target managers to influence the performance of the target firm with the purpose of maximizing or minimizing the amount of the future contingent payment, and therefore influencing the firm towards short-term rather than long-term goals (Lukas et al., 2012). Moreover, CPMs may introduce complexities on performance measurement, which may slow down post-deal integration and value creation effects, and may consequently introduce significant contracting and monitoring costs (Caselli et al., 2006; Datar et al., 2001). Barbopoulos and Sudarsanam (2012) argues that CPMs do not, in fact, provide a superior benefit against stock offers, since these may present similar contingency and value mitigation

\(^{8}\) In Entrepreneurial Financing decisions, an outside investor financially supports a given growth opportunity held by an Entrepreneurial Firm, through an equity round. The Entrepreneurial Firm is in turn owned by a wealth constrained shareholder (or set of shareholders), taken as the Entrepreneur(s), who is not able to provide the Entrepreneurial Firm with all the necessary financial resources to execute its growth strategy.
characteristics to CPMs, especially when entities have comparable sizes, a comparable contribution to post-deal value creation or when the stock of the acquirer is publicly traded and therefore allows the vendor to easily transform her or his ownership in cash. In fact, and by making use of a logistic model, these authors empirically investigated, concluding that acquisitions of privately owned firms or of subsidiaries of public firms are more prone to involve contingent payments. Lastly, one may also argue that, specifically within an Entrepreneurial Financing context, the risk-return profile of Entrepreneurial Firms, where failure rates are high, may advise investors not to reduce their potential upsides on the few successful Entrepreneurial Firms they support through CPMs or other similar mechanisms.

Empirical research on designing CPMs reveals that contingent payments may stand from 15.0% to 80.0% of total deal consideration (Bruner and Stieglir, 2014), with an average of 33.0% according to Cain et al. (2011) and acquisitions involving privately owned firms recording a 44.0% higher average contingent payment (Kohers and Ang, 2000). The CPM period ranges from one month to twenty years, with an average of 2.57 years (Cain et al., 2011), but more frequently laying on the two to five years range (Kohers and Ang, 2000).

In general, empirical research on acquirer stock returns provides support to a more widespread use of CPMs, at least from the bidder perspective. Supporting evidence includes Kohers and Ang (2000), who revealed that acquirers using CPMs recorded an abnormal return of 1.356% on the date of announcement against those that did not employ CPMs, on a sample comprising 938 deals with 82.1% Anglo-American bidders. Lukas and Heimann (2014) recorded an average 1.439% abnormal return at the date of announcement, and an average abnormal return on a three days window around the announcement date of 2.036%, in a sample exclusively involving deals in Germany. On a sample of bids announced by UK firms, Barbopoulos and Sudarsanam (2012) found that overall earn-out bids yield significantly higher returns than non earn-out bids (i.e., 1.48% against 1.07%) on a two days window around the announcement date, and that the benefits of optimal CPM use are exhausted by the second year after deal completion. In turn, unclear evidence is presented by Mantecon (2009), who found an average cumulative return of 1.01% for a three days window around the announcement date with a sample involving 2/3 of Anglo-American bidders, but weekly positive for domestic transactions and even insignificant for cross-border deals.
From an analytical point-of-view, several authors argue the existence of an analogy between CPMs and real options (Bruner and Stiegler, 2014; Caselli et al., 2006; Lukas et al., 2012). Even though contingent payments do not hold an optionality feature, they provide payoffs that mirror those of real options and might be specified as call options, as argued by Bruner and Stiegler (2014).

<table>
<thead>
<tr>
<th>Call Options on Common Stock</th>
<th>CPMs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Underlying Asset</strong></td>
<td>Shares of common stock</td>
</tr>
<tr>
<td><strong>Exercise Price</strong></td>
<td>The stated strike price of the options contract</td>
</tr>
<tr>
<td><strong>Price of the Underlying Asset</strong></td>
<td>Share price of the underlying common stock</td>
</tr>
<tr>
<td><strong>Interim Payouts</strong></td>
<td>Dividends</td>
</tr>
<tr>
<td><strong>Term of the Option</strong></td>
<td>On a pre-specified date, typically from 3 to 9 months from original issue</td>
</tr>
<tr>
<td><strong>Uncertainty</strong></td>
<td>Volatility of returns on the underlying asset</td>
</tr>
</tbody>
</table>

*Table 1. Comparison of CPMs and Call Options on Stock – adapted from Bruner and Stiegler (2014)*

Notwithstanding, there are only a few analytical papers discussing how CPMs may drive acquisitions.

Lukas et al. (2012) took a two-stage game-option approach to CPMs to examine the impact of uncertainty and of contingent payment terms on optimal M&A timing. The resulting model allowed the authors to specify a set of three empirically testable hypothesis, regarding Earn-Out ratios,
Earn-Outs premiums\textsuperscript{9} and initial deal consideration. In particular, uncertainty is argued to positively drive the initial deal consideration, the Earn-Out premium and the Earn-Out ratio, while the Earn-Out period negatively influences the initial payment and positively affect the Earn-Out premium and the Earn-Out ratio. In turn, higher performance benchmarks positively drive initial payments and negatively drive both Earn-Out premium and ratio. On this model, the contingent payment is fixed and paid at a given pre-determined date.

Lukas and Heimann (2014) also derived a set of testable empirical propositions on CPMs through a theoretical model set for a M&A context featuring information asymmetries. Grounded on a classic principal-agent model, the authors conceived a utility model in which the target firm envisages no uncertainty on product launch, while the bidder computes expected target performance through a uniform distribution. Model outputs reveal that CPMs increase the utility of buyers, by transferring some of the acquisition risk to vendors, especially when the volatility of cash flow generation of the target firm increases. Shorter Earn-Out periods are also argued to increase the utility of buyers and extremely high levels of information asymmetry may actually impede deals from taking place, by lowering the utility of both bidders and vendors. Overall, in the absence of technological information asymmetry, as CPMs allow buyers to improve their utility, such deal currency should be preferred over a classical lump sum. On this setting, the contingent payment equals a fraction (from 0.0 to 1.0) of the cash flow generated by the target firm on a pre-specified date.

Choi (2015) developed a two stage game-theoretic model for an M&A context with the purpose of addressing the question of how the post-closing stage influences the design of optimal CPMs. The authors drafted two different settings (one in which vendors holds private information, and one where bidders and sellers hold different expectations on future profit generation by the target firm) and showed that CPMs will be structured with the purpose of minimizing the deadweight loss resulting from a smaller incentive component and that, when there is a small valuation gap between acquirer and vendors, parties may actually forego from using a CPM. Similarly to Lukas et al. (2012), the contingent payment is also fixed and paid at a given pre-determined date on this model.

\textsuperscript{9}“Earn-Out Premium” is defined as the amount of the contingent payment itself that was set by the parties, while the “Earn-Out Ratio” is defined as the ratio of all contingent payments in relation to the maximum price paid.
Overall, the literature reveals that there is still room for progressing analytical research on CPMs in two different ways. Firstly, by cataloguing and valuing alternative designs for CPMs, and secondly by expanding such analytical tools to an Entrepreneurial Financing context. We intend to address these two issues on the coming sections.

3. A TAXONOMY FOR CPMs

Recent news on M&A deals reveal that CPMs may present several distinctive features. For example, on the recent acquisition of GlaxoSmithKline’s oncology products unit by Novartis, completed on March, 2015, whose total consideration amounted to $16.0 B in cash, Zephyr reported that up to $1.50 B are contingent on the results of the Combi-D trial, a Phase III study evaluating the safety and efficacy of the combination of Tafinlar and Mekinist against BRAF monotherapy. Differently, on the acquisition of the Portuguese assets of Portugal Telecom from Oi, which was completed on June, 2015, Altice offered a total consideration of €7.03 B, including €800 M contingent on revenue milestones being met. Moreover, on the acquisition of the price comparison site uSwitch that was completed on May, 2015, the property search portal Zoopla offered a total consideration of £130.00 M, including a “potential payment of up to £30.00 M for uSwitch’s management dependent on achievement of certain financial performance targets for fiscal 2016”, according to Zephyr.

Such cases illustrate that different types of CPMs should be put in place for different circumstances. On the sale of the oncology business unit held by GlaxoSmithKline to Novartis, a variable payment up to $1.50 B was put in place contingent on the accomplishment of a relevant research and development initiative, which is due when the results of such ongoing research are known. On the sale of the Portuguese assets held by Portugal Telecom to Altice, one might argue that the €800 M contingent payment served as a mechanism for narrowing a potential valuation gap. Finally, on the acquisition uSwitch by Zoopla, the contingent payment is explicitly aimed at providing an incentive to uSwitch’s management, who also sold their stakes to Zoopla, to achieve certain financial targets.

We propose a simple, but still not exhaustive, taxonomy for CPMs, which is aimed to cover the most common cases and which is essentially defined by the key financial terms of a contingent payment: its amount and its due date. On the one hand, we understand that the amount of the
contingent payment might be *fixed*, i.e., irrespective of the completion rate of the performance benchmark that triggers the contingent payment (e.g., a contingent payment of € 1 M if revenues by the end of the first twelve months after deal completion reach or surpass € 5 M) or *variable*, i.e., dependent on the completion rate of the performance benchmark (i.e., a given multiple on the excess revenue between the first twelve months after deal completion and the last twelve months prior to deal completion). On the other hand, we understand that contingent payments might be due at the *term* of the CPM period (as in the case for uSwitch, i.e., subject to the performance of the 2016 fiscal year) or at *hit* (as in the case for GlaxoSmithKleine), i.e., at the moment in which the performance benchmark is achieved\(^{10}\). Our taxonomy is then summarized on the table below.

<table>
<thead>
<tr>
<th>Due Date</th>
<th>Amount</th>
<th>Due Date</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>At Hit</td>
<td>Variable at Hit CPM</td>
<td>At Term</td>
<td>Variable at Term CPM</td>
</tr>
<tr>
<td>At Term</td>
<td>Fixed at Hit CPM</td>
<td>At Term</td>
<td>Fixed at Term CPM</td>
</tr>
</tbody>
</table>

*Table 2. A Taxonomy for CPMs*

We understand that these two CPM segmentation variables pose different challenges from a valuation and analytical perspective – as we will depict in Section 4 – and primarily serve two different purposes within the context of an M&A or Entrepreneurial Financing process.

The *due date* essentially addresses valuation gaps when parties set the contingent payment to be due at *term*, as at the moment of deal completion they may diverge on expectations regarding future

\(^{10}\) Notwithstanding, we argue that even when due at *hit*, CPMs require parties to define a given time period under which they allow the contingent payment to take place.
performance of the target firm within a specific time-frame, yielding a direct impact on settling an agreement regarding firm valuation. In this setting, parties may “agree to disagree” and wait for time to resolve their gaps on firm performance and firm valuation (Kohers and Ang, 2000). When set at *hit*, CPMs foremost privilege the provision of incentives to vendors or target management to pursue a certain goal. Alternatively, CPMs which are due at *hit* may also be suitable when the moment in which the attainment of given milestone or performance benchmark does not depend on the willingness or effort of the parties to pursue that goal (e.g., a license to operate a given plant, to which all the requirements were fulfilled by the target firm, and whose issuance is currently pending by public authorities).

The *amount* primarily addresses primarily the perception of parties regarding the attainment of the contingent payment triggers and their willingness to benefit (or lose) from potential performance upsides or downsides. When they are set in a way in which they do not replicate *fixed* contingent payments¹¹, *variable* contingent payments may allow vendors to start profiting from the contingent payment when lower levels of performance are achieved by the firm (i.e., minimizing the downside risk, if a CPM is put in place), and to exceed the payoff of a *fixed* contingent payment, when performance benchmarks are beaten by far (i.e., maximizing the upside potential, if a CPM is put in place).

Other potential variables for classifying CPMs are grounded on the underlying asset (for example, whether the CPM is based on a financial or on an operating measure or event, such as obtaining a pending license), on the underlying method of payment (for example, cash, shares of the acquiring firm or shares of the target firm), on the number of contingent payments to take place (for example, parties can set one single contingent payment, or a set of contingent payments to take place throughout several years), or on the different types of performance benchmarks – fixed, moving or cumulative – as proposed by Reum and Steele (1970).

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¹¹ For example, when a contingent payment, due on a pre-determined moment of time, equals a given multiple on the revenues in excess of a given performance benchmark, there is no difference between variable and fixed contingent payments.
4. CPMs AND ENTREPRENEURIAL FINANCING DECISIONS

In this Section, we introduce an analytical framework to support decision-making and determine investment timing in Entrepreneurial Financing processes, with the purpose of establishing the grounds for investigating how CPMs may influence their outcomes. Therefore, we first briefly present the real options model which we will make use throughout the paper and show how CPMs might be valued and introduced on such framework. We then derive the optimum investment timing conditions that allow Entrepreneurs and VCs to jointly and simultaneously support the execution of a given growth strategy by an Entrepreneurial Firm.

4.1. A real options framework for Entrepreneurial Financing decisions with CPMs

The setting comprises one Entrepreneurial Firm, owned by a single Entrepreneur, which generates positive profits and holds a growth opportunity, defined by an expansion of its current profit flow (named as $e_{\text{EXP}} > 1$) and a given capital expenditure (named as $k > 0$). Assuming that neither the Entrepreneur nor the Entrepreneurial Firm have access to debt financing, such capital expenditure should be funded through an equity round backed by the Entrepreneur, who is assumed to own limited resources, and by an external financier, who is assumed to be a VC with no funding constraints. VCs are then assumed to provide the part of the required equity that the Entrepreneur is not able to provide.

In this setting, $k^i > 0$ stands for the amount of capital initially by the Entrepreneur on the Entrepreneurial Firm, $k^a < k$ stands for the amount of additional capital that the Entrepreneur is willing to deploy on the Entrepreneurial Firm, $k > 0$ is the amount of the total capital expenditure required for executing the growth strategy and $(k - k^a)$ is the amount of capital to be deployed by the VC on the Entrepreneurial Firm. Parties are assumed to split firm ownership after carrying the equity round according to the amount of capital that each of the parties contributed to the Entrepreneurial Firm. As a result, post-equity round firm ownership held by the Entrepreneur is denoted by $0 < Q^E < 1$ and $Q^E = \frac{k^i + k^a}{k^i + k}$, while post-equity round firm ownership held by the VC is denoted by $0 < Q^{VC} < 1$ and $Q^{VC} = \frac{k - k^a}{k^i + k} = 1 - Q^E$.

The Entrepreneurial Firm generates a continuous-time profit flow ($\pi$), which is assumed to follow a Geometric Brownian Motion (GBM) diffusion process given by:
\[ d\pi = \alpha \pi dt + \sigma \pi dz \]  
(1)

where \( \pi > 0, \alpha \) and \( \sigma \) stand for the trend parameter (i.e., the drift) and to the instantaneous volatility, respectively. Additionally, assuming that agents are risk neutral, \( \alpha = r - \delta \), where \( r > 0 \) is the risk-free rate and \( \delta > 0 \) stands for the asset yield. Finally, \( dz \) is the increment of a Wiener process. Entrepreneurs and VCs are assumed to understand that the continuous profit flow (\( \pi \)) follows the same stochastic process.

4.1.1. The option to invest on the growth opportunity held by the Entrepreneur

Following the contingent-claim approached used by Dixit and Pindyck (1994), the value of the option held by the Entrepreneur to invest in the growth opportunity of the Entrepreneurial Firm, \( E(\pi) \), must satisfy the following ordinary differential equation (ODE):

\[ \frac{1}{2} \sigma^2 \pi^2 E''(\pi) + (r - \delta)\pi E'(\pi) - r E(\pi) + \pi = 0 \]  
(2)

where the last term on the left hand side of equation (2) refers to the current profit flow of the Entrepreneurial Firm and the remaining terms refer to the growth option held by the Entrepreneurial Firm. The general solution for (2) comes:

\[ E(\pi) = A\pi^{\beta_1} + B\pi^{\beta_2} + \frac{\pi}{\delta} \]  
(3)

where \( A \) and \( B \) are constants to be determined, while \( \beta_1 \) and \( \beta_2 \) are the roots of the fundamental quadratic, given by:

\[ Q_E(\beta) = \frac{1}{2} \sigma^2 \beta (\beta - 1) + (r - \delta)\beta - r = 0, \]  
(4)

i.e.

\[ \beta_1 = \frac{1}{2} - \frac{(r - \delta)}{\sigma^2} + \sqrt{\left(\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \]  
(5)

\[ \beta_2 = \frac{1}{2} - \frac{(r - \delta)}{\sigma^2} - \sqrt{\left(\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \]  
(6)
Assuming that $\pi^*_E$ stands for the optimal profit trigger to obtain Entrepreneur’s support to the growth opportunity, and considering that, in order to execute the growth strategy, $Q^E < 100\%$, and naming $CPM(\pi)$ as the contingent payment mechanism set between parties, the problem must be solved by considering the following boundary conditions:

$$E(0) = 0 \quad (7)$$

$$E(\pi^*_E) = \frac{e^{\text{EXP} \cdot \pi^*_E \cdot \delta}}{\delta} \cdot Q^E - k^a + CPM(\pi^*_E) \quad (8)$$

$$E'(\pi^*_E) = \frac{e^{\text{EXP} \cdot \delta}}{\delta} \cdot Q^E + CPM'(\pi^*_E) \quad (9)$$

Respecting condition (7) and noting that $\beta_2 < 0$, then B on the equation (3) must be equal to zero. Therefore, for the remaining of this paper, $\beta \equiv \beta_1$. The unknowns A and $\pi^*_E$ are obtained by combining conditions (8) and (9), i.e., the value matching and the smooth pasting conditions, respectively. Notice that the value matching condition held by the Entrepreneur is positively influenced by the CPM that was set between the parties. Solutions for the optimal profit trigger and for the option to invest on the growth opportunity depend on the specification defined for $CPM(\pi)$, and shall be presented on the following sub-sections for each of four major different types of CPMs we introduced in Section 3.

**4.1.2. The option to invest on the growth opportunity held by the VC**

The value of the option to invest on the growth opportunity held by the VC, given by $VC(\pi)$, should also satisfy an ODE, as shown in equation (10) below. However, unlike the Entrepreneur, this option does not include the current profit flow $\pi$ of the Entrepreneurial Firm, as VCs can only profit by undertaking the growth opportunity, and not from existing firm profitability, when they decide not to participate in this growth strategy.

$$\frac{1}{2} \sigma^2 \pi^2 \cdot VC''(\pi) + (r - \delta) \pi \cdot VC'(\pi) - r \cdot VC(\pi) = 0, \quad (10)$$

The general solution for (10) is:

$$VC(\pi) = C\pi^{\beta_1} + D\pi^{\beta_2}, \quad (11)$$
where C and D are constants to be determined, while $\beta_1$ and $\beta_2$ are the roots of the fundamental quadratic, as presented on equation (4). Similarly to the Entrepreneur, the boundary conditions are as follows:

\[
VC(0) = 0 
\]

\[
VC(\pi^*_V) = \frac{e^{\text{EXP} \cdot \pi^*_V}}{\delta} \cdot Q^{VC} - (k - k^a) - CPM(\pi^*_V) \tag{13}
\]

\[
VC'(\pi^*_V) = \frac{e^{\text{EXP}}}{\delta} \cdot Q^{VC} - CPM'(\pi^*_V). \tag{14}
\]

where $\pi^*_V$ stands for the optimal profit trigger to support the growth strategy for the VC firm. Respecting condition (12) and noting that $\beta_2 < 0$, then D on equation (11) must be equal to zero and, as before, $\beta \equiv \beta_1$. Differently to the Entrepreneur case, notice that the value matching condition stated on equation (13) will be negatively affected by the CPM.

The unknowns C and $\pi^*_V$ are obtained by combining conditions (13) and (14), i.e., the value matching and the smooth pasting conditions, respectively. Solutions for the optimal profit trigger and for the option to invest on the growth opportunity depend on the specification defined for $CPM(\pi)$.

### 4.2. Aligning Entrepreneurs, VCs and Growth Opportunities through CPMs

We are interested in determining the conditions under which Entrepreneurs and VCs would be willing to jointly support the growth opportunity held by the Entrepreneurial Firm, i.e., the conditions under which $\pi^*_E = \pi^*_V$. From a deal structuring perspective, parties can reach such an agreement either by pre-determining all deal terms – including firm ownership and any eventual up-front share consideration or premium – or by choosing to let part of definite deal terms be contingent on future performance benchmarks of the Entrepreneurial Firm.

The relevance of this issue within an Entrepreneurial Financing context is highlighted by the fact that post-equity round firm ownership is assumed to be split according to the equity contributions made by the Entrepreneur and the VC, even though the value of the assets in place held by the Entrepreneurial Firm might be greater than the face value of his equity contributions prior to
executing the growth strategy (i.e., $\pi/\delta$ might be greater than or, more generally, might be different from $k'$).

On the other hand, and differently from a typical M&A context, Entrepreneurial Financing decisions allow the Entrepreneur to retain a portion of the ownership of the Entrepreneurial Firm and, therefore, significantly profit from the value creation effects generated by the growth opportunity. In fact, without an outside investor that would allow her or his to obtain the indispensable resources to execute the envisaged growth strategy, the value of her or his option to invest in the growth opportunity would be equal to zero.

These two forces shall drive how CPMs are set, and we are specifically interested in understanding how CPMs can be designed in such a way that $\pi_E^* = \pi_{VC}^*$. With this purpose, we now analytically define each of the four major CPMs we previously introduced.

Broadly, and following the option analogy, we regard CPMs as binary call options and not as common call options on stock (Bruner and Stiegler, 2014), since their payoffs are actually discontinuous, i.e., either a fixed amount, or a variable amount linearly dependent on the value of its underlying asset.

Therefore, we analytically define fixed amount CPMs as *cash-or-nothing call* binary options, as the Entrepreneur is entitled to obtain a fixed amount of cash if the Entrepreneurial Firm achieves or exceeds a given performance benchmark. Concerning variable amount CPMs, we introduce two relevant assumptions: first, we assume that performance benchmarks (which we will name as $e_{BEN} > e_{EXP} > 1$) are grounded on the profitability of the Entrepreneurial Firm – and not on an operating measure or other financial measure different from profits – given by $\pi$ and second, we assume that the Entrepreneur will be entitled, in this case, to a multiple ($m > 0$) on the excess profit that the Entrepreneurial Firm generates over a given benchmark.

Considering these two assumptions, we define variable amount CPMs as *asset-or-nothing call* binary options, as the underlying asset of this binary option is the profitability of the Entrepreneurial Firm itself.

As a result, when computing the conditions under which $\pi_E^* = \pi_{VC}^*$, fixed amount CPMs require determining the amount of cash (named as $\theta$) that might be due to Entrepreneurs while *variable
amount CPMs require determining the multiple (named as \( m \)) on the excess profitability that will determine the amount of the contingent payment.

Concerning the due date, and taking into account the considerations for valuing fixed and variable amount CPMs we presented, when CPMs are due at term, contingent payments might be modelled as traditional binary options (Hull, 2012), while when CPMs are due at hit, contingent payments should be modelled as binary barrier options (Rubinstein, 1992; Rubinstein and Reiner, 1991). In the case of CPMs due at term, we will assume this term is exogenously determined by parties and given by \( t > 0 \). In the case of CPMs due at hit, we will assume that parties exogenously set a time period under which the Entrepreneur might be entitled to the CPM (given by \( t_{max} \)), i.e., a time period under which the parties agree that if the performance benchmark is set, the Entrepreneur is entitled to the contingent payment.

Throughout the paper, subscripts will be used to indicate the type of CPM to which a given function or variable refers to, by first indicating the acronym for the CPM amount, using an \( F \) for fixed amount CPMs and a \( V \) for variable amount CPMs, and then by indicating the acronym for the CPM due date, using a \( T \) for CPMs due at term and a \( H \) for CPMs due at hit. Subscripts are not used when such function or variable is not affected by the type of CPM it may refer to.

In Table 3, we summarize the key CPM specifications that will be used for valuing each of the four major types of CPMs we introduced, and for analyzing the conditions under which \( \pi_E^* = \pi_{V_C}^* \).
<table>
<thead>
<tr>
<th>Amount</th>
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<th>Alignment Variable</th>
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<tbody>
<tr>
<td></td>
<td>At Hit ($H$)</td>
<td>At Term ($T$)</td>
</tr>
<tr>
<td>Asset-or-Nothing Binary Barrier Option</td>
<td>$\text{CPM}_{\text{VH}}$</td>
<td>$\text{CPM}_{\text{VT}}$</td>
</tr>
<tr>
<td>(Rubinstein, 1992)</td>
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<td>(Hull, 2012)</td>
</tr>
<tr>
<td>Cash-or-Nothing Binary Barrier Option</td>
<td>$\text{CPM}_{\text{FH}}$</td>
<td>$\text{CPM}_{\text{FT}}$</td>
</tr>
<tr>
<td>(Rubinstein and Reiner, 1991)</td>
<td></td>
<td>(Hull, 2012)</td>
</tr>
</tbody>
</table>

*Table 3. Alternative CPM specifications*

Four relevant considerations should be highlighted at this point. First, based on the seminal work by Black and Scholes (1973), each of the four specifications is consistent with the underlying stochastic process that governs the profit flow of the Entrepreneurial Firm, i.e., a Geometric Brownian Motion. Second, with our approach we intend to highlight the role that uncertainty may hold on determining how parties value CPMs and not on CPMs may influence the behavior and effort of Entrepreneurs and VCs towards the accomplishment of the performance benchmarks, as in Lukas et al. (2012). Third, we do not intend to point out how asymmetric expectations on profit growth may govern the agreement between Entrepreneurs and VCs with the purpose of supporting the growth opportunity held by the Entrepreneurial Firm. Notwithstanding, the closed-form solutions which we will derive on the following sub-sections should also exist when parties hold different expectations on profit growth. Finally, within the real options framework previously presented for analyzing Entrepreneurial Financing decisions, it is assumed that, when obtaining support to the growth strategy, the Entrepreneurial Firm immediately spends the total capital expenditure requirements (named as $k$) and immediately records an increase in its profitability to $\pi^*\cdot e_{\text{EXP}}$. As a result, model
outcomes should be carefully interpreted when the underlying growth strategy is expected to be put in place throughout a long period or when its payoffs shall only become visible on a far future.

On the following sub-sections we will present, for each of the four major CPMs we introduced, how each contingent payment instrument is valued, the option to invest on the growth opportunity for Entrepreneurs and VCs, their underlying profit triggers and the optimum contingent payment \( \theta \) or optimum contingent payment multiple \( m \) that would allow Entrepreneurs and VCs to jointly support the growth opportunity held by the Entrepreneurial Firm.

### 4.2.1. Fixed Contingent Payment at Term

The value of this CPM is taken as a cash-or-nothing call (Hull, 2012) as follows:

\[
CPM_{FT} = \theta_{FT} e^{-rt} N(d2_{FT}) ,
\]

where

\[
d2_{FT} = \log \left( \frac{e^{EXP \cdot \pi}}{e^{BEN_{FT} \cdot \pi^*}} \right) + t(r - \delta - \frac{\sigma^2}{2})
\]

\[
\frac{\sigma \sqrt{t}}{2}
\]

\( N(z) \) stands for the cumulative normal density function, \( \pi^* \) stands for the profit trigger, \( \pi \) stands for the current profit, \( e^{BEN_{FT}} \) stands for the profit growth expansion benchmark for triggering the contingent payment and \( \theta_{FT} \) stands for the amount of the contingent payment.

Note that in the moment in which parties exercise their option to invest in the growth opportunity (i.e., when \( \pi_E^* = \pi_E^* \) and \( \pi_{VC}^* = \pi_{VC}^* \)), the current profitability of the Entrepreneurial Firm is \( e^{EXP \cdot \pi^*} \), and the profitability benchmark is given by \( e^{BEN_{FT} \cdot \pi^*} \).

Therefore, when computing \( \pi_E^* \) and \( \pi_{VC}^* \),

\[
\log \left( \frac{e^{EXP \cdot \pi}}{e^{BEN_{FT} \cdot \pi^*}} \right) = \log \left( \frac{e^{EXP \cdot \pi^*}}{e^{BEN_{FT} \cdot \pi^*}} \right)
\]

in equation (16), implying that \( N(d2_{FT}) \) does not depend on \( \pi \) and that \( \frac{\partial N(d2_{FT})}{\partial \pi} = 0 \). This will allow closed-form solutions to be derived for each of the value functions of the option to invest in the
growth opportunity, for each of the profit triggers and for the optimum CPM that will allow parties to jointly support the growth opportunity\(^\text{12}\).

As a result, by combining equations (8) and (15) and accordingly with (9) at this stage, we obtain the value matching and smooth pasting conditions that allow us to derive the option to invest on the growth opportunity held by the Entrepreneur in the presence of a fixed amount CPM due at term, as well as the profit trigger held by the Entrepreneur to invest in the growth opportunity given by \(\pi^*_{\text{EF}}\), i.e.

\[
E_{\text{FT}}(\pi) = \begin{cases} 
(e_{\text{EXP}} - 1) \frac{\pi^*_{\text{EF}}}{\delta} Q^E + \text{CPM}_{\text{FT}} - k^a \left( \frac{\pi}{\pi^*_{\text{EF}}} \right)^{\beta} \frac{\pi}{\delta}, & \text{for } \pi < \pi^*_{\text{EF}} \\
(e_{\text{EXP}} - 1) \frac{\pi}{\delta} Q^E + \text{CPM}_{\text{FT}} - k^a, & \text{for } \pi \geq \pi^*_{\text{EF}}
\end{cases}
\] (17)

where

\[
\pi^*_{\text{EF}} = \frac{e^{-rt}(k + k^i) \beta \delta (\theta_{\text{FT}} N(d_2) - e^{-rt}k^a)}{(k + k^i - e_{\text{EXP}}(k^a + k^i))(\beta - 1)}
\] (18)

Similarly, by combining equations (13) and (15) alongside with (14), we obtain the value matching and smooth pasting conditions that allow us to derive the option to invest on the growth opportunity held by the VC in the presence of a fixed amount CPM due at term, as well as the profit trigger held by the VC to invest in the growth opportunity given by \(\pi^*_{\text{VC}}\), i.e.

\[
V_{\text{FT}}(\pi) = \begin{cases} 
(e_{\text{EXP}} \frac{\pi^*_{\text{VC}}}{\delta} Q^C - \text{CPM}_{\text{FT}} - (k - k^a) \left( \frac{\pi}{\pi^*_{\text{VC}}} \right)^{\beta} \frac{\pi}{\delta}, & \text{for } \pi < \pi^*_{\text{VC}} \\
e_{\text{EXP}} \frac{\pi}{\delta} Q^C - \text{CPM}_{\text{FT}} - (k - k^a), & \text{for } \pi \geq \pi^*_{\text{VC}}
\end{cases}
\] (19)

where

\[
\pi^*_{\text{VC}} = \frac{e^{-rt}(k + k^i) \beta \delta (\theta_{\text{FT}} N(d_2) + e^{rt}(k - k^a))}{e_{\text{EXP}}(k - k^a)(\beta - 1)}
\] (20)

\(^\text{12}\) Note that if the profitability benchmark is exogenously determined and not dependent on \(\pi^*\) – meaning that \(e_{\text{BEN}} \pi^*\) would be replaced by a constant in equation (16) – \(\partial N(d_2, \text{FT})/\partial \pi\) would be have to be numerically determined. As a result, profit triggers both for Entrepreneurs and VCs alongside optimum CPM design would be obtained by numerical procedures. The same reasoning applies to the remaining CPMs presented on the following sections.
By equating $\pi_{E_F T} = \pi_{V_C F_T}$ – i.e., equations (18) and (20) – and solving for $\theta_{F_T}$, we obtain the optimum fixed contingent payment at term that would enable both Entrepreneurs and VCs to jointly support the growth opportunity.

$\theta_{F_T} = \frac{e^{rt}(k - k^a)(k - k^i(e_{EXP} - 1))}{(e_{EXP} - 1)(k + k^i)N(d_{2_{F_T}})}$  

(21)

### 4.2.2. Variable Contingent Payment in a Fixed Term

For valuing this CPM, we will assume that the amount of the contingent premium equals a given multiple $m$ over all the profit in excess of the existing profitability prior to executing the growth strategy. This is the reason why on equation (22), we not only introduce the variable $m$ to account for the contingent payment multiple but also the term ($e_{EXP} - 1$) to account for the fact that the contingent payment will be computed on the excess profitability prior to the growth strategy.

This CPM is then taken as an *asset-or-nothing call* (Hull, 2012), as that the Entrepreneur will be entitled to a fraction or a multiple of the underlying asset, i.e., the profit generated by the Entrepreneurial Firm.

$CPM_{VT} = m_{VT} \pi (e_{EXP} - 1)e^{-\delta t}N(d_{1_{VT}})$,  

where

$d_{1_{VT}} = \frac{Log\left(\frac{e_{EXP} \cdot \pi}{e_{BEN_{VT}} \cdot \pi^*}\right) + t(r - \delta + \frac{\sigma^2}{2})}{\sigma \sqrt{t}}$  

(23)

Following the same approach of the previous sub-section, whereby we consider that at the moment in which parties decide to invest in the growth opportunity $log\left(\frac{e_{EXP} \cdot \pi}{e_{BEN_{VT}} \cdot \pi^*}\right) = log\left(\frac{e_{EXP}}{e_{BEN_{VT}}}\right)$, we may obtain the value of the option to invest in the growth opportunity to the Entrepreneur and to the VC, alongside each of their optimum investment profit triggers, by combining equations (8), (9) and (22) for the Entrepreneur, and by combining equations (13), (14) and (22) for the VC, as follows:
\[ E_{VT}(\pi) = \begin{cases} 
(e_{\text{EXP}} - 1) \frac{\pi_{EVT}^*}{\delta} Q^E + \text{CPM}_{VT} - k^a \left( \frac{\pi}{\pi_{EVT}^*} \right)^\beta + \frac{\pi}{\pi_{EVT}^*}, & f o r \ \pi < \pi_{EVT}^* \\
(e_{\text{EXP}} - 1) \frac{\pi}{\delta} Q^E + \text{CPM}_{VT} - k^a, & f o r \ \pi \geq \pi_{EVT}^* 
\end{cases} \] 

where

\[ \pi_{EVT}^* = \frac{e^{t\delta} k^a (k + k^i)}{(-e^{t\delta}(k + k^i) - e_{\text{EXP}}(k^a + k^i)) + (e_{\text{EXP}} - 1)(k + k^i) m_{VT} \delta N(d_{1VT}))(\beta - 1)} \]

and

\[ V_{VT}(\pi) = \begin{cases} 
(e_{\text{EXP}} \frac{\pi_{VCVT}^*}{\delta} Q^c - \text{CPM}_{VT} - (k - k^a) \left( \frac{\pi}{\pi_{VCVT}^*} \right)^\beta), & f o r \ \pi < \pi_{VCVT}^* \\
e_{\text{EXP}} \frac{\pi}{\delta} Q^c - \text{CPM}_{VT} - (k - k^a), & f o r \ \pi \geq \pi_{VCVT}^* 
\end{cases} \]

where

\[ \pi_{VCVT}^* = \frac{e^{t\delta}(k - k^a) \beta \delta}{(e^{t\delta}(k - k^a) - (e_{\text{EXP}} - 1)(k + k^i) m_{VT} \delta N(d_{1VT}))(\beta - 1)} \]

Both \( \pi_{EVT}^* \) and \( \pi_{VCVT}^* \) present asymptotes dependent on \( m_{VT} \), which are obtained by finding the roots on the denominator of equations (25) and (27). For such values of \( m_{VT} \), there is no possible agreement between Entrepreneurs and VCs to support a given growth opportunity.

For the Entrepreneur, the asymptote on \( \pi_{EVT}^* \) is given by:

\[ m_{VT} = \frac{e^{t\delta}(k + k^i - e_{\text{EXP}}(k^a + k^i))}{(e_{\text{EXP}} - 1)(k + k^i) \delta N(d_{1VT})} \]

For the VC, the asymptote on \( \pi_{VCVT}^* \) is given by:

\[ m_{VT} = \frac{e^{t\delta} e_{\text{EXP}} (k - k^a)}{(e_{\text{EXP}} - 1)(k + k^i) \delta N(d_{1VT})} \]

By equating \( \pi_{EVT}^* = \pi_{VCVT}^* \) – i.e., equations (25) and (27) – and solving for \( m_{VT} \), we obtain the optimum multiple on the CPM due at term that would enable both Entrepreneurs and VCs to jointly support the growth opportunity.

\[ m_{VT} = \frac{e^{t\delta} (k - k^a) [k - k^i(e_{\text{EXP}} - 1)]}{(e_{\text{EXP}} - 1) k (k + k^i) \delta N(d_{1VT})} \]
4.2.3. Fixed Contingent Payment at Hit

Following the approach by Rubinstein and Reiner (1991), this CPM is derived as an up-and-in cash-or-nothing binary barrier option, assuming that the performance benchmark that will trigger the contingent payment is greater than or equal to its current level, as follows:

\[
CPM_{FH} = \theta_{FH} \left(\frac{e^{BEN_{FH}}}{{e^{EXP} \cdot \pi}}\right)^{a+b} N(-z) + \left(\frac{e^{BEN_{FH}}}{{e^{EXP} \cdot \pi}}\right)^{a-b} N(-z + 2b \sigma \sqrt{t_{\text{max}}}),
\]

where

\[
a = \frac{r - \delta}{\sigma^2}
\]

\[
b = \sqrt{\frac{(r - \delta)^2 + 2 \log(1 + r) \sigma^2}{\sigma^2}}
\]

\[
z = \frac{\log\left(\frac{e^{BEN_{FH}}}{\pi}\right)}{\sigma \sqrt{t_{\text{max}}}} + b \sigma \sqrt{t_{\text{max}}}
\]

As before, we assume that \(\frac{e^{BEN_{FH} \cdot \pi^*}}{e^{EXP} \cdot \pi} = \frac{e^{BEN_{FH} \cdot \pi}}{e^{EXP} \cdot \pi^*}\) and that \(\log\left(\frac{e^{BEN_{FH} \cdot \pi^*}}{e^{EXP} \cdot \pi}\right) = \log\left(\frac{e^{BEN_{FH}}}{e^{EXP}}\right)\) and obtain the value of the option to invest in the growth opportunity to the Entrepreneur and to the VC, alongside each of their optimum investment profit triggers, by combining equations (8), (9) and (34) for the Entrepreneur, and by combining equations (13), (14) and (34) for the VC, as follows:

\[
E_{FH}(\pi) = \begin{cases} 
\left(e_{\text{EXP}} - 1\right)\frac{\pi_{E_{FH}}^*}{\delta} Q^E + CPM_{FH} - k^a \left(\frac{\pi_{E_{FH}}}{\pi_{E_{FH}}^*}\right)^b + \frac{\pi}{\delta}, & \text{for } \pi < \pi_{E_{FH}}^* \\
\left(e_{\text{EXP}} - 1\right)\frac{\pi}{\delta} Q^E + CPM_{FH} - k^a, & \text{for } \pi \geq \pi_{E_{FH}}^* 
\end{cases}
\]

where

\[
\pi_{E_{FH}} = \frac{e^{BEN_{FH}}}{e^{EXP}} \left(\frac{e^{BEN_{FH}}}{e^{EXP}}\right)^a \left[\theta_{FH} \left(\frac{e^{BEN_{FH}}}{e^{EXP}}\right)^a (N(-z)\left(\frac{e^{BEN_{FH}}}{e^{EXP}}\right)^b + N(-z + 2b \sigma \sqrt{t_{\text{max}}}) - k^a \left(\frac{e^{BEN_{FH}}}{e^{EXP}}\right)^b)\right]^{-b} (k + k^i - e_{\text{EXP}}(k^a + k^i))(\beta - 1)
\]

and
\[ \begin{align*}
V_{CFH}(\pi) &= \begin{cases} 
\left( e^{\frac{\pi_{CFH} Q_{FC}}{\delta}} - CPM_{FH} - (k - k^a) \right) \left( \frac{\pi}{\pi_{CFH}^*} \right)^\beta, & \text{for } \pi < \pi_{CFH}^*, \\
\left( e^{\frac{\pi_{CFH} Q_{FC}}{\delta}} - CPM_{FH} - (k - k^a) \right), & \text{for } \pi \geq \pi_{CFH}^*,
\end{cases} 
\end{align*} \]

where

\[ \pi_{CFH}^* = \frac{(k + k^i) \beta \delta \left[ 1 + \theta_{FH} \left( \frac{e_{\text{BEN} VH} \pi}{e_{\text{EXP}} \pi} \right)^{a-b} \left( \frac{e_{\text{BEN} VH} \pi}{e_{\text{EXP}} \pi} \right)^{2b} N(-z) + N(-z + 2b \sigma \sqrt{t_{max}}) \right]}{e_{\text{EXP}} (\beta - 1)} \]

By equating \( \pi_{FH}^* = \pi_{CFH}^* \), i.e., equations (36) and (38) – and solving for \( \theta_{FH} \), we obtain the optimum fixed contingent payment at hit that would enable both Entrepreneurs and VCs to jointly support the growth opportunity.

\[ \theta_{FH}^* = \frac{\left( \frac{e_{\text{BEN} VH} \pi}{e_{\text{EXP}} \pi} \right)^{b-a} (k - k^a) \left[ k - k^i (e_{\text{EXP}} - 1) \right]}{(e_{\text{EXP}} - 1)(k + k^i) \left[ \left( \frac{e_{\text{BEN} VH} \pi}{e_{\text{EXP}} \pi} \right)^{2b} N(-z) + N(-z + 2b \sigma \sqrt{t_{max}}) \right]} \]

### 4.2.4. Variable Contingent Payment at Hit

Taking this CPM as an up-and-in asset-or-nothing binary barrier option, we take Rubinstein (1992) approach to analytically derive the value of this contingent asset, considering that the profit benchmark is equal or greater than its current value, as follows:

\[ CPM_{VF} = m_{VT} \pi (e_{\text{EXP}} - 1) \left( \frac{e_{\text{BEN} VH} \pi}{e_{\text{EXP}} \pi} \right)^{a+b} N(-z) + \left( \frac{e_{\text{BEN} VH} \pi}{e_{\text{EXP}} \pi} \right)^{a-b} N(-z + 2b \sigma \sqrt{t_{max}}), \]

where \( a, b \) and \( z \) are defined in equations (32), (33) and (34), respectively.

Assuming that \( \left( \frac{e_{\text{BEN} VH} \pi}{e_{\text{EXP}} \pi} \right) = \left( \frac{e_{\text{BEN} VH}}{e_{\text{EXP}}} \right) \) as before, we obtain the value of the option to invest in the growth opportunity to the Entrepreneur and to the VC, alongside each of their optimum investment profit triggers, by combining equations (8), (9) and (40) for the Entrepreneur, and by combining equations (13), (14) and (40) for the VC, as follows:
\[ E_{VH}(\pi) = \begin{cases} 
(e_{\text{EXP}} - 1) \frac{\pi_{E_{VH}}^\delta}{\delta} Q^E + CPM_{VH} - k^a \left( \frac{\pi}{\pi_{E_{VH}}} \right)^\beta + \pi, & \text{for } \pi < \pi_{E_{VH}} \\
(e_{\text{EXP}} - 1) \frac{\pi}{\delta} Q^E + CPM_{VH} - k^a, & \text{for } \pi \geq \pi_{E_{VH}} 
\end{cases} \]

where

\[ \pi_{E_{VH}} = \frac{(e_{\text{EXP}})^{\frac{1}{\delta}} e^k (k + k^i) \delta}{m_{VH} (e_{\text{EXP}} - 1) (k + k^i) \left( \frac{e_{\text{EXP}}}{e_{\text{EXP}}} \right)^{\frac{1}{2}}} \frac{N(-z) + N(-z + 2b \sigma \sqrt{\text{max}})}{N(-z) - N(-z + 2b \sigma \sqrt{\text{max}})} (\delta - 1) \]

and

\[ VC_{VH}(\pi) = \begin{cases} 
\left( e_{\text{EXP}} \frac{\pi_{VC_{VH}}^\delta}{\delta} Q^V - CPM_{VH} - (k - k^a) \right) \left( \frac{\pi}{\pi_{VC_{VH}}} \right)^\beta, & \text{for } \pi < \pi_{VC_{VH}} \\
e_{\text{EXP}} \frac{\pi}{\delta} Q^V - CPM_{VH} - (k - k^a), & \text{for } \pi \geq \pi_{VC_{VH}} 
\end{cases} \]

where

\[ \pi_{VC_{VH}} = \frac{\beta}{e_{\text{EXP}} \left( \frac{e_{\text{EXP}}}{e_{\text{EXP}}} \right)^{a-b} (e_{\text{EXP}} - 1) \left( \frac{e_{\text{EXP}}}{e_{\text{EXP}}} \right)^{\frac{1}{2}} N(-z) + N(-z + 2b \sigma \sqrt{\text{max}})} (\beta - 1) \]

As in the variable amount due at term CPM, there are both asymptotes on \( \pi_{E_{VH}} \) and \( \pi_{VC_{VH}} \), which are, respectively, given by:

\[ m_{VH} = \frac{(e_{\text{EXP}} - 1)(k + k^i)}{(e_{\text{EXP}} - 1)(k + k^i) \left( \frac{e_{\text{EXP}}}{e_{\text{EXP}}} \right)^{\frac{1}{2}} N(-z) + N(-z + 2b \sigma \sqrt{\text{max}})} \]

\[ m_{VH} = \frac{e_{\text{EXP}}(k + k^i)}{e_{\text{EXP}}(k + k^i) \left( \frac{e_{\text{EXP}}}{e_{\text{EXP}}} \right)^{\frac{1}{2}} N(-z) + N(-z + 2b \sigma \sqrt{\text{max}})} \]

By equating \( \pi_{E_{VH}} = \pi_{VC_{VH}} \) — i.e., equations (44) and (46) — and solving for \( m_{VH} \), we obtain the optimum multiple on the CPM due at hit that would enable both Entrepreneurs and VCs to jointly support the growth opportunity.
\[ m_{V_H}^* = \frac{(\frac{e_{BENV_H}}{e_{EXP}})^{b-a}}{\delta (e_{EXP} - 1) k (k + k_i) \left( \frac{e_{BENV_H}}{e_{EXP}} \right)^{2b} N(-z) + N(-z + 2 b \sigma \sqrt{t_{max}})} (k - k^a) \left[ k - k^i (e_{EXP} - 1) \right] \]

5. NUMERICAL EXAMPLE

We now illustrate the economic intuition behind the results introduced on the previous section through a numerical example. We start by listing the numerical assumptions we use in Table 4, and by summarizing the model outputs regarding investment timing and the design of CPMs. We conclude this section by presenting a set of sensitivities on some of the key value drivers.

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<thead>
<tr>
<th>Risk Parameters</th>
<th>Capital and Growth Opportunity</th>
<th>Contingent Payment Mechanism</th>
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<tbody>
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<td>( t_{max} )</td>
</tr>
</tbody>
</table>

*Table 4. Numerical Assumptions*

By combining the risk parameters \( r, \delta \) and \( \sigma \), we obtain \( \beta = 2.28 \) through equation (5), and by combining \( k^i, k^a \) and \( k \), we obtain \( Q^E = 65.4\% \) and \( Q^{VC} = 34.6\% \). We have also set the profit benchmark for the variable amount at term CPMs \( (e_{BENV_T}) \) equal to the expected profit growth \( (e_{EXP}) \), assuming that the amount of this contingent payment would be grounded on all excess profit above the expected growth.

5.1. CPM design and optimum investment timing

Results on the optimum CPM design – comprising both optimum amount for fixed contingent payments and optimum multiple for variable contingent payments – alongside their underlying profit triggers (\( \pi^* \)) and asymptotes are presented in Table 5.
Our numerical example shows that CPMs which are due at hit should present lower multiples or fixed amounts then those due at term, when parties understand that profit benchmarks should be attainable in the short-term or, equivalently, when they understand that the probability of profits staying below the benchmark (or reverting to levels below the benchmark) at term is significant, considering the underlying uncertainty on the profit flow of the Entrepreneurial Firm.

In fact, results reveal that $\theta^*_FT$ is more than triple than $\theta^*_FH$, while $m^*_VT$ is slightly more than double than $m^*_VH$, even when the latter comprises a higher profit benchmark (2.75 against 2.50). The practical implication of this result is that, CPMs which are due at term should lead to higher contingent payments than CPMs which are due at hit, when their underlying performance benchmarks are likely to be achieved before the term.

5.2. CPMs and the value of the investment opportunity to the Entrepreneur

In the previous sub-section, we showed that optimum investment timing is the same for each of the four alternative CPMs. However, as different CPMs are differently valued, their underlying value of the investment opportunity should differ. This argument also holds from Table 5, since the same optimum fixed contingent payments present different amounts when

<table>
<thead>
<tr>
<th>CPM</th>
<th>Supporting Equations</th>
<th>$e^{EXP}$</th>
<th>$e^{CUR}$</th>
<th>$e^{BEN}$</th>
<th>$\theta^*$</th>
<th>$m^*$</th>
<th>$\pi^*$</th>
<th>$\pi^*_E$</th>
<th>$\pi^*_VC$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CPM_{FT}$</td>
<td>(18), (20), (21)</td>
<td>2.50</td>
<td>2.50</td>
<td>2.75</td>
<td>258.57</td>
<td>-</td>
<td>47.52</td>
<td>No asymptotes</td>
<td></td>
</tr>
<tr>
<td>$CPM_{HT}$</td>
<td>(25), (27), (28)</td>
<td>2.50</td>
<td>2.50</td>
<td>2.50</td>
<td>-</td>
<td>3.65x</td>
<td>47.52</td>
<td>m = (12.18)x</td>
<td>m = 16.61x</td>
</tr>
<tr>
<td>$CPM_{FH}$</td>
<td>(36), (38), (39)</td>
<td>2.50</td>
<td>2.50</td>
<td>2.75</td>
<td>74.81</td>
<td>-</td>
<td>47.52</td>
<td>No asymptotes</td>
<td></td>
</tr>
<tr>
<td>$CPM_{VH}$</td>
<td>(42), (44), (45)</td>
<td>2.50</td>
<td>2.50</td>
<td>2.75</td>
<td>-</td>
<td>1.87x</td>
<td>47.52</td>
<td>m = (6.23)x</td>
<td>m = 8.50x</td>
</tr>
</tbody>
</table>

**Table 5. Illustration of Optimal CPM Design**
due at term or at hit, and same applies to optimum variable contingent payments, whose multiples differ when payments are due at term or at hit.

From the perspective of the Entrepreneur, we posit that CPMs which are due at hit should be more valuable than those which are due at term, controlling for the likelihood of the profit benchmark to be achieved (i.e., the closer $\frac{e_{EXP}}{e_{BEN}}$ is to 1) and that variable contingent payments should be more valuable their fixed counterparts, as they may generate a positive payoff for lower levels of profitability and may generate a higher payoff than fixed amount contingent payments when profitability exceeds its underlying threshold. From the perspective of the VC, the converse argument should hold.

In Figure 1, and further assuming that $m = 4.0$ and that $\theta = 150.00$, we illustrate this intuition on the value of the investment opportunity held by the Entrepreneur for the range $\pi < \pi^*$. Dashed lines stand for CPMs which are due at hit while normal lines stand for CPMs which are due at term.

![Figure 1. Value of the Investment Opportunity held by the Entrepreneur for each type of CPM](image)

5.3. Profit growth expectations and profit triggers

More aggressive profit growth expectations are expected to decrease profit triggers, making the investment opportunity more attractive both to the Entrepreneur and to the VC. We may observe such relationship in Figure 2, as we plot the profit trigger to invest on the growth opportunity ($\pi^*$) against the expected profit growth ($e_{EXP}$), when parties engage into optimum
CPM design. For simplicity, we plot a single curve for all the different CPMs, since all of their profit triggers are the same.

### 5.4. Volatility, value of the investment opportunity and profit triggers

Uncertainty of future business performance is taken as one of the main reasons behind the use of CPMs, as mentioned in Section 2. We will now illustrate, within the framework we introduced, how uncertainty influences investment timing and the value of option to invest in the growth opportunity for each of the CPMs we derived. For this purpose, we will keep the assumptions that $m = 4.0$, $\theta = 150.00$ and $\pi_0 = 30.00$. In Figure 3, we show how profit triggers are affected by uncertainty from the perspective of the Entrepreneur, plotting a single curve for all the different CPMs, as before. In Figure 4, we plot volatility against the value of the investment opportunity held by the Entrepreneur, for each of the four major CPMs we derived.

As expected, Figure 3 reveals that additional volatility increases the profit triggers required by Entrepreneurs and VCs to support the investment opportunity, while, from the Entrepreneur’s perspective, Figure 4 reveals that, in the presence of CPMs, growing volatility generates additional value to the investment opportunity. For lower levels of volatility, results similar to those in Section 5.2 hold, implying that CPMs which are due *at hit* are more valuable to those which are due *at term*, and *variable* amount CPMs are more valuable than *fixed* amount CPMs. However, for higher levels of volatility, we observe that the value of the investment opportunity with a *fixed* amount CPM which is due *at hit* actually converges to the value *fixed* amount CPM which is due *at term*, since for very high levels of uncertainty, the probability of the profit threshold to be hit *before* the term, converges to the probability of the profit threshold to be hit *at* the term.

### 5.5. Profit benchmarks and value of the investment opportunity

As profit benchmarks affect the likelihood of a contingent payment to become firm, a negative relationship between the both is expected to be found. Keeping the assumptions that $m = 4.0$, $\theta = 150.00$ and $\pi_0 = 30.00$ we illustrate this in Figure 5, where we plot the value of the option
held by the Entrepreneur to invest in the growth opportunity against a range of profit benchmarks above the expected profit growth (i.e., $e_{\text{BEN}} > e_{\text{EXP}}$), and in Figure 6, where we plot the relationship between a range of profit benchmarks and the optimum multiple for a variable CPM due at term. A similar relationship on optimum multiple behavior would be visible for a variable CPM due at hit.

Figure 5 shows that profit benchmarks negatively influence the value of the investment opportunity to the Entrepreneur, while, conversely, Figure 6 reveals that profit benchmarks positively affect the optimum contingent payment multiple on a variable amount CPM due at term.

5.6. Due dates and optimum contingent payments

By analyzing fixed amount CPMs, we now intend to illustrate how due dates may affect contingent payment design and, particularly, the amount of the contingent payment that should be set by Entrepreneurs and VCs so that the Entrepreneurial Firm obtains their joint support to proceed with the envisaged growth strategy. Similar results would hold to variable amount CPMs, in which we would observe analogous outputs on optimum contingent payment multiples.

Assuming that $t_{\text{max}} = t$, Figure 7 reveals that the contingent payment period holds a different effect for CPMs which are due at hit and CPMs which are due at term. While for CPMs due at term, longer contingent payment periods lead to higher optimum amounts, CPMs which are due at hit present lower optimum amounts.

The intuition behind this result lays on the fact that longer payment periods increase the probability of the profit benchmark to be achieved at any moment within the payment period, therefore making more valuable CPMs which are due at hit. When CPMs are due at term, longer payment periods actually stand for a longer deferred payment whose present value is inferior. In addition, in CPMs due at term, the underlying performance measurement is made at a specific moment of time and, therefore, in this sense less probable than for CPMs which are due at hit.
Figure 2. Profit Growth Expectations and Profit Triggers when CPMs are optimally designed

Figure 3. Volatility and Profit Triggers when CPMs are optimally designed
Figure 4. Profit Flow Volatility and Value of the Investment Opportunity held by the Entrepreneur

Figure 5. Profit Benchmarks and Value of the Investment Opportunity held by the Entrepreneur
Figure 6. Profit Growth Benchmarks and Optimum CPM Multiples in Variable CPMs due at term

Figure 7. CPM Due Dates and Optimum Contingent Payments (assuming $t_{max} = t$)
6. DISCUSSION

Model outputs illustrated in Section 5 are generally consistent with those in prior literature. The impact of uncertainty on CPM design and optimum investment timing shown in Section 5.4 and the impact of the contingent payment period on the amount of the optimum contingent payment illustrated in Section 5.6 is broadly similar to the hypothesis derived by Lukas et al. (2012). As an increase in uncertainty leads to an increased value of the option to invest in the growth opportunity held by the Entrepreneur, results in Section 5.6 are also consistent with Mantecon (2009), Ragozzino and Reuer (2009), and Lukas and Heimann (2014), in the sense that these authors predict that an increase in uncertainty increases the attractiveness of CPMs.

The impact of the profit benchmark on CPM design we introduced in Section 5.5 differs from Lukas et al. (2012), as these authors developed a framework for M&A decisions which involved an initial payment to the vendor of the target firm, instead of an equity issuance. Therefore, Lukas et al. (2012) argue that a trade-off might exist between such initial payment and the amount of a fixed contingent payment, when profit benchmarks increase. We conjecture that Lukas et al. (2012) would obtain a similar result to ours in a setting with not initial payment.

Our results show that, when optimally designed, different CPMs are equivalent when it comes to determining optimum investment timing, as profit triggers revealed to be the same for each of the four alternative CPMs we investigated\(^\text{13}\). As a result, the choice of which CPM should be set between Entrepreneurs and VCs should actually be driven by variables which are exogenous to the framework we designed, such as:

- *Liquidity constraints* on the VC side could limit the amount of funds available for deploying on a given investment opportunity or condition the timing within which such funds are available (Faccio and Masulis, 2005). Such variable could lead a preference for a mechanism whose expected amount is lower or for a mechanism that would

\(^{13}\) In unreported results, it could be shown that an optimum up-front share premium would also have a profit triggers equal to those presented for the different CPMs.
increase the chance of its underlying liability to be financed through the Entrepreneurial Firm itself;

- *Time constraints* on the VC side, given that the underlying VC cycle may condition the amount and the timing of the contingent liabilities that it may be able to accept at the moment in which the investment opportunity is being screened. In particular, a VC would not be allowed to bear a contingent liability which might be due after the fund term;

- *Liquidity preferences* on the Entrepreneur side, which may favor a deal structuring mechanism in which she or he would be entitled to an up-front cash in, instead of a contingent payment or even an up-front share premium;

- *Risk preferences* on the Entrepreneur side may drive the choice between an up-front cash in, and a fixed or a variable contingent payment. Risk-averse Entrepreneurs should prefer up-front payment mechanisms to contingent payments, settle lower benchmarks for triggering contingent payments and may reveal a preference for *fixed* amount CPMs, that protect them against down-side performance;

- *Credit risk* may play a role in analyzing a potential CPM. Entrepreneurs may regard CPMs as deferred payment mechanisms (such as vendor loans) and may therefore subject the acceptance of this contingent asset to a proper assessment of the credit risk of the VC and of the CPM that may minimize such risk;

- *Post-deal performance measurement and integration* may restraint the settlement of CPMs, as deal terms may reduce the perception of decision-control held by the Entrepreneur – meaning that potential performance might be influenced by decisions taken by the VC and affect the probability of a contingent payment to take place – and lead to the establishment of discretionary expenses, profit decisions or a new corporate organization that may affect the ability of the parties to properly measure the future performance of the Entrepreneurial Firm (Bruner and Stiegler, 2014);
- Overall deal terms require both Entrepreneurs and VCs to agree on a wide set financial and non-financial terms (including compensation, performance bonuses, value-adding roles by VCs, and corporate governance), which generate a set of negotiation trade-offs and lead to different choices of CPMs.

7. CONCLUSIONS

We showed that CPMs are becoming more relevant on M&A deal volumes and that may also be a relevant tool for Entrepreneurial Financing decisions. Our novelty approach starts by acknowledging that there are different types of CPMs, which we propose to distinguish according to their amount and due date. We then identify four major types of CPMs and separately value each one of them, following the option analogy. After introducing a real options framework for analyzing Entrepreneurial Financing decisions, we derived the optimum CPM design that would allow Entrepreneurs and VCs to jointly support a given growth opportunity and illustrated model outcomes through a numerical example, whose results allowed us to compare different CPMs and revealed to be consistent with previous literature findings.

Regarding future research paths and following the work by Cain et al. (2011), Barbopoulos and Sudarsanam (2012), and Lukas and Heimann (2014), we understand that further empirical research is needed for testing some of the theoretical propositions on CPMs within an Entrepreneurial Financing context, instead of an M&A context. In addition, and from an analytical perspective, the range of alternative CPMs extends beyond the four major types we introduced, as shown by Reum and Steele (1970), providing room for additional extensions of the framework we introduced.
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