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Productivity shocks in a union-duopoly model*

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Abstract

We consider an industry where firms are asymmetric in terms of productivity. Wages and employment are determined at the firm-level and are the result of sequential bargaining between unions and firms, with wages being negotiated first. We characterise the equilibrium and compare the outcomes in the two firms (wages, employment and surplus of all economic agents). A productivity shock affecting the most efficient firm is socially desirable, by increasing the surplus of consumers, workers and firms. The same may not be true when the productivity shock affects the least efficient firm. In this case, the consumers' gain may not be enough to outweigh the losses in aggregate worker surplus and in the industry profit.

Keywords: Asymmetric productivity; Decentralised bargaining; Duopoly; Productivity shocks.

JEL Codes: J53; L13; D60.

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1 Introduction

Empirical evidence suggests the existence of significative dispersion across firms within the same industry regarding their productivity levels (Bartelsman and Doms, 2000; Syverson, 2011). The most consensual factors that may contribute to asymmetries across firms are: (*i*) differences in the manager's profile, namely regarding the innovative profile and the degree of risk-aversion; (*ii*) differences in the access to new technology and skilled human capital; (*iii*) differences in the regulatory environment (e.g., barriers to entry affect firms' decisions regarding R&D investments); (*iv*) existence or not of R&D subsidies; and (*v*) financial market imperfections (e.g., the access to credit influences decisions to invest in R&D).

Motivated by this evidence, our main goal is to contribute to the understanding of the welfare impacts of productivity shocks in industries where firms are asymmetric in terms of productivity.[1] We will, therefore, characterise the impacts of a productivity shock on consumers, firms and workers. In particular, we will analyse how different these impacts are when the productivity shock affects an efficient or an inefficient firm.

We address these research questions using a theoretical model with the following features. Two firms with asymmetric productivity compete with each other to sell an homogeneous good. Demand is linear in prices. Labour is the only factor of production and is firm-specific. There is decentralised bargaining in the labour market, i.e., each firm only negotiates with its own workers and vice-versa (Dowrick, 1989).[2] Following Manning (1987), we consider a quite general model of labour bargaining. Unions and firms first bargain over wages, and then over employment. The parties may have different bargaining powers when negotiating over wages and over employment.[3]

We conclude that the most efficient firm pays a higher wage than the least efficient firm but incurs in a lower marginal production cost (due to its productivity advantage). Consequently, the most efficient firm profits more than the least efficient firm. We also

[1]Zhao (2001) highlighted the importance of such an analysis, for instance, when evaluating the impacts of a World Bank program with the objective of improving technology in poor countries (where productivity is lower).

[2]Despite having a distinct purpose, our model is quite similar to Petrakis and Vlassis (2000). The similarities and the differences between the two works will be explained afterwards.

[3]These assumptions make our model suitable to study competition between firms located at geographically separated regions but selling their goods in the same product market. The first contribution dealing with such markets goes back to the early 1980s to the international trade model proposed by Brander and Spencer (1985).

find that the most efficient firm produces more units of output than the least efficient firm, even when hiring less workers. Workers at the most efficient firm are better off than workers at the least efficient firm, since, even when the employment is lower, the difference in wages more than compensates the difference in employment.

We then analyse the impact of an idiosyncratic productivity shock on consumers, firms and workers. When a firm becomes more efficient, it produces more units of output while the rival produces less. However, the net effect on total output is always positive, which leads consumers to always benefit from a positive productivity shock (whichever is the firm that becomes more productive). However, consumers prefer an efficiency gain that affects the least efficient firm (since total output increases more).

If a firm becomes more productive, its profit increases while the profit of the rival decreases. A positive productivity shock affecting the most efficient firm always increases the industry profit, since the increase in the profit of the most efficient firm more than compensates the reduction in the profit of the least efficient firm. In contrast, when it is the least efficient firm that increases its productivity, the industry profit may decrease. Lahiri and Ono (1988) and Zhao (2001) arrived to a similar conclusion in the context of Cournot competition with asymmetric production costs.[4] According to these authors, when an inefficient firm becomes slightly more efficient (i.e., its marginal cost diminishes), there is a transfer of production from a more efficient firm to this firm. As a result, the average production cost of the industry increases, which may reduce the industry profit.[5]

A positive productivity shock may or may not benefit workers, depending on which firm is affected. If a firm increases its efficiency, this benefits its own workers but damages the workers of the rival firm. If it is the most efficient firm that becomes (even) more efficient, the gain of the workers at this firm more than compensates the workers' loss at the least efficient firm, and aggregate worker surplus increases. The opposite occurs if it is the least efficient firm that becomes more efficient, generating a loss of aggregate worker surplus.

If the most efficient firm increases its advantage, consumers and workers are better off, and the industry profit increases. As a result, a positive productivity shock affecting the most efficient firm is socially desirable. In contrast, if the productivity shock affects the least efficient firm, the consumers' gain may not be enough to compensate the loss in the

[4]In the models of Lahiri and Ono (1988) and Zhao (2001), production costs are exogenous. In our model, the marginal production costs have an endogenous component (wage).

[5]Wang and Zhao (2007) proved that this result still holds when firms sell differentiated products.

aggregate worker surplus and in the industry profit. This is the case when the reservation wage is sufficiently high, which allows the most efficient firm to get a big market share. In addition, we have found that the negative impact of a productivity improvement by the least efficient firm is the greater: (*i*) the lower is the unions' power in the wage negotiation; and (*ii*) the lower is the unions' power in the employment negotiation.

We also characterise the equilibrium when firms and unions simultaneously bargain over wages and employment, which we refer to as simultaneous bargaining (McDonald and Solow, 1981). We found that, when unions have the same bargaining power over wages and employment, wages are higher under simultaneous bargaining than under sequential bargaining. This results from the fact that, under sequential bargaining, setting a lower wage has a commitment value, leading to higher employment and output at the same firm and, as a result, to lower employment and output at the rival. We also found that the marginal production costs (i.e., the ratio between the wage and the efficiency parameter) end up being more asymmetric under sequential bargaining than under simultaneous bargaining. In other words, the simultaneous bargaining dissipates more the (ex-ante) asymmetry across firms than the sequential bargaining. As a result, there are situations in which the less efficient firm finds profitable to be active in the market with simultaneous bargaining, but unprofitable with sequential bargaining. We concluded that the most efficient firm always hires more workers when the bargaining is sequential (which was somehow expected, since it pays a lower wage under sequential bargaining). More surprisingly, the least efficient firm may hire more workers when the bargaining is sequential or when it is simultaneous (depending on whether its market share is above or below a certain threshold). Recall that the most efficient also pays a lower wage when the bargaining is sequential and, therefore, produces more output. Thus, on the one hand, the least efficient firm pays a lower wage when the bargaining is sequential (which may induce it to hire more workers) but, on the other hand, faces a more aggressive competitor (and, as quantities are strategic substitutes, the least efficient firm may produce less). Each of these forces may dominate, depending on the market share of the least efficient firm.

As in the baseline model of labor bargaining, we study how a productivity shock impacts on the different economic agents when there is simultaneous bargaining. We have obtained qualitatively the same results as in the scenario of sequential bargaining. In a nutshell, we found that a positive productivity shock affecting the most efficient firm increases the consumer surplus, the (aggregate) worker surplus and the industry profit.

As a result, this shock is socially desirable. A positive productivity shock affecting the least efficient firm still increases the consumer surplus, but also increases the (aggregate) worker surplus and the industry profit if and only if the reservation wage is sufficiently small. As a result, a positive productivity shock affecting the least efficient firm may or may not be socially desirable.

In order to illustrate our results, we have calibrated the parameters of our model using data concerning six OECD countries (Belgium, Canada, Netherlands, Portugal, Spain, and United States). More precisely, we have used estimates for reservation wages, efficiency levels and unions' bargaining power to calculate, according to our model, the welfare impact of an idiosyncratic productivity shock. Due to the lack of information at the firm-level, we make some approximations, namely by using values at the country level and considering an average firm as a representative firm. We have simulated our baseline bargaining model in two distinct cases: (*i*) when unions have the same bargaining power when negotiating over wages or employment (i.e., symmetric sequential bargaining); and (*ii*) when unions have no bargaining power over employment (i.e., right-to-manage).[6] In all simulated cases, we obtained that the wage increase in the firm that becomes more efficient is much more pronounced than the decrease in the wage of the rival firm. We obtained that the employment at both firms decreases in response to a positive productivity shock, whichever firm becomes more efficient.[7] We obtained further that the overall welfare increases in response to a positive productivity shock (whichever firm becomes more efficient). More precisely, a positive productivity shock increases the consumer surplus, the aggregate worker surplus and the industry profit. If the productivity shock affects the least (resp. most) efficient firm, the increase in consumer surplus is higher (resp. lower) than the increase in worker surplus or in the industry profit. We also obtained that, with symmetric sequential bargaining, a positive productivity shock increases more the overall welfare if it affects the most efficient firm. However, with right-to-manage, a positive productivity shock increases more the overall welfare if it affects the least efficient firm.

[6] Empirical evidence suggests that real-world may be somewhere in between these two scenarios. See Section 5 for a more detailed discussion.

[7] Rigorously speaking, we only found a situation where the employment of the least efficient firm increased after this firm becoming more efficient: competition between a Portuguese and a Spanish firms, and with symmetric sequential bargaining. In this case, we found that the industry profit and the worker surplus decrease if the Portuguese (less efficient) firm becomes more productive. Anyway, the increase in consumer surplus more than compensates this loss, and the overall welfare increases.

Brief literature review

There are four benchmark models of labour market negotiations, which essentially differ in the scope of the bargaining (i.e., over which variables workers and firms bargain over) and in the timing of the negotiations (i.e., if negotiations are sequential or simultaneous).[8] More precisely, the standard models are: (*i*) the *monopoly* model, in which workers unilaterally choose wages and, afterwards, firms unilaterally decide employment (Oswald, 1982); (*ii*) the *right-to-manage* model, in which unions and firms bargain over wages and, afterwards, firms unilaterally decide employment (Nickell and Andrews, 1983); (*iii*) the *efficient bargaining* model, in which unions and firms simultaneously bargain over wages and employment (McDonald and Solow, 1981); and (*iv*) the *sequential bargaining* model, in which unions and firms first bargain over wages and then over employment (Manning, 1987). When there is just one firm active in the product market, the *sequential bargaining* model captures all the other models as particular cases.[9] However, as stated by Petrakis and Vlassis (2000) and Dhillon and Petrakis (2002), the outcome from simultaneous and sequential bargaining no longer coincide when there is more than one active firm in the product market.[10] Bearing all this in mind, we consider both sequential bargaining (that has the advantage of including other models as particular cases) and simultaneous bargaining. By doing so, we cover all the benchmark bargaining models.

The previously described bargaining models were initially formulated to study negotiations involving one single firm and one single union. Dowrick (1989) pioneered the study of labour market bargaining in the context of oligopolistic product markets. The differences between the work of Dowrick (1989) and ours are clear. First, Dowrick (1989) considered symmetric firms, while we assume that firms differ in their productivity level.

[8]Models also differ in the consideration of centralised (i.e., with one representative at each side of the market) or decentralised bargaining (i.e., firm-level bargaining). Naylor (2003) compares the equilibrium under centralised and decentralised bargaining. Dobson (1994) made an intermediate assumption, by considering the existence of an industry-wide union that bargains with several firms (i.e., centralisation in just one side of the labour market). For contributions combining these three types of bargaining, see Grandner (2001) and Santoni (2014).

[9]The *monopoly model* is a particular case of the *sequential bargaining* model in which workers have all the bargaining power in the wage bargaining, while firms have all the bargaining power in the bargaining over the employment level; the *right-to-manage model* is a particular case of the *sequential bargaining* model in which firms have all the bargaining power in the bargaining over the employment level; and the outcome of the *efficient bargaining model* is the same as that of the *sequential bargaining* model in which unions have the same bargaining power in the bargaining over wage and employment (Manning, 1987).

[10]We discuss this issue in more detail in Appendix A.

Second, Dowrick (1989) focused on the cases of a simultaneous negotiation over wages and employment, and bargaining exclusively over wages (*right-to-manage* model); while we consider a more comprehensive bargaining game with two consecutive bargainings between firms and workers (Manning, 1987).

In terms of methodology, the contribution of Petrakis and Vlassis (2000) is the closest to ours. They also consider a duopoly model where firms have asymmetric productivity levels and assume that negotiations in the labour market are decentralised (i.e., negotiations between firms and unions occur at the firm-level). Nevertheless, they allowed for only two types of bargaining: right-to-manage and simultaneous bargaining; while we consider these bargaining schemes as particular cases. Anyway, what distinguishes the most the two contributions is the research question. Petrakis and Vlassis (2000) made the scope of bargaining (i.e., the variables over which firms and unions desire to bargain) endogenous, and studied whether, in equilibrium, unions and firms would bargain just over wages or also over employment. In contrast, we are mainly interested in providing an analysis of the impacts of productivity shocks on welfare.

Thus, regarding the research question, our contribution is closely related to the literature that analyses the welfare impacts of cost asymmetries across firms (Lahiri and Ono, 1988; Zhao, 2001; Wang and Zhao, 2007). These contributions studied the effects of a cost reduction in linear Cournot markets when firms incur in asymmetric production costs. One of the main conclusions is that a (small) cost reduction may be socially undesirable. More precisely, if a high-cost firm reduces its cost, there will be a transfer of production from a more efficient to a less efficient firm (which increases the average production cost of the industry) and, therefore, overall welfare may decrease. However, differently from us, this literature did not account for workers' surplus in overall welfare. More precisely, they have restricted the analysis to the (downstream) relation between firms and consumers, assuming that production costs are exogenous. We also model the (upstream) relation between firms and workers, making production costs endogenous. Our analysis contributes to the literature since, beforehand, it was not obvious whether the established welfare results would remain valid when wages are endogenously determined and the impact on worker surplus is also accounted for.

The remainder of the paper is organised in the following way. Section 2 presents the basic setting. Section 3 describes the assumptions regarding the labour market and characterises the equilibrium. Section 4 studies the welfare impacts of a productivity shock. Section 5 calibrates the parameters of the model and presents the results when simulating productivity shocks. Section 6 concludes with some remarks. Appendix A derives the equilibrium under simultaneous bargaining, and studies the welfare impact of a productivity shock in this setting. Finally, Appendix B collects most proofs.

2 Basic setting

Consider an industry with two firms (H and L) that produce homogeneous goods and sell them to consumers.[11] Total inverse demand is linear in quantities and given by $p = 1 - q_H - q_L$, where p is the market price and q_i denotes the quantity produced by firm $i \in \{H, L\}$. Labour is the only factor of production.[12] Workers are firm-specific and supply at most one unit of labour. Production exhibits constant returns to scale: with l_i workers, firm i produces $q_i(l_i) = e_i l_i$ units of output. Parameter $e_i > 0$ measures the productivity of firm i. Without loss of generality, firm H is at least as efficient as firm L, i.e., $e_H \geq e_L$. Let $r_e = \frac{e_L}{e_H} \leq 1$ denote the ratio of efficiencies. Variables e_i are exogenous, which may be suitable to cases wherein labour productivity is out of firms' reach. For example, labour productivity may be more influenced by intrinsic characteristics of workers (e.g., education) than by decisions of the firm regarding investments in more sophisticated machines or in the organisation of the plant.

Figure 1 represents this industry. Such structure may fit an industry where firms are established in different countries and sell to a third country (Brander and Spencer, 1985). In that case, a crucial assumption is the absence of labor mobility across countries.[13]

Denoting by w_i the wage at firm $i \in \{H, L\}$, the profit function of firm i can be

[11] To the best of our knowledge, only the models of Bacchiega (2007), Symeonidis (2008) and Santoni (2014) combine product differentiation with labor market bargaining. These contributions are substantially different from ours. Bacchiega (2007) considered that only the most efficient firm bargains with its workers; while Symeonidis (2008) and Santoni (2014) assumed that firms have symmetric productivity levels.

[12] Capital is assumed to be a fixed input. In our model, the output is completely determined by employment, which is the outcome of bargaining between firms and unions.

[13] For models of competition between a domestic firm and a foreign firm see, for example, Straume (2002) and Santoni (2014). Our model could also apply to the case wherein firms are established at the same country but sufficiently distant from each other. However, depending on the country, it may be more reasonable to consider collective bargaining, as in Dobson (1994).

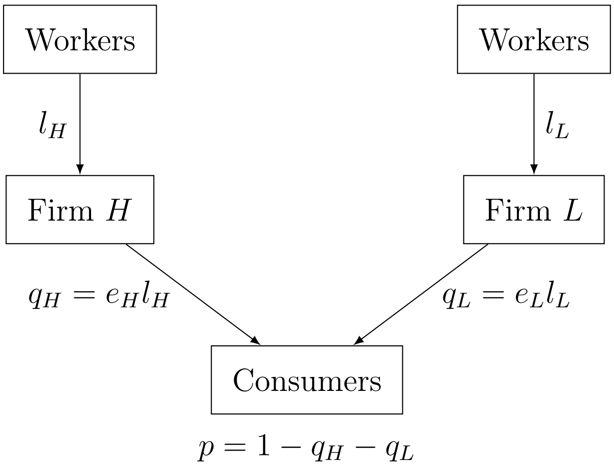


Figure 1. Industry structure.

written as:

$$\pi_i(w_i, l_i, l_j) = pq_i - w_i l_i = \Big[(1 - e_i l_i - e_j l_j)\, e_i - w_i \Big] l_i. \tag{1}$$

As products are homogeneous, consumer surplus is given by:

$$CS = \frac{(q_H + q_L)^2}{2} = \frac{(e_H l_H + e_L l_L)^2}{2}. \tag{2}$$

We assume that the reservation wage is the same for the workers at both firms, and denote it by $\bar{w} \geq 0$.[14] Thus, given the employment level, the surplus of the workers at firm $i \in \{H, L\}$ is equal to:

$$WS_i\,(w_i, l_i) = (w_i - \bar{w}) l_i.$$

Finally, total surplus is equal to the sum of the surplus of all involved economic agents:

$$TS(l_H, l_L) = CS + \sum_{i \in \{H,L\}} (\pi_i + WS_i) \;=\; \frac{1 - (1 - e_H l_H - e_L l_L)^2 - 2(l_H + l_L)\bar{w}}{2}. \tag{3}$$

Total surplus does not explicitly depend on wage levels because wages are transfers between firms and workers. However, the wages' levels affect employment levels (or,

[14]In the case of competition between firms established in different countries, it could be more reasonable to assume different reservation wages. We make this simplifying assumption in order to avoid the inclusion of additional variables.

equivalently, output levels), which affect total surplus.

3 Labour market

As pointed out by Abraham et al. (2009, p. 18), *"most European countries are characterised by labor markets where negotiations between unions and firms take place"*. Thus, we assume that the outcomes of the labor market result from the bargaining between firms and unions. Following Dowrick (1989), we consider a decentralised bargaining in the labour market,[15] in which the union at each firm bargains with the manager of that same firm.[16] Despite the negotiations at the two firms occurring independently, they are assumed to be simultaneous.[17]

3.1 Unions' preferences

Regarding the unions' utility function, we assume a particular case of the Stone-Geary specification, which is quite consensual in the literature and has been adopted since the most preliminary contributions to this topic (e.g., Dunlop, 1944).[18] More precisely, we

[15]Katz (1993, p. 3) reported evidence that *"the locus of collective bargaining is shifting downward in a number of countries, often from a national or multi-company level to the firm or plant level."*

[16]We will refer to the manager of the firm as the representative of the firm's interests. This paper is not about optimal contracts that firms' owners should offer to their managers (Fershtman and Judd, 1987; Sklivas, 1987). For a model with strategic delegation and wage bargaining, see Szymanski (1994). In his model, owners define the objective function of managers, who choose output levels and bargain with unions over wages. Szymanski (1994) concluded that, in equilibrium, owners will set the managers' incentives towards profit-maximisation (instead of sales maximisation). In a related contribution, Chatterjee and Saha (2013) considered a model of efficient bargaining where the parties (the firm and the labour union) delegate the negotiation to agents with potentially different preferences (the manager and the union leader). The authors found that, in equilibrium, the owner will orient the preferences of the manager profit maximisation, and that workers will orient the union leader towards the maximisation of the net wage bill.

[17]For models where negotiations are not simultaneous, see, for instance, Horn and Wolinsky (1988), De Fraja (1993) and Dobson (1994). Dobson (1994) considered the existence of an industry-wide union that bargains over wages with two firms. The author analysed whether the union is better off by negotiating simultaneously or sequentially with the two firms. In the latter scenario, the union uses the conditions obtained in the first negotiation to extract more surplus from the second negotiation (*pattern bargaining*). For this reason, the author concluded that, when firms are asymmetric, the union may prefer to bargain first with the "weakest" firm (i.e., the firm that will more easily abdicate from surplus). See also Banerji (2002).

[18]For more information on this topic, see the surveys conducted by Oswald (1985) and Naylor (2003).

assume that the union at firm $i \in \{H, L\}$ maximises:

$$WS_i = (w_i - \bar{w})l_i.$$

The magnitude of the reservation wage, $\bar{w}$, depends on the type of unions' outside option: if, for instance, the union has the possibility of searching for an agreement with other firms, or if the outside option is a strike. We assume that unions are equally concerned about employment and wage.[19] Note that the specified utility function represents the same preferences (rent-maximisation) as the utilitarian objective function, given by the sum of the utility of employed workers and the utility of unemployed workers, if workers are risk-neutral (Naylor, 2003).

3.2 Bargaining game

In the literature, there is an old debate on the scope of the bargaining in the labour market (i.e., about the issues included in the bargaining agendas).[20] Although there appears to be unanimity regarding the existence of wage bargaining between firms and unions, there is still no consensus about the way the employment level is determined. On the one hand, there is a branch of the literature that considers that employment is unilaterally determined by firms (e.g., Dunlop, 1944; Oswald, 1982; Nickell and Andrews, 1983); on the other hand, there are some authors considering that unions also influence the employment level (e.g., Leontief, 1946; McDonald and Solow, 1981; Manning, 1987). Real-world negotiations may be somewhere in between these two polar cases: the number of jobs may not explicitly be stated in the negotiations, but an employment bargaining may end up arising implicitly due to the negotiation of work practices that actually prevent firms from adjusting production (Clark, 1990).[21]

[19] A more general formulation would be: $[u(w_i, \bar{w})]^{\gamma} l_i$, where γ would capture the relative importance attached to employment, as considered by Petrakis and Vlassis (2000) and Dhillon and Petrakis (2002).

[20] For models with endogenous scope of bargaining, see Bughin (1999) and Petrakis and Vlassis (2000).

[21] According to some authors, a way of modelling working conditions as an issue that workers and firms bargain over could be the consideration of the bargaining over the capital-labor ratio. If capital is fixed, this will be equivalent to the bargaining over employment. However, Booth (1995) suggested that it may not be realistic to assume that firms do not adjust capital after bargaining with workers. The author suggested that the bargaining over capital-labor ratio could be interpreted as the bargaining over workers' effort, since the required effort of a given worker will be lower if there are more workers per machine.

The divergence of opinions extends to empirical contributions. For instance, Oswald (1993) conducted a (postal) inquiry to the largest US and British unions asking them if they *"normally negotiated over the number of jobs as well as over wages and conditions"*, and only 2 US (over 19) and 3 British (over 18) of the respondents answered affirmatively.[22] Nevertheless, MaCurdy and Pencavel (1986) stated that, although there is no empirical evidence that unions and firms directly bargain over employment, their data did not suggest that employment was unilaterally determined by firms. In addition, Bughin (1993) found empirical evidence that employment level is off the labor demand curve. Bughin (1996) also found that Belgian unions do not only bargain over wages but also over employment issues, like hours of work and part-time labour policies. Using data from the Belgian manufacturing industry, Dobbelaere (2004) also empirically rejected the hypothesis that workers have no influence over employment. More precisely, the author rejected the right-to-manage model in favour of the efficient-bargaining model. Using data from five EU countries, Dumont et al. (2006) empirically rejected both the efficient-bargaining and right-to-manage models, in favour of a model wherein unions and firms bargain over wages and *overhead* labour.[23] Using aggregate data of the UK labor market, Alogoskoufis and Manning (1991) empirically rejected the right-to-manage model and the efficient bargaining model over a model similar to that proposed by Manning (1987).

Bearing all this in mind, we consider a duopoly version of the model proposed by Manning (1987) and assume that firms and unions bargain over wages and employment but may have a different bargaining power in each negotiation.[24] We assume a sequential game with the following timing:[25]

[22] Layard et al. (1991, p. 91) also agreed that *"Employment is almost never bargained over as such"*, and explained that unions do not usually bargain over employment because existing workers care about their *own* job and not about the level of employment in the firm.

[23] In their model, *overhead labour* corresponds to the proportion of unproductive time that is paid to workers (e.g., corresponding to work breaks or resulting from overlarge crew sizes).

[24] Differently, McDonald and Solow (1981) assumed that unions and firms simultaneously bargain over employment and wage. Manning (1987) proved that, when there is a monopoly in the product market, the outcome from the simultaneous bargaining coincides with the outcome of the sequential bargaining when the parties have the same bargaining power in the two negotiations (i.e., $\alpha = \beta$). However, this result is no longer valid when there is a duopoly in the product market (Petrakis and Vlassis, 2000; Dhillon and Petrakis, 2002). The intuition is the following. In the case of monopoly, the two bargaining schemes (sequential and simultaneous) are equivalent due to the absence of strategic effects. In the case of a duopoly, the bargaining parties at each firm take into account that their decisions affect the decisions at the rival firm.

[25] Manning (1987, p. 124) provided reasons for assuming this timing. According to Nickell and Wadhwani (1991, p. 955): *"in U.K. industry, wages are set at discrete intervals (typically one year apart or more) whereas*

1. Unions and managers independently and simultaneously bargain over wages.
2. Unions and managers independently and simultaneously bargain over employment.

Firms behave non-cooperatively in each stage of the game, i.e., the objective of each manager is to maximise the profit his firm, given by (1).[26] To apply the bargaining analysis proposed by Nash (1950), we need to specify the disagreement payoffs, i.e., the reservation utilities in case of negotiation failure. According to Malcomson (1987), the specification of such a value is not straightforward and depends on market characteristics. For instance, the outside option of the firm may be to shut down without costs, or to hire less skilled employees. For simplicity, we assume that both parties (firm or workers) may block production and, therefore, the disagreement payoffs are null.

In the first-stage of the game, the wages are simultaneously determined at both firms. The wage at firm i is the solution of corresponding Nash bargaining game, i.e., solves the following maximisation problem:

$$\max_{w_i} \left[(w_i - \bar{w})\, l_i(w_i, w_j) \right]^{\alpha} \left[\pi_i \Big(w_i, l_i(w_i, w_j), l_j(w_i, w_j) \Big) \right]^{1-\alpha} \equiv NB_i^w (w_i, w_j)\,, \qquad (4)$$

taking as given the wage at firm $j \neq i$. Parameter $0 \leq \alpha \leq 1$ denotes the (relative) bargaining power of the unions in the wage bargaining; while $1-\alpha$ is the bargaining power of the managers. If $\alpha = 0$, managers choose wages; if $\alpha = \frac{1}{2}$, unions and managers have equal bargaining powers; while if $\alpha = 1$, unions decide wages.[27]

We are assuming that, despite the negotiations in the two firms occurring independently, the relative bargaining power of the parties (unions and managers) is assumed to be the same across firms.

Managers aim at achieving the lowest wage that allows them to hire the labour units necessary to operate the firm at the optimal level, which itself depends on wages. In contrast, unions want wages to be as high as possible, being aware that a higher wage

employment is adjusted continuously. Thus whenever employment is changed, this change generally takes place in the context of a predetermined wage."

[26] Symeonidis (2008) assumed that firms may choose employment cooperatively, despite behaving non-cooperatively in the wage bargaining stage. Straume (2002) allowed unions to collude in order to obtain wages above the equilibrium level.

[27] Binmore et al. (1986) proved that the solution of the (static) Nash bargaining game coincides with the solution of a dynamic model of alternating offers. In the context of their model, differences in the bargaining power of the parties may result either from: (i) differences in the time elapsed since one party rejects the proposal of the other party and makes a counter-offer; or (ii) differences in the beliefs of the parties regarding the probability of not reaching an agreement.

will imply lower employment.

In the second-stage of the game, given wages, the union and the manager of firm $i \in \{H, L\}$ agree on the employment level that solves:

$$\max_{l_i} \ [(w_i - \bar{w}) \, l_i]^{\beta} \, [\pi_i \, (w_i, l_i, l_j)]^{1-\beta} \equiv NB_i^l \, (w_i, l_i, l_j) \, , \tag{5}$$

taking as given the employment decision of the rival firm $j \neq i$. Again, parameter $0 \leq \beta < 1$ denotes the relative bargaining power of the union over employment; while $1 - \beta$ is the bargaining power of the manager.[28]

This bargaining game captures some models in the literature, as shown in Table 1. The only labor market bargaining model (amongst the most standard) that is not captured as a particular case of our model is the simultaneous bargaining over employment and wages (McDonald and Solow, 1981). In order to fill this gap, we analyse this bargaining scheme in Appendix A.[29]

$\alpha = 0, \ \beta = 0$	Cournot model with asymmetric costs $c_i = \frac{\bar{w}}{e_i}$.
$\alpha = 1, \ \beta = 0$	Monopoly model (Oswald, 1982).
$\alpha \in [0, 1], \ \beta = 0$	Right-to-manage model (Nickell and Andrews, 1983).
$\alpha \in [0, 1], \ \beta \in [0, 1)$	Sequential bargaining model (Manning, 1987).

Table 1. Particular cases of the sequential bargaining model.

As pointed out by Ledvina and Sircar (2012), when two firms support very asymmetric marginal production costs, only the efficient firm survives and is active in the market. To prevent the exit of the least efficient firm from the market, we make the following assumption.

[28]As we shall see later, the limit case of $\beta = 1$ is excluded to ensure that the equilibrium employment is well-defined. Recall that we have assumed that there is neither labour disutility or a limited mass of workers. As a result, if workers had all the bargaining power in the employment negotiation, they would choose an infinite employment level, as long as the wage exceeded the reservation wage, $\bar{w}$.

[29]We decided not to present it in the main body of the article, since the analysis and the main results are similar to those obtained with our baseline sequential model.

Assumption 1. *The reservation wage is sufficiently low:*

$$\bar{w} < \frac{(1-\beta)(4+\alpha-2\beta)e_L}{2(2-\beta)^2-\alpha-(2-\alpha)(2-\beta)r_e} \equiv \bar{w}^d. \tag{6}$$

For higher values of $\bar{w}$, the least efficient firm does not find profitable to be active in the market. For much higher values, none of the firms would be active in the market.

3.3 Equilibrium

To solve our two-stage game, we use backward induction. Solving the first-order condition corresponding to the maximisation problem of firm $i \in \{H, L\}$ in the second-stage of the game, given in (5):[30]

$$\frac{\partial NB_i^l}{\partial l_i} = 0 \Leftrightarrow e_i - e_i e_j l_j - w_i - (2-\beta)e_i^2 l_i = 0 \Leftrightarrow l_i(l_j) = \frac{e_i - e_i e_j l_j - w_i}{(2-\beta)e_i^2}. \tag{7}$$

As expected, the higher is the wage paid by firm i (negotiated in the first-stage of the game), the lower is the employment level at this firm. The wage paid at the rival firm j has no direct impact on the employment level at firm i. We conclude also that the levels of employment are strategic substitutes, which is a standard result when firms choose the quantities to sell of a homogeneous good. This occurs because a reduction in the employment level (and, therefore, in the output) of firm j increases the price in the product market, which leads firm i to want to produce more (and, therefore, to hire more workers).

Combining the best-response functions of the two firms, we obtain the equilibrium employment at firm $i \in \{H, L\}$:

$$l_i(w_i, w_j) = \frac{(1-\beta)e_i e_j - (2-\beta)e_j w_i + e_i w_j}{(3-4\beta+\beta^2)\, e_i^2 e_j}. \tag{8}$$

Thus, despite w_j not having a direct impact on l_i, it has an indirect positive impact on l_i. More precisely, an increase in w_j leads firm j to decrease l_j, which makes profitable for firm i to increase l_i.

[30] Second-order conditions are checked in the proof of Proposition 1.

We now solve the first-stage of the game, corresponding to wage bargaining. Using expressions (8), we can rewrite the maximisation problem (4) in the following way:

$$\max_{w_i} \left\{ \frac{1}{1-\beta} \left[\frac{(1-\beta)e_ie_j - (2-\beta)e_jw_i + e_iw_j}{(3-\beta)e_i^2e_j} (w_i - \bar{w}) \right]^{\alpha} \left[\frac{(1-\beta)e_ie_j - (2-\beta)e_jw_i + e_iw_j}{(3-\beta)e_ie_j} \right]^{2(1-\alpha)} \right\}.$$

Solving the corresponding first-order condition, we obtain the best-response function of firm $i \in \{H, L\}$:

$$\frac{\partial NB_i^w}{\partial w_i} = 0 \;\Leftrightarrow\; w_i(w_j) = \frac{\alpha e_i}{2(2-\beta)e_j} w_j + \frac{\alpha(1-\beta)e_i}{2(2-\beta)} + \frac{(2-\alpha)}{2}\bar{w}.$$

We conclude, therefore, that wages are strategic complements given the endogenous employment decisions. From the best-reply function (7), we know that a higher wage at firm j leads this firm to produce less. As a result, an increase in w_j increases the price of the good in the final market. This implies that the revenue per labor unit in firm i will increase, and, consequently, the union in firm i will be able to bargain a higher wage.

Combining the best-reply functions of the two firms, we obtain the equilibrium wage at firm i:[30]

$$w_i^* = \frac{\alpha(1-\beta)}{4-\alpha-2\beta} e_i + \frac{(2-\alpha)(2-\beta)[\alpha e_i + 2(2-\beta)e_j]}{(4-\alpha-2\beta)\,(4+\alpha-2\beta)e_j} \bar{w} \tag{9}$$

Notice that equilibrium wages depend not only on how the bargaining power is split in the wage negotiation, α, but also on how it is split in the employment negotiation, β. This has to do with the fact that the employment level (affected by β) impacts wages. As expected, when the managers have all the bargaining power in the wage negotiation, $\alpha = 0$, they set wages equal to the reservation wage: $w_L^* = w_L^* = \bar{w}$.

Replacing expressions (9) in the labour demand curve (8), we obtain the equilibrium employment level at firm $i \in \{H, L\}$:

$$l_i^* = \frac{(2-\alpha)(2-\beta)}{(3-\beta)(4-\alpha-2\beta)e_i} + \frac{(2-\alpha)(2-\beta)\left\{(2-\alpha)(2-\beta)e_i - \left[2(2-\beta)^2 - \alpha\right] e_j\right\}}{(1-\beta)(3-\beta)\left[4(2-\beta)^2 - \alpha^2\right] e_i^2 e_j} \bar{w}, \tag{10}$$

with $j \neq i$.

Proposition 1. **(Duopoly equilibrium)** *Under Assumption 1, both firms are active in the market. The equilibrium levels for wages and employment are given by (9) and*

(10), respectively.

Proof. See Appendix B. □

Multiplying expression (10) by e_i, we obtain the equilibrium output of firm $i \in \{H, L\}$:

$$q_i^* = \frac{(2-\alpha)(2-\beta)\Big\{(1-\beta)(4+\alpha-2\beta)e_i e_j + \Big[(2-\alpha)(2-\beta)e_i - \big[2(2-\beta)^2-\alpha\big]\, e_j\Big]\bar{w}\Big\}}{(1-\beta)(3-\beta)(4+\alpha-2\beta)(4-\alpha-2\beta)e_i e_j}. \tag{11}$$

Substituting (8) and (9) in (1), we obtain the equilibrium profit of firm $i \in \{H, L\}$:

$$\pi_i^* = (1-\beta)q_i^{*2}. \tag{12}$$

Total output in equilibrium is equal to:[31]

$$Q^* = \frac{(2-\alpha)(2-\beta)\left[2e_H e_L - (e_H + e_L)\bar{w}\right]}{(3-\beta)(4-\alpha-2\beta)e_H e_L}. \tag{13}$$

It is easy to show that total output increases with the unions' bargaining power over employment, β, but decreases with their bargaining power over wages, α. This is a natural result since when unions have more power in the wage negotiation (i.e., α is greater), the equilibrium wages will be higher, firms will have higher marginal production costs and will, therefore, produce less output. In contrast, as unions' bargaining power over employment increases, the employment increases (at both firms), and total output also increases.

Finally, replacing expression (13) in (2), we obtain the expression for the (equilibrium) consumer surplus:

$$CS^* = \frac{(2-\alpha)^2(2-\beta)^2\left[2e_H e_L - (e_H + e_L)\bar{w}\right]^2}{2(3-\beta)^2(4-\alpha-2\beta)^2 e_H^2 e_L^2}. \tag{14}$$

Let π_i^* and WS_i^* the profit of firm $i \in \{H, L\}$ and the surplus of its workers in equilibrium. It is straightforward to show that the ratio between the worker surplus and the firm's

[31] Note that if Assumption 1 is satisfied, the inequality $2e_H e_L - (e_H + e_L)\bar{w} > 0$ is also satisfied.

profit is the same in both firms, since:

$$\frac{WS_i^*}{\pi_i^*} = \frac{\alpha(3-\beta)}{(2-\alpha)(2-\beta)}. \tag{15}$$

Not surprisingly, the higher is the unions' bargaining power when negotiating over wages or over employment, the higher is the ratio $\frac{WS_i^*}{\pi_i^*}$.

Combining (12) and (15), we obtain:

$$WS_i^* = \frac{\alpha(1-\beta)(3-\beta)}{(2-\alpha)(2-\beta)} q_i^{*2}. \tag{16}$$

3.4 Inter-firm comparison

In this section, we compare the equilibrium outcomes of the two firms.

Lemma 1. **(Wage comparison)** *The most efficient firm pays a higher wage than the least efficient firm, $w_H^* > w_L^*$, but incurs in a lower marginal production cost, $\frac{w_H^*}{e_H} \leq \frac{w_L^*}{e_L}$.*

Proof. See Appendix B. □

Lemma 1 can be restated as:

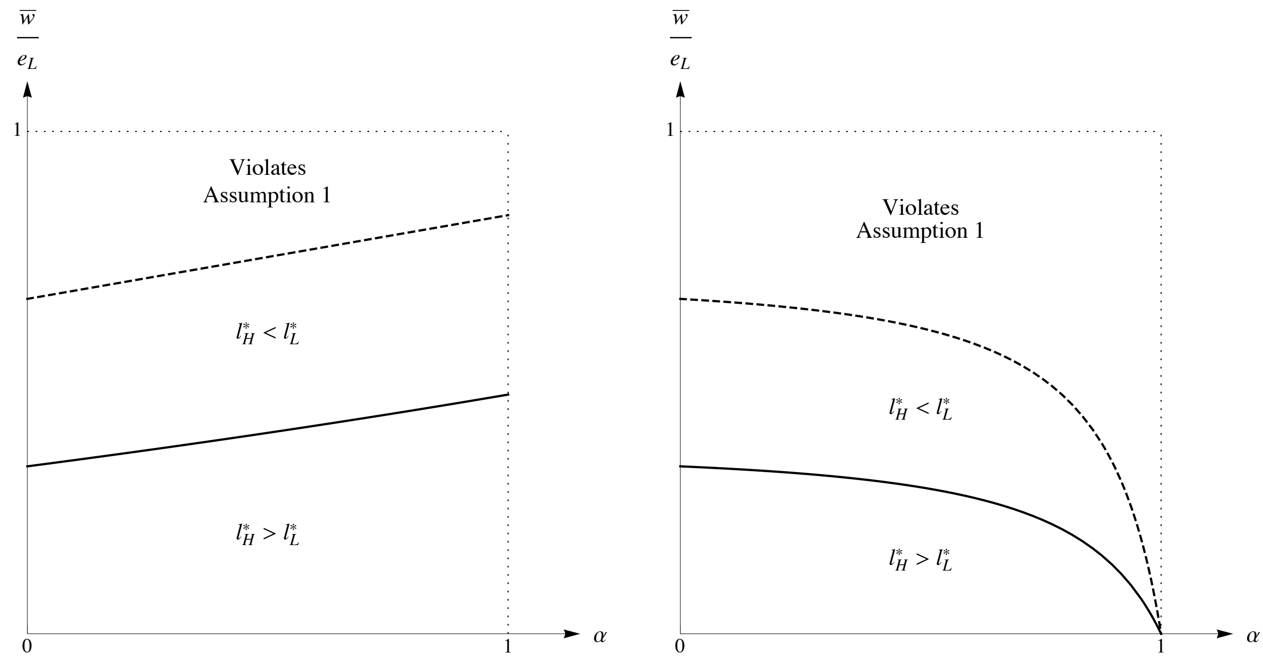
$$1 < \frac{w_H^*}{w_L^*} \leq \frac{e_H}{e_L},$$

which means that the asymmetry in (equilibrium) wages is lower than the asymmetry in efficiency levels. In other words, the bargaining in the labour market decreases the asymmetry between firms. However, looking at expressions (9), we conclude that this is due to the existence of a positive reservation wage, $\bar{w}$. If $\bar{w} = 0$, the ratios of wages and efficiency levels would be equal (i.e., $\frac{w_i^*}{w_j^*} = \frac{e_i}{e_j}$). Thus, the reservation wage works as a distortion force between the two firms that favours the least efficient firm.

In equilibrium, the most efficient firm supports a lower marginal production cost (Lemma 1). This may lead us to expect that this firm will hire more workers than the least efficient firm. However, this is not always the case.

Lemma 2. **(Employment comparison)** *The most efficient firm hires less workers than the least efficient firm if and only if the reservation wage is low enough (Figure 2):*

$$l_H^* < l_L^* \;\Leftrightarrow\; \frac{\bar{w}}{e_L} < \frac{(1-\beta)(4+\alpha-2\beta)}{\left[2(2-\beta)^2-\alpha\right](1+r_e)}. \tag{17}$$



(a) Right-to-manage: $\beta = 0$. **(b)** Symmetric bargaining power: $\alpha = \beta$.

Figure 2. Comparison of employment at the two firms when $e_H = 2e_L$.

Proof. See Appendix B. □

Even when hiring less workers, the most efficient firm produces more output, since:

$$q_H^* - q_L^* = \frac{(2-\alpha)(2-\beta)(e_H-e_L)}{(1-\beta)(4+\alpha-2\beta)e_H e_L}\bar{w} > 0.$$

Notice further that $\frac{\partial(q_H^*-q_L^*)}{\partial\bar{w}} > 0$. Thus, the higher is the reservation wage, $\bar{w}$, the higher is the difference between output of the two firms. This implies (since total output decreases with the reservation wage) that the market share of the most efficient firm is also higher.[32]

[32]See Lemma 13 in Appendix B.

The following Lemma conforms to a standard result of the Cournot model with asymmetric costs.

Lemma 3. **(Profit comparison)** *The most efficient firm profits more than the least efficient firm:* $\pi_H^* > \pi_L^*$.

Proof. Direct consequence from $\pi_i^* = (1-\beta)q_i^*$, for $i \in \{H, L\}$, and $q_H^* > q_L^*$. $\square$

Now, we compare the worker surplus at both firms. If the reservation wage is high enough, workers at firm H are surely better off than workers at firm L because the wage and the employment are the highest at firm H (Lemmata 1 and 2). When condition (17) holds, this comparison is no longer straightforward since the wage is higher at firm H but employment is higher at firm L. The following lemma states, however, that the wage-effect always more than compensates the employment-effect.

Corollary 1. **(Worker surplus comparison)** *Workers at the most efficient firm are better off than workers at the least efficient firm:* $WS_H^* > WS_L^*$.

Proof. Direct consequence of (15) and Lemma 3. $\square$

4 Welfare impact of a productivity shock

In this section, we study the impacts of a positive idiosyncratic productivity shock on the different economic agents (i.e., firms, workers and consumers). More precisely, we analyse how a technical innovation (translated into a decrease in the marginal cost) affecting one firm impacts on the equilibrium outcomes. We assume that the productivity shock has no direct impacts on the rival firm. We can think, for instance, about situations wherein the workers at one firm become more efficient (e.g., for attending to training courses, for the introduction of more efficient machines, the redesign of the production process within the firm) while, due to the absence of labor mobility, the rival firm's efficiency is not affected. If firms are established in different countries, this shock may be due to external factors to the industry (e.g., changes in regulatory environment, education policy, attribution/restriction of R&D subsidies, access to the credit market, etc.), in which case the idiosyncrasy of the shock is even more natural.

Lemma 4. **(Impact of a productivity shock on wages)** *If firm $i \in \{H, L\}$ increases its productivity, the wage at firm i increases while the wage at firm j decreases:*

$$\frac{\partial w_i^*}{\partial e_i} > 0 \quad \textit{and} \quad \frac{\partial w_j^*}{\partial e_i} < 0, \quad \textit{with } j \neq i.$$

Proof. See Appendix B. □

Thus, if a firm becomes more efficient, it will pay a higher wage to its workers. In addition, the increase in the efficiency of firm i decreases the marginal production cost of firm j (i.e., there is a positive spillover). The magnitude of this effect depends: (i) positively on the efficiency level of firm j; and (ii) negatively on the efficiency of firm i.

Lemma 5. **(Impact of a productivity shock on employment)**

For $i, j \in \{H, L\}$ and $j \neq i$:

1. *A positive productivity shock affecting firm i decreases employment at firm j:*

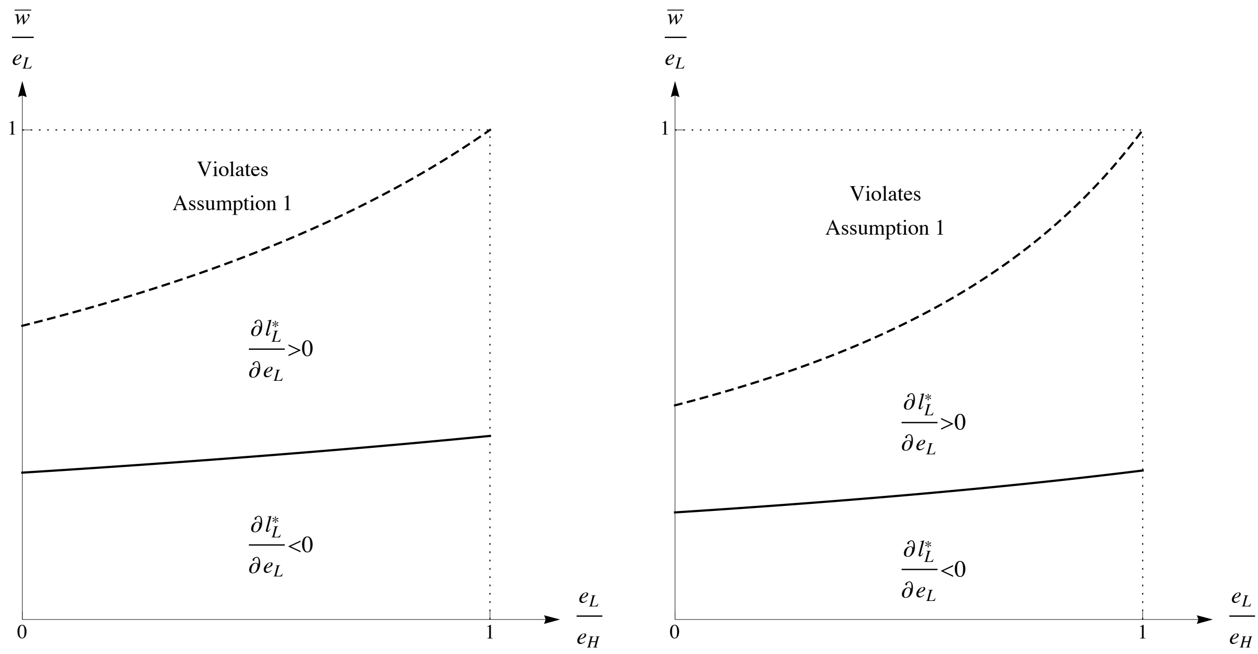
$$\frac{\partial l_j^*}{\partial e_i} < 0.$$

2. *A positive productivity shock affecting firm i decreases employment at firm i if and only if the reservation wage is sufficiently low (see Figures 3 and 4):*

$$\exists \tilde{w}_i < \bar{w}^d : \quad \frac{\partial l_i^*}{\partial e_i} < 0 \;\Leftrightarrow\; \bar{w} < \tilde{w}_i.$$

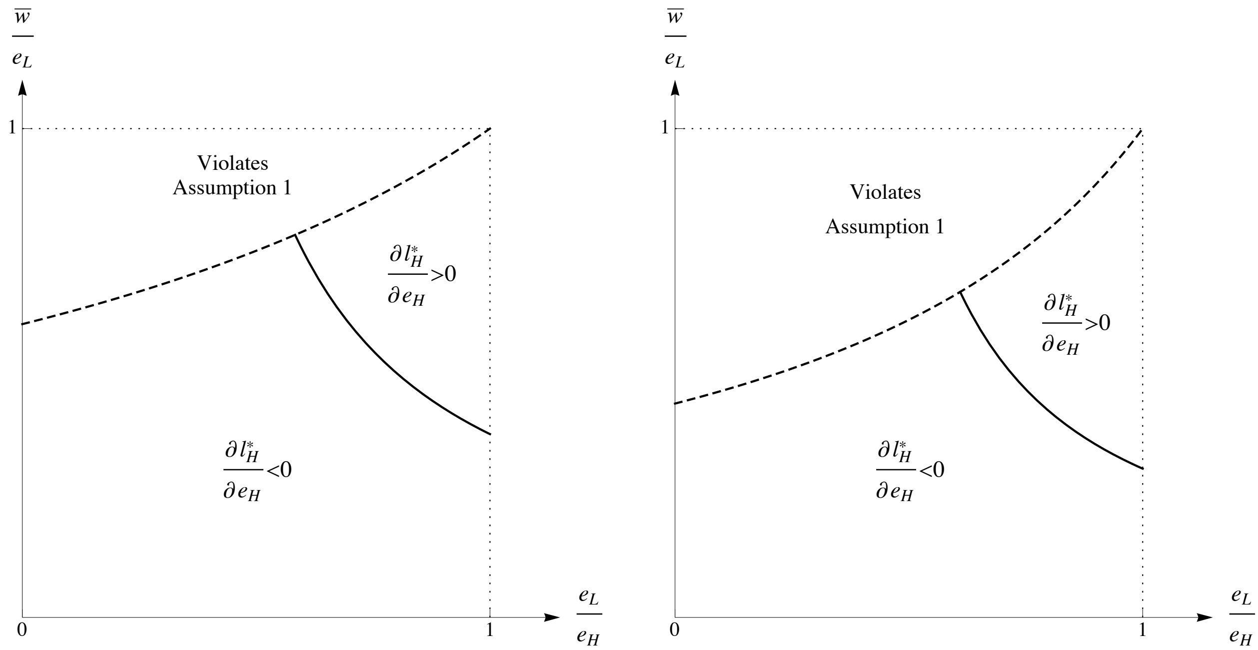
Proof. See Appendix B. □

When a firm becomes more efficient, it can produces the same output with lower workers. Thus, this direct effect may lead a firm to reduce the number of workers after becoming more efficient. However, when firms compete in quantities, they increase their output in response to a reduction in production costs (see the Lemma 6 and the subsequent discussion). If the productivity shock does not increase output too much, the firm will use less workers. If, instead, the productivity shock impacts output sufficiently, the existing workers may not be sufficient to expand production as much as desired by



(a) Right-to-manage: $\alpha = \frac{1}{2}$, $\beta = 0$. **(b)** Symmetric bargaining power: $\alpha = \beta = \frac{1}{2}$.

Figure 3. Impact of a positive productivity shock affecting firm L on the employment at firm L.



(a) Right-to-manage: $\alpha = \frac{1}{2}$, $\beta = 0$. **(b)** Symmetric bargaining power: $\alpha = \beta = \frac{1}{2}$.

Figure 4. Impact of a positive productivity shock affecting firm H on the employment at firm H.

the firm, and the firm hires more workers. Which of these cases end up occurring depends on the magnitude of the reservation wage relatively to the asymmetry between firms.

An efficiency gain by firm i always induces firm j to hire less workers because, no matter what happens to l_i, the output of firm i will increase and (as output decisions are strategic substitutes), firm j reduces production. As the productivity level of firm j is unchanged, the firm will necessarily need less workers to produce less output.

Lemma 6. **(Impact of a productivity shock on output)**

1. *A positive productivity shock affecting firm $i \in \{H, L\}$ increases the output of firm i and reduces output of firm $j \neq i$:*

$$\frac{\partial q_i^*}{\partial e_i} > 0 \quad \textit{and} \quad \frac{\partial q_j^*}{\partial e_i} < 0.$$

2. *A positive productivity shock affecting firm $i \in \{H, L\}$ increases total output:* $\frac{\partial Q^*}{\partial e_i} > 0$

Proof. See Appendix B. □

Thus, if the (positive) productivity shock affects firm H, the output gap between the two firms increases; while if it affects firm L, the output gap decreases. Dixit (1986) and Zhao (2001) have reached similar conclusions in the context of Cournot duopoly with asymmetric production costs. We prove, therefore, that their conclusions remain valid when there is bargaining in the labor market.

An idiosyncratic efficiency gain increases total output, since:

$$\frac{\partial Q^*}{\partial e_i} = \frac{(2-\alpha)(2-\beta)\bar{w}}{(3-\beta)(4-\alpha-2\beta)e_i^2} > 0, \tag{18}$$

which means that the expansion in the output of the firm that becomes more efficient outweighs the output contraction by the rival. As firms produce homogeneous goods, what matters for consumers is the (total) output level. Thus, the consumers always benefit from a technological improvement, whichever firm gains in efficiency.[33]

Corollary 2. **(Impact of a productivity shock on consumers)**

[33]Dixit (1986), Lahiri and Ono (1988) and Zhao (2001) have also concluded that, in an oligopoly with asymmetric production costs, a reduction in the marginal cost of any firm benefits consumers.

1. *If firm $i \in \{H, L\}$ becomes more efficient, consumers are better off:* $\frac{\partial CS^*}{\partial e_i} > 0$.
2. *An increase in the efficiency of firm L benefits more consumers than an increase in the efficiency of firm H:* $\frac{\partial CS^*}{\partial e_L} > \frac{\partial CS^*}{\partial e_H}$.

As we have seen before, a positive productivity shock affecting firm i decreases wage and employment at firm j. As a result, workers at firm j are worse off if firm i becomes more productive. Furthermore, if the reservation wage is sufficiently high, the wage and the employment at firm i increase if this firm becomes more efficient and, therefore, the workers at firm i become better off. When the reservation wage is low enough, it is not obvious whether a positive productivity shock affecting firm i may be beneficial or detrimental for its workers (on the one hand, the wage increases and, on the other hand, the employment decreases).

Lemma 7. (Impact of a productivity shock on workers)

For $i, j \in \{H, L\}$ and $j \neq i$:

1. *The more efficient is firm i, the higher is the surplus of its workers:* $\frac{\partial WS_i^*}{\partial e_i} > 0$.
2. *The more efficient is firm i, the lower is worker surplus at the rival firm:* $\frac{\partial WS_j^*}{\partial e_i} < 0$.
3. *A positive productivity shock affecting firm H increases the aggregate worker surplus:* $\frac{\partial WS^*}{\partial e_H} > 0$, *with* $WS^* = WS_H^* + WS_L^*$.
4. *A positive productivity shock affecting firm L increases the aggregate worker surplus if and only if the reservation wage is sufficiently low (See Figure 5):*

$$\exists \hat{w} < \bar{w}^d : \frac{\partial WS^*}{\partial e_L} > 0 \;\Leftrightarrow\; \bar{w} < \hat{w}$$

Proof. See Appendix B. □

A positive productivity shock is beneficial for workers at the firm that becomes more efficient but detrimental for workers at the rival firm. When the productivity shock affects the most efficient firm, the increase in worker surplus at the most efficient firm more than compensates the decrease at the least efficient firm, and the aggregate worker surplus increases. In contrast, when the productivity shock affects the least efficient firm, the aggregate worker surplus may increase or decrease.

Using the fact that the ratio between worker surplus and firm's profit, at firm $i \in \{H, L\}$, does not depend on e_i, we conclude that the impact of a productivity shock on profits is exactly the same as on worker surplus.

Corollary 3. **(Impact of a productivity shock on profits)**

For $i, j \in \{H, L\}$ and $j \neq i$:

1. *The more efficient firm i is, the higher is its individual profit and the lower is the profit of the rival: $\frac{\partial \pi_i^*}{\partial e_i} > 0$ and $\frac{\partial \pi_j^*}{\partial e_i} < 0$.*
2. *A positive productivity shock affecting firm H increases the industry profit: $\frac{\partial \Pi^*}{\partial e_H} > 0$, with $\Pi^* = \pi_H^* + \pi_L^*$.*
3. *A positive productivity shock affecting firm L increases the industry profit if and only if the reservation wage is sufficiently low (see Figure 5), i.e.:*

$$\exists \hat{w} < \bar{w}^d : \frac{\partial \Pi^*}{\partial e_L} > 0 \;\Leftrightarrow\; \bar{w} < \hat{w}$$

Proof. Follows straightforwardly from equality (15) and Lemma 7. □

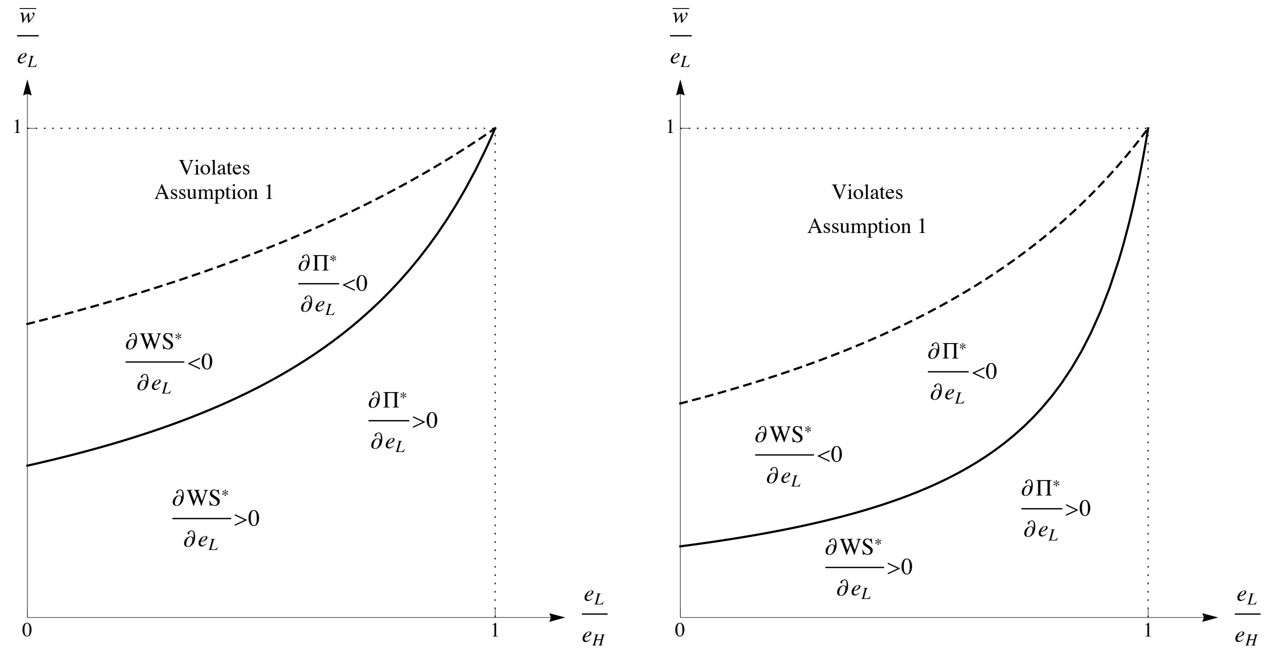
As a result of an idiosyncratic positive productivity shock, a firm expands production and gets a higher profit (while the opposite occurs at the rival firm). When it is the most efficient firm that becomes (even) more efficient, its gain more than compensates the loss of the least efficient firm, and the industry profit increases. However, it is the least efficient firm that improves its efficiency, the net impact on the industry profit depends on the magnitude of the reservation wage.

Lemma 8. **(Impact of a productivity shock on total surplus)**

1. *A positive productivity shock affecting firm H increases total surplus: $\frac{\partial TS^*}{\partial e_H} > 0$.*
2. *A positive productivity shock affecting firm L increases total surplus if and only if the reservation wage is sufficiently small (Figure 6):*

$$\exists \check{w} : \frac{\partial TS^*}{\partial e_L} > 0 \;\Leftrightarrow\; \bar{w} < \check{w}$$

Proof. See Appendix B. □



(a) Right-to-manage: $\beta = 0$.

(b) Symmetric bargaining power: $\alpha = \beta = \frac{1}{2}$.

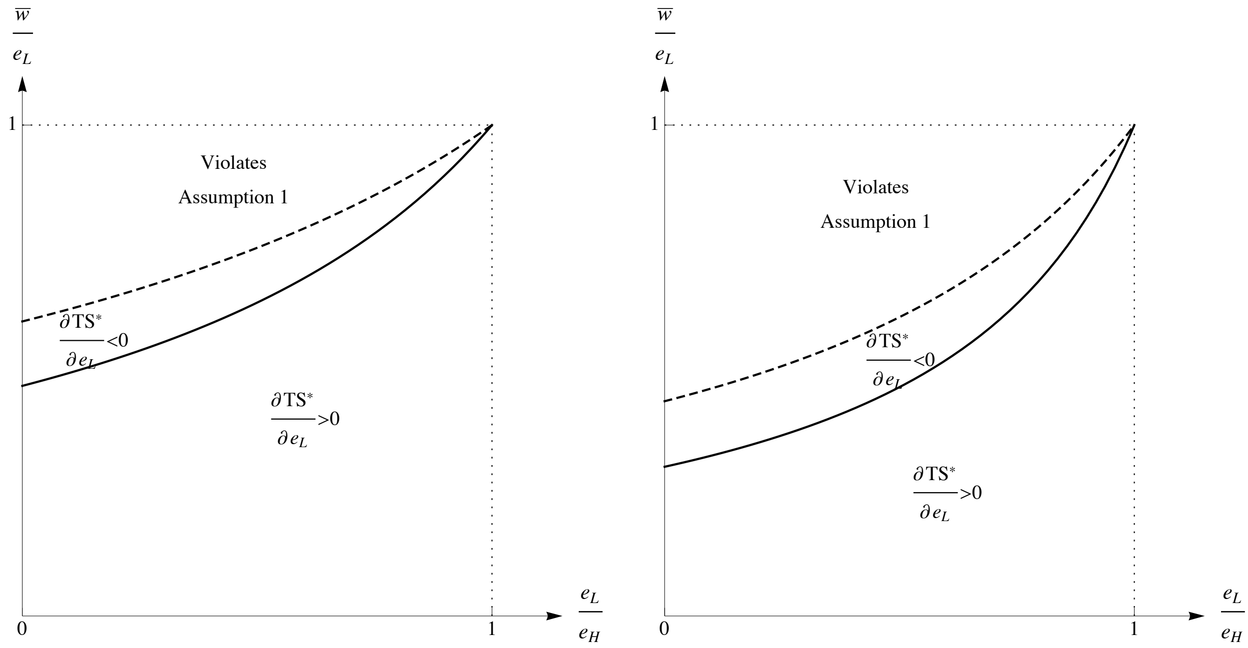
Figure 5. Impact of a positive productivity shock affecting firm L on the industry profit and on the aggregate worker surplus.

In Appendix A, we prove that these results remain qualitatively valid when wages and employment are bargained simultaneously, i.e., there is a single negotiation at each firm (McDonald and Solow, 1981).

Lahiri and Ono (1988), Zhao (2001), and Wang and Zhao (2007) have already found that an improvement in the technology of the least efficient firm may reduce total surplus. Intuitively, the reason is that this improvement leads the least efficient firm to produce more and obtain a higher market share. As a result, there is a shift in production from a more efficient to a less efficient firm, generating a loss of total surplus. Nevertheless, these contributions neglected the interactions in the labor market and, beforehand, it was not obvious whether their result would remain valid when considering the workers' surplus in the welfare of the economy. We contribute, therefore, to the literature on this aspect.

Lemma 9. **(Impact of unions' bargaining power on the social damages caused by a positive productivity shock affecting the least efficient firm)**

1. *The higher the unions' bargaining power over wages, the less likely is a reduction in total surplus as a result of an increase in* e_L.



(a) Right-to-manage: $\beta = 0$.

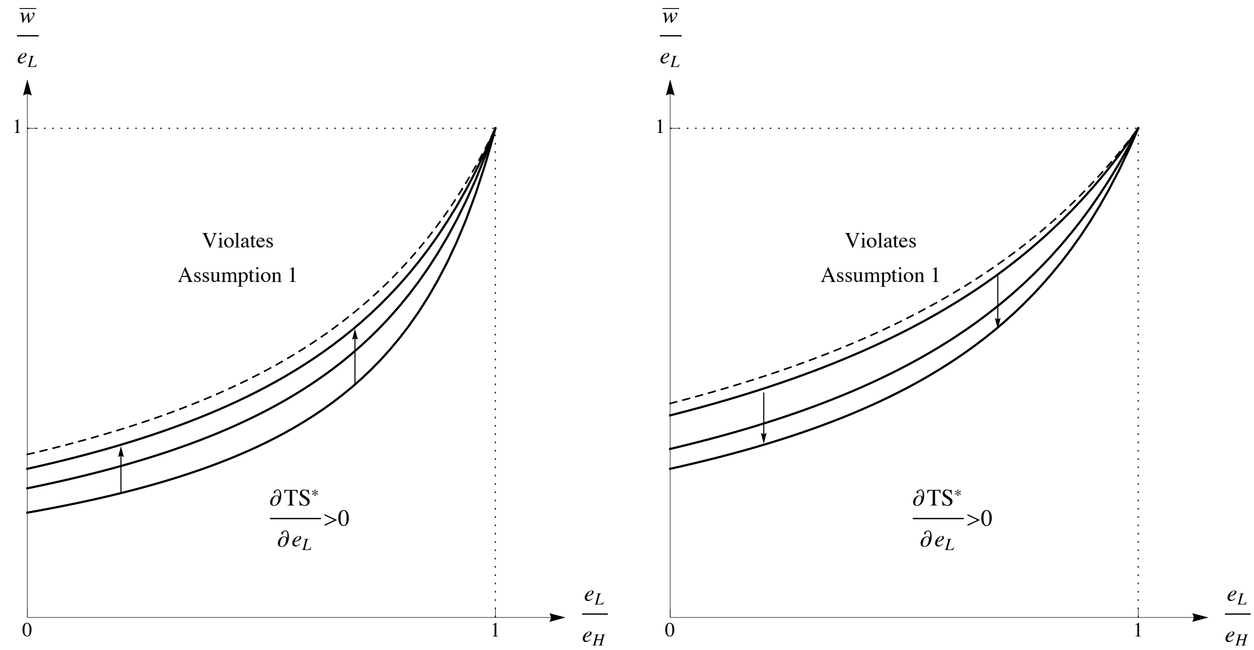
(b) Symmetric bargaining power: $\alpha = \beta = \frac{1}{2}$.

Figure 6. Impact of a positive productivity shock affecting by L on total surplus.

2. *The higher the unions' bargaining power over employment, the more likely is a reduction in total surplus as a result of an increase in* e_L.

Proof. See Appendix B. □

As illustrated in Figure 7, Lemma 9 states that an increase in α shifts the solid line in Figures 6 upwards; while an increase in β shifts the solid line downwards. Thus, the social welfare loss generated by an efficiency gain in firm L is: (i) the greatest when wages are unilaterally determined by firms and employment is unilaterally determined by unions (i.e., $\alpha = 0$ and $\beta \to 1$); and (ii) the smallest when unions choose wages, $\alpha = 1$, and firms choose employment, $\beta = 0$. As a result, the welfare loss estimated by Zhao (2001) in the context of Cournot competition without labor market modelling (i.e., when $\alpha = \beta = 0$) is an intermediate value for the loss when making the input prices endogenous. In other words, the existence of labor negotiations may amplify or reduce the welfare loss generated from the reduction in the costs of the least efficient firm.



(a) Increase in α (for $\beta = 0.5$). **(b)** Increase in β (for $\alpha = 0.5$).

Figure 7. Impact of increasing α and β on the region for which an efficiency gain by firm L is socially detrimental.

5 Model simulation

In this section, we present the results obtained from the calibration and simulation of our theoretical model. Due to the lack of data, we calibrate the model parameters for the case wherein the two competing firms are established in different countries. Owing to the absence of labor market data at the firm-level (e.g., reservation wage, unions' bargaining power, efficiency), we use average estimates at the country level. More precisely, we will use estimates for 1997 concerning an average firm in six different countries: Belgium (BEL), Canada (CAN), Netherlands (NLD), Portugal (PRT), Spain (ESP), and the United States of America (USA). Despite these data limitations, we believe that the simulation exercise is a good illustration of our theoretical results.

5.1 Calibration

We start by describing and justifying the calibration of our model parameters.

Reservation wage ($\bar{w}$)

Authors have adopted different measures of the workers' outside option, i.e., the monetary compensation that workers receive from being unemployed. Following Layard et al. (1991) and Dumont et al. (2006), we assume that the reservation wage is given by:[34]

$$\bar{w} = (1 - \phi u)w^e + \phi u b, \tag{19}$$

where: u denotes the unemployment rate, w^e the expected outside wage, b the unemployment benefit, and ϕ the employment-unemployment transition rate. Following Dumont et al. (2006), we proxy the expected outside wage, w^e, by the minimum wage in the country and assume $\phi = 0.2$. Table 2 sums up the values for these variables for the countries we are considering in 1997.

	Annual wages	Annual worked hours per worker	Hourly wage (w)	Minimum hourly wage (w^e)	Unemployment rate (u)	Replacement rate	Unemployment benefit (b)	Reservation wage ($\bar{w}$)
Belgium	43826	1572	27.88	10.49	0.13	0.40	11.06	10.49
Canada	36907	1785	20.68	7.08	0.09	0.15	3.08	7.08
Netherlands	45311	1480	30.62	9.98	0.06	0.52	15.97	9.98
Portugal	23827	1890	12.61	4.06	0.07	0.35	4.36	4.06
Spain	35406	1750	20.23	5.18	0.21	0.35	7.16	5.18
United States	46141	1846	25.00	7.20	0.05	0.14	3.47	7.20

Wages and worked hours are the averages per full-time and full-year worker in the economy. Wages are expressed in 2014 constant prices at 2014 USD PPPs (OECD Statistics). Unemployment rate is the ratio between unemployed people and the labour force, which is defined as the sum of unemployed and the people in paid or self-employment (OECD Statistics). The replacement rate corresponds to the fraction of the (gross) unemployment benefit over previous (gross) earnings (OECD Statistics). It could be preferable to use the net replacement rate. However, as far as we know, this variable is only available in the OECD statistics since 2001. Unemployment benefit is expressed in 2014 USD PPPs per hour and was obtained multiplying the replacement rate by the average hourly wage. The reservation wage was obtained by applying formula (19) with $\phi = 0.2$.

Table 2. Labor statistics (1997).

[34] Some authors proxied the reservation wage by the minimum wage (e.g., Veugelers, 1989; Brock and Dobbelaere, 2006). This is a particular case of (19) when $\phi = b = 0$ and w^e is the minimum wage. Differently, Bughin (1996) considered the reservation wage as *"the wage the unionized worker can receive outside the firm"* (p. 293), and computed it as one minus the unemployment rate (proxy of the probability of being employed) times the industry wage. Again, this is a particular case of (19) when $\phi = 1$, $b = 0$ and w^e is the industry wage.

Unions' bargaining power (α, β)

In the literature, there are several approaches to empirically measure the unions' bargaining power. As pointed out by Dowrick (1990), some initial approaches considered the union coverage as the main (or even unique) determinant of unions' bargaining power. However, according to the author, such an assumption *"ignores the complex range of factors which may in fact determine unions' ability to increase wages [...] Union coverage may be a pre-condition for wage-bargaining success, but organisational and market considerations are likely to be important too for both unions and employers."* Meanwhile, alternative and more robust approaches have been adopted.[35] Botero et al. (2004) obtained estimates for the unions' bargaining power by building averages of dummy variables that equal one if the labor market exhibits some characteristic that may influence the unions' bargaining power. We use the estimates obtained by Botero et al. (2004) to calibrate our model parameters regarding the unions' bargaining power.[36] We have chosen to use their estimates, since they were obtained taking into account an extensive number of characteristics of the labor market that may influence the way how wages and employment are determined. In addition, Botero et al. (2004) provided estimates for a big range of countries in 1997. Table 3 presents rounded estimates obtained by the authors for the unions' bargaining power (in our set of countries we are considering).

Before proceeding, we would like to refer some difficulties that we faced when trans-

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[35] Layard and Nickell (1986) measured the unions' bargaining power based on the difference between the wages of unionised and non-unionised workers. Veugelers (1989) and Dowrick (1990) adopted a similar approach, by proxying the unions' bargaining power by the premium of (average) wages over the reservation wage (after adjusting for other workers' characteristics that may influence the wage premium and that are not related to differences in bargaining power). Differently, Bughin (1993) estimated the unions' bargaining power in a production function framework. His approach has the advantage of allowing to statistically test the validity of different models of union behaviour.

[36] More precisely, we use the estimates that the authors obtained for a variable called *Collective Relations Laws Index*. This measure is obtained by averaging two other variables: *Labor Union Power* and *Collective Disputes*. The variable *Labor union power* is obtained by averaging of 7 dummy variables, which take value 1: *"(1) if employees have the right to unionise, (2) if employees have the right to collective bargaining, (3) if employees have the legal duty to bargain with unions, (4) if collective contracts are extended to third parties by law, (5) if the law allows closed shops, (6) if workers, or unions, or both have a right to appoint members to the Boards of Directors, and (7) if workers' councils are mandated by law."* (Botero et al., 2004, Table 1, p. 1349). Similarly, the variable *Collective Disputes* is obtained by averaging 8 dummy variables *"which equal one: (1) if employer lockouts are illegal, (2) if workers have the right to industrial action, (3) if wildcat, political, and sympathy/solidarity/secondary strikes are legal, (4) if there is no mandatory waiting period or notification requirement before strikes can occur, (5) if striking is legal even if there is a collective agreement in force, (6) if laws do not mandate conciliation procedures before a strike, (7) if third-party arbitration during a labor dispute is mandated by law, and (8) if it is illegal to fire or replace striking workers."*

Country	Belgium	Canada	Netherlands	Portugal	Spain	United States
Unions' bargaining power	0.42	0.20	0.46	0.65	0.59	0.26

Table 3. Unions' bargaining power in 1997 (Botero et al., 2004).

lating these estimates into our model. First, our model considers the interaction between two firms within the same industry, while Botero et al. (2004) provided estimates at the country level. As pointed out by several authors, empirical evidence suggests the existence of variation in the unions' bargaining power across industries (see, e.g., Svejnar, 1986; Veugelers, 1989; Dobbelaere, 2004; Dumont et al., 2006).[37] Despite being aware of this limitation, the lack of estimates at the industry-level makes the estimates obtained by Botero et al. (2004) the best fit to our calibration exercise.[38] Second, Botero et al. (2004) obtained an unique estimate for the unions' bargaining power, while, in our theoretical model, unions may have different bargaining power over wages and employment. Empirical evidence suggests that unions tend to attach less weight to employment than to wages (e.g., Andrews and Harrison, 1998). When translated to our model, this means that $\beta < \alpha$. Nevertheless, Dumont et al. (2006) found evidence that, despite unions tending to weight more gains in wages than in employment, *"employment or working conditions are generally part of the bargaining process"* (p. 94). Several other empirical contributions also rejected the hypothesis of employment being unilaterally determined by firms (see, e.g., MaCurdy and Pencavel, 1986; Bean and Turnbull, 1988). Alogoskoufis and Manning (1991) empirically rejected *"both the labour demand model and the hypothesis that wage employment bargains are efficient, [being] in favour of a generalised model of inefficient bargaining for wages and employment."* (p. 23) Again, the translation of this evidence into our model would be that $\beta \neq 0$. Taking this into account, we decided to simulate our model for two extreme scenarios (regarding the union's bargaining power over employment): (i) the case in which unions have equal bargaining power over wages and employment, i.e., $\alpha = \beta$; and (ii) the case in which unions have no bargaining power over the employment, i.e., $\beta = 0$).

[37]Veugelers (1989) found empirical evidence that, even within the same industry, there may be dispersion in the bargaining power (e.g., between blue and white collar workers).

[38]Actually, Dumont et al. (2006) estimated the bargaining power at the sector level for 5 EU countries (Germany, France, Italy, Belgium, and the UK). However, the authors obtained average estimates for the time period between 1994 and 1998. Thus, the adoption of their estimates would imply an average of the remaining parameters over that time period. Between averaging over years and averaging over sectors, we considered that the second option is more suitable for our purpose.

Efficiency levels (e_H, e_L)

To finish our calibration exercise, we need estimates for firms' efficiency levels, e_i with $i \in \{H, L\}$. The values presented in Table 4 correspond to the OECD estimates for GDP per hour worked, measured in USD at current prices and current PPPs.

	Canada	Belgium	Netherlands	Portugal	Spain	United States
GDP per hour worked	29.9	40.3	35.4	17.0	28.3	35.9

Table 4. GDP per hour worked (OECD Statistics).

In the theoretical model, we assumed that, despite firms being different in terms of efficiency, the unions' bargaining power and the reservation wage were equal at both firms. Under these assumptions, estimates for the model parameters suggest the division of our set of countries in three groups: Belgium/Netherlands, Canada/United States, and Spain/Portugal (Table 5).

	Country	$\bar{w}$	Unions' bargaining power	e_i	e_H/e_L
Group I	United States	7.14	0.26	35.9	1.20
	Canada			29.9	
Group II	Belgium	10.24	0.44	40.3	1.14
	Netherlands			35.4	
Group III	Spain	4.62	0.62	28.3	1.66
	Portugal			17.0	

Reservation wages and unions' bargaining power correspond to the average of the corresponding values for the two countries in each group.

Table 5. Calibrated parameters of the model.

5.2 Results

Tables 6 and 7 present the impact of a 5% idiosyncratic positive productivity shock on the equilibrium outcomes. Results in Table 6 were obtained assuming $\alpha = \beta$ and both equal to the estimate for the union's bargaining power presented in Table 5); while results in Table 7 were obtained assuming $\beta = 0$.

	w_{USA}	w_{CAN}	l_{USA}	l_{CAN}	CS	π_{USA}	π_{CAN}	Π	WS_{USA}	WS_{CAN}	WS	TS
$\uparrow e_{USA}$	1.51%	-0.22%	-2.53%	-1.44%	1.22%	4.75%	-2.87%	1.56%	4.75%	-2.87%	1.56%	1.38%
$\uparrow e_{CAN}$	-0.30%	1.29%	-1.47%	-1.60%	1.46%	-2.92%	6.75%	1.13%	-2.92%	6.75%	1.13%	1.30%

(a) USA *vs.* Canada

	w_{BEL}	w_{NLD}	l_{BEL}	l_{NLD}	CS	π_{BEL}	π_{NLD}	Π	WS_{BEL}	WS_{NLD}	WS	TS
$\uparrow e_{BEL}$	1.90%	-0.39%	-1.52%	-2.21%	1.67%	6.93%	-4.37%	2.21%	6.93%	-4.37%	2.21%	1.91%
$\uparrow e_{NLD}$	-0.47%	1.72%	-2.13%	-0.40%	1.90%	-4.22%	9.37%	1.46%	-4.22%	9.37%	1.46%	1.70%

(b) Belgium *vs.* Netherlands

	w_{ESP}	w_{PRT}	l_{ESP}	l_{PRT}	CS	π_{ESP}	π_{PRT}	Π	WS_{ESP}	WS_{PRT}	WS	TS
$\uparrow e_{ESP}$	2.83%	-0.38%	-2.92%	-2.03%	1.00%	3.91%	-4.02%	1.97%	3.91%	-4.02%	1.97%	1.41%
$\uparrow e_{PRT}$	-0.77%	2.05%	-1.92%	0.63%	1.66%	-3.81%	11.65%	-0.03%	-3.81%	11.65%	-0.03%	0.95%

(c) Spain *vs.* Portugal

Table 6. Impact of a 5% positive idiosyncratic productivity shock if $\alpha = \beta$.

	w_{USA}	w_{CAN}	l_{USA}	l_{CAN}	CS	π_{USA}	π_{CAN}	Π	WS_{USA}	WS_{CAN}	WS	TS
$\uparrow e_{USA}$	1.62%	-0.19%	-2.80%	-1.06%	1.22%	7.97%	-2.11%	1.45%	4.17%	-2.11%	1.45%	1.34%
$\uparrow e_{CAN}$	-0.25%	1.40%	-1.11%	-2.06%	1.46%	-2.22%	5.75%	1.23%	-2.22%	5.75%	1.23%	1.34%

(a) USA *vs.* Canada

	w_{BEL}	w_{NLD}	l_{BEL}	l_{NLD}	CS	π_{BEL}	π_{NLD}	Π	WS_{BEL}	WS_{NLD}	WS	TS
$\uparrow e_{BEL}$	2.13%	-0.28%	-2.22%	-1.24%	1.67%	5.42%	-2.46%	1.93%	5.42%	-2.46%	1.93%	1.82%
$\uparrow e_{NLD}$	-0.34%	1.95%	-1.26%	-1.51%	1.90%	-2.50%	6.95%	1.68%	-2.50%	6.95%	1.68%	1.77%

(b) Belgium *vs.* Netherlands

	w_{ESP}	w_{PRT}	l_{ESP}	l_{PRT}	CS	π_{ESP}	π_{PRT}	Π	WS_{ESP}	WS_{PRT}	WS	TS
$\uparrow e_{ESP}$	3.24%	-0.22%	-3.45%	-0.70%	1.00%	2.77%	-1.40%	1.31%	2.77%	-1.40%	1.31%	1.19%
$\uparrow e_{PRT}$	-0.43%	2.56%	-0.86%	-1.79%	1.66%	-1.71%	6.33%	1.12%	-1.71%	6.33%	1.12%	1.32%

(c) Spain *vs.* Portugal

Table 7. Impact of a 5% positive idiosyncratic productivity shock if $\beta = 0$.

Comparing Tables 6 and 7, the first conclusion we can draw is that the qualitative impact of a productivity shock on the equilibrium is the same for the three pairs of countries and does not depend on whether or not unions bargain over employment.[39]

We obtained that a positive productivity shock affecting firm $i \in \{H, L\}$ increases the wage at firm i but decreases the wage at firm $j \neq i$ (which confirms the general statements in Lemma 4). We have also obtained that the wage increase at the firm i is notably higher (in absolute value) than the wage decrease at firm j.

In all simulated scenarios, a positive efficiency shock affecting the most efficient firm decreases the employment at both firms (corresponding to the region of Figure 4 where $\frac{\partial l_H^*}{\partial e_H} < 0$). With the exception of the Portuguese firm when $\alpha = \beta$, employment also decreases if the least efficient firm becomes more efficient (corresponding to region where $\frac{\partial l_L^*}{\partial e_L} < 0$ of Figure 3).

Regarding profits, our simulation exercise illustrates that a positive productivity shock increases the profit of the firm that becomes more efficient but decreases the profit of the rival (as proved, in general, in Corollary 3). Except for the case of competition between a Portuguese and a Spanish firms and $\alpha = \beta$,[40] we obtained that a positive productivity shock increases the industry profit whichever firm becomes more efficient (corresponding to the region of Figure 5 where $\frac{\partial \Pi^*}{\partial e_L} > 0$).

As expected from equality (15), the impact of a productivity shock on workers of firm i coincides with the impact on the profit of this firm. And, as a result, the impact on the aggregate worker surplus is the same as on the industry profit.

Tables 6 and 7 also suggest that a productivity shock (whichever the firm is affected) is beneficial for consumers (as proved, in general, in Lemma 2). Interestingly, the impact of a productivity shock on consumer surplus does not seem to depend on whether the unions directly affect the employment choice.

Finally, we obtained that total surplus increases in response to a positive productivity shock affecting the most efficient firm (as proved, in general, in Lemma 8) or the least efficient firm (corresponding to the region where $\frac{\partial TS^*}{\partial e_L} > 0$ of Figure 6). Our results

[39] The only exceptions occur for the case of Portugal/Spain if $\alpha = \beta$.

[40] In this case, the significant increase in the profit of the Portuguese firm (if becoming 5% more efficient) was not sufficient to outweigh the profit loss of the Spanish firm. This is due do to the fact that the two firms are very asymmetric in terms of productivity (recall that $\frac{eESP}{ePRT} = 1.66$), which generates a great asymmetry in market shares (respectively: 63.7% *vs.* 36.3%, before the shock; and 62% *vs.* 38%, after the shock).

suggest further that if the productivity shock affects the least (resp. most) efficient firm, the increase in consumer surplus is higher (resp. lower) than the increase in worker surplus or in the industry profit. Finally, when $\alpha = \beta$, a positive productivity shock increases more the overall welfare if affecting the most efficient firm; while, when $\beta = 0$, the overall welfare increases more if the productivity shock affects the least efficient firm.

6 Conclusions

We built a theoretical model to study how asymmetries in firms' efficiency level affect workers, firms and consumers. We considered that two firms compete to sell an homogeneous good. Labour is the only factor of production, but firms differ in their productivity. Wages and employment result from sequential bargaining at the firm-level, i.e., between the manager and the workers at each firm.

We found that the most efficient firm pays a higher wage but profits more than the least efficient firm (due to its efficiency advantage). The most efficient firm produces more output even when hiring less workers (again, due to its efficiency advantage). Interestingly, this difference is the greater the higher is the reservation wage. Workers at the most efficient firm are better off than those working at the least efficient firm, since differences in wages more than compensate potentially lower employment level.

We have also studied the welfare impacts of an idiosyncratic productivity shock. Consumers always benefit from an efficiency gain, no matter whether it is the most efficient firm or the least efficient firm that becomes more efficient. This occurs because an efficiency gain always increases aggregate output level, which is the only issue that matters for consumers, since products are homogeneous. Workers at a given firm become better off in response to a productivity shock if and only if the profit of this firm increases. Thus, when the most efficient firm increases its productivity: the aggregate worker surplus and the industry profit increase (i.e., the increase at the most efficient firm more than compensates the decrease at the least efficient firm); and total surplus increases. When the least efficient firm increases its productivity: (i) the aggregate worker surplus and the industry profit may increase or decrease; and total surplus may increase or decrease (due to the transfer of production from a more efficient to a less efficient firm, which increases the average production cost).

In future research, we would like to extend this contribution in several directions. In particular, we would like to make the efficiency levels endogenous, e.g., by adding an initial stage wherein firms invest in R&D. Firms will choose different efficiency levels if they are subject to different market conditions, for example as those suggested in the empirical surveys of Bartelsman and Doms (2000) and Syverson (2011).

A Simultaneous bargaining

Suppose now that the employment and the wage levels at each firm result from a single and simultaneous negotiation (McDonald and Solow, 1981). More precisely, they are the solution of the following maximisation problem:

$$\begin{aligned}\max_{(l_i,w_i)} \ & \left[(w_i-\bar{w})\,l_i\right]^{\theta}\left[\pi_i\left(w_i,l_i,l_j\right)\right]^{1-\theta} \\ &= l_i(w_i-\bar{w})^{\theta}\left[(1-e_il_i-e_jl_j)\,e_i-w_i\right]^{1-\theta}\equiv NB_i^s\left(w_i,l_i,l_j\right),\end{aligned} \tag{20}$$

where $0\leq\theta<1$ measures the relative bargaining power of unions. As in the baseline model, we make the following assumption to ensure that both firms are active in the market.

Assumption 2. *The reservation wage is sufficiently low:* $\bar{w}<\frac{e_He_L}{2e_H-e_L}\equiv\bar{w}^{ds}$.

Assumption 2 is weaker than Assumption 1, since the upper bound that it imposes for $\bar{w}$ is higher:[41]

$$\bar{w}^{ds}>\bar{w}^{d} \ \Leftrightarrow \ \frac{\left[2(2-\beta)^2-\alpha\right]e_L(e_H-e_L)}{\left[2(2-\beta)^2-\alpha\right]e_H-(2-\alpha)(2-\beta)e_L}>0.$$

In other words, the maximum reservation wage for the least efficient firm to be active in the market under simultaneous bargaining, $\bar{w}^{ds}$, is higher than under sequential bargaining, $\bar{w}^{d}$ given in (6). As a result, in the following comparisons of the outcomes of the sequential and the simultaneous bargaining we will impose that Assumption 1 is satisfied.

We have also concluded that are a range of parameters (i.e., for $\bar{w}^{d}<\bar{w}<\bar{w}^{ds}$) for which the least efficient firm is not active in the market if there is sequential bargaining over wages and employment, but is active if there is simultaneous bargaining.[42]

[41] In the proof of Proposition 1 in Appendix B, we prove that $\left[2(2-\beta)^2-\alpha\right]e_H-(2-\alpha)(2-\beta)e_L>0$.

[42] *A priori*, it is not obvious which scenario is socially preferable. On the one hand, under simultaneous bargaining, we would have two firms (instead of one) active in the market. On the other hand, production would be borne by a more efficient (i.e. a low-cost) firm. Such an analysis would be interesting but is out of the scope of this work.

Proposition 2. **(Simultaneous bargaining equilibrium)** *If $\bar{w} < \bar{w}^{ds}$, the equilibrium wage and employment at firm $i \in \{L, H\}$ with simultaneous bargaining are:*

$$w_i^s = \frac{\theta e_i}{3} + \frac{(3-2\theta)e_j + \theta e_i}{3e_j}\bar{w} \quad \textit{and} \quad l_i^s = \frac{1}{3e_i} + \frac{e_i - 2e_j}{3e_i^2 e_j}\bar{w}. \tag{21}$$

Proof. The best-reply functions of firm $i \in \{H, L\}$ obtained from the first-order conditions of the maximisation problem (20) are:

$$l_i(l_j, w_i) = \frac{(1 - e_j l_j)\, e_i + w_i}{(2-\theta)e_i^2}$$

$$w_i(l_i, l_j) = (1-\theta)\bar{w} - \theta(1 - e_i l_i - e_j l_j)e_i$$

Combining them with the best-reply function of firm j, we obtain expressions (21). In equilibrium, firm L is active in the market iff:

$$l_L^s > 0 \;\Leftrightarrow\; \bar{w} < \frac{e_H e_L}{2e_H - e_L} \;\;\Leftrightarrow\;\; \frac{\bar{w}}{e_L} < \frac{1}{2 - r_e}, \tag{22}$$

with $r_e = \frac{e_L}{e_H}$. It is straightforward that if condition (22) is satisfied, we have $l_H^s > 0$. $\square$

Notice that the employment level does not depend on the bargaining power of the parties, θ. Actually, the output levels coincide with the equilibrium values under Cournot competition with asymmetric production costs equal to $\frac{\bar{w}}{e_i}$. This occurs because, under simultaneous bargaining, at each firm, both the union and the manager aim at maximising (total) surplus. It is the parameter θ that, then, determines how this joint surplus is shared between the parties: the higher is θ, the higher is the wage and, therefore, the better off is the union.

As in the sequential bargaining game, in equilibrium, the ratio between worker surplus and profit at firm $i \in \{H, L\}$ does not depend on e_i. More precisely, the share that workers at firm i receive of the surplus generated by the activity of firm i is equal to the unions' bargaining power (and, therefore, is the same in the two firms): $\frac{WS_i^s}{WS_i^s + \pi_i^s} = \theta$. This is a property of the Nash bargaining solution, according to which the surplus generated from the agreement is divided proportionally to the bargaining power of the involved parties. Thus, when unions set wages and employment (i.e., $\theta = 1$), workers get all the surplus generated from the firms' activity; while, when firms unilaterally set wages and employment (i.e., $\theta = 0$), firms get all the generated surplus.

Manning (1987) proved that, when there is a monopoly in the product market, simultaneous bargaining (over employment and wage) is equivalent to sequential bargaining if unions and firms have the same bargaining power in the two negotiations, i.e., $\alpha = \beta$. However, when replacing $\alpha = \beta = \theta$ in (9) and (10), we obtain:

$$w_i^* = \frac{\theta(1-\theta)e_i}{4-3\theta} + \frac{(2-\theta)^2[2(2-\theta)e_j + \theta e_i]}{(4-\theta)(4-3\theta)e_j}\bar{w}, \tag{23}$$

$$l_i^* = \frac{(2-\theta)^2}{(3-\theta)(4-3\theta)e_i} + \frac{(2-\theta)^2\left[e_i(2-\theta)^2 - e_j\left(8-9\theta+2\beta^2\right)\right]}{(1-\theta)(3-\theta)(4-\theta)(4-3\theta)e_i^2 e_j}\bar{w},$$

which are different from (21). As stated by Petrakis and Vlassis (2000) and Dhillon and Petrakis (2002), in the case of duopoly, the sequential and simultaneous bargaining no longer generate the same outcome due to the existence of strategic effects. For instance, when bargaining over the wage, a firm-union unit takes into account not only the impact on the employment at that firm, but also the impact on the wage and employment at the rival firm.

Lemma 10. **(Wage comparison)** *The wage at firm $i \in \{H, L\}$ under simultaneous bargaining is higher than under sequential bargaining (with $\alpha = \beta = \theta$), i.e., $w_i^s > w_i^*$.*

Proof. See Appendix B. □

The previous result is quite natural since, in the simultaneous bargaining, firms and unions do not internalise the response of their rival to the wage they set (as they do in the case of sequential bargaining).

Using expressions (23) and (21), we obtain that the advantage of the most efficient firm (over the least efficient firm) in terms of marginal production cost is higher under sequential bargaining than under simultaneous bargaining, since:

$$\left(\frac{w_L^*}{e_L} - \frac{w_H^*}{e_H}\right) - \left(\frac{w_L^s}{e_L} - \frac{w_H^s}{e_H}\right) = \frac{\theta\bar{w}(e_H - e_L)}{(4-\theta)e_H e_L} > 0.$$

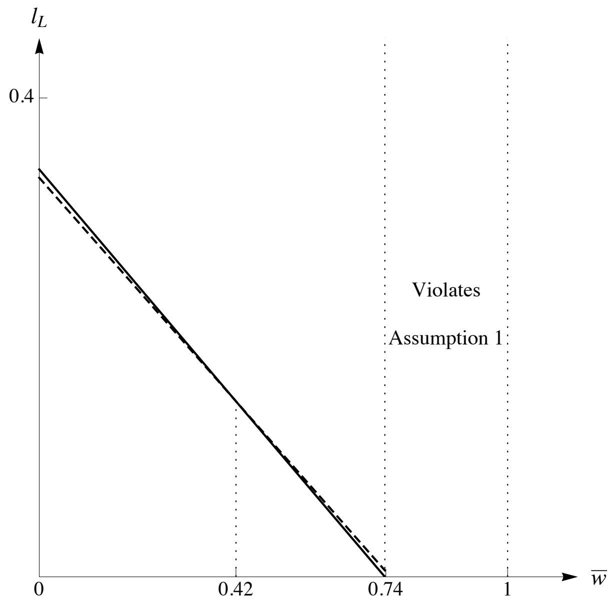
This provides a rationale for the existence of a region of parameters for which the least efficient firm is active under simultaneous bargaining but inactive under sequential bargaining.

Lemma 11. **(Employment comparison)**

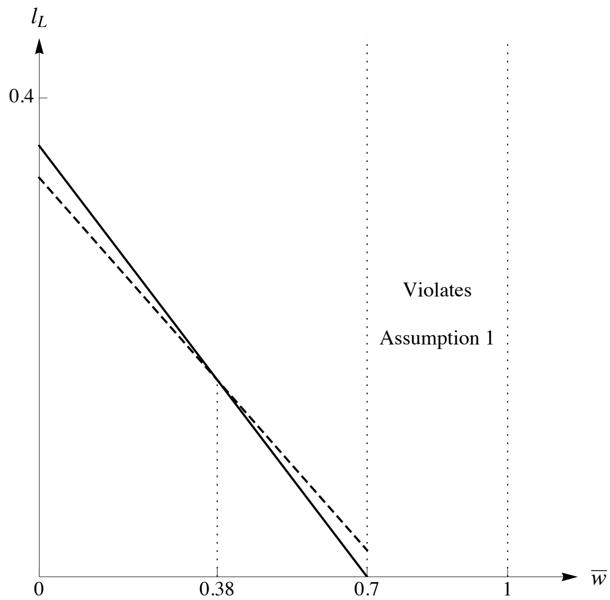
1. *The employment at the most efficient firm is higher under sequential bargaining (with $\alpha = \beta = \theta$) than under simultaneous bargaining: $l_H^* > l_H^s$.*
2. *The employment at the least efficient firm is higher under sequential bargaining (with $\alpha = \beta = \theta$) than under simultaneous bargaining if and only if the reservation wage is sufficiently small (see Figure 8):*

$$\exists \tilde{w} > 0 : l_L^* > l_L^s \Leftrightarrow \bar{w} < \tilde{w}.$$

Proof. See Appendix B. □



(a) $\alpha = \beta = \theta = 0.2$



(b) $\alpha = \beta = \theta = 0.5$

Figure 8. Comparison of employment at firm L under simultaneous (dashed line) and sequential (solid line) bargaining, when $e_L = 1$ and $e_H = 1.5$.

As the wage at firm $i \in \{H, L\}$ is higher with simultaneous bargaining than with sequential bargaining, it is not surprising that firm H hires less workers when bargaining is simultaneous. What is a bit counterintuitive is the fact of the same might not be true regarding the least efficient firm. Notice, however, that: as the most efficient also pays a lower wage when the bargaining is sequential, it also produces more output with this bargaining scheme. Thus, when the bargaining is sequential, the least efficient firm pays a lower wage (which may induce it to hire more workers) but competes against a more aggressive rival (and, as quantities are strategic substitutes, the least efficient firm may contract output). Lemma 11 states that each of these opposite forces may dominate.

Lemma 12. **(Welfare impacts of a positive productivity shock)**

1. *A positive productivity shock affecting firm H increases the consumer surplus, the aggregate workers surplus and the industry profit:*

$$\frac{\partial CS^s}{\partial e_H} > 0, \qquad \frac{\partial WS^s}{\partial e_H} > 0 \qquad \textit{and} \qquad \frac{\partial \Pi^s}{\partial e_H} > 0.$$

2. *A positive productivity shock affecting firm L always increases consumer surplus. It also increases the aggregate worker surplus and the industry profit if and only if the reservation wage is sufficiently small:*

$$\frac{\partial CS^s}{\partial e_L} > 0 \qquad \textit{and} \qquad \left.\begin{array}{l} \frac{\partial WS^s}{\partial e_L} > 0 \\ \frac{\partial \Pi^s}{\partial e_L} > 0 \end{array}\right\} \Leftrightarrow \bar{w} < \frac{e_H e_L}{5e_H - 4e_L}.$$

3. *A positive productivity shock affecting firm H always increases total surplus, while affecting by firm L may or may not increase total surplus:*

$$\frac{\partial TS^s}{\partial e_H} > 0 \qquad \textit{and} \qquad \frac{\partial TS^s}{\partial e_L} > 0 \Leftrightarrow \bar{w} < \frac{4e_H e_L}{11e_H - 7e_L}.$$

Proof. See Appendix B. □

Notice that we have qualitatively recovered the results regarding the welfare impact of a productivity shock derived for the case of sequential bargaining (see Lemmata 2-8). This works, therefore, as a robustness check of our previous results.

B Proofs

Proof of Proposition 1

The second-order conditions are satisfied at the second-stage of the game since:

$$\frac{\partial (NB_i^l)^2}{\partial^2 l_i} = -(1-\beta)e_i^2 \left[(w_i - \bar{w})\, l_i\right]^{\beta} \left[\pi_i(l_i, l_j, w_i)\right]^{-1-\beta} \left[2\pi_i(l_i, l_j, w_i) + \beta e_i^2 l_i^2\right] < 0,$$

for $i \in \{H, L\}$. Furthermore, the sign of $\left.\frac{\partial (NB_i^w)^2}{\partial^2 w_i}\right|_{(w_i^*, w_j^*)}$ is the same as the sign of:

$$-2\alpha(2-\alpha)\left[\frac{(2-\beta)\left\{(4+\alpha-2\beta)(1-\beta)e_i e_j + \left[(2-\alpha)(2-\beta)e_i - \left[2(2-\beta)^2-\alpha\right] e_j\right]\bar{w}\right\}}{(4+\alpha-2\beta)(4-\alpha-2\beta)}\right]^2 < 0,$$

which implies that second-order conditions are satisfied in the first-stage of the game.

Using (10), the least efficient firm is active in the market if and only if:

$$\begin{aligned} l_L^* > 0 \;&\Leftrightarrow 1 + \frac{(2-\alpha)(2-\beta)e_L - \left[2(2-\beta)^2 - \alpha\right] e_H}{(1-\beta)(4+\alpha-2\beta)e_H e_L}\bar{w} > 0 \\ &\Leftrightarrow \{(2-\alpha)(2-\beta)e_L - \left[2(2-\beta)^2-\alpha\right] e_H\}\bar{w} > -(1-\beta)(4+\alpha-2\beta)e_H e_L. \end{aligned} \tag{24}$$

The RHS of the last inequality is negative if and only if:

$$(2-\alpha)(2-\beta)e_L - \left[2(2-\beta)^2-\alpha\right] e_H < 0 \;\Leftrightarrow\; \frac{e_L}{e_H} < \frac{2(2-\beta)^2-\alpha}{(2-\alpha)(2-\beta)}, \tag{25}$$

which is always satisfied since:

$$\frac{2(2-\beta)^2-\alpha}{(2-\alpha)(2-\beta)} \geq 1 \;\Leftrightarrow\; (1-\beta)(4+\alpha-2\beta) \geq 0. \tag{26}$$

Thus, inequality (24) is equivalent to:

$$\bar{w} < \frac{(1-\beta)(4+\alpha-2\beta)e_H e_L}{\left[2(2-\beta)^2-\alpha\right] e_H - (2-\alpha)(2-\beta)e_L}.$$

□

Proof of Lemma 1

Comparing the equilibrium expressions for wages, given in (9), we obtain:

$$w_H^* - w_L^* = \frac{\alpha(e_H - e_L)\left[(1-\beta)(4+\alpha-2\beta)e_H e_L + (2-\alpha)(2-\beta)(e_H+e_L)\bar{w}\right]}{(4+\alpha-2\beta)(4-\alpha-2\beta)e_H e_L} > 0.$$

In addition,

$$\frac{w_H^*}{e_H} - \frac{w_L^*}{e_L} = -\frac{(2-\alpha)(2-\beta)(e_H-e_L)}{(4+\alpha-2\beta)e_H e_L}\bar{w} < 0.$$

$\square$

Proof of Lemma 2

Using the expressions for the employment level in equilibrium, given in (10), we obtain:

$$l_H^* < l_L^* \;\Leftrightarrow\; \frac{(2-\alpha)(2-\beta)\left\{\left[2(2-\beta)^2-\alpha\right](e_H+e_L)\bar{w} - (1-\beta)(4+\alpha-2\beta)e_H e_L\right\}(e_H-e_L)}{(3-\beta)(1-\beta)(4-\alpha-2\beta)(4+\alpha-2\beta)e_H^2 e_L^2} < 0$$

$$\Leftrightarrow\; \left[2(2-\beta)^2-\alpha\right](e_H+e_L)\bar{w} - (1-\beta)(4+\alpha-2\beta)e_H e_L < 0$$

$$\Leftrightarrow \bar{w} < \frac{(1-\beta)(4+\alpha-2\beta)e_H e_L}{\left[2(2-\beta)^2-\alpha\right](e_H+e_L)}. \tag{27}$$

This condition is compatible with Assumption 1 if:

$$\frac{(1-\beta)(4+\alpha-2\beta)e_H e_L}{\left[2(2-\beta)^2-\alpha\right](e_H+e_L)} < \bar{w}^d$$

$$\Leftrightarrow \frac{(3-\beta)(1-\beta)(4+\alpha-2\beta)(4-\alpha-2\beta)e_H e_L^2}{\left[2(2-\beta)^2-\alpha\right](e_H+e_L)\left\{(2-\alpha)(2-\beta)e_L - \left[2(2-\beta)^2-\alpha\right]e_H\right\}} < 0$$

$$\Leftrightarrow (2-\alpha)(2-\beta)e_L - \left[2(2-\beta)^2-\alpha\right]e_H < 0,$$

which, from (25), is satisfied. $\square$

Lemma 13. *The equilibrium market share of the most efficient firm, σ_H^*, is strictly increasing in the reservation wage, while the market share of the least efficient firm, σ_L^*, is strictly decreasing in the reservation wage:*

$$\frac{\partial \sigma_H^*}{\partial \bar{w}} > 0 \qquad \textit{and} \qquad \frac{\partial \sigma_L^*}{\partial \bar{w}} > 0.$$

Proof. Using (11) and (13), we can obtain the equilibrium market share of firm $i \in \{H, L\}$:

$$\sigma_i^* = \frac{q_i^*}{Q^*} = \frac{(1-\beta)(4+\alpha-2\beta)e_ie_j + \left[(2-\alpha)(2-\beta)e_i - \left(2(2-\beta)^2-\alpha\right)e_j\right]\bar{w}}{(1-\beta)(4+\alpha-2\beta)\left[2e_ie_j-(e_i+e_j)\bar{w}\right]}.$$

The derivative of this expression with respect to the reservation wage is:

$$\frac{\partial \sigma_i^*}{\partial \bar{w}} = \frac{(3-\beta)(4-\alpha-2\beta)e_j(e_i-e_j)}{(1-\beta)(4+\alpha-2\beta)\left[2e_ie_j-(e_i+e_j)\bar{w}\right]^2}.$$

Thus:

$$\frac{\partial \sigma_i^*}{\partial \bar{w}} > 0 \;\Leftrightarrow\; e_i > e_j \;\Leftrightarrow\; i = H.$$

□

Proof of Lemma 4

The derivative of the equilibrium expression of wage at firm $i \in \{H, L\}$ with respect to e_i and e_j, respectively, is:

$$\frac{\partial w_i^*}{\partial e_i} = \frac{\alpha(1-\beta)(4+\alpha-2\beta)e_j + \alpha(2-\alpha)(2-\beta)\bar{w}}{(4+\alpha-2\beta)(4-\alpha-2\beta)e_i} > 0$$

and:

$$\frac{\partial w_i^*}{\partial e_j} = -\frac{\alpha(2-\alpha)(2-\beta)e_i}{(4+\alpha-2\beta)(4-\alpha-2\beta)e_j^2}\bar{w} < 0.$$

□

Proof of Lemma 5

1. Impact of e_i on l_i^*.

The derivative of expression (10) with respect to e_i is:

$$\frac{\partial l_i^*}{\partial e_i} = -\frac{(2-\alpha)(2-\beta)}{(1-\beta)(3-\beta)(4+\alpha-2\beta)(4-\alpha-2\beta)e_j e_i^3} \times \left\{(1-\beta)(4+\alpha-2\beta)e_i e_j + \Big[(2-\alpha)(2-\beta)e_i - 2\left[2(2-\beta)^2-\alpha\right]e_j\Big]\bar{w}\right\}. \tag{28}$$

Thus:

$$\frac{\partial l_i^*}{\partial e_i} < 0 \;\Leftrightarrow\; (1-\beta)(4+\alpha-2\beta)e_i e_j + \Big[(2-\alpha)(2-\beta)e_i - 2\left[2(2-\beta)^2-\alpha\right]e_j\Big]\bar{w} > 0. \tag{29}$$

1.1. Substituting $i = j = L$ in (29):

$$\frac{\partial l_L^*}{\partial e_L} < 0 \Leftrightarrow \Big[(2-\alpha)(2-\beta)e_L - 2\left[2(2-\beta)^2-\alpha\right]e_H\Big]\bar{w} > -(1-\beta)(4+\alpha-2\beta)e_L e_H. \tag{30}$$

The LHS of the last inequality is negative since:

$$(2-\alpha)(2-\beta)e_L - 2\left[2(2-\beta)^2-\alpha\right]e_H < 0 \;\Leftrightarrow\; \frac{e_L}{e_H} < \frac{2\left[2(2-\beta)^2-\alpha\right]}{(2-\alpha)(2-\beta)}$$

and, by assumption, $\frac{e_L}{e_H} < 1$ and:

$$\frac{2\left[2(2-\beta)^2-\alpha\right]}{(2-\alpha)(2-\beta)} > 1 \;\Leftrightarrow\; 12-(14+\alpha)\beta+4\beta^2 > 0,$$

is satisfied for $(\alpha,\beta) \in [0,1]\times[0,1)$. Thus, inequality (30) is equivalent to:

$$\frac{\partial l_L^*}{\partial e_L} < 0 \;\Leftrightarrow\; \bar{w} < \frac{(1-\beta)(4+\alpha-2\beta)e_L e_H}{2\left[2(2-\beta)^2-\alpha\right]e_H - (2-\alpha)(2-\beta)e_L}.$$

This condition is compatible with Assumption 1 since:

$$\frac{(1-\beta)(4+\alpha-2\beta)e_L e_H}{2\left[2(2-\beta)^2-\alpha\right]e_H - (2-\alpha)(2-\beta)e_L} < \frac{(1-\beta)(4+\alpha-2\beta)e_H e_L}{e_H\left(2(2-\beta)^2-\alpha\right) - (2-\alpha)(2-\beta)e_L}$$

$$\Leftrightarrow\; \left[2(2-\beta)^2-\alpha\right]e_H > 0.$$

1.2. Substituting $i = j = H$ in (29):

$$\frac{\partial l_H^*}{\partial e_H} < 0 \;\Leftrightarrow\; \Big[(2-\alpha)(2-\beta)e_H - 2\left[2(2-\beta)^2 - \alpha\right] e_L\Big]\bar{w} > -(1-\beta)(4+\alpha-2\beta)e_H e_L$$

The RHS of the last inequality is clearly negative. The coefficient of $\bar{w}$ is positive if and only if:

$$(2-\alpha)(2-\beta)e_H - 2\left[2(2-\beta)^2 - \alpha\right] e_L > 0 \;\Leftrightarrow\; \frac{e_H}{e_L} > \frac{2\left[2(2-\beta)^2 - \alpha\right]}{(2-\alpha)(2-\beta)}. \tag{31}$$

The expression in the RHS is always greater than one since:

$$\frac{2\left[2(2-\beta)^2 - \alpha\right]}{(2-\alpha)(2-\beta)} > 1 \;\Leftrightarrow\; 12 - 14\beta - \alpha\beta + 4\beta^2 > 0$$

is satisfied for $(\alpha, \beta) \in [0,1] \times [0,1)$. Hence if condition (31) is satisfied, we conclude that $\frac{\partial l_H^*}{\partial e_H} < 0$. If, instead, $\frac{e_H}{e_L} < \frac{2\left[2(2-\beta)^2-\alpha\right]}{(2-\alpha)(2-\beta)}$, we have that:

$$\frac{\partial l_H^*}{\partial e_H} < 0 \;\Leftrightarrow\; \bar{w} < \frac{(1-\beta)(4+\alpha-2\beta)e_H e_L}{2\left[2(2-\beta)^2-\alpha\right] e_L - (2-\alpha)(2-\beta)e_H}. \tag{32}$$

This condition is compatible with Assumption 1 if and only if:

$$\frac{(1-\beta)(4+\alpha-2\beta)e_H e_L}{2\left[2(2-\beta)^2-\alpha\right] e_L - (2-\alpha)(2-\beta)e_H} < \frac{(1-\beta)(4+\alpha-2\beta)e_H e_L}{\left[2(2-\beta)^2-\alpha\right] e_H - e_L(2-\alpha)(2-\beta)} \tag{33}$$

$$\Leftrightarrow \frac{(1-\beta)(4+\alpha-2\beta)\Big[(3-\beta)(4-\alpha-2\beta)e_H - \left(20-4\alpha-18\beta+\alpha\beta+4\beta^2\right) e_L\Big] e_H e_L}{\Big[(2-\alpha)(2-\beta)e_H - 2\left[2(2-\beta)^2-\alpha\right] e_L\Big]\Big[(2-\alpha)(2-\beta)e_L - \left[2(2-\beta)^2-\alpha\right] e_H\Big]} < 0.$$

Recall that we are assuming that condition (31) is not satisfied. Thus, using conditions (25) and (26), we conclude that the denominator of the last fraction is positive. As a result, the last inequality is equivalent to:

$$(3-\beta)(4-\alpha-2\beta)e_H - \left(20-4\alpha-18\beta+\alpha\beta+4\beta^2\right) e_L < 0 \Leftrightarrow \frac{e_H}{e_L} < \frac{20-4\alpha-18\beta+\alpha\beta+4\beta^2}{(3-\beta)(4-\alpha-2\beta)}$$

Now, we need to check the compatibility with the assumption that $\frac{e_H}{e_L} < \frac{2\left[2(2-\beta)^2-\alpha\right]}{(2-\alpha)(2-\beta)}$:

$$\frac{20-4\alpha-18\beta+\alpha\beta+4\beta^2}{(3-\beta)(4-\alpha-2\beta)} < \frac{2\left[2(2-\beta)^2-\alpha\right]}{(2-\alpha)(2-\beta)}$$

$$\Leftrightarrow \frac{\alpha^2\left(2-4\beta+\beta^2\right)-4(2-\beta)^2\left(7-8\beta+2\beta^2-\alpha\right)}{(2-\alpha)(2-\beta)(3-\beta)(4-\alpha-2\beta)} < 0.$$

The denominator of the LHS is clearly positive. Regarding the numerator, we have that, for $(\alpha,\beta) \in [0,1] \times [0,1)$, it is always lower than -1. This implies that the inequality is always satisfied.

Hence, we conclude that, if $\frac{20-4\alpha-18\beta+\alpha\beta+4\beta^2}{(3-\beta)(4-\alpha-2\beta)} \leq \frac{e_H}{e_L} < \frac{2\left[2(2-\beta)^2-\alpha\right]}{(2-\alpha)(2-\beta)}$, condition (33) is violated, which implies that condition (32) is surely satisfied under Assumption 1.

If, instead, $1 \leq \frac{e_H}{e_L} \leq \frac{20-4\alpha-18\beta+\alpha\beta+4\beta^2}{(3-\beta)(4-\alpha-2\beta)}$, we conclude that condition (32) is compatible with Assumption 1.

In short:

$$\frac{\partial l_H^*}{\partial e_H} > 0 \;\Leftrightarrow\; \begin{cases} \frac{\bar{w}}{e_L} > \frac{(1-\beta)(4+\alpha-2\beta)}{2[2(2-\beta)^2-\alpha]r_e-(2-\alpha)(2-\beta)}. \\ \\ r_e > \frac{(3-\beta)(4-\alpha-2\beta)}{20-4\alpha-18\beta+\alpha\beta+4\beta^2}. \end{cases}$$

2. Impact of e_j on l_i^*.

The derivative of l_i^*, given in (10), with respect to e_j is negative since:

$$\frac{\partial l_i^*}{\partial e_j} = -\frac{(2-\alpha)^2(2-\beta)^2}{(1-\beta)(3-\beta)(4-\alpha-2\beta)(4+\alpha-2\beta)e_i e_j^2}\bar{w} < 0.$$

$\square$

Proof of Lemma 6

The derivative of expression (11) for q_i^*, $i \in \{H, L\}$, with respect to e_i is:

$$\frac{\partial q_i^*}{\partial e_i} = \frac{(2-\alpha)(2-\beta)\left[2(2-\beta)^2-\alpha\right]}{(1-\beta)(3-\beta)(4+\alpha-2\beta)(4-\alpha-2\beta)e_i^2}\bar{w} > 0;$$

while with respect to e_j, $j \neq i$ it is:

$$\frac{\partial q_i^*}{\partial e_j} = -\frac{(2-\alpha)^2(2-\beta)^2}{(1-\beta)(3-\beta)(4+\alpha-2\beta)(4-\alpha-2\beta)e_j^2}\bar{w} < 0.$$

$\square$

Proof of Lemma 7

From (16) and Lemma 6, it follows straightforwardly that:

$$\frac{\partial WS_i^*}{\partial e_i} > 0 \quad \text{and} \quad \frac{\partial WS_i^*}{\partial e_j} < 0.$$

for $i, j \in \{H, L\}$ and $j \neq i$. Let us now evaluate the impact of e_i on the aggregate worker surplus, WS^*. Using (16), we have:

$$\frac{\partial WS^*}{\partial e_H} = \frac{\alpha(1-\beta)(3-\beta)}{(2-\alpha)(2-\beta)} \frac{\partial \left({q_H^*}^2 + {q_L^*}^2\right)}{\partial e_H} = \frac{2\alpha(1-\beta)(3-\beta)}{(2-\alpha)(2-\beta)} \left(q_H^* \frac{\partial q_H^*}{\partial e_H} + q_L^* \frac{\partial q_L^*}{\partial e_H}\right).$$

From (18) we have that:

$$\frac{\partial q_H^*}{\partial e_H} + \frac{\partial q_L^*}{\partial e_H} > 0.$$

Thus, as $q_H^* > q_L^*$:

$$\frac{\partial WS^*}{\partial e_H} > \frac{2\alpha(1-\beta)(3-\beta)}{(2-\alpha)(2-\beta)}\, q_L^* \left(\frac{\partial q_H^*}{\partial e_H} + \frac{\partial q_L^*}{\partial e_H}\right) > 0.$$

The analysis regarding the impact of e_L on WS^* is not so easy because we still have $q_H^* > q_L^*$ but $\left|\frac{\partial q_H^*}{\partial e_L}\right| < \left|\frac{\partial q_L^*}{\partial e_L}\right|$. The derivative of WS^* with respect to e_i, for $i \in \{H, L\}$, is:

$$\begin{aligned}
\frac{\partial WS^*}{\partial e_i} =& \frac{2\alpha(2-\alpha)(2-\beta)\bar{w}}{(1-\beta)(3-\beta)\left[4(2-\beta)^2-\alpha^2\right]^2 e_i^3 e_j} \times \\
&\times \Big[(4+\alpha-2\beta)^2(1-\beta)^2 e_i e_j + \; + \Big\{2(2-\beta)\left[\alpha^2 + 4(2-\beta)^2 - 2\alpha\left(5-4\beta+\beta^2\right)\right] e_i - \\
&- \left[4(2-\beta)^2\left(5-4\beta+\beta^2-2\alpha\right) + \alpha^2\left(5-4\beta+\beta^2\right)\right] e_j\Big\}\bar{w}\Big],
\end{aligned}$$

As a result, $\frac{\partial WS^*}{\partial e_L} > 0$ if and only if:

$$\Big\{2(2-\beta)\left[\alpha^2+4(2-\beta)^2-2\alpha\left(5-4\beta+\beta^2\right)\right]e_L- \\ -\left[4(2-\beta)^2\left(5-4\beta+\beta^2-2\alpha\right)+\alpha^2\left(5-4\beta+\beta^2\right)\right]e_H\Big\}\bar{w} > -(1-\beta)^2(4+\alpha-2\beta)^2 e_H e_L. \tag{34}$$

The RHS of the last inequality is negative, while the LHS is positive iff:

$$\frac{e_H}{e_L} < \frac{2(2-\beta)\left[\alpha^2+4(2-\beta)^2-2\alpha\left(5-4\beta+\beta^2\right)\right]}{4(2-\beta)^2\left(5-4\beta+\beta^2-2\alpha\right)+\alpha^2\left(5-4\beta+\beta^2\right)}. \tag{35}$$

The expression in the RHS always represents a number lower than 1, since:

$$\frac{4(2-\beta)^2\left(5-4\beta+\beta^2-2\alpha\right)+\alpha^2\left(5-4\beta+\beta^2\right)}{2(2-\beta)\left[\alpha^2+4(2-\beta)^2-2\alpha\left(5-4\beta+\beta^2\right)\right]} > 1 \;\Leftrightarrow\; (1-\beta)^2(4+\alpha-2\beta)^2 > 0. \tag{36}$$

Thus, condition (35) is never satisfied. As a result, the inequality (34) is equivalent to:

$$\frac{\partial WS^*}{\partial e_L} > 0 \;\Leftrightarrow\; \bar{w} < \tilde{w} \tag{37}$$

with:

$$\tilde{w} = \frac{(1-\beta)^2\left(4+\alpha-2\beta\right)^2 e_H e_L}{\left[4(2-\beta)^2\left(5-4\beta+\beta^2-2\alpha\right)+\alpha^2\left(5-4\beta+\beta^2\right)\right]e_H-2(2-\beta)\left[\alpha^2+4(2-\beta)^2-2\alpha\left(5-4\beta+\beta^2\right)\right]e_L}.$$

We need now to check whether this condition is compatible with Assumption 1:

$$\tilde{w} < \bar{w}^d \;\Leftrightarrow\; -\frac{(2-\alpha)(1-\beta)(4+\alpha-2\beta)(4-\alpha-2\beta)\left(6-5\beta+\beta^2\right)e_H e_L(e_H-e_L)}{\left\{\left[2(2-\beta)^2-\alpha\right]e_H-(2-\alpha)(2-\beta)e_L\right\}\times B} < 0 \tag{38}$$

with:

$$B = -\Big\{2(2-\beta)\left[\alpha^2+4(2-\beta)^2-2\alpha\left(5-4\beta+\beta^2\right)\right]e_L- \\ -\Big[4(2-\beta)^2\left(5-4\beta+\beta^2-2\alpha\right)+\alpha^2\left(5-4\beta+\beta^2\right)\Big]e_H\Big\}$$

Using (25), we know that the first term in the denominator of (38) is positive. In addition, we have proved that the coefficient of $\bar{w}$ in (34) is negative. As this coefficient is equal to $-B$, we conclude that $B > 0$. Thus, inequality (38) is surely satisfied, meaning that condition $\bar{w} < \tilde{w}$ is compatible with Assumption 1.

$\square$

Proof of Lemma 8

1. Impact of e_H on total surplus, TS^*

Recalling that $TS^* = CS^* + \Pi^* + WS^*$ and using Lemmata 2-3, it follows immediately that $\frac{\partial TS^*}{\partial e_H} > 0$.

2. Impact of e_L on total surplus, TS^*.

The derivative of the equilibrium total surplus, TS^* with respect to e_i, $i \in \{H, L\}$, is:

$$\begin{aligned}\frac{\partial TS^*}{\partial e_i} =& \frac{(2-\alpha)(2-\beta)\bar{w}}{(1-\beta)(4+\alpha-2\beta)(3-\beta)^2\left(4-\alpha-2\beta\right)^2 e_i^3 e_j}\times \\ &\times\Bigg[2(1-\beta)\left[4(2-\beta)^3+2\alpha\left(2-3\beta+\beta^2\right)-\alpha^2\right]e_ie_j+ \\ &+\Big\{(2-\alpha)(2-\beta)\left(28-5\alpha-26\beta+\alpha\beta+6\beta^2\right)e_i- \\ &-\Big[\alpha^2\left(8-5\beta+\beta^2\right)+4(2-\beta)^2\left(11-9\beta+2\beta^2\right)-2\alpha(2-\beta)\left(17-10\beta+\beta^2\right)\Big]e_j\Big\}\bar{w}\Bigg],\end{aligned} \tag{39}$$

with $j \neq i$. This expression is positive if and only if:

$$\begin{aligned}&\Big\{(2-\alpha)(2-\beta)\left(28-5\alpha-26\beta+\alpha\beta+6\beta^2\right)e_i- \\ &-\Big[\alpha^2\left(8-5\beta+\beta^2\right)+4(2-\beta)^2\left(11-9\beta+2\beta^2\right)-2\alpha(2-\beta)\left(17-10\beta+\beta^2\right)\Big]e_j\Big\}\bar{w} > \\ &> -2(1-\beta)\left[4(2-\beta)^3+2\alpha\left(2-3\beta+\beta^2\right)-\alpha^2\right]e_ie_L\end{aligned} \tag{40}$$

As $4(2-\beta)^3 + 2\alpha\left(2-3\beta+\beta^2\right) - \alpha^2 \geq 3$ for $(\alpha, \beta) \in [0,1] \times [0,1)$, we conclude that the RHS of the last inequality is negative. The coefficient of $\bar{w}$ is positive if and only if:

$$\begin{aligned}&(2-\alpha)(2-\beta)\left(28-5\alpha-26\beta+\alpha\beta+6\beta^2\right)\frac{e_i}{e_j} > \\ &> \alpha^2\left(8-5\beta+\beta^2\right)+4(2-\beta)^2\left(11-9\beta+2\beta^2\right)-2\alpha(2-\beta)\left(17-10\beta+\beta^2\right).\end{aligned}$$

For $(\alpha, \beta) \in [0,1] \times [0,1)$, both sides of the last inequality are positive. Thus, the inequality is equivalent to:

$$\frac{e_i}{e_j} > \frac{\alpha^2\left(8-5\beta+\beta^2\right)+4(2-\beta)^2\left(11-9\beta+2\beta^2\right)-2\alpha(2-\beta)\left(17-10\beta+\beta^2\right)}{(2-\alpha)(2-\beta)\left(28-5\alpha-26\beta+\alpha\beta+6\beta^2\right)}. \tag{41}$$

Substituting $i = H$ and $j = L$ in the last inequality, we obtain a condition for $\frac{e_L}{e_H}$ that is incompatible with the assumption that $e_H > e_L$. Thus, inequality (40) for $i = L$ and $j = H$ is equivalent to:

$$\frac{\partial TS^*}{\partial e_L} > 0 \;\Leftrightarrow\; \bar{w} < \frac{2(1-\beta)\left[4(2-\beta)^3+2\alpha\left(2-3\beta+\beta^2\right)-\alpha^2\right]e_L e_H}{A}, \tag{42}$$

with:

$$A = \Big[\alpha^2\left(8-5\beta+\beta^2\right)+4(2-\beta)^2\left(11-9\beta+2\beta^2\right)-2\alpha(2-\beta)\left(17-10\beta+\beta^2\right)\Big]e_H - \\ -(2-\alpha)(2-\beta)\left(28-5\alpha-26\beta+\alpha\beta+6\beta^2\right)e_L.$$

We need now to check the compatibility of condition (42) with Assumption 1:

$$\frac{2(1-\beta)\left[4(2-\beta)^3+2\alpha\left(2-3\beta+\beta^2\right)-\alpha^2\right]e_H e_L}{A} > \bar{w}^d \tag{43}$$

$$\Leftrightarrow \frac{(2-\alpha)(1-\beta)\left(6-5\beta+\beta^2\right)\left[4(2-\beta)^2-\alpha^2\right]e_H e_L(e_H-e_L)}{(2-\alpha)(2-\beta)e_L-\left[2(2-\beta)^2-\alpha\right]e_H}\times\frac{1}{B} > 0, \tag{44}$$

with:

$$B = \Big[\alpha^2\left(8-5\beta+\beta^2\right)+4(2-\beta)^2\left(11-9\beta+2\beta^2\right)-2\alpha(2-\beta)\left(17-10\beta+\beta^2\right)\Big]e_H - \\ -(2-\alpha)(2-\beta)\left(28-5\alpha-26\beta+\alpha\beta+6\beta^2\right)e_L.$$

From (25), we know that the denominator of the first fraction in (44) is negative. The numerator of this fraction is clearly positive. Thus, inequality (43) is equivalent to $B < 0$, which occurs if and only if:

$$\left[\alpha^2\left(8-5\beta+\beta^2\right)+4(2-\beta)^2\left(11-9\beta+2\beta^2\right)-2\alpha(2-\beta)\left(17-10\beta+\beta^2\right)\right]\frac{e_H}{e_L} < \\ < (2-\alpha)(2-\beta)\left(28-5\alpha-26\beta+\alpha\beta+6\beta^2\right).$$

As, for $(\alpha, \beta) \in [0,1] \times [0,1)$, both sides of the inequality are positive, it is equivalent to:

$$\frac{e_H}{e_L} < \frac{(2-\alpha)(2-\beta)\left(28-5\alpha-26\beta+\alpha\beta+6\beta^2\right)}{\alpha^2\left(8-5\beta+\beta^2\right)+4(2-\beta)^2\left(11-9\beta+2\beta^2\right)-2\alpha(2-\beta)\left(17-10\beta+\beta^2\right)}.$$

This condition is, however, incompatible with the assumption that $e_H > e_L$, since:

$$\frac{(2-\alpha)(2-\beta)\left(28-5\alpha-26\beta+\alpha\beta+6\beta^2\right)}{\alpha^2\left(8-5\beta+\beta^2\right)+4(2-\beta)^2\left(11-9\beta+2\beta^2\right)-2\alpha(2-\beta)\left(17-10\beta+\beta^2\right)} < 1 \;\Leftrightarrow$$

$$\Leftrightarrow \; -2(1-\beta)(4+\alpha-2\beta)\left(8-\alpha-8\beta+2\beta^2\right) < 0$$

is always satisfied for $(\alpha, \beta) \in [0,1] \times [0,1)$. We conclude, therefore, that $B > 0$ and that condition (43) is never satisfied. As a result, $\frac{\partial TS^*}{\partial e_L} > 0$ if and only if:

$$\frac{\bar{w}}{e_L} < \frac{2(1-\beta)\left[4(2-\beta)^3+2\alpha\left(2-3\beta+\beta^2\right)-\alpha^2\right]}{\alpha^2\left(8-5\beta+\beta^2\right)+4(2-\beta)^2\left(11-9\beta+2\beta^2\right)-2\alpha(2-\beta)\left(17-10\beta+\beta^2\right)-(2-\alpha)(2-\beta)\left(28-5\alpha-26\beta+\alpha\beta+6\beta^2\right)r_e}.$$

Proof of Lemma 9

Let $\bar{w}_n$ denote the value for the reservation wage for which the (marginal) impact on TS^* of a change in e_L is null. More precisely: $\left.\frac{\partial TS^*}{\partial e_L}\right|_{\bar{w}=\bar{w}_n} = 0$. The expression for $\bar{w}_n$ coincides with the RHS of inequality (42).

1. Impact of α on $\bar{w}_n$.

The derivative of $\bar{w}_n$ with respect to α is:

$$\frac{\partial \bar{w}_n}{\partial \alpha} = \frac{4(1-r_e)(3-\beta)^2(2-\beta)(1-\beta)}{D^2}\Big\{\alpha^2(1+\beta)+4(2-\beta)\left[10-11\beta+3\beta^2-\alpha(3-\beta)\right]\Big\},$$

where:

$$\begin{aligned} D = {} & \alpha^2\left(8-5\beta+\beta^2\right)+4(2-\beta)^2\left(11-9\beta+2\beta^2\right) \\ & -2\alpha\left(34-37\beta+12\beta^2-\beta^3\right)-r_e(2-\beta)\left[56+\alpha^2(5-\beta)-52\beta+12\beta^2-\alpha\left(38-28\beta+6\beta^2\right)\right] \end{aligned}$$

Hence:

$$\frac{\partial \bar{w}_n}{\partial \alpha} > 0 \;\Leftrightarrow\; \alpha^2(1+\beta)+4(2-\beta)\left[10-11\beta+3\beta^2-\alpha(3-\beta)\right] > 0.$$

Notice that $10-11\beta+3\beta^2-\alpha(3-\beta) > 0$ is a sufficient condition for $\frac{\partial \bar{w}_n}{\partial \alpha} > 0$. We have that:

$$10-11\beta+3\beta^2-\alpha(3-\beta)>0 \;\Leftrightarrow\; \alpha<\frac{10-11\beta+3\beta^2}{3-\beta}$$

is surely satisfied since $\alpha \leq 1$ and the RHS of the last inequality is greater or equal than 1 for $\beta \in [0,1)$.

2. Impact of β on $\bar{w}_n$.

The derivative of $\bar{w}_n$ with respect to β is:

$$\frac{\partial \bar{w}_n}{\partial \beta}=\frac{2(3-\beta)(1-r_e)\times N}{D^2},$$

where:

$$\begin{aligned}N=&-\alpha^3\Big[18(1-\beta)+2\beta^2(5-\beta)-\alpha(1+\beta)\Big]-8\alpha^2(2-\beta)(1-\beta)\left(5-4\beta+\beta^2\right)-\\&-8(2-\beta)^2\Big[2(5-3\beta)(2-\beta)^2-\alpha(3-\beta)\left(9-10\beta+3\beta^2\right)\Big]\end{aligned}$$

and:

$$\begin{aligned}D=&\alpha^2\left(8-5\beta+\beta^2\right)+4(2-\beta)^2\left(11-9\beta+2\beta^2\right)-2\alpha\left(34-37\beta+12\beta^2-\beta^3\right)-\\&-r_e(2-\beta)\Big[56+\alpha^2(5-\beta)-52\beta+12\beta^2-\alpha\left(38-28\beta+6\beta^2\right)\Big].\end{aligned}$$

Thus: $\frac{\partial \bar{w}_n}{\partial \beta}<0 \;\Leftrightarrow\; N<0$. Notice that if: (i) $18(1-\beta)+2\beta^2(5-\beta)-\alpha(1+\beta)>0$, and (ii) $2(5-3\beta)(2-\beta)^2-\alpha(3-\beta)\left(9-10\beta+3\beta^2\right)>0$, we have proven that $N<0$. But:

$$18(1-\beta)+2\beta^2(5-\beta)-\alpha(1+\beta)>0 \;\Leftrightarrow\; \alpha<\frac{18(1-\beta)+2\beta^2(5-\beta)}{1+\beta}$$

is satisfied, since the RHS of the last inequality is greater than 1 for $\beta \in [0,1)$. Moreover:

$$2(5-3\beta)(2-\beta)^2-\alpha(3-\beta)\left(9-10\beta+3\beta^2\right)>0 \;\Leftrightarrow\; \alpha<\frac{2(5-3\beta)(2-\beta)^2}{(3-\beta)\left(9-10\beta+3\beta^2\right)}$$

is surely satisfied because:

$$\frac{2(5-3\beta)(2-\beta)^2}{(3-\beta)\left(9-10\beta+3\beta^2\right)}>1 \;\Leftrightarrow\; (1-\beta)\left(13-12\beta+3\beta^2\right)>0$$

holds for $\beta \in [0,1)$. $\square$

Proof of Lemma 10

Assuming $\alpha = \beta = \theta$ and subtracting (21) to (23):

$$w_i^* - w_i^s = \frac{\theta}{3e_j(4-\theta)(4-3\theta)}\Big[-e_ie_j(4-\theta) + [e_j(8-5\theta) - 4e_i(1-\theta)]\,\bar{w}\Big].$$

Thus:

$$w_i^* - w_i^s < 0 \;\Leftrightarrow\; [(8-5\theta)e_j - 4(1-\theta)e_i]\,\bar{w} < e_ie_j(4-\theta). \tag{45}$$

If $(8-5\theta)e_j - 4(1-\theta)e_i < 0$, the last inequality is surely satisfied (since the RHS of the inequality is positive). Suppose, instead, that: $(8-5\theta)e_j - 4(1-\theta)e_i > 0$. In this case:

$$w_i^* < w_i^s \;\Leftrightarrow\; \bar{w} < \frac{e_ie_j(4-\theta)}{(8-5\theta)e_j - 4(1-\theta)e_i}. \tag{46}$$

We will now prove that, under Assumption 2, this condition is always satisfied. If $i = L$ and $j = H$:

$$\frac{e_Le_H(4-\theta)}{(8-5\theta)e_H - 4(1-\theta)e_L} > \frac{e_Le_H}{2e_H - e_L} \;\Leftrightarrow\; \frac{3e_H(e_H - e_L)e_L\theta}{(2e_H - e_L)\,[e_H(8-5\theta) - 4e_L(1-\theta)]} > 0$$

is always satisfied, since we are assuming that $e_H(8-5\theta) - 4e_L(1-\theta) > 0$ (otherwise, condition (45) would be trivially satisfied). Thus, if Assumption 2 holds, condition (46) is satisfied, and we conclude that $w^* < w^s$. Following the same steps, it can be shown that: if $i = H$ and $j = L$ and Assumption 2 is satisfied, condition (46) holds. $\square$

Proof of Lemma 11

The equilibrium employment level at firm $i \in \{H, L\}$ is higher under symmetric sequential bargaining (with $\alpha = \beta = \theta$) than under simultaneous bargaining iff:

$$l_i^* > l_i^s \;\Leftrightarrow\; \frac{\theta\Big[e_ie_j\,(4-5\theta+\theta^2) + [e_j\,(-20+22\theta-5\theta^2) + e_i\,(16-17\theta+4\theta^2)]\,\bar{w}\Big]}{3e_i^2e_j(4-\theta)(3-\theta)(1-\theta)(4-3\theta)} > 0$$

$$\Leftrightarrow\; e_ie_j\left(4-5\theta+\theta^2\right) + \Big[e_j\left(-20+22\theta-5\theta^2\right) + e_i\left(16-17\theta+4\theta^2\right)\Big]\bar{w} > 0$$

$$\Leftrightarrow\; \Big[e_j\left(-20+22\theta-5\theta^2\right) + e_i\left(16-17\theta+4\theta^2\right)\Big]\bar{w} > -e_ie_j\left(4-5\theta+\theta^2\right)$$

As the RHS of the last inequality is surely positive, the inequality is surely satisfied if $e_j\left(-20+22\theta-5\theta^2\right)+e_i\left(16-17\theta+4\theta^2\right)>0$. Otherwise, the inequality is equivalent to:

$$\bar{w}<\frac{e_ie_j\left(4-5\theta+\theta^2\right)}{e_j\left(20-22\theta+5\theta^2\right)-e_i\left(16-17\theta+4\theta^2\right)}. \tag{47}$$

We need to check the compatibility with Assumption 2. Let us first assume that $i=H$ and $j=L$. In this case, condition (47) is surely satisfied (under Assumption 2), since:

$$\frac{e_He_L(4-\theta)(1-\theta)}{e_L\left(20-22\theta+5\theta^2\right)-e_H\left(16-17\theta+4\theta^2\right)}>\frac{e_Le_H}{2e_H-e_L}$$

$$\Leftrightarrow\ \frac{3e_H(e_H-e_L)e_L\left(8-9\theta+2\theta^2\right)}{(2e_H-e_L)\Big[e_L\left(20-22\theta+5\theta^2\right)-e_H\left(16-17\theta+4\theta^2\right)\Big]}>0$$

$$\Leftrightarrow e_L\left(-20+22\theta-5\theta^2\right)+e_H\left(16-17\theta+4\theta^2\right)<0$$

is surely satisfied (under the assumption that expression (47) is valid). Thus: $l_H^*>l_H^s$.

Consider now that $i=L$ and $j=H$. Start by noticing that the condition for the validity of condition (47) is always satisfied:

$$e_H\left(-20+22\theta-5\theta^2\right)+e_L\left(16-17\theta+4\theta^2\right)>0\ \Leftrightarrow\ \frac{e_H}{e_L}>\frac{16-17\theta+4\theta^2}{20-22\theta+5\theta^2}$$

and the expression in the RHS is lower than 1 for $\theta\in(0,1)$. Checking the compatibility of condition (47) with Assumption 2:

$$\frac{e_Le_H(4-\theta)(1-\theta)}{e_H\left(20-22\theta+5\theta^2\right)-e_L\left(16-17\theta+4\theta^2\right)}<\frac{e_Le_H}{2e_H-e_L}$$

$$\Leftrightarrow\ \frac{3e_He_L(e_H-e_L)(2-\theta)^2}{(2e_H-e_L)\Big[e_H\left(-20+22\theta-5\theta^2\right)+e_L\left(16-17\theta+4\theta^2\right)\Big]}<0$$

$$\Leftrightarrow e_H\left(-20+22\theta-5\theta^2\right)+e_L\left(16-17\theta+4\theta^2\right)<0.$$

As the last inequality is always satisfied, we conclude that condition (47) is compatible with Assumption 2. In sum:

$$l_L^*>l_L^s\ \Leftrightarrow\ \bar{w}<\frac{e_He_L\left(4-5\theta+\theta^2\right)}{e_H\left(20-22\theta+5\theta^2\right)-e_L\left(16-17\theta+4\theta^2\right)}.$$

$\square$

Proof of Lemma 12

Welfare impact of an increase in e_H

- As $\frac{\partial CS^s}{\partial e_H} = \frac{\bar{w}}{2e_H^2} > 0$, consumers become better off if e_H increases.

- The derivative of the aggregate worker surplus, $WS^s = (w_L^s - \bar{w})l_L^s + (w_H^s - \bar{w})l_H^s$, with respect to e_H is positive iff:

$$\frac{\partial WS^s}{\partial e_H} > 0 \;\Leftrightarrow\; \frac{2\theta\bar{w}[e_H e_L + (4e_H - 5e_L)\bar{w}]}{9e_H^3 e_L} > 0 \;\Leftrightarrow\; (4e_H - 5e_L)\bar{w} > -e_H e_L.$$

  If $\frac{e_H}{e_L} > \frac{5}{4}$, the last inequality is surely satisfied and, therefore, $\frac{\partial WS^s}{\partial e_H} > 0$. Otherwise:

$$\frac{\partial WS^s}{\partial e_H} > 0 \;\Leftrightarrow\; \bar{w} < \frac{e_H e_L}{5e_L - 4e_H}.$$

  Under Assumption 2, the last condition always holds, and, therefore, $\frac{\partial WS^s}{\partial e_H} > 0$.

- Start by noticing that $\frac{WS^s}{\Pi^s} = \frac{\theta}{1-\theta}$ does not depend on e_L and e_H. Thus, the derivative of the equilibrium industry profit with respect to e_H, $\frac{\partial \Pi^s}{\partial e_H}$, is positive whenever the derivative of the equilibrium aggregate worker surplus with respect to e_H, $\frac{\partial WS^s}{\partial e_H}$, is positive.

- An increase in e_H increases total surplus, $TS^s = CS^s + WS^s + \Pi^s$, iff:

$$\frac{\partial TS^s}{\partial e_H} > 0 \;\Leftrightarrow\; \frac{\bar{w}[4e_H e_L + (7e_H - 11e_L)\bar{w}]}{9e_H^3 e_L} > 0.$$

  If $\frac{e_H}{e_L} > \frac{11}{7}$, the last inequality is surely satisfied. Otherwise, it is equivalent to:

$$\frac{\partial TS^s}{\partial e_H} > 0 \;\Leftrightarrow < \frac{4e_H e_L}{11e_L - 7e_H}.$$

  Under Assumption 2, the last inequality is satisfied, which implies that $\frac{\partial TS^s}{\partial e_H} > 0$.

Welfare impact of an increase in e_L

- As $\frac{\partial CS^s}{\partial e_L} = \frac{\bar{w}}{2e_L^2} > 0$, consumers become better off if e_L increases.

- The derivative of the aggregate worker surplus with respect to e_L is positive iff:

$$\frac{\partial WS^s}{\partial e_L} > 0 \Leftrightarrow \frac{2\theta\bar{w}[e_H e_L - (5e_H - 4e_L)\bar{w}]}{9e_H e_L^3} > 0 \Leftrightarrow \bar{w} < \frac{e_H e_L}{5e_H - 4e_L},$$

  and it is easy to check that the last condition is compatible with Assumption 2.

- Again, as $\frac{WS^s}{\Pi^s} = \frac{\theta}{1-\theta}$, the sign of $\frac{\partial \Pi^s}{\partial e_L}$ is the same as that of $\frac{\partial WS^s}{\partial e_L}$.

- The derivative of total surplus with respect to e_L is positive iff:

$$\frac{\partial TS^s}{\partial e_L} > 0 \Leftrightarrow \frac{\bar{w}[4e_H e_L - (11e_H - 7e_L)\bar{w}]}{9e_H e_L^3} \Leftrightarrow \bar{w} < \frac{4e_H e_L}{11e_H - 7e_L},$$

  and it is easy to check that this condition is not implied by Assumption 2.

□

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