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ABSTRACT

The aim of this work is to determine how well criteria designed to help the selection of the adequate number of mixture components perform in mixture regressions of normal data. We address this research question based on results of an extensive experimental design. The simulation experiment compares several criteria (26), including information criteria and classification-based criteria. In this full factorial design we manipulate 9 factors and 22 levels, namely: true number of segments (2 or 3), mean separation between segments (low, medium or high), number of consumers (100 or 300), number of observations per consumer (5 or 10), number of predictors (2, 6 or 10), measurement level of predictors (binary, metric or mixed), error variance (20% or 60%), minimum segment size (5-10%, 10-20% or 20-30%) and error distribution (normal versus uniform). The performance of the segment retention criteria is evaluated by their success rates; we also investigate the influence of experimental factors and their levels on success rates. The best results were obtained for the criteria AIC_3, AIC_4, HQ, ICLBIC and ICOMPLBIC. BIC and CAIC also perform well with large samples and a large number of market segments.

KEY-WORDS: Market segmentation, information criteria, classification criteria, experimental design, simulation.

1 INTRODUCTION

Despite the popularity of mixture regression models for normal data in market segmentation literature (DeSarbo & Cron, 1988; Ramaswamy, *et al.,* 1993; Helsen *et al*, 1993; DeSarbo, *et al.*, 1992; Wedel & DeSarbo, 1994,1995; Jedidi *et al.,* 1996; DeSarbo et al., 2001; Bowman *et al.*, 2004), the decision of how many market segments to keep for managerial decisions is, according to many authors (DeSarbo *et al.* 1997; Wedel & DeSarbo, 1995; Wedel e Kamakura, 2000; Hawkins *et al.*, 2001; Andrews & Currim, 2003a,b), an open issue. To assess the true number of market segments is essential because many marketing decisions - segmentation, targeting, positioning, marketing mix - depended on it.

As the true number of market segments in real-world data is unknown, the evaluation of the effectiveness of segment retention criteria is usually accomplished through an experimental design.

Few studies have been published focusing on the segment retention problem in mixture regression models of normal data. The first work by Hawkins *et al.* (2001), examines the performance of 12 base criteria, namely AIC (Akaike, 1973), AIC_3 (Bozdogan, 1990), MDL (Rissanen, 1986, 1987), ICOMP (Bozdogan, 1993), CL, NEC (Celeux & Soromenho, 1996), PC (Bezdek, 1981), AWE (Banfield & Raftery, 1993), MIR (Windham & Cutler, 1992), ALL, ANC, WID (Cutler & Windham, 1994) by varying the number of mixture components, the degree of separation between components and the mixing proportions. The authors conclude that PC was the least successful criterion and report good results for MDL and AWE. The study by Andrews and Currim (2003a) compares de performance of AIC (Akaike, 1993), AIC3 (Bozdogan, 1994), BIC (Schwartz, 1978), CAIC (Bozdogan, 1987), ICOMP (Bozdogan, 1990), NEC (Celeux & Soromenho, 1996) and the validation sample log likelihood (Andrews & Currim, 2003a) manipulating eight data characteristics, namely: true number of segments, mean separation between segment coefficients, number of individuals, number of predictors, error variance, minimum segment size and measurement level of predictors. The authors find that AIC_3 is the best criterion to use with mixture regression models of normal data.

This work extends previous studies in two ways. Fist, we intend to include an experimental condition not explored yet in previous studies, namely the distributional misspecification. According to Andrews e Currim (2003a: 242), "*for analysts, another potentially important factor that was not studied directly in these simulations was distributional misspecification, that is, when the distribution of the data does not match that of the model*". Furthermore, as a large number of criteria were not considered before, we aim at comparing the performance of 26 criteria.

The plan of this work is as follows: we start briefly describing the multivariate normal mixture regression and the information and classification criteria, next we present the experimental design used to generate the simulated data, then discuss the findings of the study and finish with a conclusion.

1 BACKGROUND

1.1 2.1. Multivariate Normal Mixture Regression

The latent regression model simultaneously estimate separate regression functions and memberships in S clusters

Let:

$s = 1,...,S$ indicate derived segments;

$n = 1,...,N$ indicate consumers;

$k = 1,...,K$ indicate repeated observations from consumer *n*;

$j = 1,...,J$ indicate explanatory variables;

β_{js} = be the value of j - *th* regression coefficient for the s - *th* cluster;

$\boldsymbol{\beta}_s = (\beta_j)$;

$\boldsymbol{\Sigma}_s =$ be the covariance matrix for segment *s*;

$y_{nk} =$ be the value of the dependent variable for repeated measure k on consumer *n*;

$\mathbf{y}_n = (y_k)$;

$x_{njk} =$ be the value of the j - *th* independent variable for repeated measure k on consumer *n*;

$\mathbf{x}_n = ((x_{jk}))$.

Assume that the metric dependent vector $\mathbf{y}_n = ((y_{nk}))$ is distributed as a finite mixture of S conditional multivariate normal densities (1):

$$\mathbf{y}_n \sim \sum_{s=1}^{S} \lambda_s f_s\left(\mathbf{y}_n \left| \mathbf{x}_n, \boldsymbol{\beta}_s, \boldsymbol{\Sigma}_s\right.\right), \tag{1}$$

where f_s is defined by the expression (2)

$$f_s\left(\mathbf{y}_n \left| \mathbf{x}_n, \boldsymbol{\beta}_s, \boldsymbol{\Sigma}_s\right.\right) = \left(2\pi\right)^{-K/2} \left|\Sigma_s\right|^{-1/2} \exp\left[-1/2\left(\mathbf{y}_n - \mathbf{X}\boldsymbol{\beta}_s^{'}\right)\Sigma_s^{-1}\left(\mathbf{y}_n - \mathbf{X}\boldsymbol{\beta}_s^{'}\right)'\right] \tag{2}$$

and λ_s, $s = 1,...,S$ are independent mixing proportions satisfying the following restrictions:

$$0 \leq \lambda_s \leq 1 \tag{3}$$

$$\sum_{s=1}^{S} \lambda_s = 1. \tag{4}$$

Given a sample of N independent consumers, one can thus form the likelihood (5) and the log-likelihood (6) expressions:

$$L = \prod_{n=1}^{N}\left[\sum_{s=1}^{S} \lambda_s f_s\left(\mathbf{y}_n \left| \mathbf{x}_n, \boldsymbol{\beta}_s, \boldsymbol{\Sigma}_s\right.\right)\right] \tag{5}$$

$$\ln L = \sum_{n=1}^{N} \ln \sum_{s=1}^{S} \lambda_s f_s\left(\mathbf{y}_n \left| \mathbf{x}_n, \boldsymbol{\beta}_s, \boldsymbol{\Sigma}_s\right.\right). \tag{6}$$

The implementation of the maximum likelihood procedure is done by using an Expectation-Maximization – EM type framework (Dempster *et al.*, 1977). To derive the EM algorithm is necessary to introduce non-observed data via the indicator function: $z_{ns} = 1$ if *n* comes from latent class *s* and $z_{ns} = 0$, otherwise; it is assumed that z_{ns} are i.i.d multinomial. So, the joint likelihood of the "complete data" $\mathbf{y}_n = \left(y_{nk}\right)$ and $\mathbf{z}_n = \left(z_{ns}\right)$ for all consumers is:

$$\ln L_c = \sum_{n=1}^{N}\sum_{s=1}^{S} z_{ns} \ln\left[f_s\left(\mathbf{y}_n \left| \mathbf{x}_n, \boldsymbol{\beta}_s, \boldsymbol{\Sigma}_s\right.\right)\right] + \sum_{n=1}^{N}\sum_{s=1}^{S} z_{ns} \ln \lambda_s . \tag{7}$$

To give starting values of the parameters, the expectation (E step) and maximization (M step) of this algorithm are alternated until convergence of a sequence of log-likelihood values is obtained. Once estimates of λ, Σ and β are obtained for any M-step procedure, one can assign each consumer n to each latent class or market segment s via estimated posterior probability (applying Bayes' rule), providing a fuzzy clustering (E-step):

$$p_{ns} = \frac{\lambda_s f_s\left(\mathbf{y}_n \left|\mathbf{X}, \boldsymbol{\beta}_s, \boldsymbol{\Sigma}_s\right.\right)}{\sum_{s=1}^{S} \lambda_s f_s\left(\mathbf{y}_n \left|\mathbf{X}, \boldsymbol{\beta}_s, \boldsymbol{\Sigma}_s\right.\right)}, \tag{8}$$

where $\sum_{s=1}^{S} p_{ns} = 1$, and $0 \le p_{ns} \le 1$.

2.2. The Criteria

We intend to compare the performance of 12 information criteria and 14 classification-based criteria, described subsequently, through a simulation experiment. As the likelihood increases with the addition of a component to a mixture model, some heuristics, called Information Criteria, attempt to balance the increase in fit obtained against the larger number of parameters estimated to models with more clusters. Information Criteria are a general family, including criteria that are estimates of (relative) Kullback-Leibler distance, approaches who have been derived within a Bayesian framework for model selection and those named consistent criteria. Although Information Criteria account for over-parameterization, as large number of clusters is derived, is also important to ensure that the segments are sufficiently separated to the selected solution. To access the ability of a mixture model in providing well-separated clusters, an entropy statistic can be used to evaluate the degree of separation in the estimated posterior probabilities. This approach yields the Classification Criteria. Some measures are derived in the context of mixture models and other are "imported" from the fuzzy literature (Bezdek *et al.*, 1997). These criteria are named respectively as probabilistic indices and fuzzy indices[1]. Table 1 presents all criteria compared in this study.

The reader is referred to the author's previous work (Brochado e Martins, 2005) and to the references cited below (Table 1) for detailed discussions of the theoretical underpinnings of these criteria.

[1] The quantities p_{ns} are interpreted as partial memberships in the context of fuzzy clustering and as probabilities of membership in the context of mixture models.

2 EXPERIMENTAL DATA

In this experimental design we manipulate 9 factors and 22 levels, namely: (1) true number of mixture components (2 or 3); (2) mean separation between segments (low – 0,5, medium – 1,0 or high – 1,5), (3) number of consumers (100 or 300), (4) number of observations per consumer (5 or 10), (5) number of predictors (2, 6 or 10), (6) measurement level of predictors (binary, metric or mixed), (7) error variance (20% or 60%), (8) minimum segment size (5-10%, 10-20% or 20-30%) and (9) error distribution (normal versus uniform). As the design is factorial, with tree replications (datasets) per cell, a total of $7776=2^5 3^5$ is generated using the Gauss 6. package.

Table 1. Information Criteria and Classification Criteria

Criteria	Description	Reference
	Information Criteria	
	Kullback-Leible Estimators	
Akaike Information Criteria	$\text{AIC} = -2\ln L + 2k$	Akaike (1973)
Modified AIC 3	$\text{AIC}_3 = -2\ln L + 3k$	Bozdogan (1994)
Modified AIC 4	$\text{AIC}_4 = -2\ln L + 4k$	Bozdogan (1994)
Small sample AIC	$\text{AIC}_c = \text{AIC} + \left[2k\left(k+1\right)\right]/\left(N-k-1\right)$	Hurvich & Tsai (1989, 1995)
	Bayesian Criteria	
Bayesian Information Criteria	$\text{BIC} = -2\ln L + k\ln N$	Schwartz (1978)
Consistent Criteria		
Consistent AIC	$\text{CAIC} = -2\ln L + k\left[\left(\ln N\right)+1\right]$	Bozdogan (1987)
CAIC with Fisher Information	$\text{CAICF} = \text{AIC} + k\log N + \log\lvert\mathbf{F}\rvert$	Bozdogan (1987)
Information Complexity Criterion	$\text{ICOMP} = -2\ln L + k\ln\left[\frac{tr\left(\text{F}^{-1}\right)}{k}\right] - \ln\lvert\mathbf{F}^{-1}\rvert$	Bozdogan (1994)
Hannan –Quinn	$\text{HQ} = -2\ln L + 2k\ln\left(\ln N\right)$	Hannan & Quinn (1979)
Minimum Description Length 2	$\text{MDL}_2 = -2\ln L + 2k\ln N$	Liang *et al.* (1992)
Minimum Description Length 5	$\text{MDL}_5 = -2\ln L + 5k\ln N$	Liang *et al.* (1992)

Table 1. Information Criteria and Classification Criteria (cont.)

Criteria	Description	Reference
	Classification Criteria	
	Fuzzy Indices	
Partition Coeficient	$\text{PC} = \sum_{n=1}^{N} \sum_{s=1}^{S} p_{ns}^{2} \Big/ N$	Bezdek (1981)
Partition Entropy	$\text{PE} = \left[\sum_{n=1}^{N} \sum_{s=1}^{S} p_{ns} \ln p_{ns} \right] \Big/ N$	Bezdek (1981)
Normalized Partition Entropy	$\text{NPE} = \text{PE} \big/ \left[1 - S/N \right]$	Bezdek (1981)
Nonfuzzy Index	$\text{NFI} = \left[S \left(\sum_{n=1}^{N} \sum_{s=1}^{S} p_{ns}^{2} \right) - N \right] \Big/ \left[N \left(S - 1 \right) \right]$	Roubens (1978)
Minimum Hard Tendency	$\text{Min}_{ht} = \max_{1 \le s \le S} \left\{ -\log_{10} \left(T_s \right) \right\}$	Rivera, *et al.* (1990)
Mean Hard Tendency	$\text{Mean}_{ht} = \sum_{s=1}^{S} -\log_{10} \left(T_s \right) \Big/ S$	Rivera, *et al.* (1990)
	Probabilistic Indices	
Entropy Measure	$\text{Es} = 1 - \left[\sum_{n=1}^{N} \sum_{s=1}^{S} -p_{ns} \ln p_{ns} \right] \Big/ N \ln S$	DeSarbo *et al.* (1992)
Logarithm of the partition Probability	$\text{LP} = -\sum_{n=1}^{N} \sum_{s=1}^{S} z_{ns} \ln p_{ns}$	Biernacki (1997)
Entropy	$\text{E} = -\sum_{n=1}^{N} \sum_{s=1}^{S} p_{ns} \ln p_{ns}$	Biermacki (1997)
Normalized Entropy Criterion	$\text{NEC}(s) = \text{E}(s) \big/ \ln L(s) - \ln L(1)$	Celeux & Soromenho (1996)
Classification Criterion	$\text{C} = -2 \ln L + 2\text{E}$	Biernacki & Govaert (1997)
Classification Likelihood Criterion	$\text{CLC} = -2 \ln L + 2\text{LP}$	Biernacki & Govaert (1997)
Approximate Weight of Evidence	$\text{AWE} = -2 \ln L_c + 2k \left(3/2 + \ln N \right)$	Banfied & Raftery (1993)
Integrated Completed Likelihood – BIC	$\text{ICL-BIC} = -2 \ln L + 2\text{LP} + k \ln N$	Biernacki & Celeux (1998)
ICL with BIC approximation	$\text{ICOMPLBIC} = -2 \ln L + 2\text{E} + k \ln N$	Dias (2004)

To improve general inference, we generate parameter values randomly for each segment and each data set. We follow the approach proposed by Andrews and Currim (2003)[2] to generate the coefficients for explanatory variables for each group, $\boldsymbol{\beta}_s = \left(\beta_{sp}\right)$, $s = 1,...,S$,. The variance of the error term was obtained trough the approach proposed by Vriens *et al.* (1996) and Andrews *et al* (2002)[3]. Consumers are assigned to segments on the basis of randomly determined segment sizes for each data set. The smallest segment consists of 5-10%, 10-20% or 20-30% of the sample, depending on the level of factor 7. Random and uniform distributed errors (factor 9) are generated with average 0 and standard deviation 1.

3 RESULTS

We evaluate the performance of segment retention criteria by their hit rate, or percentage of datasets in which the criteria identify the correct number of segments; we also considered the over fitting rate and the under fitting rate; given two criteria with similar success rates, we prefer under fitting to over fitting; this argument is due to two empirical arguments presented in literature: first, empirical results show that over fitting produces larger parameters bias than under fitting does (Andrews & Currim, 2003a,b); second over fitting sometimes produce very small segments with large or unstable parameter values (Cutler & Windham, 1994).

Table 1 and Table 2 show the success rates (S) and rates of over fitting (O) by experimental condition and by criteria. As example, AIC correctly identified the true number of segments in 63% of data sets with two components (Factor 1), over fitted the number of components in 31% of data sets and under fitted the number of components in 6% of these data sets.

The best overall success rates were obtained for both information criteria, as AIC_3 (71%), HQ (69%) and AIC_4 (68%), and classification criteria, as ICLBIC (70%) and ICOMPLBIC (68%).

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2 The vector of parameters for the first segment $\boldsymbol{\beta}_1$ is random generated within the interval -1,5 to 1,5; next, a vector of separations $\boldsymbol{\delta}_i$ ($i = L, M, H$) is generated, with average 0,5, 1,0 or 1,5 (and standard devation equal to 10% of the average, i.e., 0,05, 0,1 and 0,15) and a vector of signs $\mathbf{S}^{+}_{-}$, positives (1) and negatives (-1), for $\boldsymbol{\delta}_i$ ($i = L, M, H$); the vector with low (0,5), medium (1,0) and large (1,5) separations are denoted, respectively, by $\boldsymbol{\delta}_S$, $\boldsymbol{\delta}_M$ and $\boldsymbol{\delta}_E$. Vector coefocients for segment 2 and 3 are obtained with $\boldsymbol{\beta}_2 = \boldsymbol{\beta}_1 + \mathbf{S}^{+}_{-}\boldsymbol{\delta}_i$, $i = S, M, L$ and $\boldsymbol{\beta}_3 = \boldsymbol{\beta}_1 - \mathbf{S}^{+}_{-}\boldsymbol{\delta}_i$, $i = B, M, E$, respectively.

3 The Percentage Error Variance (*PEV*) is defined as $PEV = \sigma^2_\varepsilon \big/ \sigma^2_\varepsilon + \sigma^2_u$, where σ^2_ε represents the variance of the error teem and σ^2_u the variance of the vector $\mathbf{U} = \left(U_{nk} = \mathbf{X}_{nk}\boldsymbol{\beta}_{ns}\right)$, $n = 1,...,N$, $k = 1,...,K$. Thus, defining PEV as 20% or 60%, $\sigma^2_\varepsilon = \left(PEV \big/ 1 - PEV\right)\sigma^2_u$.

This result is consistent with (Hawkins, 1999: 70) who stated that "*augmented complete log likelihood functions may be the next generation of measures for investigation*".

Although some information criteria - AIC, AIC_c e ICOMP - registered a tendency to over estimate the number of mixture components, others, as MDL_5, MDL_2, CAICF, CAIC e BIC presented higher rates of under fitting than over fitting. Almost all classification criteria (E_s, E, LP, AWE, NEC, ICL, ICLBIC; ICOMPLBIC, PC, PE, NPE, NFI e $MEAN_{ht}$) presented high rates of under fitting.

Table 2. Rates of sucess (S) and overfitting (O) by information criterion and experimental condition

	AIC		AIC_3		AIC_4		AIC_c		BIC		CAIC		CAICF		ICOMP		MDL_2		MDL_5		HQ	
	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O
Factor 1																						
2	63%	31%	81%	5%	81%	1%	66%	26%	71%	0%	68%	0%	60%	0%	66%	26%	57%	0%	38%	0%	81%	1%
3	64%	21%	60%	8%	55%	5%	64%	18%	45%	3%	42%	3%	34%	2%	64%	17%	34%	2%	20%	1%	56%	5%
Factor 2																						
0,5	53%	27%	56%	5%	51%	2%	54%	23%	39%	1%	36%	0%	25%	0%	57%	20%	25%	0%	12%	0%	52%	2%
1	67%	24%	76%	6%	73%	3%	69%	21%	64%	2%	61%	1%	54%	1%	67%	21%	52%	1%	32%	0%	73%	3%
1,5	69%	26%	81%	8%	80%	4%	72%	22%	71%	3%	69%	3%	62%	2%	71%	22%	59%	2%	42%	1%	81%	5%
Factor 3																						
100	61%	24%	65%	6%	61%	3%	63%	19%	58%	2%	47%	1%	38%	1%	62%	21%	37%	1%	20%	0%	62%	3%
300	66%	27%	77%	6%	75%	3%	67%	26%	50%	2%	64%	2%	55%	2%	69%	22%	54%	1%	37%	1%	75%	3%
Factor 4																						
5	54%	33%	63%	8%	60%	3%	56%	28%	49%	1%	48%	1%	39%	1%	58%	27%	38%	1%	21%	0%	61%	3%
10	73%	19%	79%	4%	76%	3%	74%	16%	66%	2%	63%	2%	55%	2%	73%	15%	53%	1%	36%	1%	76%	3%
Factor 5																						
2	67%	22%	76%	7%	77%	4%	68%	21%	69%	2%	67%	2%	60%	2%	74%	13%	59%	2%	44%	1%	77%	4%
6	63%	26%	71%	6%	66%	3%	64%	22%	56%	2%	55%	2%	46%	1%	63%	25%	44%	1%	25%	0%	67%	3%
10	60%	29%	66%	6%	61%	3%	62%	23%	49%	1%	45%	1%	35%	1%	58%	25%	34%	1%	17%	0%	62%	3%
Factor 6																						
metric	64%	22%	68%	6%	64%	3%	65%	19%	53%	2%	51%	2%	41%	2%	66%	17%	41%	2%	25%	1%	64%	3%
binary	63%	30%	75%	7%	72%	3%	65%	26%	64%	2%	61%	1%	53%	1%	64%	24%	50%	1%	34%	0%	73%	4%
mixed	64%	24%	70%	6%	68%	2%	65%	21%	56%	1%	54%	1%	47%	1%	65%	22%	45%	1%	27%	0%	69%	3%
Factor 7																						
20%	69%	29%	87%	8%	88%	5%	72%	26%	83%	3%	81%	3%	72%	2%	75%	21%	72%	2%	51%	1%	88%	5%
60%	58%	22%	55%	4%	48%	1%	58%	18%	33%	0%	29%	0%	22%	0%	55%	21%	19%	0%	7%	0%	49%	1%
Factor 8																						
5-10%	55%	31%	61%	10%	58%	7%	56%	27%	47%	5%	45%	4%	38%	3%	57%	25%	36%	3%	22%	1%	59%	7%
10-20%	68%	22%	75%	5%	72%	1%	70%	19%	61%	1%	58%	1%	49%	1%	67%	21%	48%	1%	30%	0%	72%	1%
20-30%	67%	24%	77%	4%	74%	1%	69%	20%	66%	0%	63%	0%	54%	0%	72%	18%	53%	0%	35%	0%	75%	1%
Factor 9																						
normal	65%	25%	72%	6%	69%	3%	67%	21%	58%	2%	55%	2%	46%	1%	67%	20%	46%	1%	29%	1%	69%	4%
uniform	61%	26%	70%	6%	67%	3%	63%	23%	58%	2%	56%	1%	47%	1%	63%	22%	45%	1%	28%	1%	68%	3%
Overrall	63%	26%	71%	6%	68%	3%	65%	22%	58%	2%	55%	2%	47%	1%	65%	21%	45%	1%	29%	1%	69%	3%

Table 3. Rates of sucess (S) and overfitting (O) by classification criterion and experimental condition

	E_s		E		LP		AWE		NEC		CL		CLC		ICLBIC		ICOMPL		PC		PE		NPE		NFI		$MEAN_{ht}$		MIN_{ht}	
	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O
Factor 1																														
2	74%	26%	93%	7%	92%	8%	99%	1%	83%	17%	61%	39%	74%	26%	98%	2%	98%	2%	92%	8%	96%	4%	96%	4%	61%	39%	90%	10%	76%	24%
3	23%	23%	12%	26%	14%	27%	30%	2%	25%	30%	47%	36%	44%	31%	42%	3%	39%	4%	12%	31%	5%	15%	6%	12%	25%	16%	24%	9%	19%	49%
Factor 2																														
0,5	42%	42%	48%	16%	48%	18%	57%	1%	45%	29%	47%	44%	53%	34%	61%	3%	60%	4%	47%	19%	48%	9%	48%	8%	31%	11%	50%	15%	45%	36%
1	50%	40%	54%	15%	55%	17%	68%	1%	57%	21%	55%	37%	60%	28%	73%	2%	72%	2%	54%	18%	52%	9%	52%	8%	32%	8%	58%	8%	49%	34%
1,5	54%	39%	57%	18%	57%	18%	69%	2%	62%	19%	61%	32%	65%	24%	75%	3%	73%	3%	55%	21%	52%	10%	53%	7%	33%	5%	62%	6%	48%	39%
Factor 3																														
100	48%	11%	53%	19%	53%	20%	61%	1%	54%	26%	50%	44%	56%	34%	67%	2%	66%	3%	52%	22%	51%	10%	51%	9%	32%	8%	57%	12%	46%	42%
300	49%	11%	53%	14%	53%	15%	68%	2%	55%	21%	58%	31%	62%	24%	73%	3%	71%	4%	52%	17%	51%	8%	51%	7%	32%	9%	56%	7%	49%	31%
Factor 4																														
5	46%	11%	51%	17%	51%	18%	61%	1%	51%	27%	50%	39%	55%	30%	65%	2%	65%	4%	50%	20%	49%	10%	49%	9%	29%	8%	54%	13%	46%	37%
10	52%	12%	55%	16%	56%	17%	68%	1%	58%	20%	59%	36%	64%	27%	74%	2%	72%	3%	54%	19%	52%	9%	53%	7%	35%	8%	60%	7%	49%	35%
Factor 5																														
2	52%	16%	56%	8%	56%	9%	70%	1%	58%	14%	62%	27%	66%	19%	74%	2%	72%	3%	54%	11%	52%	5%	53%	4%	39%	11%	58%	8%	51%	28%
6	47%	9%	51%	19%	52%	20%	63%	1%	53%	25%	53%	39%	59%	29%	69%	3%	68%	4%	51%	21%	49%	11%	50%	9%	30%	8%	56%	10%	44%	40%
10	46%	8%	52%	22%	52%	24%	60%	1%	51%	31%	47%	47%	53%	38%	66%	2%	65%	3%	52%	25%	50%	13%	50%	10%	28%	6%	57%	11%	48%	41%
Factor 6																														
metric	42%	12%	50%	21%	50%	22%	62%	2%	49%	28%	54%	37%	56%	32%	68%	3%	66%	5%	50%	23%	50%	12%	50%	10%	29%	8%	54%	11%	47%	38%
binary	53%	11%	56%	10%	56%	11%	67%	1%	60%	18%	53%	40%	60%	29%	73%	2%	72%	2%	54%	14%	52%	6%	53%	4%	35%	9%	59%	9%	48%	35%
mixed	50%	11%	53%	18%	53%	20%	64%	1%	54%	24%	55%	35%	61%	26%	68%	2%	68%	3%	52%	21%	50%	11%	50%	9%	32%	7%	57%	9%	47%	36%
Factor 7																														
20%	61%	14%	60%	6%	61%	7%	78%	2%	67%	9%	59%	39%	68%	28%	86%	3%	85%	3%	59%	9%	54%	4%	55%	2%	40%	11%	66%	5%	53%	29%
60%	36%	8%	46%	27%	45%	28%	51%	1%	41%	37%	49%	36%	50%	30%	54%	2%	52%	3%	45%	30%	47%	15%	47%	14%	24%	6%	48%	14%	42%	43%
Factor 8																														
5-10%	53%	11%	54%	16%	54%	16%	60%	3%	55%	23%	52%	40%	57%	31%	64%	5%	63%	6%	53%	18%	52%	9%	52%	7%	35%	7%	58%	11%	48%	37%
10-20%	51%	9%	54%	16%	55%	17%	66%	1%	56%	23%	57%	36%	62%	27%	72%	2%	70%	3%	53%	19%	51%	9%	51%	7%	33%	8%	58%	9%	48%	36%
20-30%	42%	14%	51%	18%	51%	20%	68%	0%	51%	24%	53%	37%	58%	29%	73%	0%	72%	1%	50%	20%	50%	10%	50%	9%	28%	10%	54%	9%	45%	37%
Factor 9																														
normal	51%	11%	54%	14%	55%	16%	65%	1%	55%	23%	56%	36%	60%	28%	70%	3%	69%	3%	53%	18%	50%	0%	50%	0%	17%	0%	58%	10%	48%	35%
uniform	47%	11%	51%	19%	51%	20%	64%	1%	54%	24%	52%	39%	58%	30%	69%	2%	68%	3%	51%	21%	51%	19%	52%	15%	47%	16%	56%	9%	47%	38%
Overall	49%	11%	53%	16%	53%	18%	64%	1%	54%	23%	54%	38%	59%	29%	70%	2%	68%	3%	52%	19%	51%	9%	51%	8%	32%	8%	57%	10%	47%	36%

When we analyse success rates by experimental condition, we observe that no one criterion is best for all experimental conditions. For example, relatively to Factor 1 AWE registered the best success rate for a 2 mixture component solution, while AIC, AIC_c e ICOMP presented the best results for a 3 segment solution.

In order to investigate the influence of experimetal factors and their levels on sucess rates we estimate logit regressions (where 1 = sucess), with the predictors being binary variables representing the simulation design factors.

The logit meta-analysis found that the criteria generally have higher success rates when there are 2 mixture components, less explanatory variables, binary variables relatively to metric and mixed, larger separation between segments, larger samples sizes and smaller error variance. Surprisingly, we also found that data uniform distributed errors have a small negative effect on segment retention criteria success rates.

Table 5. Meta-Analysis of results: Logit resuts by information criterion and experimental condition

Factor/ level	AIC	AIC_3	AIC_4	AIC_c	BIC	CAIC	CAICF	ICOMP	MDL_2	MDL_5	HQ
C	1,147	1,257	0,897	1,191	-0,089	-0,500	-1,253	0,902	-1,545	-3,828	1,052
	(0,163) *	(0,203) *	(0,220) *	(0,166) *	(0,229)	(0,231) **	(0,237) *	(0,166) *	(0,246) *	(0,316) *	(0,219) *
F1 (2)	-0,041	1,569	2,330	0,130	2,636	2,619	2,725	0,082	2,595	2,559	2,268
	(0,086)	(0,116) *	(0,139) *	(0,087)	(0,153) *	(0,154) *	(0,158) *	(0,088)	(0,161) *	(0,181) *	(0,137) *
F2 (2)	0,360	0,759	1,480	0,293	2,040	2,260	2,670	0,803	2,778	3,775	1,360
	(0,106) *	(0,135) *	(0,155) *	(0,107) *	(0,170) *	(0,174) *	(0,182) *	(0,111) *	(0,189) *	(0,238) *	(0,152) *
F2 (6)	0,130	0,338	0,482	0,107	0,762	0,975	1,179	0,226	1,150	1,252	0,476
	(0,104)	(0,130) *	(0,143) *	(0,106)	(0,154) *	(0,157) *	(0,162) *	(0,105) **	(0,167) *	(0,198) *	(0,142) *
F3 (metric)	-0,017	-0,153	-0,311	-0,017	-0,317	-0,263	-0,581	0,082	-0,405	-0,395	-0,398
	(0,105)	(0,131)	(0,144) **	(0,107)	(0,154) **	(0,155) ***	(0,159) *	(0,109)	(0,162) **	(0,186) **	(0,144) *
F3 (binary)	-0,055	0,406	0,425	0,006	0,805	0,799	0,624	-0,012	0,620	0,888	0,355
	(0,105)	(0,135) **	(0,148) *	(0,107)	(0,158) *	(0,158) *	(0,158) *	(0,108)	(0,162) *	(0,183) *	(0,147) **
F4 (0,5)	-0,735	-1,861	-2,554	-0,869	-3,240	-3,383	-3,859	-0,724	-3,798	-4,417	-2,501
	(0,105) *	(0,141) *	(0,166) *	(0,107) *	(0,189) *	(0,194) *	(0,207) *	(0,109) *	(0,212) *	(0,259) *	(0,164) *
F4 (1,0)	-0,098	-0,463	-0,698	-0,170	-0,823	-0,822	-0,852	-0,202	-0,741	-1,286	-0,746
	(0,108)	(0,141) *	(0,154) *	(0,110)	(0,158) *	(0,158) *	(0,156) *	(0,111) ***	(0,160) *	(0,177) *	(0,154) *
F5 (100)	-0,224	-0,951	-1,337	-0,221	-1,534	-1,670	-1,800	-0,339	-1,883	-2,369	-1,220
	(0,086) *	(0,111) *	(0,126) *	(0,087) **	(0,136) *	(0,139) *	(0,142) *	(0,089) *	(0,147) *	(0,177) *	(0,124) *
F6 (5)	-0,898	-1,201	-1,419	-0,905	-1,719	-1,608	-1,642	-0,760	-1,766	-2,092	-1,370
	(0,087) *	(0,113) *	(0,127) *	(0,089) *	(0,139) *	(0,138) *	(0,140) *	(0,089) *	(0,146) *	(0,171) *	(0,126) *
F7 (20%)	0,516	2,311	3,364	0,664	4,484	4,612	4,578	0,967	5,025	5,703	3,291
	(0,086) *	(0,125) *	(0,157) *	(0,088) *	(0,189) *	(0,193) *	(0,196) *	(0,090) *	(0,212) *	(0,271) *	(0,154) *
F8 (5-10%)	-0,547	-1,148	-1,385	-0,614	-1,882	-1,837	-1,621	-0,727	-1,930	-1,903	-1,427
	(0,104) *	(0,135) *	(0,151) *	(0,106) *	(0,167) *	(0,167) *	(0,167) *	(0,109) *	(0,176) *	(0,199) *	(0,150) *
F8 (10-20%)	0,046	-0,171	-0,146	0,024	-0,557	-0,474	-0,499	-0,239	-0,599	-0,678	-0,244
	(0,107)	(0,138)	(0,150)	(0,110)	(0,157) *	(0,157) *	(0,157) *	(0,111) **	(0,161) *	(0,180) *	(0,149) ***
F9 (normal)	0,180	0,147	0,156	0,168	0,040	-0,032	-0,099	0,223	0,017	0,078	0,112
	(0,086) **	(0,108)	(0,119)	(0,087) ***	(0,126)	(0,127)	(0,128)	(0,088) **	(0,132)	(0,149)	(0,118)
% Correct	66%	83%	85%	69%	87%	87%	88%	68%	88%	90%	85%
LR	260	1.007	1470	296	1924	1976	2.029	351	2092	1938	1425

* α<1%; ** α<5%; *** α<10%

4 CONCLUSION

As we rely on heuristics as information and classification based criteria to guide us on the selection of the model to pick, is important to understand how segment retention criteria behave. As in real world data the true number of market segments is unknown, we address this question with an experimental design by manipulating nine factors. We conclude that the number of mixture, the mean separation between segments, number of consumers, number of observations per consumer, number of predictors , measurement level of predictors, error variance, minimum segment size and error distribution influence the performance of both information and classification criteria. The best overall success rates were obtained for both information criteria, as AIC_3 (71%), HQ (69%) and AIC_4 (68%), and classification criteria, as ICLBIC (70%) and ICOMPLBIC (68%).

Table 6. Meta-Analysis of results: Logit results by classification criterion and experimental condition

Factor/ Level	E_s	E	LP	AWE	NEC	CL	CLC	ICLBIC	ICOMPLBIC	PC	PE	NPE	NFI	$MEAN_{ht}$
C	**-2,297**	**-3,422**	**-3,439**	**-2,562**	**-2,039**	**-0,026**	**-0,306**	**-1,094**	**-1,324**	**-3,231**	**-3,974**	**-3,986**	**-1,238**	**-1,769**
	(0,195) *	*(0,304)* *	*(0,291)* *	*(0,365)* *	*(0,208)* *	*(0,153)*	*(0,162)* **	*(0,272)* *	*(0,279)* *	*(0,288)* *	*(0,406)* *	*(0,423)* *	*(0,175)* *	*(0,214)* *
F1 (2)	**2,743**	**5,855**	**5,545**	**1,073**	**3,402**	**0,577**	**1,378**	**6,694**	**6,747**	**5,453**	**7,238**	**7,587**	**0,746**	**3,849**
	(0,111) *	*(0,220)* *	*(0,206)* *	*(0,557)* *	*(0,130)* *	*(0,082)* *	*(0,089)* *	*(0,303)* *	*(0,298)* *	*(0,196)* *	*(0,297)* *	*(0,334)* *	*(0,094)* *	*(0,136)* *
F2 (2)	**0,404**	**0,557**	**0,522**	**2,136**	**0,546**	**0,646**	**0,610**	**1,142**	**1,066**	**0,335**	**0,571**	**0,537**	**0,590**	**0,104**
	(0,122) *	*(0,177)* *	*(0,169)* *	*(0,246)* *	*(0,132)* *	*(0,101)* *	*(0,108)* *	*(0,191)* *	*(0,191)* *	*(0,171)* **	*(0,236)* **	*(0,234)* **	*(0,114)* *	*(0,138)*
F2 (6)	**0,088**	**-0,032**	**-0,029**	**0,706**	**0,178**	**0,272**	**0,262**	**0,366**	**0,367**	**-0,118**	**-0,140**	**-0,111**	**0,121**	**-0,095**
	(0,121)	*(0,178)*	*(0,169)*	*(0,224)* *	*(0,130)*	*(0,100)* *	*(0,106)* **	*(0,183)* **	*(0,183)* **	*(0,172)*	*(0,237)*	*(0,236)*	*(0,116)*	*(0,138)* *
F3 (metric)	**-0,485**	**-0,349**	**-0,273**	**-0,441**	**-0,372**	**-0,076**	**-0,251**	**-0,050**	**-0,352**	**-0,192**	**-0,166**	**-0,111**	**-0,165**	**-0,209**
	(0,122) *	*(0,179)* **	*(0,170)* **	*(0,223)* **	*(0,131)* *	*(0,100)*	*(0,107)* **	*(0,183)*	*(0,184)* **	*(0,172)*	*(0,236)*	*(0,236)*	*(0,115)*	*(0,138)*
F3 (binary)	**0,205**	**0,354**	**0,464**	**0,639**	**0,411**	**-0,086**	**-0,086**	**0,650**	**0,575**	**0,320**	**0,434**	**0,537**	**0,196**	**0,217**
	(0,121) **	*(0,176)* **	*(0,169)* *	*(0,224)* *	*(0,131)* *	*(0,100)*	*(0,107)*	*(0,187)* *	*(0,188)* *	*(0,171)* ***	*(0,234)* ***	*(0,234)* **	*(0,113)* ***	*(0,138)*
F4 (0,5)	**-0,792**	**-1,212**	**-1,195**	**-2,565**	**-1,226**	**-0,630**	**-0,613**	**-1,885**	**-1,970**	**-1,091**	**-1,100**	**-1,269**	**-0,065**	**-0,967**
	(0,123) *	*(0,187)* *	*(0,177)* *	*(0,254)* *	*(0,135)* *	*(0,101)* *	*(0,108)* *	*(0,199)* *	*(0,201)* *	*(0,179)* *	*(0,248)* *	*(0,251)* *	*(0,114)*	*(0,141)* *
F4 (1,0)	**-0,278**	**-0,290**	**-0,322**	**-0,291**	**-0,368**	**-0,283**	**-0,256**	**-0,197**	**-0,257**	**-0,203**	**-0,218**	**-0,375**	**-0,013**	**-0,301**
	(0,121) **	*(0,175)* ***	*(0,168)* **	*(0,221)*	*(0,131)* *	*(0,101)* *	*(0,108)* **	*(0,189)*	*(0,192)*	*(0,170)*	*(0,234)*	*(0,232)* ***	*(0,113)*	*(0,137)* **
F5 (100)	**-0,044**	**0,062**	**0,094**	**-1,567**	**-0,045**	**-0,334**	**-0,266**	**-0,883**	**-0,764**	**0,078**	**0,036**	**0,000**	**-0,047**	**0,095**
	(0,099)	*(0,144)*	*(0,137)*	*(0,197)* *	*(0,107)*	*(0,082)* *	*(0,087)* *	*(0,155)* *	*(0,155)* *	*(0,140)*	*(0,190)*	*(0,190)*	*(0,093)*	*(0,112)*
F6 (5)	**-0,367**	**-0,647**	**-0,626**	**-1,631**	**-0,499**	**-0,387**	**-0,440**	**-1,318**	**-1,156**	**-0,588**	**-0,664**	**-0,773**	**-0,288**	**-0,523**
	(0,099) *	*(0,147)* *	*(0,140)* *	*(0,198)* *	*(0,108)* *	*(0,082)* *	*(0,088)* *	*(0,161)* *	*(0,160)* *	*(0,142)* *	*(0,197)* *	*(0,199)* *	*(0,093)* *	*(0,114)* *
F7 (20%)	**1,540**	**2,128**	**2,141**	**5,233**	**1,898**	**0,414**	**0,868**	**3,889**	**4,036**	**1,866**	**1,957**	**2,330**	**0,904**	**1,492**
	(0,107) *	*(0,196)* *	*(0,184)* *	*(0,318)* *	*(0,123)* *	*(0,082)* *	*(0,088)* *	*(0,199)* *	*(0,206)* *	*(0,178)* *	*(0,266)* *	*(0,298)* *	*(0,095)* *	*(0,125)* *
F8 (5-10%)	**0,712**	**0,329**	**0,356**	**-1,811**	**0,298**	**-0,045**	**-0,057**	**-1,377**	**-1,352**	**0,382**	**0,437**	**0,381**	**0,395**	**0,265**
	(0,123) *	*(0,178)* **	*(0,169)* **	*(0,239)* *	*(0,131)* **	*(0,100)*	*(0,106)*	*(0,192)* *	*(0,193)* *	*(0,172)* **	*(0,235)* **	*(0,234)* ***	*(0,114)* *	*(0,138)* **
F8 (10-20%)	**0,595**	**0,391**	**0,441**	**-0,565**	**0,349**	**0,156**	**0,184**	**-0,210**	**-0,231**	**0,324**	**0,247**	**0,301**	**0,251**	**0,312**
	(0,122) *	*(0,178)* **	*(0,170)* *	*(0,224)* *	*(0,131)* *	*(0,101)*	*(0,107)* ***	*(0,187)*	*(0,189)*	*(0,172)* ***	*(0,235)*	*(0,234)*	*(0,115)* **	*(0,138)* **
F9 (normal)	**0,250**	**0,416**	**0,511**	**0,065**	**0,080**	**0,168**	**0,099**	**0,136**	**0,125**	**0,352**	**-0,364**	**-0,583**	**-1,565**	**0,183**
	(0,099) *	*(0,145)* *	*(0,139)* *	*(0,181)*	*(0,107)*	*(0,082)* **	*(0,087)*	*(0,150)*	*(0,151)*	*(0,141)* **	*(0,192)*	*(0,195)* *	*(0,098)* *	*(0,112)*
% Correct	77%	91%	90%	94%	80%	63%	70%	91%	92%	90%	95%	95%	78%	84%
LR	1094	2256	2.150	2575	1377	199	443	2015	2075	2169	2.734	2750	481	1512

* α<1%; ** α<5%; *** α<10%

We also found that data distributional misspecification has a small negative effect on segment retention criteria success rates; however, we intend to extend this work by considering different scenarios for distribution misspecification. As no criterion has been perfectly capable of identifying the correct number of segments, additional research should continue to search for better critera for segment retention.

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