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n. 443 Jan. 2012

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João Correia da Silva [1]
Carlos Hervés-Beloso [2]

[1] CEF.UP, Faculdade de Economia, Universidade do Porto
[2] Universidad de Vigo

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João Correia-da-Silva

CEF.UP and Faculdade de Economia, Universidade do Porto.

Carlos Hervés-Beloso

RGEA, Facultad de Económicas. Universidad de Vigo.

January 11th, 2012.

Abstract. Private state-verification is introduced in a two-period economy with spot markets in both periods and complete futures markets for contingent delivery in the second period. Existence of equilibrium is established, under standard assumptions. The equilibrium allocation is shown to be generically efficient if the number of states is not greater than the number of goods.

Keywords: General equilibrium, Differential information, Private state-verification, Two-period economies, Existence of equilibrium, Generic efficiency.

JEL Classification Numbers: C62, C72, D51, D82.

We thank Alberto Bennardo, Andrés Carvajal, Herakles Polemarchakis, Marco LiCalzi and the participants in the CSEF Workshop on Equilibrium Analysis Under Uncertainty at U. Napoli, the 4th Economic Theory Workshop in Vigo, the 2nd UECE Lisbon Meeting on Game Theory and Applications, the 20th European Workshop on General Equilibrium, the 10th SAET Meeting on Current Trends in Economics and the 5th Meeting of the Portuguese Economic Journal, for useful comments and suggestions. João Correia-da-Silva (joao@fep.up.pt) acknowledges support from CEF.UP, Fundação para a Ciência e Tecnologia and FEDER (PTDC/EGE-ECO/111811/2009). Carlos Hervés-Beloso (cherves@uvigo.es) acknowledges support from Ministerio de Educaci on y Ciencia, Xunta de Galicia and FEDER (Projects ECO2009-14457-C0401ECON and 10PXIB300141PR).

1 Introduction

Agents frequently wish to make contracts that are contingent on a future event (like any insurance contract), but the enforcement of such contracts may be problematic if only one party is able to observe the event. Even if both parties observe the event, this may not be sufficient to enforce a contract. It may be necessary to prove to a third party that the event has occurred.

The first attempt to incorporate this kind of information asymmetries in general equilibrium theory was made by Radner (1968), who restricted agents to make contracts that are contingent on events that they can observe. This is too restrictive, as the other party may find it to be in his interest to honor the contract, even if a violation of the contract could be concealed. Such contracts are said to be incentive compatible (Hurwicz, 1972).

Allowing agents to make any incentive compatible contract, Prescott and Townsend (1984a, 1984b) showed the existence of optimal allocations and sought to decentralize them through a price system. However, to induce agents to self-select incentive compatible contracts, such decentralization may require non-linear prices (Jerez, 2005; Rustichini and Siconolfi, 2008).

Our purpose is to investigate the economic effects of asymmetries in the ability to verify the occurrence of events, in the context of competitive markets (with linear price systems). Our framework may be described as a model of general equilibrium with private and incomplete state-verification. While Townsend (1979) studied the effects of costly state-verification, we assume that to verify the occurrence of an event is either free or impossible. State-verification is incomplete, and this incompleteness varies across agents.

We consider a two-period economy with spot markets in both periods, present and future, and complete futures markets (in the first period) for contingent delivery (in the second period). In the first period, being uncertain about the future state of nature, agents trade in the spot markets and in the futures markets. The trade in the spot markets determines present consumption (goods are assumed to be non-durable). The contracts made in the futures markets determine the bundle that the agents have the right to receive in each of the possible future states of nature. In the second-period spot markets, agents sell the bundle that is delivered to them, together with their second-period endowments, to acquire their second-period consumption bundle. It is assumed that agents trade in the present anticipating the future spot prices and, therefore, the bundle that they will be able to consume in the future.[1]

This market structure coincides with that of Arrow (1953) and Debreu (1959). The difference here is that agents are assumed to have incomplete, and differential, abilities to verify the future state of nature. Each agent has an exogenously given information structure, which is a partition of the set of possible states of nature. In the future, with the objective of enforcing the contracts made in the present, all that agents can verify is that the state of nature belongs to a certain element of their information partition.

We assume that trade in the futures markets is mediated by profit-maximizing firms.[2] In the first period, each agent makes a contract with one of these firms, stipulating a net trade for each of the possible future states of nature. In the second period, given the agent's incomplete ability to verify the states of nature, the firm may have the opportunity to deliver a less valuable net trade. We assume that, in case of litigation between the agent and the firm, it is the agent that bears the burden of proof, and that the agent's ability to prove that a certain state of nature has occurred or not is exogenous and described by her information partition. The firm may choose, therefore, among the net trades that correspond to states of nature that the agent cannot distinguish from the true state of nature. As a result, the agent always receives, in each state of nature, the less valuable (according to the spot prices in that state of nature) of the possible net trades. In the spirit of the revelation principle (Myerson, 1979), we restrict agents to make trades which induce truthful deliveries.

It is assumed that agents cannot use prices to prove to a third party that a certain state has occurred. This contrasts with what is assumed in the works of Radner (1979)

[1]This modifies the model of an economy with uncertain delivery (Correia-da-Silva and Hervés-Beloso, 2008, 2009, 2011) by opening spot markets in the second period.

[2]This was also assumed by Prescott and Townsend (1984a, 1984b), Jerez (2005), Bisin and Gottardi (2006) and Rustichini and Siconolfi (2008).

and Allen (1981). We rule out revelation through prices because it is known that this would eliminate the informational asymmetries, rendering the model useless to explain their economic effects. Allowing revelation through prices would imply that every contract could be enforced. We assume that even if prices allow an agent to infer the true state of nature, this is useless as a means of enforcing contracts.[3]

In our framework, there are contracts which cannot be enforced because agents have incomplete information. Markets are, therefore, incomplete. But in a fundamentally different way from that considered by Radner (1972) and Magill and Quinzii (1996). Here, each agent faces different trade possibilities, that are, in addition, endogenous.

The "Hidden Information Economy" of Bisin and Gottardi (1999) is closely related to ours. The main differences are the following: (i) they consider an aggregate shock that is publicly observed and an idiosyncratic shock that is privately observed, while we consider the more general case of uncertainty described by a set of future states of nature and private state-verification described by agent-specific information partitions; (ii) we allow for state-dependent preferences; (iii) we consider a single agent of each type, instead of countably many; (iv) we consider a complete set of markets for contingent delivery, while they only consider securities that are payable in a numeraire good; (v) we suppose that each agent uses her verifiable information to enforce contracts, while they allow the outcome of trade for one agent to depend on non-verifiable messages that are sent by the others. In their model, existence of equilibrium requires a minimal form of non-linearity of prices (a bid-ask spread). The origin of the non-existence issue is the fact that agents are able to influence the payoff of securities via their choice of message. In our model, this is not possible. Agents need to provide verifiable evidence about the state of nature instead of sending a message which may be true or false.

We establish the existence of equilibrium, under standard assumptions. The standard proof of existence does not apply because, as a consequence of restricting agents to make

[3]In spite of ruling out revelation through prices (the use of prices as proof in court), we will conclude that the equilibrium allocation coincides with that of the symmetric information model. The information structures of the agents will turn out to be generically irrelevant, as in the models of Radner (1979) and Allen (1981), as long as the number of states of nature is not greater than the number of goods.

trades that induce truthful delivery, the choice set is not lower hemicontinuous with respect to prices. Adapting a technique used in a related contribution (Correia-da-Silva and Hervés-Beloso, 2011), we start by constructing a sequence of economies in which a violation of the truthful delivery restrictions is possible, but implies utility penalties that are increasingly harsh along the sequence. After obtaining the corresponding sequence of equilibria, we prove that an accumulation point of this sequence is an equilibrium of the economy under study.

In the case in which the number of goods is greater than the number of states of nature, we obtain a strong characterization result. Generically, i.e., in almost all economies with private state-verification, an equilibrium allocation would also be an equilibrium allocation under complete state-verification.[4] Surprisingly, incomplete state-verification does not imply any loss of efficiency. The agents are able to overcome their incomplete abilities to verify the occurrence of events by selecting an appropriate bridge portfolio, which guarantees truthful delivery of the desired wealth transfers across states of nature.

The paper is organized as follows. In Section 2, we present the model of a two-period economy with private state-verification and establish existence of equilibrium. In Section 3, we show that the equilibrium allocation is generically optimal in the sense of Pareto. In Section 4, we conclude the paper with some remarks.

2 The model

We consider an economy that extends over two time periods, the present ($\tau = 0$) and the future ($\tau = 1$), in which a finite number of agents, $\mathcal{I} = \{1, ..., I\}$, trade a finite number of commodities, $\mathcal{L} = \{1, ..., L\}$.

In the present, there is uncertainty about the state of the environment that will prevail in the future. There is a finite set of possible states of nature, $\mathcal{S} = \{1, ..., S\}$, and agents

[4] As most genericity results, this characterization requires a differentiability assumption. We assume that the preferences of the agents are well-behaved in the sense of Debreu (1972), which implies that the demand functions are continously differentiable.

agree that the probabilities of occurrence of each state are given by $\mu \in \Delta^S$.

Each agent's private information is described by a partition of $\mathcal{S}$. Agent i knows that if state s occurs, she will only be able to prove that the state of nature belongs to the element of her information partition that contains s, which is denoted by $P^i(s)$.

The initial endowments of agent i are $e_0^i \in \mathbb{R}_+^L$ and $e_1^i \in \mathbb{R}_+^{SL}$.

Assumption 1 (Endowments)**.**

The endowments of each agent are strictly positive: $e_0^i \gg 0$ *and* $e_1^i \gg 0, \ \forall i \in \mathcal{I}$.

The agent's preferences about consumption in both periods, (x_0^i, x_1^i), are described by an utility function, $U^i : \mathbb{R}_+^L \times \mathbb{R}_+^{SL} \to \mathbb{R}$.

Assumption 2 (Preferences)**.**

The utility functions of the agents are continuous, concave and strictly increasing.[5]

There are spot markets at $\tau = 0$ and at $\tau = 1$, and futures markets at $\tau = 0$ for contingent delivery at $\tau = 1$. The deliveries contracted in the futures markets may be conditional on the occurrence of any event (set of states of nature), thus each agent i chooses a plan of net deliveries, specifying what she should receive in each state of nature, $y^i = (y^i(1), ..., y^i(s), ..., y^i(S)) \in \mathbb{R}^{SL}$.

The prices in the spot markets at $\tau = 0$ and $\tau = 1$ are denoted by p_0 and p_1, respectively, and the prices in the futures markets are denoted by q. We normalize prices by imposing that $(p_0, q) \in \Delta^{L+SL}$ and that $p_1(s) \in \Delta^L$, for each $s \in \mathcal{S}$.

At $\tau = 0$, agent i trades her endowments, e_0^i, for a consumption bundle, $x_0^i \in \mathbb{R}_+^L$, and a plan of future net deliveries, $y^i \in \mathbb{R}^{SL}$. The corresponding budget restriction is:

$$\left(x_0^i, y^i\right) \in B^i\left(p_0, q\right) = \left\{(z_0, w) \in \mathbb{R}_+^L \times \mathbb{R}^{SL} : \ p_0 \cdot z_0 + q \cdot w \leq p_0 \cdot e_0^i\right\}.$$

[5]By strictly increasing, it is meant that an increase in consumption of any of the goods is strictly desired by the agents: $(x_0^i, x_1^i) \geq (z_0^i, z_1^i)$ and $(x_0^i, x_1^i) \neq (z_0^i, z_1^i)$ implies that $U^i(x_0^i, x_1^i) > U^i(z_0^i, z_1^i)$.

Trade in the futures markets is mediated by profit-maximizing intermediaries, who are also price takers. The relationship between agents and financial intermediaries is asymmetric, as it is the agent that bears the burden of proof. At $\tau = 1$, if state s occurs, each agent i can only prove that the state of nature belongs to $P^i(s)$, therefore, the financial intermediaries decide which of the alternatives among $\{y^i(t)\}_{t \in P^i(s)}$ is delivered to each agent i.[6] Profit maximization by the financial intermediaries implies that only the cheapest alternatives, according to $p_1(s)$, may be delivered.

Hence, agents receive, in each state s, one of the cheapest bundles among those that they cannot prove, using only $P^i(s)$, that do not correspond to the truthful delivery.[7] Accordingly, we can restrict (without loss of generality) the choice of agent i to satisfy the following restrictions, which induce truthful delivery:[8]

$$y^i \in D^i(p_1) = \left\{ z \in \mathbb{R}^{SL} : \ p_1(s) \cdot z(s) \leq p_1(s) \cdot z(t), \ \forall t \in P^i(s), \forall s \in \mathcal{S} \right\}.$$

At $\tau = 1$, in state s, agent i receives $y^i(s)$ (truthful delivery), which she trades, together with her endowments, $e_1^i(s)$, for a consumption bundle, $x_1^i(s) \in \mathbb{R}_+^L$. The corresponding budget restriction is:

$$x_1^i(s) \in B_s^i(p_1(s), y^i(s)) = \left\{ z \in \mathbb{R}_+^L : \ p_1(s) \cdot z \leq p_1(s) \cdot \left[y^i(s) + e_1^i(s) \right] \right\}.$$

The budget set for future consumption in all states, $B_1^i(p_1, y^i)$, is defined as follows:

$$x_1^i \in B_1^i(p_1, y^i) \ \Leftrightarrow \ x_1^i(s) \in B_s^i(p_1(s), y^i(s)), \forall s \in \mathcal{S}.$$

Let $x^i = (x_0^i, y^i, x_1^i)$, $e^i = (e_0^i, 0, e_1^i)$ and $p = (p_0, q, p_1)$. We write $x^i \in B^i(p)$ whenever $(x_0^i, y^i) \in B_0^i(p_0, q)$ and $x_1^i \in B_1^i(p_1, y^i)$.

[6] It is assumed that the information conveyed by prices cannot be used to enforce contracts.

[7] See Correia-da-Silva and Hervés-Beloso (2008, 2009, 2011) for a more detailed justification.

[8] The choice of $y^i \notin D^i(p_1)$ would never be optimal, as it would lead to the delivery of some $z^i \in D^i(p_1)$, cheaper than y^i. The agent would be better off by choosing z^i instead of y^i.

The choice set of agent i is, therefore:

$$C^i(p) = \left\{x^i = \left(x_0^i, y^i, x_1^i\right) \in \mathbb{R}_+^L \times \mathbb{R}^{SL} \times \mathbb{R}_+^{SL} : \; x^i \in B^i(p) \wedge y^i \in D^i(p_1)\right\}.$$

In sum, the problem of agent i can be written as:

$$\begin{aligned} \max \quad & U^i(x_0^i, x_1^i) \\ \text{s.t.} \quad & p_0 \cdot x_0^i + q \cdot y^i \leq p_0 \cdot e_0^i, \\ & p_1(s) \cdot x_1^i(s) \leq p_1(s) \cdot y^i(s) + p_1(s) \cdot e_1^i(s), \; \forall s \in \mathcal{S}, \\ & p_1(s) \cdot y^i(s) \leq p_1(s) \cdot y^i(t), \; \forall t \in P^i(s), \; \forall s \in \mathcal{S}. \end{aligned}$$

Or, equivalently, as:

$$\max \quad U^i(x_0^i, x_1^i) \;\; \text{s.t.} \;\; x^i \in C^i(p).$$

The choice of the financial intermediaries is denoted $x^f = (x_0^f, y^f, x_1^f) \in B^f(p)$, where the choice set, $B^f(p)$, is defined as $B^i(p)$ but with null endowments.[9] We assume that they wish to maximize an objective function that is strictly increasing:[10]

$$\max \quad U^f(x_0^f, x_1^f) \;\; \text{s.t.} \;\; x^f \in B^f(p).$$

The demand of the financial intermediaries becomes unbounded whenever, for some state s, the relative prices in the spot markets at $\tau = 1$ are different from the relative prices in the futures markets for contingent delivery in this state. That is, there are arbitrage opportunities unless we have $q(s)$ parallel to $p_1(s)$, for all $s \in \mathcal{S}$. If, for every state of nature, the prices in the futures markets and the prices in the spot markets are parallel,

[9] As long as free entry is allowed, the number of financial intermediaries is irrelevant. We can assume that they behave as a single price-taking intermediary.

[10] Our results do not depend on the actual specification of the firm's objective function. With incomplete markets, the difficulties in defining an appropriate objective function for a firm are well-known. See, for example, Drèze (1985).

the financial intermediaries cannot obtain any positive consumption plan, $(x_0^f, x_1^f) \neq 0$, and are, therefore, indifferent among any alternative in their choice set:

$$q(s) \parallel p_1(s),\ \forall s \in \mathcal{S} \ \Rightarrow\ q \cdot y^f = 0, \forall x^f \in B^f(p).$$

Hence, from now on, we will restrict our search for equilibrium prices to the following set of *no arbitrage* price systems:

$$\mathcal{P} = \left\{(p_0, q, p_1) \in \Delta^{L+SL} \times (\Delta^L)^S :\ \forall s \in \mathcal{S},\ q(s) \parallel p_1(s)\right\},$$

and suppose that the financial intermediaries clear the futures markets by choosing:

$$x^f = \left(0, y^f, 0\right), \quad \text{with} \ \ y^f = -\sum_{i \in \mathcal{I}} y^i,$$

which is an optimal choice that belongs to their choice set.

If agents make optimal choices and markets clear, the economy is in equilibrium.

Definition 1 (Equilibrium)**.**

An equilibrium of an economy, $E = \{e^i, U^i, P^i\}_{i \in \mathcal{I}}$, is a pair (x^*, p^*), where x^* is a vector of individual choices, $x^* = \{x^{i*}\}_{i \in \mathcal{I}}$, and $p^* \in \mathcal{P}$ is a price system, satisfying:

(i) $x^{i*} \in \underset{z \in C^i(p^*)}{\operatorname{argmax}}\ U^i(z_0, z_1),\ \forall i \in \mathcal{I}$ *[individual optimality];*

(ii) $\sum_{i \in \mathcal{I}} \left(x_0^{i*}, x_1^{i*}\right) = \sum_{i \in \mathcal{I}} \left(e_0^i, e_1^i\right)$ *[feasibility].*

To establish existence of equilibrium, we construct a sequence of economies in which the choice set of each agent i is $B_i(p)$ instead of $C_i(p)$. However, the choice of an $x_i \notin C_i(p)$ implies a utility penalty that is increasingly harsher along the sequence of economies. After obtaining a corresponding sequence of equilibria, we prove that an accumulation point (which exists) is an equilibrium of the original economy.[11] Assumptions 1 and 2

[11] If the correspondences from prices to the choice sets, $C_i(p)$, were continuous, it would be straightfor-

guarantee the existence of an equilibrium.

Theorem 1 (Existence)**.**

Under Assumptions 1 and 2, there exists an equilibrium of the economy $E = \{e^i, U^i, P^i\}_{i\in\mathcal{I}}$.

The welfare theorems do not necessarily hold. The information asymmetries may generate an inefficient allocation of risk-bearing, because the uninformed agents may not be able to make the desired wealth transfers across states and time.

Interestingly, the existence of markets for the future delivery of various goods (as opposed to contingent claims that are only payable in the numeraire good) generates additional possibilities for the transference of wealth across states and time. It is easy to construct an example in which a complete set of contingent markets allows agents to arrive at the optimal allocation of risk-bearing (the same as in the case of complete information), while securities are not sufficient.[12] This conclusion contrasts with the equivalence result obtained by Arrow (1953) for the case of public state-verification.

3 Generic efficiency

If there were no deliverability restrictions (as in the case of complete state-verification), the equilibrium allocation would be optimal in the sense of Pareto, because our model would coincide with the classical general equilibrium model as presented by Debreu (1959, chapter 7).

On the other hand, in the case in which there is a single good in each state, our model coincides with the model of Radner (1968). In that case, the incompleteness of state-

ward to establish existence of equilibrium (Debreu, 1952). But the choice correspondences are not lower hemicontinuous. This property fails when prices in some state are null ($\exists s: \; p_1(s) = 0$) or when prices in two indistinguished states are collinear ($\exists s, t \in P_i(s), k \in \mathbb{R}_{++}: \; p_1(s) = kp_1(t)$).

[12] An example can be found in a preliminary version of this work (Correia-da-Silva and Hervés-Beloso, 2010).

verification abilities implies efficiency losses and there is only constrained efficiency.[13]

In this section, we show that if there are at least as many goods as states of nature, then, generically, the equilibrium allocation is optimal in the sense of Pareto. In order to prove this, we proceed in two steps. First, we show that if the state-contingent spot price systems are linearly independent, then the agents are able to attain any consumption plan that is in their budget set (i.e., the deliverability constraints are not relevant) by choosing an appropriate "bridge portfolio". Afterwards, we show that, generically (i.e., in almost all economies), the state-contingent equilibrium price systems are linearly independent.

Lemma 1 (Wealth transfers)**.**

Consider a vector of strictly positive and linearly independent state-contingent spot price systems, $p_1 = (p_1(1), ...p_1(S)) \in \Delta_{+}^{SL}$, *and a vector of desired wealth transfers,* $w = (w(1), ...w(S)) \in \mathbb{R}_{++}^{S}$.

If $L \geq S$, *there exists a portfolio,* $y = (y(1), ...y(S)) \in \mathbb{R}^{SL}$, *that implements the desired wealth transfers,* $p_1(s) \cdot y(s) = w(s), \ \forall s \in \mathcal{S}$, *and satisfies the possible deliverability constraints,* $p_1(s) \cdot y(s) \leq p_1(s) \cdot y(t), \ \forall s, t \in \mathcal{S}$.

Proof. Let $\bar{w} = \max_{s \in \mathcal{S}} w(s)$. Observe that there exists a $y(s)$ such that:

$$\begin{bmatrix} p(1,1) & ... & p(1,L) \\ ... & ... & ... \\ p(s,1) & ... & p(s,L) \\ ... & ... & ... \\ p(S,1) & ... & p(S,L) \end{bmatrix} \begin{bmatrix} y(s,1) \\ ... \\ y(s,L) \end{bmatrix} = \begin{bmatrix} \bar{w} \\ ... \\ w(s) \\ ... \\ \bar{w} \end{bmatrix},$$

because the number of equations is not greater than the number of variables, $S \leq L$, and the equations are not inconsistent (they could be if the rows of the price matrix were linearly dependent).

[13]Constrained efficiency in the sense of Pareto-optimality among the allocations in which agents consume the same in states of nature that belong to the same set of their information partitions.

This means that $p_1(s) \cdot y(s) = w(s)$ while $p_1(t) \cdot y(s) = \bar{w} \geq w(t),\ \forall t.$ $\square$

To study the generic welfare properties of equilibria, we impose further restrictions on the preferences of the agents by assuming that they are *well-behaved* in the sense of Debreu (1972, p. 613). That is, if the preference relations are monotone, convex, continuous, complete preorderings of class C^2, and if the indifference hypersurfaces have everywhere a non-zero curvature and have their closures contained in $\mathbb{R}^{SL}_{++}$.[14] This assumption is not too strong, since any monotone, convex, continuous and complete preference relation can be approximated by a sequence of *well-behaved* preference relations (Mas-Colell, 1974).

Assumption 3 (Demand)**.**

The preferences of the agents are well-behaved *in the sense of Debreu (1972, p. 613).*

Assumption 3 implies that the preferences of the agents can be described by demand functions that are continuously differentiable (Debreu, 1972). Obviously, the resulting aggregate excess demand function is also continuously differentiable.

An additional property of the aggregate excess demand function that is required is that it converges to infinity when the price system converges to the boundary of the simplex. This is also guaranteed under Assumption 3.

We consider a space of economies, $\mathcal{E} = \mathbb{R}^{IN}_{++}$, in which preferences are kept fixed and satisfy Assumption 3. In this space, the vector of initial endowments, $e \in \mathbb{R}^{IN}_{++}$, completely characterizes an economy.

Let $N \equiv L + SL$ and define the space of admissible price systems, $\mathcal{P}' \subset \mathbb{R}^{N-1}$, as the open set that is obtained by removing the last coordinate from the interior of Δ^N.

The aggregate excess demand function, $Z : \mathcal{P}' \times \mathcal{E} \to \mathbb{R}^{N-1}$, is defined as the difference between the sum of the individual demands and the aggregate endowment. Again, we omit the last coordinate as it can be obtained from the others using Walras' Law.

[14] Preferences that satisfy this additional requirement of having indifference hypersurfaces with non-zero curvature were designated as *strongly convex* by Malinvaud (1972) and as *differentiably strictly convex* by Mas-Colell (1985).

We want to show that, generically (i.e., in an open and dense subset of $\mathcal{E}$), equilibrium prices for state-contingent delivery, $p_1(s)$ for each $s \in \mathcal{S}$, are linearly independent. In this case, we say that $p \in \mathcal{P}^*$.

Lemma 2 (Prices)**.**

There exists an open and dense set $\mathcal{E}^* \subset \mathcal{E}$ *such that,* $\forall e \in \mathcal{E}^* : \; Z(p, e) = 0 \Rightarrow p \in \mathcal{P}^*$.

Proof. Since Z has no critical point, by the regular value theorem, the set $M \equiv Z^{-1}(0)$ is a differentiable manifold of dimension IN (the equilibrium price manifold).[15]

Let $pr : M \to \mathcal{E}$ be the projection of the equilibrium price manifold to the parameter space $\mathcal{E}$. An economy is regular, $e \in \mathcal{R}$, if and only if it is a regular value of the projection $pr : M \to \mathcal{E}$. Otherwise, it is a critical economy. It is well-known that the set of critical economies, $\mathcal{C} = \mathcal{E} \setminus \mathcal{R}$, is null (Debreu, 1970).

A price system is a regular equilibrium price system of the economy $e \in \mathcal{E}$ if and only if $Z(p, e) = 0$ and $\partial_p Z(p, e)$ has full rank. If an economy is regular, all its equilibrium price systems are regular (Dierker, 1982).

Therefore, for $\bar{e} \in \mathcal{R}$, we can apply the implicit function theorem to obtain the following result. Given a point of the equilibrium manifold, $(\bar{p}, \bar{e}) \in M$, there are open sets, $\mathcal{P}'' \subset \mathcal{P}'$ and $\mathcal{E}' \subset \mathcal{E}$, and a C^1 function $g : \mathcal{E}' \to \mathcal{P}''$ such that $g(\bar{e}) = \bar{p}$ and, for $(p, e) \in \mathcal{P}'' \times \mathcal{E}'$, $Z(p, e) = 0$ if and only if $g(e) = p$. Moreover, $\partial g(\bar{e}) = - \left[\partial_p Z(\bar{p}, \bar{e})\right]^{-1} \partial_e Z(\bar{p}, \bar{e})$, which implies that $\partial g(\bar{e})$ has full rank.

This means that we can move the equilibrium price in any direction by perturbing the initial endowments. Any neighborhood of $(\bar{p}, \bar{e}) \cap M$ contains, therefore, equilibrium price systems for which the prices in different states are not linearly dependent.

On the other hand, since $\partial g(\bar{e})$ has finite values, if $\bar{p} \in \mathcal{P}^*$, then there is a neighborhood of $(\bar{p}, \bar{e}) \cap E$ such that prices in different states are also in $\mathcal{P}^*$. □

[15] See, for example, Balasko (2009, p. 28).

The main result of this section is a straightforward consequence of Lemmas 1 and 2.

Theorem 2 (Optimality)**.**

If $L \geq S$, there exists an open and dense set $\mathcal{E}^* \subset \mathcal{E}$ such that, $\forall e \in \mathcal{E}^*$, the equilibria of the economy with complete state-verification are also equilibria of any economy with the same endowments and preferences but with incomplete state-verification.

This surprising result establishes that, if $L \geq S$, the information partitions of the agents are irrelevant. The equilibrium allocation is independent of the information structure.

4 Conclusion

We have shown that the classical general equilibrium model of trade under uncertainty (Debreu, 1959, chapter 7) can be extended to the case in which agents have incomplete and differential abilities to verify the occurence of events. In contrast with our previous work, we allowed agents to trade in the second period. This setup is more realistic because, with private state-verification, the second-period spot markets are not redundant. In fact, the combination of futures markets (that open in the first-period) with second-period spot markets expands the possibilities for wealth transfers across states and time.

We concluded that the opening of spot markets in the second period guarantees existence of equilibrium (under the standard assumptions). To establish existence, it is no longer necessary to make the assumption (needed in Correia-da-Silva and Hervés-Beloso, 2011) that every state of nature can be verified by at least one agent.

In this model, market incompleteness arises endogenously as a consequence of incomplete state-verification, if the number of states of nature is greater than the number of goods. In this case, the equilibrium allocation is typically inefficient. It should be clear, however, that in comparison with the model of Radner (1968) and the huge literature on differential information economies that followed (see, for example, Glycopantis and

Yannelis, 2005), the Pareto-frontier is expanded because there are additional trade possibilities in the first period and because agents also benefit from the possibility of trading in the second period.

If the number of states is not greater than the number of goods, markets actually become complete (in spite of incomplete state-verification). Agents are able to induce truthful delivery of the desired wealth transfers by choosing, for delivery in each state, goods that are relatively cheap in this state but relatively expensive in the other states. This strong and surprising result suggests that the information of the agents (with information being an exogenously given ability to verify the occurrence of events) is irrelevant. The equilibrium allocation is independent of the information structure of the economy.

We remark that, despite the fact that the relative prices for future delivery in a given state coincide with the relative prices in the future spot markets in the same state, the agents do not buy, in the futures markets, the bundle that they desire to consume in the future (this would render the future spot markets irrelevant). In the futures markets, agents select a "bridge portfolio", not intended for consumption, but to induce the desired wealth transfers in the absence of complete state-verification.

The optimal allocation of risk-bearing cannot, however, be achieved by a system of securities and commodity markets, with securities being payable in money (Arrow, 1953).[16] It may be the case that a complete set of contingent markets allows agents to arrive at an optimal allocation of risk-bearing, while a system of securities and commodity markets does not. If agents have incomplete abilities to verify the occurrence of relevant events, what was a redundancy in the ways of transferring wealth across states becomes useful as a means of enforcing truthful deliveries.

[16]In a seminal work, Arrow (1953) has shown that an optimal allocation of risk-bearing could be achieved by a system of securities and commodity markets, with securities being payable in money. This permits economizing on markets. Only $S + L$ markets (where S is the number of states of nature, and L is the number of commodities) are needed, instead of a complete set of markets for contingent claims on commodities, which totals a number of SL markets.

5 Appendix

Proof of Theorem 1.

We start by constructing a sequence of Arrow-Debreu economies (i.e., with public state-verification), $\{E_n\}_{n\in\mathbb{N}}$. In each economy of the sequence, agents have the same endowments as in the economy under study, but modified utility functions. The choice set of each agent i is $B^i(p)$ instead of $C^i(p)$, but agent i suffers a utility penalty if she chooses an $x^i \notin C^i(p)$. These penalties become harsher along the sequence.

In the economy $E_n = \{e^i, U_n^i\}_{i\in\mathcal{I}}$, the utility functions of the agents are:[17]

$$U_n^i(x^i, p_1) = U^i(x_0^i, x_1^i) - n \sum_{s\in\mathcal{S}} \mu(s) \max_{t\in P^i(s)} \{p_1(s)\cdot y^i(s) - p_1(s)\cdot y^i(t)\}.$$

It is obvious that, for any $n \in \mathbb{N}$, the utility functions, U_n^i, are continuous. The maximum of linear functions is a convex function, and multiplying a convex function by a negative constant, $-n$, yields a concave function. Hence, the objective functions, $U_n^i(x^i, p_1)$, are concave in the first variable. Observe also that the utility penalty preserves *no satiation.* The plan $x^i + \epsilon\bar{1}$ is always preferred to x^i (the utility penalty is kept constant).

To show existence of competitive equilibrium in E_n, consider, for now, the following convex and bounded choice space:

$$\bar{X} = \{z \in \mathbb{R}_+^L \times \mathbb{R}^{SL} \times \mathbb{R}_+^{SL} : \ \big(0, -2e_1^T, 0\big) \leq (z_0, w, z_1) \leq \big(2e_0^T, 2e_1^T, 2e_1^T\big)\}.$$

The budget correspondence of agent i, in this bounded economy, is:

$$\bar{B}^i(p) = B^i(p) \cap \bar{X}.$$

[17]Notice that, since $s \in P^i(s)$, penalties are never negative.

For each $i \in \mathcal{I}$, let $\psi_n^i(x,p) = \underset{z^i \in \bar{B}^i(p)}{\operatorname{argmax}} \left\{ U_n^i(z^i, p_1) \right\}$.

By Lemma 3, the budget correspondences, $\bar{B}^i(p)$, are continuous with nonempty compact values. Hence, by Berge's Maximum Theorem, the demand correspondence, $\psi_n^i(x,p)$, is u.h.c. with nonempty compact values.[18] It is also convex-valued, because U_n^i is concave in the first variable.

An auctioneer chooses a price system with the objective of maximizing the value of excess demand. Since $\mathcal{P}$ is not convex, let the auctioneer choose prices with $(p_0, p_1) \in \Delta^{L+SL}$ and $q = p_1$, and denote this space by $\hat{\mathcal{P}}$.

Let $\psi_n^p(x,p) = \underset{p \in \hat{\mathcal{P}}}{\operatorname{argmax}} \left\{ p_0' \cdot \sum_{i \in \mathcal{I}} (x_0^i - e_0^i) + p_1' \cdot \sum_{i \in \mathcal{I}} (x_1^i - e_1^i) \right\}$.

This correspondence is also u.h.c. with nonempty compact and convex values. Therefore, the product correspondence, $\psi_n = \Pi_{i \in \mathcal{I}} \psi_n^i \times \psi_n^p$, also is. Applying the Theorem of Kakutani, we find that there exists a fixed point of ψ_n, that we denote by (x_n, p_n). To prove that it is an equilibrium of E_n, we must show that it satisfies feasibility.

Suppose that there is excess demand for some good. If another good does not have excess demand, its price must be zero, which, in turn, implies excess demand. Hence, there must be excess demand for all the goods in the spot markets (at $\tau = 0$ and at $\tau = 1$).

Aggregating the budget restrictions at $\tau = 0$, we obtain (recall that $q = p_1$):

$$\sum_{i \in \mathcal{I}} p_1 \cdot y^i \leq \sum_{i \in \mathcal{I}} p_0 \cdot \left(e_0^i - x_0^i \right) \leq 0.$$

On the other hand, aggregating the budget restrictions at $\tau = 1$, we obtain:

$$\sum_{i \in \mathcal{I}} p_1(s) \cdot y^i(s) \geq \sum_{i \in \mathcal{I}} p_1(s) \cdot \left[x_1^i(s) - e_1^i(s) \right] \geq 0, \ \forall s \in \mathcal{S}.$$

[18] See, for example, Aliprantis and Border (2006).

This implies that:

$$\sum_{i\in\mathcal{I}} p_1(s)\cdot y^i(s) = \sum_{i\in\mathcal{I}} p_1(s)\cdot \left[e_1^i(s) - x_1^i(s)\right] = 0,\ \forall s\in\mathcal{S}.$$

Therefore, $p_0 = 0$ and $p_1 = 0$. Contradiction. There is no excess demand.

The usual extension to the unbounded choice set applies, therefore, (x_n, p_n) is an equilibrium of $E_n = \{e^i, U_n^i\}_{i\in\mathcal{I}}$. Convert the price system from $\hat{\mathcal{P}}$ to $\mathcal{P}$, dividing each $p_{1n}(s)$ by $\|p_{1n}(s)\|_1$.

The resulting sequence of equilibria, $\{(x_n, p_n)\}_{n\in\mathbb{N}}$, which is contained in a compact set, has an accumulation point, denoted by (x^*, p^*). This is our candidate for an equilibrium of the original economy.

It is straightforward to see that x^* is feasible, $\sum_{i\in\mathcal{I}} x^{i*} \le \sum_{i\in\mathcal{I}} e^i$, and that it satisfies the budget restrictions, $x^{i*}\in B^i(p^*),\ \forall i\in\mathcal{I}$.

Suppose that x^{i*} violated one of the delivery restrictions, $x^{i*}\notin D^i(p_1^*)$, by more than $\delta > 0$. Then, for sufficiently high n, x_n^i would also violate the corresponding restriction by more than δ. For $t\in P^i(s)$, $\exists n_0\in\mathbb{N}$ such that, for all $n > n_0$:

$$p_1^*(s)\cdot y^{i*}(s) > p_1^*(s)\cdot y^{i*}(t) + \delta \ \Rightarrow\ p_{1n}(s)\cdot y_n^i(s) > p_{1n}(s)\cdot y_n^i(t) + \delta.$$

Utility among feasible allocations is bounded by $U^i(e^T)$, so we can consider a n_0 that is sufficiently high for $n_0\delta > U^i(e^T) - U^i(e^i)$. It would follow that $U_n^i(x_n^i, p_n) < U^i(x_n^i) - n_0\delta < U^i(x_n^i) - U^i(e^T) + U^i(e^i) < U^i(e^i) = U_n^i(e^i, p_n)$. Contradiction.

To establish that (x^*, p^*) is an equilibrium, we only need to prove that each x^{i*} is individually optimal at prices p^*.

Individual optimality of x^{i*}.

Assume (by way of contradiction) that there exists $x'\in C^i(p^*)$ such that $U^i(x') > U^i(x^{i*})$.

We will show that this implies that (x_n, p_n) is not an equilibrium of E_n, for large n.

Observe that if $p_1(s) = p_1(t)$ with $t \in P^i(s)$, the deliverability conditions imply:

$$\begin{cases} p_1^*(s) \cdot [y'(s) - y'(t)] \leq 0 \\ p_1^*(t) \cdot [y'(t) - y'(s)] \leq 0 \end{cases} \Rightarrow \begin{cases} p_1^*(s) \cdot [y'(s) - y'(t)] = 0 \\ p_1^*(t) \cdot [y'(s) - y'(t)] = 0 \\ q^*(s) \cdot [y'(s) - y'(t)] = 0 \\ q^*(t) \cdot [y'(s) - y'(t)] = 0. \end{cases}$$

Therefore, the agent obtains the same utility by choosing $y''(s) = y''(t) = \frac{y'(s)+y'(t)}{2}$ instead of $y'(s)$ and $y'(t)$. Define $w \in C^i(p^*)$ by modifying y' in this way.

By continuity of U^i, there exists $\delta > 0$ such that $x'' = (1-\delta)w$ is strictly preferred to x^{i*}, belongs to $C^i(p^*)$, is in the interior of $B^i(p^*)$, and is also in the interior of $B^i(p_n)$, for n greater than some n_0.

Furthermore, there exists $\epsilon > 0$ such that $d(z, x'') < \epsilon$ implies that $U^i(z) > U^i(x_i^*)$, with z in the interior of $B^i(p^*)$. There also exists $n_1 > n_0$ such that $d(z, x'') < \epsilon$ implies that z is in the interior of $B^i(p_n)$ and that $U^i(z) > U_i(x_n^i)$ (notice that we are considering U^i and not U_n^i), for all $n > n_1$.

Let $n_2 > n_1$ be sufficiently large for $d(p_n, p^*) < \epsilon, \forall n > n_2$.

To finish the proof, we will construct $\hat{x} \in B(x'', \epsilon)$ that belongs to $C^i(p_n)$, contradicting the fact that x_n^i maximizes U_n^i at prices p_n.

Let $k^{(s,t)} = p_1^*(s) \cdot [y''(t) - y''(s)]$. Since $x'' \in C^i(p^*)$:

$$t \in P^i(s) \;\Rightarrow\; p_1^*(s) \cdot [y''(t) - y''(s)] = k^{(s,t)} \geq 0.$$

Let $d\hat{x} = \hat{x} - x''$ and $dp_n = p_n - p^*$. Manipulating a deliverability condition:

$$p_1^*(s) \cdot [y''(t) - y''(s)] = k^{(s,t)} \Leftrightarrow$$
$$\Leftrightarrow [p_{1n}(s) - dp_{1n}(s)] \cdot [\hat{y}(t) - d\hat{y}(t) - \hat{y}(s) + d\hat{y}(s)] = k^{(s,t)} \Leftrightarrow$$
$$\Leftrightarrow p_{1n}(s) \cdot [\hat{y}(t) - \hat{y}(s)] = k^{(s,t)} + p_{1n}(s) \cdot [d\hat{y}(t) - d\hat{y}(s)] + dp_{1n}(s) \cdot [y''(t) - y''(s)] \Leftrightarrow$$
$$\Leftrightarrow p_{1n}(s) \cdot [\hat{y}(t) - \hat{y}(s)] > k^{(s,t)} - 2\epsilon - 2\epsilon\|e_T\|.$$

Define k^{min} as the minimum among the strictly positive $k^{(s,t)}$.

Choose a smaller $\epsilon > 0$, if necessary, to make $2\epsilon(\|e_T\| + 1) < k^{min}$. This guarantees that the strict inequalities for x'' and p_1^* remain strict for any $\hat{x} \in B(x'', \epsilon)$ and p_{1n} with $n > n_2$. If all $k^{(s,t)}$ were strictly positive, then $\hat{x}$ would have no utility penalty. We would have $U_n^i(\hat{x}) > U_n^i(x_n^i)$, which would be a contradiction (the consumption plan in the equilibrium sequence, x_n^i, would not be a maximizer of U_n^i).

If some inequalities are not strict for x'' and p_1^*, we need to guarantee that they are still satisfied for some $\hat{x} \in B(x'', \epsilon)$ and some p_{1n} with $n > n_2$.

Select displacements from y'' to $\hat{y}$ that are parallel to p_1^*, choosing:

$$d\hat{y}(s) = -\frac{\epsilon}{2} \frac{p_1^*(s)}{\|p_1^*(s)\|}.$$

Now define $\gamma^{(s,t)} = \left(1 - \frac{p_1^*(s) \cdot p_1^*(t)}{\|p_1^*(s)\|\|p_1^*(t)\|}\right) \|p_1^*(s)\|$. Notice that $\gamma^{(s,t)} = 0$ if and only if $p_1^*(s) = p_1^*(t)$. Let γ^{min} as the lowest of the strictly positive $\gamma^{(s,t)}$.

Let $\epsilon_2 = \frac{\epsilon \gamma^{min}}{4\|e^T\|}$, and consider some $n_3 > n_2$ that is large enough for:

$$d(p_n, p^*) < \min\{\epsilon_2, \epsilon\}, \forall n > n_3.$$

Consider an inequality that is not strict for p^* and w'', i.e., some $k^{ab} = 0$. If $p_1(a) \neq p_1(b)$, we have $\gamma^{ab} \geq \gamma^{min}$. This inequality still holds for p_n, with $n > n_3$, and $\hat{y}$:

$$\begin{aligned}
&p_{1n}(a) \cdot [\hat{y}(b) - \hat{y}(a)] = \\
&= p_1^*(a) \cdot [w''(b) + d\hat{y}(b) - w''(a) - d\hat{y}(a)] + dp_{1n}(a) \cdot [\hat{y}(b) - \hat{y}(a)] = \\
&= p_1^*(a) \cdot [d\hat{y}(b) - d\hat{y}(a)] + dp_{1n}(a) \cdot [\hat{y}(b) - \hat{y}(a)] > \\
&> p_1^*(a) \cdot [d\hat{y}(b) - d\hat{y}(a)] - 2\epsilon_2 \|e_T\| = \\
&= p_1^*(a) \cdot \frac{\epsilon}{2} \left[\frac{p_1^*(a)}{\|p_1^*(a)\|} - \frac{p_1^*(b)}{\|p_1^*(b)\|} \right] - 2\epsilon_2 \|e_T\| = \\
&= \frac{\epsilon}{2} \frac{p_1^*(a) \cdot p_1^*(a)}{\|p_1^*(a)\| \|p_1^*(a)\|} \|p_1^*(a)\| - \frac{\epsilon}{2} \frac{p_1^*(a) \cdot p_1^*(b)}{\|p_1^*(a)\| \|p_1^*(b)\|} \|p_1^*(a)\| - \frac{\epsilon}{2} \gamma^{min} = \\
&= \frac{\epsilon}{2} \gamma^{ab} - \frac{\epsilon}{2} \gamma^{min} \geq 0.
\end{aligned}$$

If $p^1(a) = p^1(b)$, then $x''^1(a) = x''^1(b)$ and $d\hat{y}(a) = d\hat{y}(b)$. In this case, $\hat{y}(a) = \hat{y}(b)$ and the deliverability condition is also satisfied.

Hence, $U_i^n(\hat{x}) > U_i^n(x_i^n)$. Contradiction. □

Lemma 3.

In the bounded economy, the budget correspondence, $\bar{B}^i$, is continuous.

Proof of Lemma 3:

It is easy to see that $\bar{B}^i(p)$ is upper hemicontinuous, as the inequalities which must be satisfied are not strict.

Let $x \in \bar{B}^i(p)$ and consider a ball centered at x with radius $\epsilon > 0$, denoted $B(x, \epsilon)$. To prove that $\bar{B}^i$ is lower hemicontinuous, we need to show that $\exists \delta > 0$ such that, for a given $p' \in B(p, \delta)$, there exists $z \in B(x, \epsilon) \cap \bar{B}^i(p')$.

Observe that $(x_0', y', x_1') = (0, 0, 0)$ strictly satisfies all the budget restrictions. Therefore, any convex combination of x and x' also does. Let x'' be a convex combination of x and x' with enough weight on x so that it belongs to $B(x, \epsilon)$.

We have $p_0 \cdot x''_0 + q \cdot y'' - p_0 \cdot e^i_0 < 0$ and $p_1(s) \cdot x''_1(s) - p_1(s)\,[y''(s) + e^i_1(s)] < 0,\ \forall s \in \mathcal{S}$. By continuity, for sufficiently small δ, any $p' \in B(p, \delta)$ preserves the inequalities. $\square$

References

Allen, B. (1981), "Generic Existence of Completely Revealing Equilibria for Economies with Uncertainty when Prices Convey Information", *Econometrica*, 49 (5), 1173-1199.

Arrow, K.J. (1953), "The Role of Securities in the Optimal Allocation of Risk-Bearing", *Econometrie*, translated and reprinted in 1964, *Review of Economic Studies*, 31 (2), 91-96.

Balasko, Y. (2009), "The Equilibrium Manifold", MIT Press, Cambridge.

Bisin, A. and P. Gottardi (1999), "Competitive Equilibria with Asymmetric Information", *Journal of Economic Theory*, 87 (1), 1-48.

Bisin, A. and P. Gottardi (2006), "Efficient Competitive Equilibria with Adverse Selection", *Journal of Political Economy*, 114 (3), 485-516.

Correia-da-Silva, J. and C. Hervés-Beloso (2008), "Subjective Expectations Equilibrium in Economies with Uncertain Delivery", *Journal of Mathematical Economics*, 44 (7-8), 641-650.

Correia-da-Silva, J. and C. Hervés-Beloso (2009), "Prudent Expectations Equilibrium in Economies with Uncertain Delivery", *Economic Theory*, 39 (1), 67-92.

Correia-da-Silva, J. and C. Hervés-Beloso (2010), "Two-period economies with private state verification", FEP Working Papers 374.

Correia-da-Silva, J. and C. Hervés-Beloso (2011), "General Equilibrium with Uncertain Delivery", *Economic Theory*, forthcoming.

Debreu, G. (1952), "A Social Equilibrium Existence Theorem", *Proc. Natl. Acad. Sci. USA*, 38, 803-886.

Debreu, G. (1959), "Theory of Value", Wiley, New York.

Debreu, G. (1970), "Economies with a finite set of equilibria", *Econometrica*, 38 (3), 387-392.

Debreu, G. (1972), "Smooth Preferences", *Econometrica*, 40 (4), 603-615.

Dierker, E. (1982), "Regular Economies", in *Handbook of Mathematical Economics*, Vol. 2, Chapter 17, edited by K.J. Arrow and M.D. Intriligator, North-Holland, Amsterdam.

Drèze, J.H. (1985), "(Uncertainty and) The Firm in General Equilibrium Theory", *Economic Journal*, 95, 1-20.

Glycopantis, D. and N.C. Yannelis (2005), "Differential Information Economies", *Studies in Economic Theory*, 19, Springer, New York.

Hurwicz, L. (1972), "On Informationally Decentralized Systems", in *Decision and Organization*, edited by M. McGuire and R. Radner, North Holland, Amsterdam.

Jerez, B. (2005), "Incentive Compatibility and Pricing Under Moral Hazard", *Review of Economic Dynamics*, 8, 28-47.

Magill, M. and M. Quinzii (1996), "Theory of Incomplete Markets", MIT Press, Cambridge.

Malinvaud, E. (1972), "Prices for Individual Consumption: Quantity Indicators for Collective Consumption", *Review of Economic Studies*, 39 (4), 385-405.

Mas-Colell, A. (1974), "Continuous and Smooth Consumers: Approximation Theorems", *Journal of Economic Theory*, 8 (3), 305-336.

Mas-Colell, A. (1985), "The Theory of General Economic Equilibrium - A Differentiable Approach", Cambridge University Press, Cambridge.

Myerson, R.B. (1979), "Incentive Compatibility and the Bargaining Problem", *Econometrica*, 47 (1), 61-74.

Prescott, E.C. and R.M. Townsend (1984a), "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard", *Econometrica*, 52 (1), 21-46.

Prescott, E.C. and R.M. Townsend (1984b), "General Competitive Equilibria Analysis in an Economy with Private Information", *International Economic Review*, 25 (1), 1-20.

Radner, R. (1968), "Competitive Equilibrium Under Uncertainty", *Econometrica*, 36 (1), 31-58.

Radner, R. (1972), "Existence of equilibrium of plans, prices, and price expectations in a sequence of markets", *Econometrica*, 40 (2), 289-304.

Radner, R. (1979), "Rational Expectations Equilibrium: Generic Existence and the Information Revealed by Prices", *Econometrica*, 47 (3), 655-678.

Rustichini, A. and P. Siconolfi (2008), "General Equilibrium in Economies with Adverse Selection", *Economic Theory*, 37 (1), 1-29.

Townsend, R.M. (1979), "Optimal Contracts and Competitive Markets with Costly State Verification", *Journal of Economic Theory*, 21 (2), 265-293.

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Faculdade de Economia da Universidade do Porto
Rua Dr. Roberto Frias, 4200-464 Porto | Tel. 225 571 100
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