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in Progress

n. 617 Dez 2019

ISSN: 0870-8541

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Hysteresis and Sources of Aggregate Employment Inertia

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Abstract

It is widely recognized that aggregate employment dynamics is characterized by hysteresis. In the presence of hysteresis the long run level of employment instead of being unique, and history-independent depends on the adjustment path that is taken, which includes the monetary and fiscal measures. It is thus important to study the presence of hysteresis in the macrodynamics of employment to understand whether the recession followed 2007's financial crisis will have permanent effects, and prospectively to conduct fiscal and monetary policies. The main contribution of this paper is to analyse the relative contribution of the main sources hysteresis (non-convex adjustment costs, uncertainty, and the flexibility of working time arrangements) to the width of the employment band of inaction. For that purpose, a switching employment equation was estimated from a computational implementation of the linear play model of hysteresis. From our results we found significant hysteresis effects in the aggregate employment dynamics caused by the presence of non-convex adjustment costs as uncertainty. We also found that the flexibility firms may have to adjust labour input by varying the number of hours of work per employee helps to mitigate the effect of uncertainty upon the band of inaction.

JEL Classification E24; J23.

Keywords: employment, hysteresis, adjustment costs, uncertainty, hours of work

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[†] Center for Economics and Finance at the University of Porto. This research has been financed by Portuguese public funds through FCT - Fundação para a Ciência e a Tecnologia, I.P., in the framework of the project UID/ECO/04105/2019.

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1. Introduction

Hysteresis is a unifying mathematical concept, firstly applied in the field of physics of magnetism, where the dynamics of an input-output system has the properties of: *a*) non-linearity, in the sense that reversing a shock does not bring back the system to its initial state; *b*) *remanence*, since temporary and sufficiently large shocks may have permanent effects; and *c*) selectivity, because only non-dominated shocks have permanent effects (see Mayergoyz, 1986; 2003).

These properties are typically observed in the macrodynamics of employment due to the presence of non-convex adjustment costs, such as in firm's investment projects (associated with physical and human capital) and market entry (see *e.g.*, Cross, 1994, Belke and Göcke, 1999, Mota and Vasconcelos, 2012, and Mota *et al*, 2012; 2015).

Non-convex adjustment costs creates a wedge between the marginal revenue of the employment adjustment and the corresponding cost, giving rise to different firms' employment demand thresholds for upward and downward adjustment. The employment band of inaction is the difference between these two thresholds. Inside this band, aggregate demand changes are not enough to induce firms to adjust the number of employees (see, *e.g.*, Bertola, 1990, Dixit 1991, Belke and Göcke 1999).

In addition to non-convex adjustment costs, other sources of employment inertia are firms' demand uncertainty and the capability of firms to adjust labour input along the intensive margin (*i.e.*, fluctuations in hours worked per employee).

The band of inaction is a positive function of the magnitude of the non-convex adjustment costs, the level of demand uncertainty, and the flexibility of working time adjustment in response to changing demand conditions.

In the presence of hysteresis there is no unique and predetermined long-run equilibrium. The long run outcome of the economic system depends on the adjustment path that is taken, which includes monetary and fiscal measures (see, *e.g.*, Cross *et al.*, 2005, Arestis and Sawyer, 2009, Setterfield, 2009, Cross, 2014, Göcke and Matulaityte, 2015, and Bassi and Lang, 2016).

The presence of hysteresis in the macrodynamics of employment is a key issue. Noteworthy, to understand whether the recession that have followed 2007 financial crisis will have permanent effects (see Cross, 2014, Ball, 2014, Bassi and Lang, 2016, and Yagan, 2017) and, prospectively, to conduct fiscal and monetary policies (see DeLong and Summers, 2012).

The main contribution of this paper is to analyse the relative contribution of non-convex adjustment costs, uncertainty, and the flexibility of working time arrangements to the width of the employment band of inaction.

For this purpose, a switching employment equation is estimated from a computational implementation of the linear play model of hysteresis (also known as the friction-backlash model[4]). On one hand, this equation describes the behaviour upon which for small changes of aggregate demand there is a weak reaction of employment (due to the presence of adjustment costs, uncertainty and the possibility that firms have to adjust labour input by varying the number of hours of work per employee). On the other hand, for large changes there is a strong reaction. Portuguese monthly data from industry spanning from January 2000 to December 2018 is used.

The remainder of the paper is organised as follows. Section 2 presents the microfoundations of employment hysteresis together with the sources of inertia at the firm level and describes the aggregation procedure up to the macro level. Section 3 presents the data set and the empirical strategy proposed. Section 4 presents our empirical results, and Section 5 concludes.

2. Micro foundations of Hysteresis and Aggregate Behaviour

Depending on the level of aggregation and on the degree of heterogeneity across agents, a weak (micro) form of hysteresis and a strong (macro) one can be distinguished. Both types of hysteresis are characterized as an input-output system with specific properties and non-linear response to shocks (Amable *et al*., 1994).

2.1. Weak Hysteresis

It is widely recognized in the literature that firms face costs to adjust their level of employment (see *e.g*. Hamermesh 1989, Caballero *et al*. 1997, Varejão and Portugal 2007, and Mota *et al*. 2012).

[4] Linear play models, although not standard in the economic literature, are well suited to describe the aggregate dynamics of employment (see, *e.g.*, Belke and Göcke, 1999; 2001, Göcke, 2002, Mota and Vasconcelos, 2012, and Mota *et al*, 2012; 2015).

Employment adjustment costs are incurred when workers are hired or dismissed, and take the form of irreversible costs, *i.e.*, they are sunk costs. These costs may be *fixed (convex)* or *variable (non-convex)* depending on how they vary with the size of employment adjustment.

The costs of advertising, recruiting and training the new workers, including the costs resulting from disruption in production when the new workers are hired are examples of adjustment costs on the hiring side. On the firing side, adjustment costs include mandatory procedural requirements for employers initiating a dismissals process, including the notice period for dismissal, and severance payments determined by employment protection legislation.

An important part of these adjustment costs are non-convex, *i.e.*, independent of the number of workers that are hired or dismissed (see, *e.g.*, Nickel, 1978, and Hamermesh and Pfann, 1996). They are mainly related to personnel and legal departments to deal with hires and fires.

The presence of non-convex employment adjustment costs, at the firm level, leads to an employment decision conditional on two separate thresholds of aggregate demand, $P_{exit,j}$ and $P_{entry,j}$. The former induces firm j to exit the market (or to contract the level of employment) and the latter to enter the market (or to expand the level of employment).

The consequence is that employment adjustments should be made relatively infrequently and in large amounts, rather than continuously responding to small changes of firms' product demand. This is precisely what is found in the empirical literature (see, *e.g.*, Caballero and Engel, 1993, Caballero *et al*., 1997, Rota, 2004, King and Thomas, 2006, Nilsen *et al*., 2007, Varejão and Portugal, 2007, and Mota *et al*., 2012).

This behaviour can be well described by a non-ideal relay hysteresis operator, $\mathcal{R}_{Y_{exit,j},Y_{entry,j}}$.

Let us assume that: *i*) the level of employment of every heterogeneous firm j is $\mathcal{R}_{Y_{exit,j},Y_{entry,j}} = 1$, if the firm is active (*i.e.*, in the market), and $\mathcal{R}_{Y_{exit,j},Y_{entry,j}} = 0$, if the firm is inactive (*i.e.*, out of the market); *ii*) when active each firm produces one unit of output, with a variable costs w_j, which it sells at a price P_t; *iii*) every individual firm must pay a fixed and constant cost in time to hire a worker (enter), H_j, or to fire its single worker (leave the market), F_j. In this setting, the employment demand function of firm j,

derived from a profit maximizing problem with discrete time and an infinite plan horizon is[5]

$$n_{j,t} = \mathcal{R}_{P_{exit,j},P_{entry,j}}(t_0, n_{j,t_0})[P_t] = \begin{cases} 1, if\ n_{j,t-1} = 0\ and\ P_t \geq w_j + \frac{i}{i+1}H_j \\ \quad or\ n_{j,t-1} = 1\ and\ P_t > w_j - \frac{i}{i+1}F_j \\ \\ 0, if\ n_{j,t-1} = 0\ and\ P_t < w_j + \frac{i}{i+1}H_j \\ \quad or\ n_{j,t-1} = 1\ and\ P_t \leq w_j - \frac{i}{i+1}F_j \end{cases} \quad (1)$$

where $n_{j,t}$ denotes the employment of firm j in period t, $P_{exit,j} = w_j - \frac{i}{i+1}F_j$ and $P_{entry,j} = w_j + \frac{i}{i+1}H_j$ ($P_{exit,j} < P_{entry,j}$) are the threshold values for exit and entry, P_t ($t \geq t_0$) is the price level (considered as a proxy of aggregate demand), and n_{j,t_0} is the initial state that can take the values of 1 or 0. The values of the operator are defined for $n_{j,t_0} = 0$ if $P_{t_0} < P_{exit,j}$, for $n_{j,t_0} = 1$ if $P_{t_0} > P_{entry,j}$, and both for $n_{j,t_0} = 0$ and $n_{j,t_0} = 1$ if $P_{entry,j} < P_{t_0} < P_{exit,j}$.

A situation where firm j enters or stays active produces $n_{j,t} = 1$ while a situation where firm j stays inactive or exits produces $n_{j,t} = 0$. The difference between the two trigger points $P_{entry,j} - P_{exit,j} = \frac{i}{i+1}(H_j + F_j)$ creates an employment band of inaction (see Bertola, 1990, Dixit, 1992 and Belke and Göcke, 2001).

Each firm requires an aggregate positive demand shock $P_t > P_{entry,j}$ to hire its workforce (or enter) and an aggregate negative demand shock $P_t < P_{exit,j}$ to fire (or exit). Demand shocks within the range $P_{exit,j} < P_t < P_{entry,j}$ do not change employment.

Furthermore the current level of demand, P_t, is not sufficient to determine the firm's state of employment and the whole history of P_t must be taken into account. Therefore the system exhibits path dependence and 'multibranch non-linearity'.[6]

[5] We assume a discount factor is $\delta = \frac{1}{1+i}$. See Göcke (2002) and Mota *et al.* (2012) for a complete description of the model.

[6] Note that the decision to enter the market is akin to the hiring decision, and the decision to exit is akin to the firing decision. This simplification does not restricts the application of the model as we can consider a firm disaggregated into single production units, each of them represented by a non-ideal relay (see Belke and Göcke, 1999).

In addition, the literature shows that aggregate demand uncertainty interacts with the non-convex employment adjustment costs, widening the employment band of inaction and reinforcing the hysteresis effects (see, *e.g.*, Dixit, 1989, Dixit and Pindyck, 1994 and Belke and Göcke, 1999).

Modelling uncertainty in a simple way by assuming a nonrecurring single stochastic change in the output price, which can be either positive, $+\mu$, or negative,$-\mu$, with the same probability, Belke and Göcke (1999) showed that the employment band of inaction widens by a factor of $\frac{2\mu}{1+2i}$.

It is also documented in the literature that hysteresis effects in the dynamics of employment are enlarged by firms' ability to adjust labour input along the intensive margin - the hours adjustment margin (see Mota and *et al.*, 2012). The flexibility employers have to adjust working time in response to changes in labour demand depends on the characteristics of short-time working schemes (including work-sharing requirements, eligibility requirements, conditionality requirements and the generosity of short-time working compensation), the restrictions on overtime work and wage premiums, and the possibility of using working-time accounts.[7]

Together with employment non-convex adjustment costs and uncertainty these regulations affect the upward and downward flexibility of hours of work per employee and determine the firm's response to demand shocks.

2.2. Strong Hysteresis

An aggregation approach based on the Preisach model of hysteresis[8] is now exposed.

Taking $P_{exit,min}$ as the exiting threshold for the less demanding firm and $P_{entry\,max}$ as the entering threshold of the most demanding firm, the Preisach triangle is defined by

$$T = \{(P_{exit,j}, P_{entry,j}): P_{entry,j} \geq P_{exit,j}\ and\ P_{exit,j} \geq P_{exit,min} \\ and\ P_{entry,j} \leq P_{entry\,max}\}$$

[7] See OECD (2017).

[8] The Preisach model is a procedure for aggregating non-ideal relays of the type described by Equation (1) developed in general terms by Krasnosel'skii and Pokrovskii (1989) and Mayergoyz (1986), and introduced to economics by Amable *et al.* (1993) and Cross (1994).

The aggregate economy is represented as a set of the potential number of active $J \in T$ firms (or units of labour) acting according to Equation (1), whereby entry and exit trigger prices are assumed to be firm specific.[9]

Given that the level of employment of every active firm is one, the aggregate employment at time t, N_t, is fully described by sum of the active firms in the market:[10]

$$N_t = N_t[t_0, N_{t_0}] = \iint_T u(P_{exit,j}, P_{entry,j}) \mathcal{R}_{P_{exit,j}, P_{entry,j}}[t_0, n_{j,t_0}] P_t dP_{exit,j} dP_{entryj} \quad (2)$$

where $u(P_{exit,j}, P_{entry,j})$ is the weight (density) function of the individual firms in T where the pairs $(P_{exit,j}, P_{entry,j})$ are distributed with some integral density:

$$\int_T u(P_{exit,j}, P_{entry,j})\, dP_{exit,j} dP_{entryj} = 1$$

In the Preisach Triangle the distance of the relays from the origin is determined by the variable cost, w_j, and the orthogonal distance of the relays from the 45° line is a positive function of: *i*) the non-convex-costs of adjusting the number or employees; *ii*) the degree of uncertainty; *iii*) the flexibility of adjusting labour input through changes in the number of hours of work per employee.

For cycles of variation in aggregate demand, a continuous macroeconomic hysteresis loop for aggregate employment occurs (see Mota and Vasconcelos, 2012, p. 99).[11] Differently from what happens at the firm level, at the aggregate level every reversal of the direction of P_t leads to a structural break in the employment-aggregate demand relationship. This break is represented by a continuous transition between different curves (branches). After a path reversal followed by N_t a weak response of employment occurs and it will evolve into a strong one, once the entry or exit thresholds of many firms are passed. Whenever direction of the aggregate demand changes, a continuous branch-to-branch transition arises, and transitory changes in P_t can lead to permanent variations in N_t.

[9] The heterogeneity in the way firms react to aggregate demand shocks arises from specific cost structures including nonconvex adjustment costs (that may depend on firms' age, size, ownership, average work skill level and innovation), and from the way firms deal with uncertainty (see, Gaëlle and Scarpetta, 2006).

[10] For a complete explanation of the Preisach model of hysteresis see Mayergoyz (2003) and Mota and Vasconcelos (2012).

[11] Note that the branches of the hysteresis loop are quadratic functions if an uniform Preisach function, $u(P_{exit,j}, P_{entry,j})$, is considered.

The Preisach model of hysteresis provides a path-dependent equilibrium where the current state of aggregate employment is determined by the historical sequence of employment adjustments. In this sense, all values of the aggregate employment may be regarded as equilibrium values.

3. Empirical Application and Data

4.1. Empirical Model

For the empirical part of the work we apply the linear play model of hysteresis, which can be viewed as a piecewise-linear approximation of the Preisach hysteresis loop.[12] An important advantage of this model is that it allows to quantify the effect of the different sources of employment inertia.

The graphical representation of the play model of hysteresis is illustrated in Figure 1. The model contains two steep lines: a downward spurt line and an upward spurt line, along with many flatter lines (play lines) connecting the two spurt lines. The horizontal distance between the spurt lines in called the play interval.

We describe now, following Belke and Göcke (2001), the change in aggregate employment, N_t, induced by a change in aggregate demand, as separated among a weak reaction along a *play line* and a strong reaction along a *spurt line*, whenever the aggregate demand changes sufficiently.

For the estimation we use real production in industry, Y_t, as a proxy for the aggregate demand represented in the theoretical models of Section 2 by P_t.

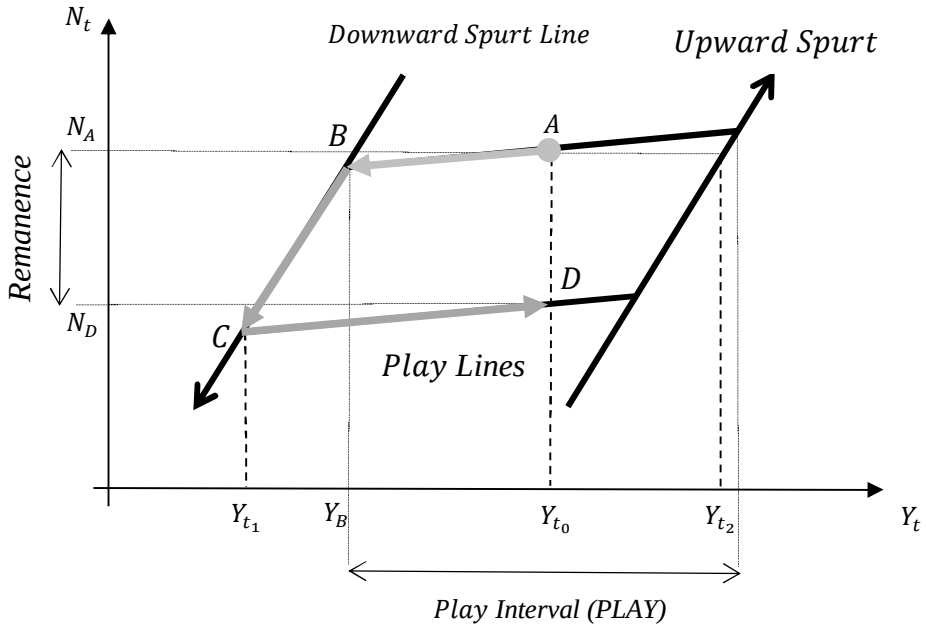
Assume $Y_t = Y_{t_0}$ and $N_t = N_A$ (the system starts at point A in Figure 1). A decrease in the level of aggregate demand to Y_{t_1} causes, initially, a weak response of the aggregate employment (along a *play line*) until a threshold value Y_B is surpassed (the system is at point B). This is due to the factors of inertia described in the previous sections. When the fall in aggregate demand becomes sufficiently large (*i.e.*, when the threshold value Y_B is surpassed), the employment responds strongly, decreasing along a *downward*

[12] See Kranosel'skii and Pokrovskii (1989) and Visitin (1994) for a general description of the model. See also Göcke (2001) and Mota *et al.* (2012; 2015), for an application to the dynamics of aggregate employment.

spurt line (the level of employment follows the sequence ABC). If the level of aggregate demand returns to the initial value, Y_{t_0}, employment increases weakly again along a *play line*, which was vertically shifted downward. Employment follows the sequence CD reaching the level N_D.

The transitory negative shock leaves a permanent effect on the level of aggregate employment (remanence). To bring back the employment to its initial level, a further increase (coercivity) of aggregate demand up to Y_{t_2} is needed.

Figure 1: *Play Hysteresis Loop*



The linear play model is implemented through a linear switching employment equation with an unknown and a time varying splitting factor (play interval), $PLAY_t$, that captures the hysteresis effects.

The system to be solved takes the form

$$\begin{cases} N_t = \beta_0 + \beta_1 Y_t + \beta_2 SPURT_t + \beta_3 W_t + \varepsilon_t \\ SPURT_t = f(PLAY_t) \\ PLAY_t = \gamma + \delta CISS_t \end{cases} \quad (3)$$

and is going to be explained in the following.

The first equation of system (3) is the employment equation. N_t stands for the aggregate level of employment, Y_t for the proxy of aggregate demand, W_t for the real wage, and ε_t for a random disturbance term.

The $SPURT_t$ is an artificial hysteresis transformed input variable that results from the Y_t series filtered out of all changes that occur inside the play interval ($\Delta Y_t < PLAY_t$). In this model β_1 gives the reaction of aggregate employment, N_t, along the *play line*, while $(\beta_1 + \beta_2)$ gives the reaction of N_t along the *spurt lines*:

$$\frac{dN_t}{dY} = \beta_1 + D\beta_2, \quad with\ D = \begin{cases} 0, & on\ the\ play\ lines \\ 1, & on\ the\ spurt\ lines \end{cases} \tag{4}$$

The presence of hysteresis is corroborated if the memory or remanence parameter, β_2, is significantly greater than zero.

The $SPURT_t$, is computed following the algorithm described in Belke and Göcke (2001) and a posteriori developed in MATLAB by Mota *et al.* (2015).

Finally, the play interval, $PLAY_t$, instead of constant, as it is assumed in Figure 1 to facilitate the exposition, is modelled as a positive function of the non-convex employment adjustment costs, captured by parameter γ, and as a positive function of uncertainty captured by parameter δ.

The Sovereign Systemic Stress Composite Indicator, $CISS_t$, is used as a proxy of uncertainty. It is an indicator of the contemporaneous instability or "stress" in the financial system. It comprises five segments of the financial system: *i*) the bank sector; *ii*) the non-bank financial institutions sector; *iii*) money markets; *iv*) securities markets; and *v*) foreign exchange markets. The current level of stress in each of these segments is measured on the basis of three indicators capturing agents' uncertainty, investor disagreement and information asymmetries. Higher values indicate more stress.

Overall, the Play algorithm works as follows:

1. Execute a grid search over a set of admissible values of the parameters of the play equation, γ and δ.
2. For each pair (γ, δ):
    2.1. Compute the time varying play interval: $PLAY_t = \gamma + \delta CISS_t$
    2.2. Compute the spurt variable: $SPURT_t = f(PLAY_t)$
    2.3. Estimate the employment equation by fully modified least squares (FM-OLS)
3. Compute the corresponding R^2 and Select the pair (γ, δ) that maximizes the R^2 of the employment equation. Build $N_t = \beta_0 + \beta_1 Y_t + \beta_2 SPURT_t + \beta_3 W_t + \varepsilon_t$

Further details, particularly about the construction of the spurt variable from a given play interval (step 2.2) can be found in Belke and Göcke (2001) and in Mota *et al.* (2012; 2015).

In order to analyse the effect of the possibility firms have to adjust labour input by varying the number of hours of work per employee, we also estimate system (3) with the total hours of work as a dependent variable.

4.2. Data

Industrial monthly data from the EUROSTAT – General Statistics, Industry Commerce and Services is used in the estimation. Aggregate employment, N_t, is measured by the index of the number of employees in industry. The real production in industry adjusted by the number of working days, Y_t, is used as the proxy of aggregate demand. Real wages, W_t, are measured by the index of gross wages in industry deflated by the general consumer price index. We also use as a dependent variable the index of the volume of work done, hours worked, H_t. The series are seasonally adjusted and the data covers the period from January 2000 to December 2018.

4. Estimation Results

As we are dealing with aggregate time series, we start by analysing the stationarity of the series. For the variables in levels the augmented Dickey-Fuller test statistic is larger than the 1% critical value (-3.355) indicating that we do not reject the non-stationary of the series (see Table 1). For the first difference of the series we reject the hypothesis of the existence of a unit root (test statistic is smaller than the 1% critical value for all the variables). We conclude that the variables are integrated of order one, I(1).

Table 1: *Augmented Dickey-Fuller Test Statistics*

Variable	Level	1st Difference
N_t	-2.768	-3.374
TH_t	-2.260	-22.013
Y_t	-1.787	-17.376
W_t	-1.573	-3.682
$SPURT_N_t$	-3.301	-14.355
$SPURT_TH_t$	-3.0968	-14.0257

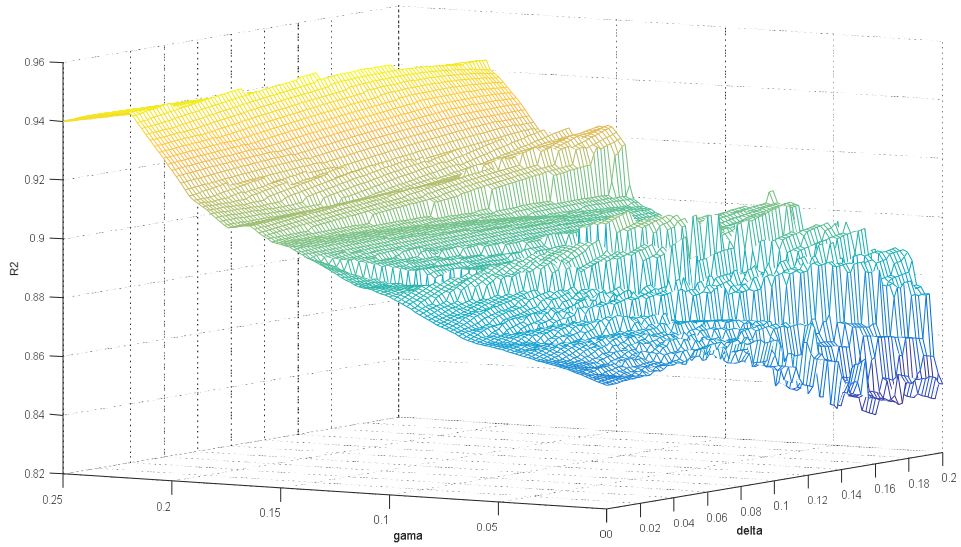
(1% critical value: -3.355)

The Johansen Test Procedure was also used for cointegration testing. Based on the trace test performed with four lags in the VAR representation and with an intercept and time trend in the cointegrating employment equation (3), the hypothesis of a single cointegrating vector relating the variables is not rejected. For both the employment equation and total hours equation the trace test statistic is greater than the 5 per cent critical value (see Table 2).

As the series are non-stationary we estimate the employment equation using fully modified least Squares (FM-OLS), to obtain asymptotically unbiased estimates of the parameters. FM-OLS estimator modifies least squares with semiparametric corrections that account for serial correlation effects and for endogeneity in the regressors that result from the existence of cointegrating relationships (see Phillips and Hansen, 1990, and Phillips, 1995).

Through the process of two-dimensional grid search described in Section 4.1, the estimated values obtained for the parameter of the play equation are $\gamma = 0.214$ and $\delta = 0.104$ (see Figure 2 and Table 2).

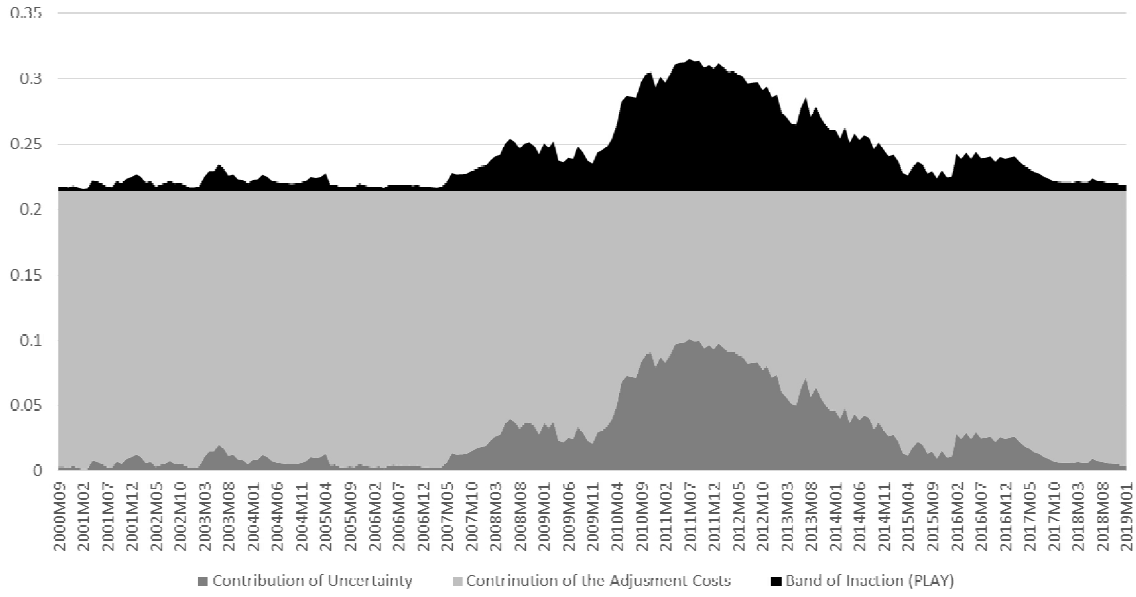
Figure 2: *R^2 of the Employment Equation Resulting from Two-dimensional Grid Search Over Play Interval Parameters*



These results are consistent with the presence of an employment band of inaction that depends positively on the magnitude of employment adjustment costs, but which is also found to be wider when uncertainty is high.

Figure 3 shows that non-convex adjustment costs is the major determinant of the employment band of inaction. Nonetheless, uncertainty is also important. The percentage of the band of inaction that is determined by uncertainty ranges from 0.68% to 32.05%.

Figure 3: *The Contribution of Non-convex Adjustment Costs and Uncertainty for the Width of the Band of Inaction*



The original industrial production series, Y_t, is plotted in Figure 4 a) along with its transformation according to the model of hysteresis, $SPURT_N_t$ (Figure 4 b)).

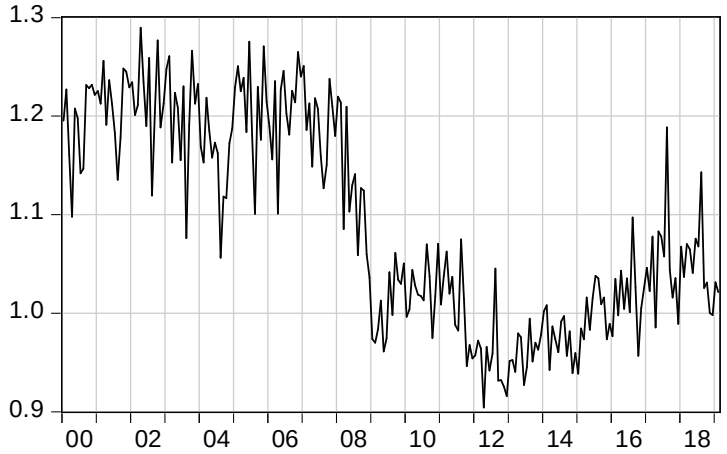
The estimates of the employment elasticity, obtained by FM-OLS, along the play lines, β_1, and the increment of this same elasticity along the spurt lines, β_2, are displayed in Table 2 (column 2). The employment elasticity along the spurt lines is given by $\beta_1 + \beta_2$.

Applying a fully-modified Wald test, we find that the reaction along the play lines is positive ($\beta_1 = 0.217$) and significant at 1%, as expected. The parameter β_2 is equal to 1.789 and significant at 1%. This implies that employment responds strongly to sufficiently large demand shocks (the reaction along the spurt lines is 2.006), corroborating the predictions of the play model of hysteresis.[13]

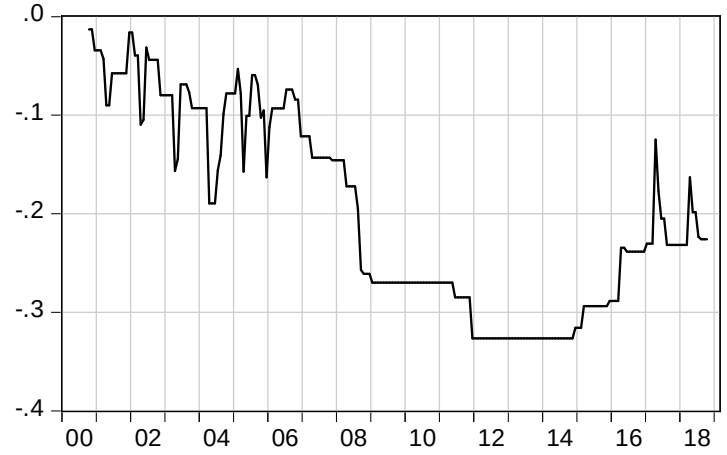
[13] The coefficient associated to the real wage, β_1, is negative and significant at 5%, as expected.

Figure 4: *Original Industrial Production Series and the Hysteresis Variable (Spurt)*

a) Production in Industry



b) Hysteresis Variable (Spurt Variable) - Employment Equation



c) Hysteresis Variable (Spurt Variable) - Total Hours Equation

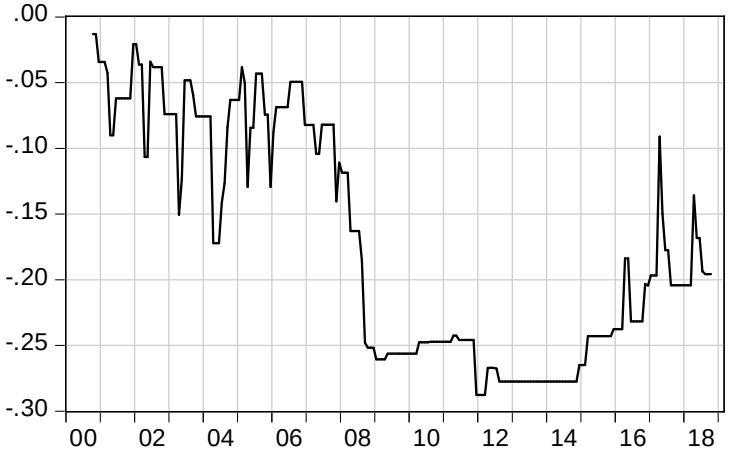


Table 2: *FM-OLS Estimation Results*

Independent Variables	Dependent Variable	
	Aggregate Employment in Industry (N_t)	Total Hours of Work in Industry (H_t)
$\boldsymbol{PLAY_t}$	γ = 0.214 δ = 0.104	γ = 0.214 δ = 0.062
$\boldsymbol{C}$	1.531*** (10.671)	1.607*** (11.293)
Y_t	0.217*** (2.674)	0.269*** (3.248)
$SPURT_N_t$	1.789*** (15.701)	-
$SPURT_H_t$	-	1.709*** (16.200)
W_t	-0.233** (-2.133)	-0.354*** (-3-336)
R^2	0.946	0.945
Trace Test Stat	73.344 (Critical Value: 47.856)	69.132 (Critical Value: 47.856)

t-statistics are in parentheses.

***, **,* Significant at 1, 5, and 10 per cent respectively

We repeat the exercise with labour input (total hours of work) as a dependent variable.

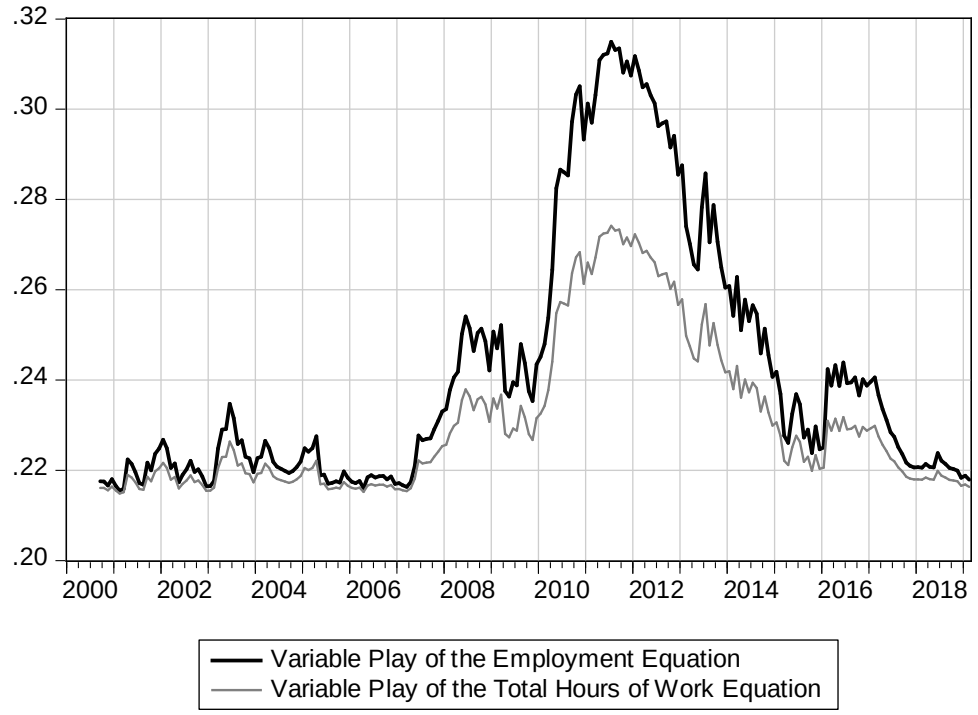
Through the process of two-dimensional grid search the estimated values obtained for the parameter of the play equation are γ = 0.214 and δ = 0.062 (the results are in Table 2, column 3). The correspondent hysteresis variable, $SPURT_H_t$, is displayed in Figure 4 c).

It is worth mention that the estimated value of parameter γ that captures the effect the non-convex employment adjustment costs to the width of the band of inaction is the same for employment and total hours of work. However, the estimated value of the parameter that captures the effect of uncertainty, δ, is higher in the case of employment. This means than uncertainty has a higher impact on the adjustment of employment in comparison with the adjustment in hours of work. This may be justified by the fact that employees do not see the change in the number of hours of work per employee as irreversible.

Figure 5 shows that the band of inaction, *i.e.*, the play interval, $PLAY_t$, is systematically higher for employment in comparison with total hours of work, and the difference between the bands of inaction increases in periods where uncertainty is higher.

The results in Table 2, column 3, show that labour input (total hours of work) is more responsive to the proxy of aggregate demand than employment along the play lines (β_1 increases from 0.217 to 0.269), but the contrary happens along the spurt lines.

Figure 5: *Original Industrial Production Series and the Hysteresis Variable (Spurt):*



Just as a robustness check of the results, particularly to discard any simultaneity bias problem that may exist, we also reestimate the employment switching equation using GMM, considering as instruments four lags of the independent variables. We concluded that the results do not change significantly (see Appendix).

Conclusion

In this paper we estimate a switching employment equation based on the play hysteresis operator to analyse the relative contribution of the factors of inertia to the width of the employment band of inaction.

We found significant hysteresis effects in the aggregate employment dynamics caused by the presence of non-convex adjustment costs and uncertainty.

In the case of employment, the width of the band of inaction in mainly determined by non-convex adjustment costs. In periods where uncertainty is high, the band of inaction increases significantly.

We also found that the flexibility firms may have to adjust labour input by varying the number of hours of work per employee helps to mitigate the effect of uncertainty upon the band of inaction.

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Appendix.

GMM Estimation Results

Independent Variables	Dependent Variable	
	Aggregate Employment in Industry (N_t)	Total Hours of Work in Industry (H_t)
$\boldsymbol{C}$	1.6131*** (11.966)	1.713*** (12.031)
Y_t	0.198** (2.052)	0.142 (1.2828)
$SPURT_N_t$	1.825*** (13.202)	-
$SPURT_H_t$	-	1.865*** (14.106)
W_t	-0.285*** (-4.289)	-0.286 (-3.953)
R^2	0.944	0.940

t-statistics are in parentheses.
***, **,* Significant at 1, 5, and 10 per cent respectively

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